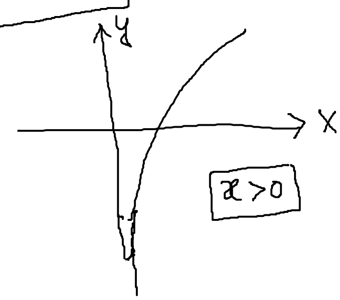
DOMAIN OF A FUNCTION

- "SET OF ALL INPUTS OVER WHICH THE FUNCTION HAS DEFINED OUTPUTS"

- THE DOMAIN Y=F(X) IS THE SET OF ALL REAL X FOR WHICH F(X) IS DEFINED(REAL).

- RULES FOR FINDING DOMAIN :-

$\boxed{y = \ln x}$



$\boxed{x > 0}$

1. EXPRESION UNDER EVEN ROOT(square root , fourth root etc.) ≥ 0

$\sqrt{A}$ or $(A)^{\frac{1}{\text{Even}}} \equiv (A)^{\frac{1}{2m}}$ | $(A)^{\frac{1}{\text{odd}}} \equiv (A)^{\frac{1}{2m+1}}$

$\Rightarrow \boxed{A \geq 0}$ | $\Rightarrow \boxed{A \in R}$

Eg:-1 $f(x) = \sqrt{1 - \sqrt{1 - x^2}}$ Ans:- $[-1, 1]$

$\Rightarrow 1 - \sqrt{1 - x^2} \geq 0$ — ① $\Rightarrow 1 \geq \sqrt{1 - x^2} \Rightarrow 1 \geq 1 - x^2$

$\Rightarrow x^2 \geq 0 \Rightarrow \boxed{x \in R}$ — ①

$\Rightarrow$ ② $1 - x^2 \geq 0$ — ②

$\Rightarrow x^2 - 1 \leq 0$

$\Rightarrow (x-1)(x+1) \leq 0$

$+ \quad - \quad +$; $-1 \quad 1$

$x \in [-1, 1]$ — ②

$\Rightarrow$ Ans $x \in [-1, 1]$

2. 1/A , DENOMINATOR ≠ 0

Eg:- $f(x) = \dfrac{1}{\sqrt{16 - 4^{x^2 - x}}}$

$\Rightarrow \dfrac{1}{A} \Rightarrow \boxed{A \neq 0}$

$f(x) = \dfrac{1}{\sqrt{A}} \Rightarrow \begin{cases} \sqrt{A} \neq 0 \\ A \geq 0 \end{cases} \Rightarrow \boxed{A > 0}$

$\Rightarrow 16 - 4^{x^2 - x} > 0$

$\Rightarrow 16 > 4^{x^2 - x}$

$\Rightarrow 4^2 > 4^{x^2 - x}$

$\Rightarrow 2 > x^2 - x$

$\Rightarrow x^2 - x - 2 < 0$

$\Rightarrow (x-2)(x+1) < 0$

$\Rightarrow \boxed{x \in (-1, 2)}$

$a^x > a^y \Rightarrow \begin{cases} x > y & \text{if } a > 1 \\ x < y & \text{if } 0 < a < 1 \end{cases}$

3. $\log_b(a)$

$\Rightarrow \begin{cases} a > 0 & \text{— ①} \\ b > 0 & \text{— ②} \\ b \neq 1 & \text{— ③} \end{cases}$

Eg $y = \log_x (x^2 - 4)$

$\Rightarrow \begin{cases} x^2 - 4 > 0 & \text{— ①} \\ x > 0 & \text{— ②} \\ x \neq 1 & \text{— ③} \end{cases}$

Ans $\Rightarrow$ ① ∩ ② ∩ ③

Eg:-1 $f(x) = \sqrt{\log_{1/4} \left(\dfrac{5x - x^2}{4} \right)}$ Ans:- $(0, 1] \cup [4, 5)$

$\Rightarrow \begin{cases} \log_{1/4} \dfrac{5x - x^2}{4} \geq 0 & \text{— ①} \\ \dfrac{5x - x^2}{4} > 0 & \text{— ②} \end{cases}$

