Date 12/09/2024

Thursday

D-H Parameters

☆ Standardize and automate coding.

0 to n links
(0 - base/ground)

1 to n joints (i^{th} joint connects link $i-1$ and i)

0 to n coordinate frames - i^{th} coordinate rigidly connected to i^{th} link.

i^{th} joint variable $\longrightarrow q_i$

Z_i are the Z's along joint axis (axis of rotation for R joint and axis of motion for P joint)

Instead of descriptive representation, need compact tabular representation.

Ideally need 6 parameters to represent any arbitrary homogeneous transformation.
For DH, we standardize convention so that we only need 4. (r - [illegible] origin -)

Recall

$$H = \begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix}$$

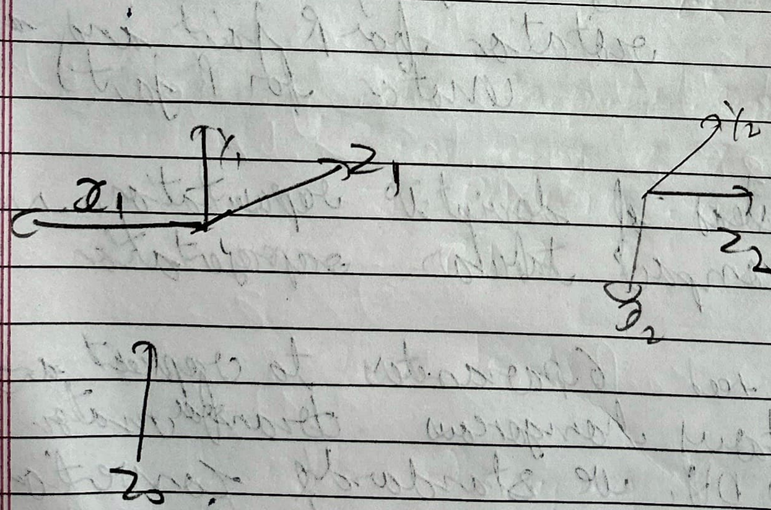
(DH parameters for each joint)

pair of frames

(θ, d, a, α)

~~A = Rot...~~

$$A = R_{z,\theta} \cdot Trans_{z,d} \cdot Trans_{x,a} \cdot Rot_{x,\alpha}$$



$\cancel{A = R_{z,\theta}}$:

$$\therefore A = \begin{bmatrix} \cos\theta & -\sin\theta\cos\alpha & \sin\theta\sin\alpha & a\cos\theta \\ \sin\theta & \cos\theta\cos\alpha & -\cos\theta\sin\alpha & a\sin\theta \\ 0 & \sin\alpha & \cos\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Chapter 6 - Dynamics

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = \tau_i$$

Equations of Motion

$$\sum_j d_{kj}(q)\,\ddot{q}_j + \sum_{i,j} c_{ijk}(q)\,\dot{q}_i\dot{q}_j + \phi_k(q) = \tau_k,$$

$$k = 1, 2, 3 \ldots n$$

$$\phi_k(q) = \frac{\partial V}{\partial q_k}$$

$V \rightarrow$ Potential Energy

P.T.O

More common to write in matrix form

$$\boxed{D(q)\cdot\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau}$$

$\dot{D}q - 2C(q,\dot{q})$ should be skew symmetric

Chapter 4 - Inverse Kinematics

☆ Given end-effector position and orientation find joint positions/variables.

$$H_0^n(q_1, q_2 \cdots q_n) = \begin{bmatrix} R_0^n & d_0^n \\ 0 & 1 \end{bmatrix}$$

$R = R_0^n$, $d = d_0^n$ → given

find $q_1, q_2 \cdots q_n$

Prefer closed-form solutions

☆ Computational reasons

☆ Resolving redundancies

Two tricks

(i) Kinematic Decoupling

Manipulator with atleast 6 DOF and last 3 joints intersecting at a point (spherical wrist), ~~joints~~ axes

☆ Let us assume exactly 6 joints and last 3 joint axis intersect at a point O.

z_4, z_5, z_6 intersect at O.

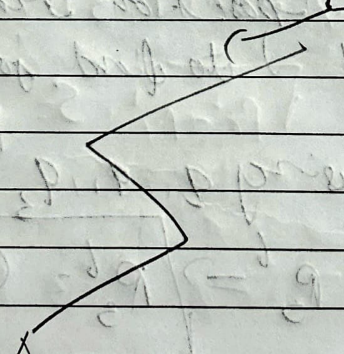
O_4, O_5 are at O.

