~~[illegible]~~

~~4.~~

5. A singular configuration is when

☆ Configurations from which certain directions of motion may be unattainable.

☆ Bounded gripper velocities may correspond to unbounded joint velocities.

☆ Usually (not always) singularities correspond to points on the boundary of the manipulator workspace, i.e the maximum reach of manipulator.

→ We can find whether the configuration is singular or not by checking the Jacobian's Rank. Jacobian has shape $(6, n)$

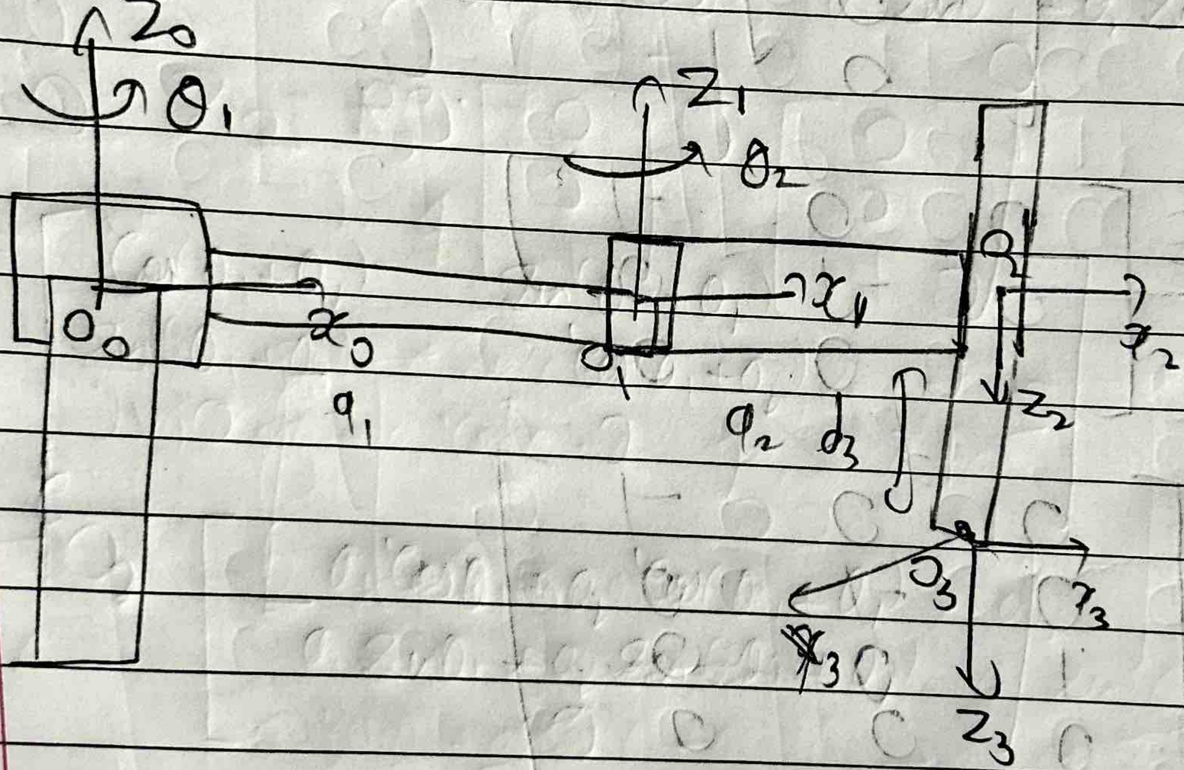
∴ If Rank $< \min(6, n)$ it is a singular config.

~~☆ Also, we can equate determinant (Jacobian) to 0 and find singularity point~~

☆ Consider a manipulator with 3-DOF arm and 3-DOF wrist. We can find the singular config by $J(q) = 0$.

8.