$\Rightarrow \dfrac{5x - x^2}{4} \leq 1$

$\Rightarrow 5x - x^2 \leq 4$

$\Rightarrow x^2 - 5x + 4 \leq 0$

$\Rightarrow \dfrac{5x - x^2}{4} > 0$

$\Rightarrow 5x - x^2 > 0$

$\Rightarrow x^2 - 5x < 0$

Ans:- ① ∩ ②

$x \in (0, 1] \cup [4, 5)$

$\frac{5x-x^2}{4} > 0$ —② | $\Rightarrow x^2-5x+4 \le 0$ | $\Rightarrow 5x-x^2 > 0$ | Ans:- ①∩②

$\Rightarrow \log_b x \geq K$

$\Rightarrow \begin{cases} x \geq b^K & \text{if } b>1 \\ x \le b^K & \text{if } 0<b<1 \end{cases}$

$\Rightarrow (x-4)(x-1) \le 0$

$\Rightarrow x \in [1,4]$ —①

$\Rightarrow x^2-5x<0$

$\Rightarrow x(x-5)<0$

$\Rightarrow x \in (0,5)$ —②

$x \in (0,1] \cup [4,5)$

4. (A) $\sin^{-1}(A)$, $\cos^{-1}(A)$ to be defined

$\Rightarrow A \in [-1,1]$

$\Rightarrow -1 \le A \le 1$

Eg:- $f(x) = \sin^{-1}(x^2-4)$

$\Rightarrow -1 \le x^2-4 \le 1$

$3 \le x^2 \le 5$

$\Rightarrow \sqrt{3} \le \sqrt{x^2} \le \sqrt{5}$

$\sqrt{3} \le |x| \le \sqrt{5} \Rightarrow$ $\sqrt{3} \le x \le \sqrt{5}$ OR $-\sqrt{5} \le x \le -\sqrt{3}$

$\Rightarrow x \in [-\sqrt{5},-\sqrt{3}] \cup [\sqrt{3},\sqrt{5}]$

(B) $\operatorname{cosec}^{-1}(A)$ and $\sec^{-1}(A)$

$\Rightarrow A \in (-\infty,-1] \cup [1,\infty)$

$\Rightarrow$ $A \le -1$ OR $A \ge 1$

$\cup$

Final Ans

(C) $\sin x$ / $\cos x$ / $\tan^{-1}x$ / $\cot^{-1}x$ / e^x / a^x / $|x|$, any Polynomial Function / $[x]$ / $\{x\}$ / $\operatorname{sgn}(x)$ / $x^{\frac{1}{2n+1}}$

$\Rightarrow$ Will have Domain as R.

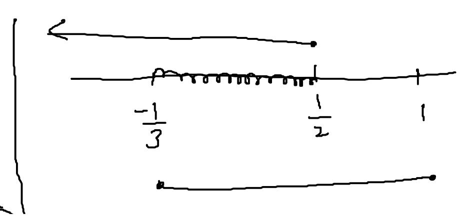
(D) $\tan A$ / $\sec A$ $\Longrightarrow$ $\boxed{A \in R - (2n+1)\frac{\pi}{2}}$

$\frac{\sin A}{\cos A}$ $\Rightarrow \cos A \neq 0$ $A \neq (2n+1)\frac{\pi}{2}$

$\cot A$ / $\operatorname{cosec} A$ $\Rightarrow A \in R - n\pi$

Eg:- $f(x) = \sqrt{1-2x} + 3\sin^{-1}\left(\frac{3x-1}{2}\right)$ Ans:- $\left[-\frac{1}{3}, \frac{1}{2}\right]$

$\Rightarrow \begin{cases} 1-2x \geq 0 & \text{—①} \\ -1 \le \frac{3x-1}{2} \le 1 & \text{—②} \end{cases}$



(&) $-1 \le \frac{3x-1}{2} \le 1$ — (2)

$\Rightarrow \begin{cases} \boxed{x \le 1/2} \text{ — (1)} \\ -2 \le 3x-1 \le 2 \end{cases}$ $\quad -\frac{1}{3} \le x \le 1$ — (2) $\quad x \in \left[-\frac{1}{3}, \frac{1}{2}\right]$ Ans

$\Rightarrow -1 \le 3x \le 3$

Eg:- Domain of $f(x) = \underbrace{\cos^{-1}x}_{\Downarrow} + \underbrace{\cot^{-1}x}_{\Downarrow} + \underbrace{\operatorname{cosec}^{-1}x}_{\Downarrow}$ is- Ans :- $\{-1, 1\}$