O_6 can be in general be d_6 away from O in z_6 direction

$$O_6 = O + d_6 R \hat{k}$$

$$\vec{O_6} - \vec{O} = d_6 R \hat{k}$$

R gives orientation of end-effector



P.T-O

We know $d_6^7 = O_6$, also we know R, d_6

$$\therefore O = O_6 - d_6 R \hat{k}$$

$$\vec{O} = \vec{p} : \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} d_x - d_6 r_{13} \\ d_y - d_6 r_{23} \\ d_z - d_6 r_{33} \end{bmatrix}$$

1. Use equation above to find p (or O) to ~~speed~~.

2. Solve inverse position kinematics (as 4.1.6 for standard) to find joint variables q_1, q_2, q_3

3. Calculate R_0^3 using q_1, q_2, q_3

4. Since $R = R_0^3 \cdot R_3^6 \Rightarrow \boxed{R_3^6 = (R_0^3)^T R}$

5. Find three wrist angles (Euler angles) that give R_3^6.

Consider

$$R_3^{L} = \begin{bmatrix} n_1 & n_2 & n_3 \\ n_4 & n_5 & n_6 \\ n_7 & n_8 & n_9 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} C\phi C\theta C\psi - S\phi S\psi & -C\phi C\theta S\psi - S\phi C\psi & \cos\phi \sin\theta \\ S\phi C\theta C\psi - C\phi S\phi & -S\phi C\theta S\psi + C\phi C\psi & \sin\phi \sin\theta \\ -S\theta C\psi & S\theta S\psi & \cos\theta \end{bmatrix}$$

euler angle (θ, ϕ, ψ)

$R_{z\theta} \, R_{y\phi} \, R_{z\psi}$

$$V = J_V \dot{q}$$

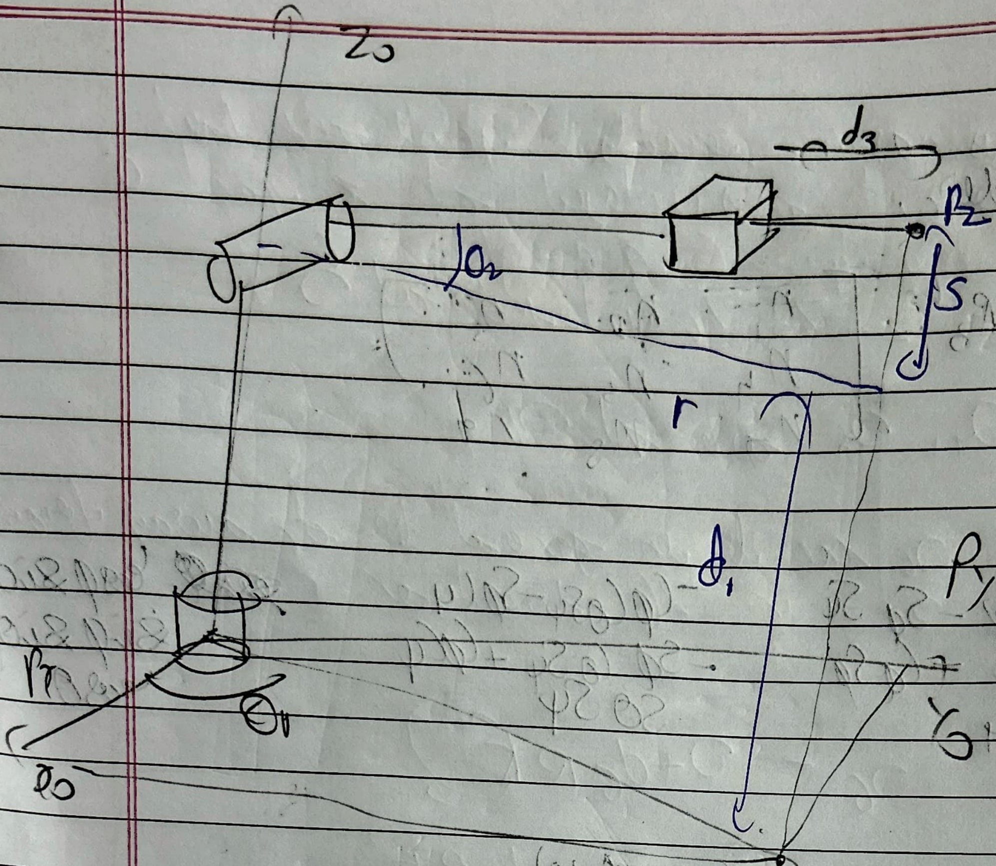
$$\delta V = J_V \delta q$$

$$\delta q = J_V \cdot \delta q$$

$$\therefore \delta q = J_V^{-1} \delta x$$

$$q_{t+1} - q_t = J_V^{+} \delta x$$

$$\boxed{\therefore q_{t+1} = q_t + J_V^{+} \delta x}$$



$\theta_1 = A\tan(P_x, P_y)$

$\theta_2 = A\tan(r, S)$

$r = \sqrt{P_x^2 + P_y^2}, \; S = P_z - d_1$