16720: Computer Vision

[redacted]

Q1.1 Suppose two cameras fixate on a point x (see Figure 1) in space such that their principal axes intersect at that point

$p1 = [0 \quad 0 \quad 1]'$

$p2 = [0 \quad 0 \quad 1]'$ ...(1)

It should follow the epipolar correspondence

$p1' * F * p2 = 0$ ...(2)

Here, the value of F is

$$F = \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} \quad ...(3)$$

Substitute (1) and (3) in (2):

$$[0 \quad 0 \quad 1] \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$$

$\implies \quad 0 * F_{31} + 0 * F_{32} + 1 * F_{33} = 0$

$\implies \quad F_{33} = 0$

Q 1.2 Let the coordinates of the image points be

$p_l = [x_l \quad y_l \quad 1]'$ ...(1)

$p_r = [x_r \quad y_r \quad 1]'$ ...(2)

Consider the case of two cameras viewing an object such that the second camera differs from the first by a pure translation y that is parallel to the x-axis

$t = [t1 \quad 0 \quad 0]'$

There is no rotation

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The essential matrix will be:

$E = t' \times R$

$$E = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t1 \\ 0 & t1 & 0 \end{bmatrix}$$

The epipolar line and its left image point must satisfy

$$E * p_r = 0 \quad \Longrightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t1 \\ 0 & t1 & 0 \end{bmatrix} * \begin{bmatrix} x_r \\ y_r \\ 1 \end{bmatrix} = 0$$

$\Longrightarrow \quad -t1 * Y + t1 * y_r \ \Longrightarrow Y = y_r$

Similarly for the point on the right image point. Hence proved, the epipolar lines in the two cameras are also parallel to the x-axis as is shown by the y coordinate

Q 1.3 Suppose we have an inertial sensor which gives us the accurate positions (Ri and ti, the rotation matrix and translation vector) of the robot at time i.

Let G_1 be the homogeneous transformation of frame 1

$G_1 = [R_1 \mid t_1]$ ...(1)

Let G_2 be the homogeneous transformation of frame 2

$G_2 = [R_2 \mid t_2]$ ...(2)

Relative homogenous transformation between the 2 frames is given by

$$G_{rel} = [R_{rel}|t_{rel}] = G_1^{-1} * G_2$$

$$\Longrightarrow G_{rel} = [R_{rel}|t_{rel}] = [R_1|t_1]^{-1}[R_2|t_2]$$

$$\Longrightarrow G_{rel} = \begin{bmatrix} R_1^{-1}R_2 & R_1^{-1}t_1 - R_1^{-1}t_2 \\ 0 & 1 \end{bmatrix}$$

Here $R_{rel} = R_1^{-1}R_2$ *and* $t_{rel} = R_1^{-1}t_1 - R_1^{-1}t_2$

Suppose the camera intrinsics (K) are known, express the essential matrix (E) and the fundamental matrix (F) in terms of K, Rrel and trel

$$E = [t_{rel}]R_{rel}$$

The fundamental matrix F

$$F = KE = K\,[t_{rel}]R_{rel}$$

Q 1.4

Assume 3D point P and its reflection P’ in the mirror. There is no rotation, only translation between the two points. They are related by the homogeneous transformation

$$P = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} P'$$

The image points p and p’ corresponding to P and P’ must satisfy the epipolar correspondence.

$p'Fp = 0$ ...(1)

Here F = KE, where E is the essential matrix

$p'KEp = 0$ ...(2)

Here
$E = [t]R$ where t is the translation and R is the rotation

$$t = [t_x \quad t_y \quad t_z]'$$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow E = \begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix}$$

Taking the transpose of E

$$E^T = \begin{bmatrix} 0 & t_z & -t_y \\ -t_z & 0 & t_x \\ t_y & -t_x & 0 \end{bmatrix}$$