$x \in [-1, 1] \quad x \in R \quad x \le -1$ or $x \ge 1$

$\Rightarrow$ $x \in \{-1, 1\}$ Ans

For ^nC_r / ^nP_r to be defined

$\Rightarrow \begin{cases} n, r \in I \checkmark \\ \text{(\&) } n \ge 1 \checkmark \\ \text{(\&) } r \ge 0 \checkmark \end{cases}$

(&) $n \ge r \checkmark$

Eg:- Find domain of $^{x-3}C_{20-x}$ is- Ans $x \in \{12, 13, 14, \cdots, 19, 20\}$

$\Rightarrow x-3 \in I$, $20-x \in I$

$x-3 \ge 1$ — (1) $\Rightarrow x \ge 4$ — (1)

(&) $20-x \ge 0$ — (2) $\Rightarrow x \le 20$ — (2)

(&) $x-3 \ge 20-x$ — (3) $\Rightarrow 2x \ge 23$

$x \ge 11.5$ — (1)

$\Rightarrow x \in \{11, 12, 13, \cdots 20\}$

6. For Domain of $\varphi(x) = (f(x))^{g(x)} = (\text{var.})^{\text{Var.}}$

$\Rightarrow \begin{cases} \boxed{f(x) > 0} \\ \text{(\&) } \boxed{g(x) \text{ must be Real.}} \end{cases}$

⇒ $\left\{ \begin{array}{l} f(x) > 0 \\ \& \ \boxed{g(x) \text{ must be Real.}} \end{array} \right.$

Eg:- $f(x) = (\ln x)^{\sin x}$

⇒ $\ln x > 0$ —(1) ⇒ $\boxed{x > 1}$ ⇒ $x \in (1, \infty)$

& $\sin x \in R$ —(1) ⇒ $x \in R$

☺ NOTE:-

Algebraic Operations on Functions :

If f and g are real valued functions of x with domain set A and B respectively, then both f and g are defined in $A \cap B$. Now we define $f + g$, $f - g$, $(f \,.\, g)$ and (f/g) as follows:

(i) $(f \pm g)(x) = f(x) \pm g(x)$ — domain in each case is $A \cap B$

(ii) $(f.g)(x) = f(x).\, g(x)$

(iii) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ domain is $\{x \mid x \in A \cap B$ such that $g(x) \neq 0\}$.

$y = f(x) \pm g(x)$ → D_1, D_2 → $D_1 \cap D_2$

$y = f(x) \cdot g(x)$ → $D_1 \cap D_2$

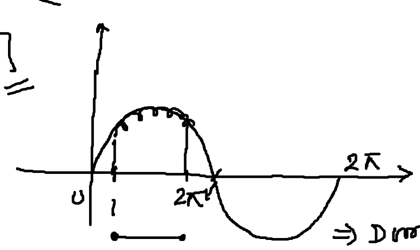
$y = \frac{f(x)}{g(x)}$ → $\boxed{D_1 \cap D_2}$ & $g(x) \neq 0$

Eg:-2 $f(x) = \sqrt{\sin x} + \sqrt{(x-1)(2-x)}$

⇒ $\left\{ \begin{array}{l} \sin x \geq 0 \text{ —(1)} \\ (x-1)(2-x) \geq 0 \text{ —(2)} \end{array} \right.$ ⇒ $(x-1)(x-2) \leq 0$

⇒ $x \in [1, 2]$

⇒ $\left\{ \begin{array}{l} \sin x \geq 0 \text{ —(1)} \\ \Rightarrow -(x-1)(x-2) \geq 0 \end{array} \right.$



⇒ Domain ⇒ $x \in [1, 2]$

Eg:-3 $f(x) = \sqrt{x-3} \cdot \sqrt{x+3}$

$\Rightarrow$ Domain $\Rightarrow x \in [\ldots$

Eg:-3 $f(x) = \sqrt{x-3} \cdot \sqrt{x+3}$

$$\Rightarrow \begin{cases} x-3 \geq 0 \text{ --- (1)} \Rightarrow \boxed{x \geq 3} \\ x+3 \geq 0 \text{ --- (2)} \Rightarrow \boxed{x \geq -3} \end{cases} \Rightarrow \boxed{x \in [3, \infty)}$$

wrong method: $f(x) = \sqrt{x^2 - 3^2} \Rightarrow (x^2 - 3^2) \geq 0 \Rightarrow (x-3)(x+3) \geq 0$
$\Rightarrow x \in (-\infty, -3] \cup [3, \infty)$