SCARA



Link	a	α	d	θ	Type
1	a_1	0	0	θ_1	R
2	a_2	π	0	θ_2	R
3	0	0	d_3	0	R

Link - 1

$a = a_1,\ \alpha = 0,\ d = 0,\ \theta = \theta_1$

$$\therefore A_1 = T_0^1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & a_1\cos\theta_1 \\ \sin\theta_1 & \cos\theta_1 & 0 & a_1\sin\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore R_0^1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\vec{O}_1 = \begin{bmatrix} a_1\cos\theta_1 \\ a_1\sin\theta_1 \\ 0 \end{bmatrix}$$ (Position of joint 1 w.r.t base frame)

Link - 2

$a = a_2,\ \alpha = \pi,\ d = 0,\ \theta = \theta_2$

$$A_2 = \begin{bmatrix} \cos\theta_2 & \sin\theta_2 & 0 & a_2\cos\theta_2 \\ \sin\theta_2 & -\cos\theta_2 & 0 & a_2\sin\theta_2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

P.T.O

$$T_0^2 = A_1 A_2$$

$$= \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & a_1\cos\theta_1 \\ \sin\theta_1 & \cos\theta_1 & 0 & a_1\sin\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & \sin\theta_2 & 0 & a_2\cos\theta_2 \\ \sin\theta_2 & -\cos\theta_2 & 0 & a_2\sin\theta_2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^2 = \begin{bmatrix} \cos(\theta_1+\theta_2) & \sin(\theta_1+\theta_2) & 0 & a_1\cos\theta_1 + a_2\cos(\theta_1+\theta_2) \\ \sin(\theta_1+\theta_2) & -\cos(\theta_1+\theta_2) & 0 & a_1\sin\theta_1 + a_2\sin(\theta_1+\theta_2) \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore R_0^2 = \begin{bmatrix} \cos(\theta_1+\theta_2) & \sin(\theta_1+\theta_2) & 0 \\ \sin(\theta_1+\theta_2) & -\cos(\theta_1+\theta_2) & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

&

$$O_2 = \begin{bmatrix} a_1\cos\theta_1 + a_2\cos(\theta_1+\theta_2) \\ a_1\sin\theta_1 + a_2\sin(\theta_1+\theta_2) \\ 0 \end{bmatrix}$$

P.T.O

classmate

Date ____
Page ____

~~$T_0^3 = A_1 A_2 A_3$~~

~~$= T_0^2 \cdot A_3$~~

$$\cancel{= \begin{bmatrix} \cos(\theta_1+\theta_2) & \sin(\theta_1+\theta_2) & 0 & a_1\cos\theta_1 + a_2\cos(\theta_1+\theta_2) \\ \sin(\theta_1+\theta_2) & -\cos(\theta_1+\theta_2) & 0 & a_1\sin\theta_1 + a_2\sin(\theta_1+\theta_2) \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot A_3}$$

where ~~A_3~~ =

Link - 3

$a = 0, \quad \alpha = 0, \quad d = d_3, \quad \theta = 0$

$$\therefore A_3 = T_2^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$T_0^3 = T_0^2 \cdot A_3$

$$= \begin{bmatrix} \cos(\theta_1+\theta_2) & \sin(\theta_1+\theta_2) & 0 & a_1\cos\theta_1 + a_2\cos(\theta_1+\theta_2) \\ \sin(\theta_1+\theta_2) & -\cos(\theta_1+\theta_2) & 0 & a_1\sin\theta_1 + a_2\sin(\theta_1+\theta_2) \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta_1+\theta_1) & \sin(\theta_1+\theta_1) & 0 & a_1\cos\theta_1 + a_2\cos(\theta_1+\theta_1) \\ \sin(\theta_1+\theta_1) & -\cos(\theta_1+\theta_1) & 0 & a_1\sin\theta_1 + a_2\sin(\theta_1+\theta_2) \\ 0 & 0 & -1 & -d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ End-effector coordinates wrt ground frame $= \begin{bmatrix} a_1\cos\theta_1 + a_2\cos(\theta_1+\theta_2) \\ a_1\sin\theta_1 + a_2\sin(\theta_1+\theta_2) \\ -d_3 \end{bmatrix}$