Since $E^T = -E$

E is a skew symmetric matrix

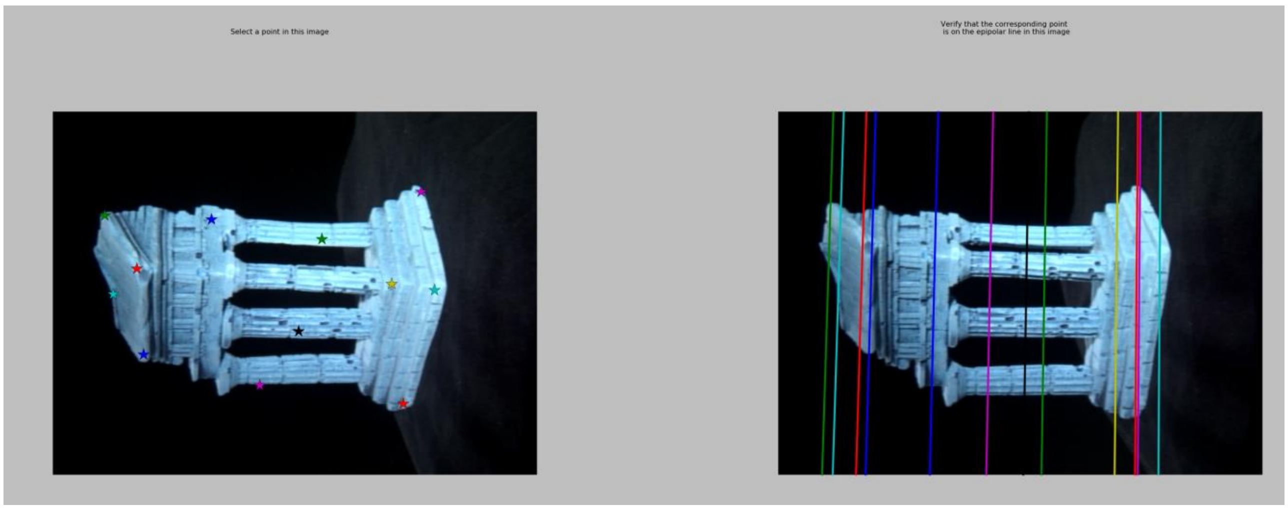
F is also a skew symmetric matrix

Hence, this situation is equivalent to having two images of the object which are related by a skew-symmetric fundamental matrix.

Q2.1 Eight Point Algorithm

The F matrix

$$\begin{bmatrix} -8.33149227\mathrm{e}-09 & 1.29538462\mathrm{e}-07 & -1.17187851e-03 \\ 6.51358337\mathrm{e}-08 & 5.70670059\mathrm{e}-09 & -4.13435037\mathrm{e}-05 \\ 1.13078765\mathrm{e}-03 & 1.91823637\mathrm{e}-05 & 4.16862081\mathrm{e}-03 \end{bmatrix}$$



Q2.2 Seven Point Algorithm

The F matrix

$$\begin{bmatrix} 1.11455058e-06 & -4.84002674e-08 & -2.57821524e-03 \\ 5.08132974e-07 & -5.70699106e-07 & -9.87502208e-05 \\ 1.83560473e-03 & 2.82991210e-04 & 6.35483250e-02 \end{bmatrix}$$

Points1

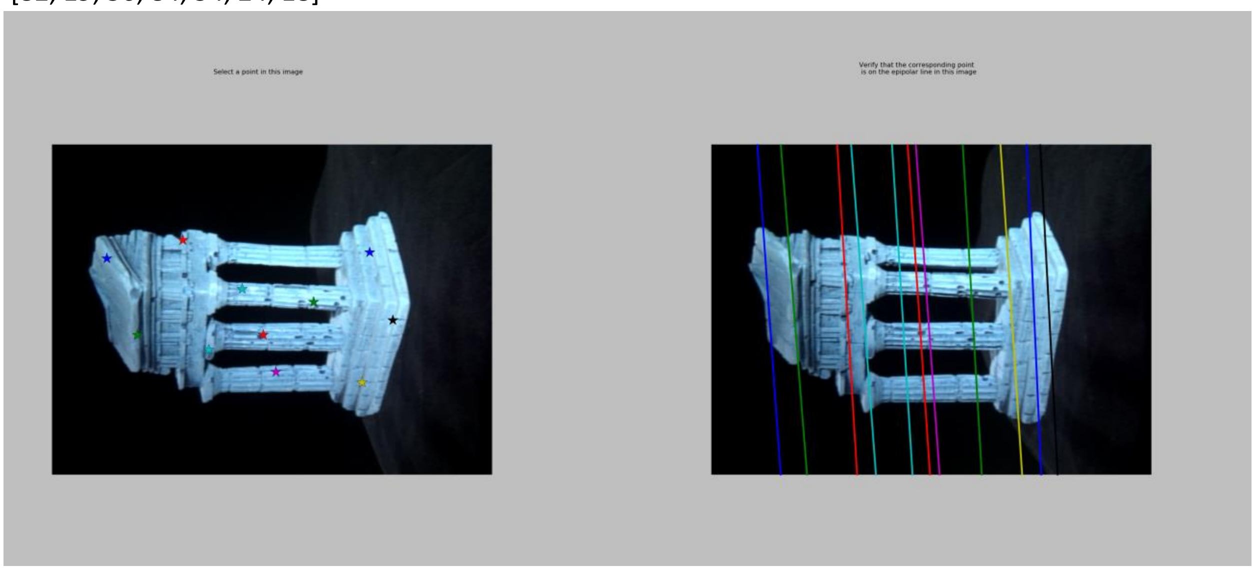
[[123, 126], [474, 384], [501, 152], [473, 103], [255, 203], [152, 225], [156, 305]]