Note $\Rightarrow$ Any operation of undefined will be undefined

Eg :- $\sqrt{x-3} \cdot \sqrt{\sin x - 2}$

$y = \sqrt{x-3} \cdot \sqrt{\sin x - 2}$

$$\Rightarrow \begin{cases} x-3 \geq 0 \text{ --- (1)} \Rightarrow \boxed{x \geq 3} \\ \sin x - 2 \geq 0 \text{ --- (2)} \Rightarrow \boxed{\sin x \geq 2} \end{cases} \Rightarrow \boxed{x \in \phi}$$

$\boxed{\sin x \geq 2} \Rightarrow \boxed{x \in \phi}$

$\sin x \in [-1, 1]$

Eg:- $y = \sqrt{(x-4)(\sin x - 2)}$

$\Rightarrow (x-4)(\sin x - 2) \geq 0$

$\Rightarrow -(x-4)\underbrace{(2 - \sin x)}_{+ve} \geq 0$

$\Rightarrow -(x-4) \geq 0$

$\Rightarrow \boxed{x - 4 \leq 0} \Rightarrow \boxed{x \leq 4}$ Ans $x \in (-\infty, 4]$

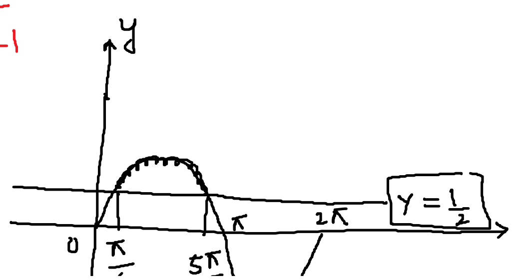
SOLVING INEQUALITIES INVOLVING PERIODIC FUNCTIONS

- IF SOLUTION OCCUR IN PERIODIC SENSE THE WRITE SOLUTION FOR ANY ONE OF THE CYCLE (PARTICULAR SOLN) AND THEN ADD INTEGRAL MULTIPLE OF PERIOD.

Ex:-1 $f(x) = \sqrt{2\sin x - 1}$

$\Rightarrow 2\sin x - 1 \geq 0$

$\Rightarrow \boxed{\sin x \geq \frac{1}{2}}$

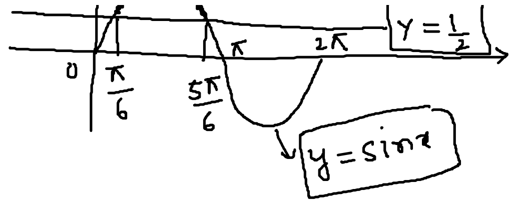


P.S

$x \in \left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$

$\Rightarrow \boxed{\sin x \geq \frac{1}{2}}$

$\boxed{T = 2\pi}$



$x \in \left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$

Domain

$x \in \left[\frac{\pi}{6} + nT, \frac{5\pi}{6} + nT\right]$

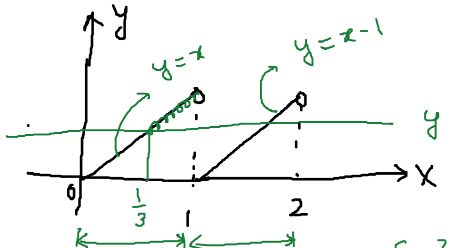
$x \in \left[\frac{\pi}{6} + 2n\pi, \frac{5\pi}{6} + 2n\pi\right]$

Ex:-2. $f(x) = \sqrt{3\{x\} - 1}$

$\Rightarrow 3\{x\} - 1 \geq 0$

$\Rightarrow \boxed{\{x\} \geq \frac{1}{3}}$

$\boxed{T = 1}$



$\boxed{\{I\} = 0}$

$\{x\} \geq \frac{1}{3}$

$\;\;\| \quad\;\; \|$

$\;\;y \quad\;\; y$

$\{x\} \geq \frac{1}{3}$

$\Rightarrow$ P.S $\Rightarrow x \in \left[\frac{1}{3}, 1\right)$

G.S $\Rightarrow x \in \left[\frac{1}{3} + nT, 1 + nT\right) \Rightarrow x \in \left[\frac{1}{3} + n, 1 + n\right)$ Ans

DOMAIN OF COMPOSITE FUNCTION

When Finding the Domain of a Composite Function You must Consider the Domain of the Inside Function and the Domain of the Composite Function.