Now, for Jacobian

$$J_{vi} = \begin{cases} Z_{i-1} \times (\vec{O_e} - \vec{O_{i-1}}), & i \in \text{revolute} \\ Z_{i-1}, & i \in \text{prismatic} \end{cases}$$

$$\therefore J_{v1} = Z_0 \times (\vec{O_e} - \vec{O_0})$$

$$\therefore Z_0 = R_0^0 \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\therefore J_{v_1} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ a_1\cos\theta_1 + a_2\cos(\theta_1+\theta_2) & a_1\sin\theta_1 + a_2\sin(\theta_1+\theta_2) & -d_3 \end{vmatrix}$$

$$J_{v_1} = \begin{bmatrix} -a_1\sin\theta_1 - a_2\sin(\theta_1+\theta_2) \\ a_1\cos\theta_1 + a_2\cos(\theta_1+\theta_2) \\ 0 \end{bmatrix}$$

RTO

$J_{v_2} = Z_1 \times (\overrightarrow{O_P} - \overrightarrow{O_1})$

$$Z_1 = R_0^1 \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\therefore J_{v_2} = \begin{vmatrix} i & j & k \\ 0 & 0 & 1 \\ a_2\cos(\theta_1+\theta_2) & a_2\sin(\theta_1+\theta_2) & -d_3 \end{vmatrix}$$

$$J_{v_2} = \begin{bmatrix} -a_2\sin(\theta_1+\theta_2) \\ a_2\cos(\theta_1+\theta_2) \\ 0 \end{bmatrix}$$

$$J_{v_3} = Z_2 = R_0^2 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

$$\therefore J_v = \begin{bmatrix} -a_1\sin\theta_1 - a_2\sin(\theta_1+\theta_2) & -a_2\sin(\theta_1+\theta_2) & 0 \\ a_1\cos\theta_1 + a_2\cos(\theta_1+\theta_2) & a_2\cos(\theta_1+\theta_2) & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$J_w = [Z_0 \quad Z_1 \quad 0]$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

P.T.O

$$J = \begin{bmatrix} -a_1 \sin\theta_1 - a_2 \sin(\theta_1+\theta_2) & -a_2 \sin(\theta_1+\theta_2) & 0 \\ a_1 \cos\theta_1 + a_2 \cos(\theta_1+\theta_2) & a_2 \cos(\theta_1+\theta_2) & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Consider $\dot{q} = \begin{bmatrix} \theta_1' \\ \theta_2' \\ d' \end{bmatrix}$

$\therefore \vec{V_e} = J_v \dot{q}$

$$= \begin{bmatrix} -a_1 \sin\theta_1 - a_2 \sin(\theta_1+\theta_2) & -a_2 \sin(\theta_1+\theta_2) & 0 \\ a_1 \cos\theta_1 + a_2 \cos(\theta_1+\theta_2) & a_2 \cos(\theta_1+\theta_2) & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \theta_1' \\ \theta_2' \\ d' \end{bmatrix}$$

$$\vec{V_e} = \begin{bmatrix} (-a_1 \sin\theta_1 - a_2 \sin(\theta_1+\theta_2))\theta_1' - a_2 \sin(\theta_1+\theta_2)\theta_2' \\ (a_1 \cos\theta_1 + a_2 \cos(\theta_1+\theta_2))\theta_1' + a_2 \cos(\theta_1+\theta_2)\theta_2' \\ -d' \end{bmatrix}$$

5. A singular configuration is when

- ☆ Configurations from which certain directions of motion may be unattainable.

- ☆ Bounded gripper velocities may correspond to unbounded joint velocities.

- ☆ Usually (not always) singularities correspond to points on the boundary of the manipulator workspace, i.e. the maximum reach of manipulator.

— We can find whether the configuration is singular or not by checking the Jacobian's Rank. Jacobian has shape $(6, n)$

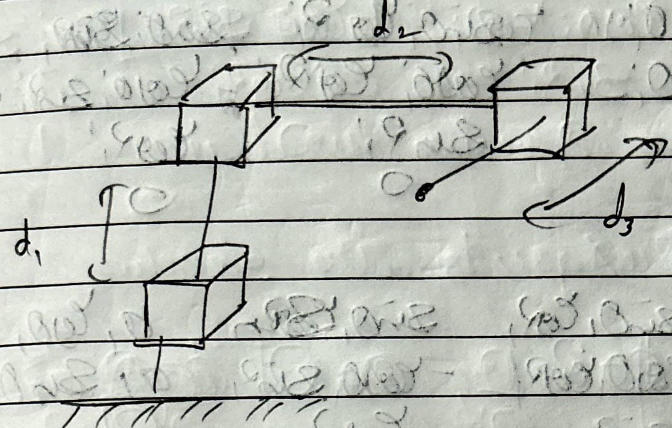
$\therefore$ If Rank $< \min(6, n)$ it is a singular config.