Points2

[[123, 131], [484, 386], [494, 130], [465, 105], [254, 187], [151, 199], [157, 302]]

Corresponding indices

[82, 19, 56, 84, 54, 24, 18]



Q3.1 Estimated E using F from the eight-point algorithm

$$\begin{bmatrix} -1.92592122e-02 & 3.00526429e-01 & -1.73693252e+00 \\ 1.51113725e-01 & 1.32873151e-02 & -3.08885271e-02 \\ 1.73986815e+00 & 9.11774761e-02 & 3.90697725e-04 \end{bmatrix}$$

Q3.2

Let U and U' be the 2D image points of 3D point X in 2 images. M and M' are the projection matrices.

$U_i = M\,X_i \quad \Longrightarrow \quad U_i \times MX_i = 0$ ....(1)

$U'_i = M'X_i \quad \Longrightarrow \quad U'_i \times M'X_i = 0$ ....(2)

Substituting the values of U_i *and* U'_i and stacking them vertically

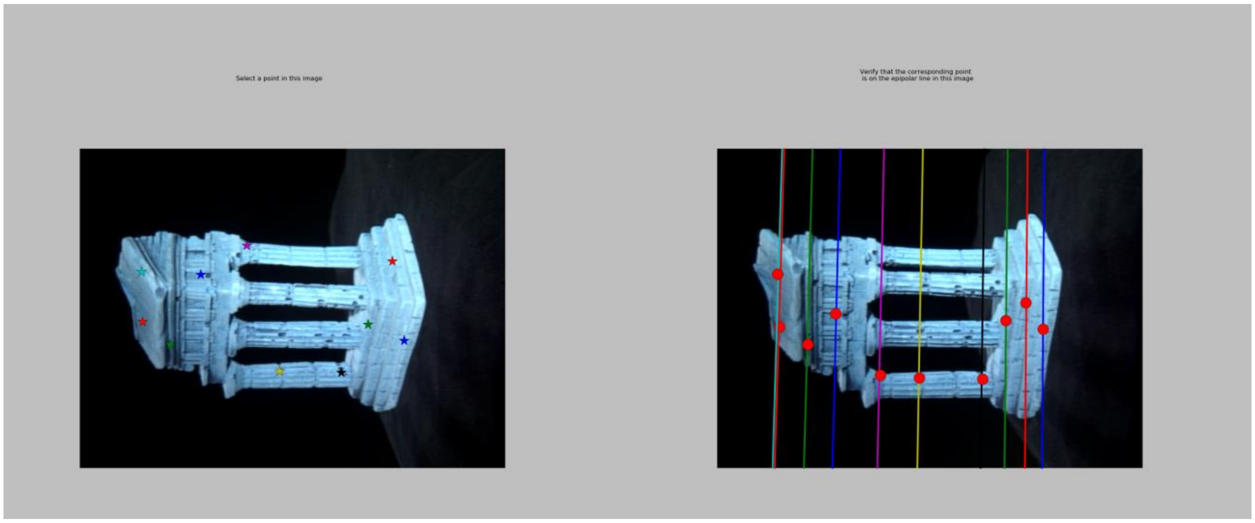
$$U_i = \begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix},\ U'_i = \begin{bmatrix} u'_i \\ v'_i \\ 1 \end{bmatrix}, \qquad M = \begin{bmatrix} p^{1T} \\ p^{2T} \\ p^{3T} \end{bmatrix} \ and\ M' = \begin{bmatrix} p'^{1T} \\ p'^{2T} \\ p'^{3T} \end{bmatrix}$$

Also considering more than 2 sets of points. Here T stands for column number

$$\begin{bmatrix} v_i p^{3T} - p^{2T} \\ u_i p^{3T} - p^{1T} \\ v'_i p'^{3T} - p'^{2T} \\ u'_i p'^{3T} - p'^{1T} \\ . \\ . \end{bmatrix} X = 0 \qquad \Longrightarrow \qquad A = \begin{bmatrix} v_i p^{3T} - p^{2T} \\ u_i p^{3T} - p^{1T} \\ v'_i p'^{3T} - p'^{2T} \\ u'_i p'^{3T} - p'^{1T} \\ . \\ . \end{bmatrix}$$

Q4.1

A screenshot of epipolarMatchGUI with some detected correspondences.



Q4.2