- STEP FOR FINDING DOMAIN OF COMPOSITE FUNCTION :-

  h(x)=(fog)(x)

→ Find the domain of the inside function, g(x) , and the composite function, h(x).

→ The domain of the composite function is the intersection of the domains of both g(x) and h(x)

☺ NOTE:- For Domain $\Rightarrow$ "Will go outside to inside"

EX:-1. (i) $y = e^{\sin x}$

$\Rightarrow e^{\square} \longrightarrow \sin x \Rightarrow \boxed{x \in R}$

$\uparrow x \in R \quad \uparrow x \in R$

(ii) $y = e^{\sin^{-1} x}$

$\Rightarrow e^{\square} \longrightarrow \sin^{-1} x \Rightarrow x \in [-1, 1]$

$\Downarrow R \qquad \hookrightarrow -1 \leq x \leq 1$

(iii) $y = \sin^{-1}(e^x)$

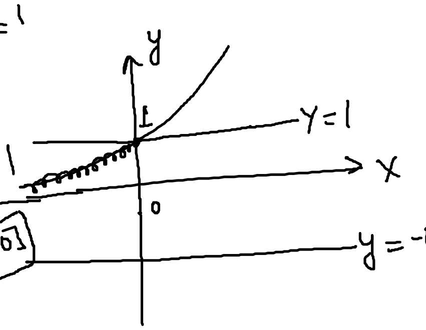
$\sin^{-1}\square \longrightarrow e^x$
$\hookrightarrow x \in R$

$-1 \leq \square \leq 1$

$\Rightarrow -1 \leq e^x \leq 1$

$\Downarrow$

$\boxed{x \in (-\infty, 0]}$



(iv) $y = \tan^{-1}\left(e^{\cos(\sqrt{x})}\right)$

$\tan^{-1}\square \longrightarrow e^{\square} \longrightarrow \cos\square \longrightarrow \sqrt{x}$

$\;\;\| \qquad\quad \Downarrow \qquad\quad \Downarrow \qquad\quad \Downarrow$

$\tan^{-1}\square \longrightarrow e^{\square} \longrightarrow \cos\square \longrightarrow \sqrt{x}$

$\Downarrow \quad R \qquad \Downarrow \quad R \qquad \Downarrow \quad R \qquad \Downarrow \quad x \geq 0$

$\Rightarrow \boxed{x \in R}$

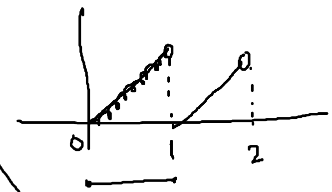
(v) $y = \ln(\{x\})$

$\ln\square \longrightarrow \{x\}$

$\Downarrow \quad \square > 0 \qquad \Downarrow \quad \underline{\underline{x \in R}}$

$\boxed{\{x\} > 0}$ $\Rightarrow \not\triangleleft$

$\boxed{\{I\} = 0}$



$x \in (0, 1)$

$\underline{\underline{G.S}}$ $x \in (0 + nT, 1 + nT]$

$x \in (0+n, 1+n) \Rightarrow x \in (n, n+1)$

(vi) $y = \sin^{-1}(\underline{\underline{\sin x}})$

$\sin^{-1}\square \longrightarrow \sin x$

$\Downarrow \quad \boxed{-1 \leq \square \leq 1} \qquad \Downarrow \quad R$

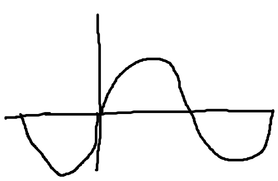
$-1 \leq \sin x \leq 1$

$\hookrightarrow \boxed{x \in R}$

(V) $f(x) = \sqrt{\sin(\cos x)}$ $\longrightarrow$

Ans :- $2n\pi - \frac{\pi}{2} \leq x \leq 2n\pi + \frac{\pi}{2}$, $n \in I$

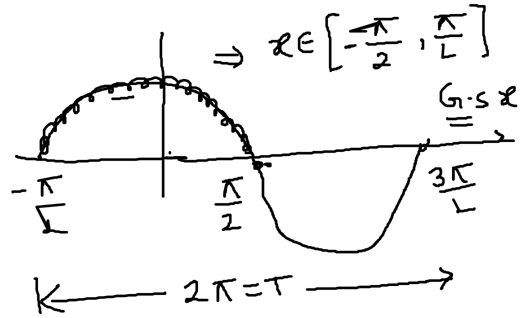
$\sin(\underline{\cos x}) \geq 0$



$\boxed{-1 \leq \cos x \leq 1}$

$\sin\square \geq 0$

$\uparrow [0, 1]$



$\Rightarrow x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\underline{\underline{G.S}}$ $x \in \left[-\frac{\pi}{2} + 2n\pi, \frac{\pi}{2} + 2n\pi\right]$