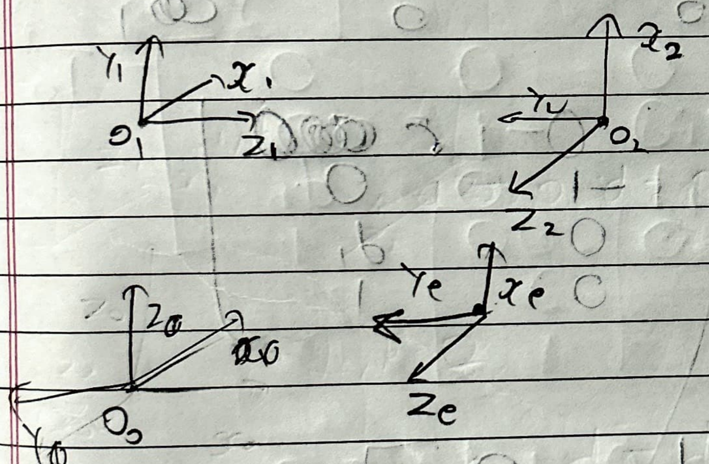
~~☆ Also, we can equate determinant of Jacobian to 0 and find singularity point~~

- ☆ Consider a manipulator with 3-DOF arm and 3-DOF wrist. We can find the singular config by $J(q) = 0$.

Assignment - 3

9.





We have the parameters

Link	a	α	d	θ	Type
1	0	$\pi/2$	d_1	0	P
2	0	$-\pi/2$	d_2	$\pi/2$	P
3	0	0	d_3	0	P

P.T.O

$$A_i = T_{i-1}^{i} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For A_1

$a = 0,\ \alpha = \frac{\pi}{2},\ \theta = 0,\ d_1$

$$\therefore A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = T_0^1$$

$\therefore$ For A_2

$a = 0,\ \alpha = -\frac{\pi}{2},\ d = d_2,\ \theta = \frac{\pi}{2}$

$$\therefore A_2 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For A_3

$a=0,\ \alpha=0,\ d=d_3,\ \theta=0$

$$\therefore A_3=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&d_3\\0&0&0&1\end{bmatrix}$$

$$\therefore T_0^2=\begin{bmatrix}1&0&0&0\\0&0&-1&0\\0&1&0&d_1\\0&0&0&1\end{bmatrix}\begin{bmatrix}0&0&-1&0\\1&0&0&0\\0&-1&0&d_2\\0&0&0&1\end{bmatrix}$$

$$=\begin{bmatrix}0&0&-1&0\\0&1&0&-d_2\\1&0&0&d_1\\0&0&0&1\end{bmatrix}$$

$T_0^3=T_0^2\cdot A_3$

$$=\begin{bmatrix}0&0&-1&0\\0&1&0&-d_2\\1&0&0&d_1\\0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&d_3\\0&0&0&1\end{bmatrix}$$

$$T_0^3=\begin{bmatrix}0&0&-1&-d_3\\0&1&0&-d_2\\1&0&0&d_1\\0&0&0&1\end{bmatrix}$$

P.T.O

$\therefore$ end-effector coordinates $\equiv \begin{bmatrix} -d_3 \\ -d_2 \\ d_1 \end{bmatrix}$

Now, for jacobian, we have

* All three joints are revolute

$J = \begin{bmatrix} J_v \\ J_w \end{bmatrix}$, where $J_v = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}^T$

$J_{vi} = Z_{i-1}$

$Z_{i-1} = R_0^{i-1} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$Z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$Z_1 = R_0^1 \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$

$$J_v = R_0^3 \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore J_v = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\therefore J = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore \vec{V_e} = J_V \cdot \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

V_e.   P-J-O

$$\vec{V_e} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{d}_1 \\ \dot{d}_2 \\ \dot{d}_3 \end{bmatrix}$$

$$\vec{V_e} = \begin{bmatrix} -\dot{d}_3 \\ -\dot{d}_2 \\ \dot{d}_1 \end{bmatrix}$$