(vi) $f(x) = \sqrt{\dfrac{\log_2(x-2)}{\log_{1/2}(3x-1)}}$ $\longrightarrow$

Ans :- $(2, 3]$

$$\frac{\log_2(x-2)}{\log_{1/2}(3x-1)} \geq 0 \Rightarrow \frac{\log_2(x-2)}{-\log_2 3x-1} \geq 0 \Rightarrow \frac{\log_2(x-2)}{\boxed{\log_2 3x-1}} \leq 0$$

(1) $x - 2 > 0 \Rightarrow \boxed{x > 2}$

(2) $3x - 1 > 0 \Rightarrow \boxed{x > \frac{1}{3}}$

$\Rightarrow \boxed{x > 2}$

$\Rightarrow$

$\log_2(x-2) \leq 0 \Rightarrow x - 2 \leq 1$

$\boxed{x \leq 3}$ — (1)

(#) $3x-1>0 \Rightarrow \boxed{x>\frac{1}{3}}$

(#) $\log_{1/2}(3x-1) \neq 0 \Rightarrow 3x-1 \neq 1$ $\boxed{x \neq \frac{2}{3}}$

$\log_2(x-2) \leq 0 \Rightarrow x-2 \leq 1$

$\boxed{x \leq 3}$ — ①

$\Rightarrow$ Ans $\Rightarrow x \in (2,3]$

(vii) $f(x) = \dfrac{\sqrt{\cos x - \frac{1}{2}}}{\sqrt{6+35x-6x^2}}$ Ans:- $\left(-\frac{1}{6}, \frac{\pi}{3}\right] \cup \left[\frac{5\pi}{3}, 6\right)$

$\cos x - \frac{1}{2} \geq 0$ — ① $\Rightarrow \boxed{\cos x \geq \frac{1}{2}}$

$6+35x-6x^2 > 0$ — ②

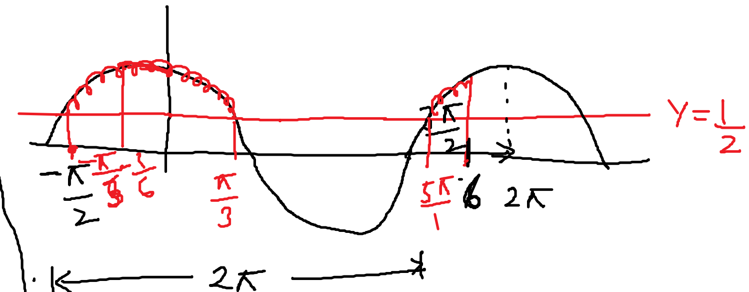
$\Rightarrow 6x^2 - 35x - 6 < 0$

$\Rightarrow 6x^2 - 36x + x - 6 < 0$

$\Rightarrow 6x(x-6) + (x-6) < 0$

$\Rightarrow (6x+1)(x-6) < 0$

$\Rightarrow x \in \left(-\frac{1}{6}, 6\right)$ — ②



$x \in \left(-\frac{1}{6}, \frac{\pi}{3}\right] \cup \left[\frac{5\pi}{3}, 6\right)$ Ans

EX:-2. If domain of f(x) is (–∞ , 0], then domain of $f(6\{x\}^2 - 5\{x\} + 1)$ is (where {·} represents fractional part function).

(A*) $\bigcup_{n\in I}\left[n+\frac{1}{3}, n+\frac{1}{2}\right]$ (B) $(-\infty, 0)$ (C) $\bigcup_{n\in I}\left[n+\frac{1}{6}, n+1\right]$ (D) $\bigcup_{n\in I}\left[n-\frac{1}{2}, n-\frac{1}{3}\right]$

$f(x)$ $\hookrightarrow \leq 0$

$f(\square)$ $\hookrightarrow \leq 0$

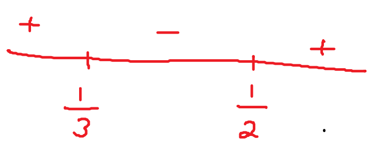
$f(\underbrace{6\{x\}^2 - 5\{x\} + 1}_{\leq 0})$

$\Rightarrow 6\{x\}^2 - 5\{x\} + 1 \leq 0$

Let $\{x\} = t$

$6t^2 - 5t + 1 \leq 0$

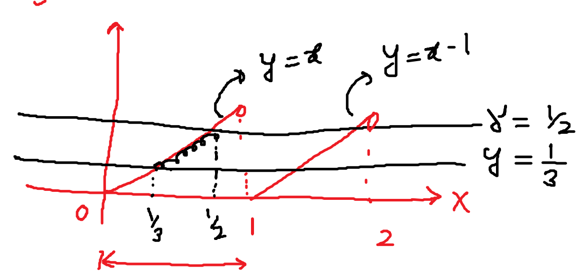
$\Rightarrow (3t-1)(2t-1) \leq 0$



$\frac{1}{3} \leq t \leq \frac{1}{2}$

$\frac{1}{3} \leq t \leq \frac{1}{2}$

$\frac{1}{3} \leq \{x\} \leq \frac{1}{2}$



$x \in \left[\frac{1}{3}, \frac{1}{2}\right]$

G.S $x \in \left[\frac{1}{3}+nT, \frac{1}{2}+nT\right]$, $n \in I$

$T = 1$

$x \in \left[\frac{1}{3}+n, \frac{1}{2}+n\right]$ Ans

EX:-3. If $\boxed{q^2 - 4pr = 0}$, $p > 0$, then the domain of the function $f(x) = \log(px^3 + (p + q)x^2 + (q + r)x + r)$ is:

(A) $R - \left\{-\frac{q}{2p}\right\}$ (B*) $R - \left[(-\infty, -1] \cup \left\{-\frac{q}{2p}\right\}\right]$

(C) $R - \left[(-\infty, -1) \cap \left\{-\frac{q}{2p}\right\}\right]$ (D) R

$f(x) = \log(\underline{px^3 + (p+q)x^2 + (q+r)x + r})$

$\Rightarrow px^3 + (p+q)x^2 + (q+r)x + r > 0$

$\Rightarrow$ Sum of Coeff. $= 0$

$\Rightarrow (x+1)$

Put $\boxed{x=-1}$ $\cancel{-p} + \cancel{p} + \cancel{q} - \cancel{q} - \cancel{r} + \cancel{r} = 0$

$px^3 + (p+q)x^2 + (q+r)x + r \equiv (x+1)(px^2 + \alpha x + r)$

$\Rightarrow$ Coeff. of x^2

L.H.S = R.H.S

$\equiv (x+1)(px^2 + qx + r)$

$\Rightarrow \cancel{p} + q = \alpha + \cancel{p}$

$\boxed{\alpha = q}$

$\Rightarrow (x+1)(px^2 + qx + r) > 0$

given $\boxed{q^2 - 4pr = 0}$ $\leftarrow \boxed{D=0}$

$\boxed{p>0}$

$ax^2 + bx + c$

$\hookrightarrow \boxed{D=0} \equiv a\left(x - \frac{b}{2a}\right)^2$

$px^2 + qx + r \equiv p\left(x - \frac{q}{2p}\right)^2$

$\Rightarrow (x+1)p\left(x - \frac{q}{2p}\right)^2 > 0$

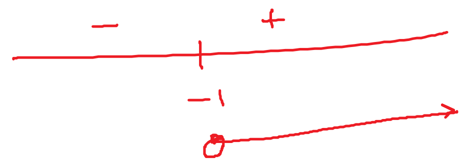
$\Rightarrow (x+1)\left(x - \frac{q}{2p}\right)^2 > 0$

$\neq 0$ $\neq 0$

$$(x+1)\left(x-\frac{q}{2p}\right)^2 > 0$$

$$\boxed{(x+1) > 0} \quad \& \quad \boxed{x \neq \frac{q}{2p}}$$

$x+1 > 0$



$$x \in (-1, \infty) - \left\{\frac{q}{2p}\right\}$$

or $x \in R - \left\{(-\infty, -1] \cup \left\{\frac{q}{2p}\right\}\right\}$