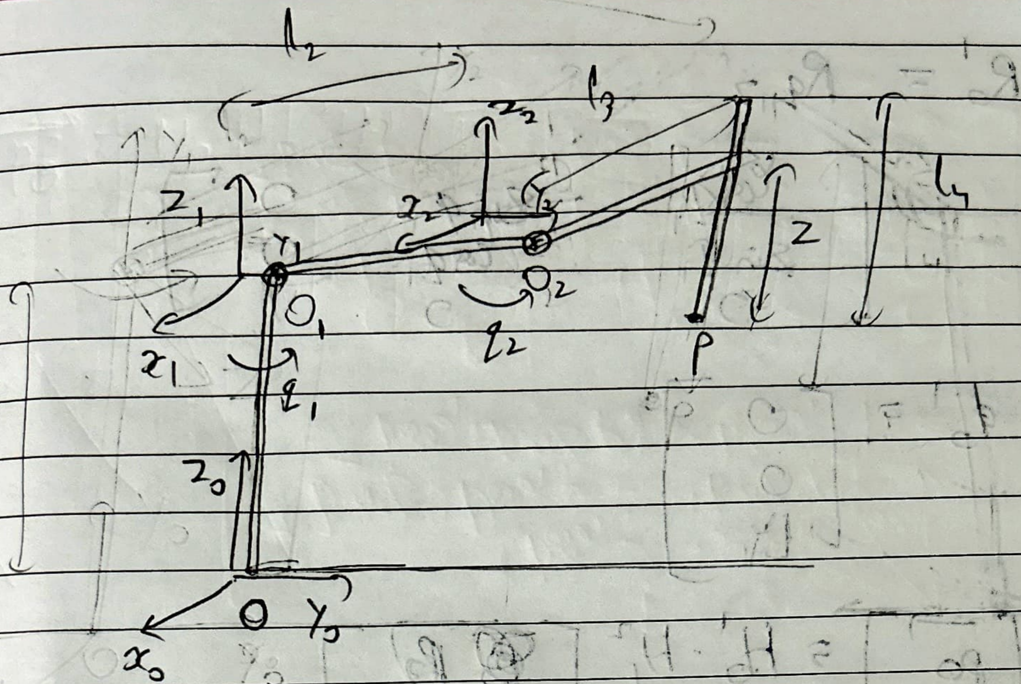
7.



$$\cancel{\vec{p_2} = l_3\hat{k} - z}$$

$$\vec{p_2} = l_3\hat{j} - z\hat{k}$$

$$R_1^2 = R_{z,q_2}$$

$$d_1^2 = l_2\hat{j}$$

$$\therefore R_1^2 = \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 \\ \sin q_2 & \cos q_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$d_1^2 = \begin{bmatrix} 0 \\ l_2 \\ 0 \end{bmatrix}, \quad \vec{p_2} = \begin{bmatrix} 0 \\ l_3 \\ -z \end{bmatrix}$$

$$R_0^1 = R_{q_1, z}$$

$$= \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 \\ \sin q_1 & \cos q_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$d_0^1 = \begin{bmatrix} 0 \\ 0 \\ L_1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} p_0 \\ 1 \end{bmatrix} = H_0^1 \cdot H_1^2 \begin{bmatrix} p_2 \\ 1 \end{bmatrix}$$

$$\therefore H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix}$$

$$\therefore H_0^1 = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 & 0 \\ \sin q_1 & \cos q_1 & 0 & 0 \\ 0 & 0 & 1 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 & 0 \\ \sin q_2 & \cos q_2 & 0 & L_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_0^1 \cdot H_1^2 = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 & 0 \\ \sin q_1 & \cos q_1 & 0 & 0 \\ 0 & 0 & 1 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 & 0 \\ \sin q_2 & \cos q_2 & 0 & l_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore H_0^2 = \begin{bmatrix} \cos(q_1+q_2) & -\sin(q_1+q_2) & 0 & -l_2 \sin q_1 \\ \sin(q_1+q_2) & \cos(q_1+q_2) & 0 & l_2 \cos q_1 \\ 0 & 0 & 1 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

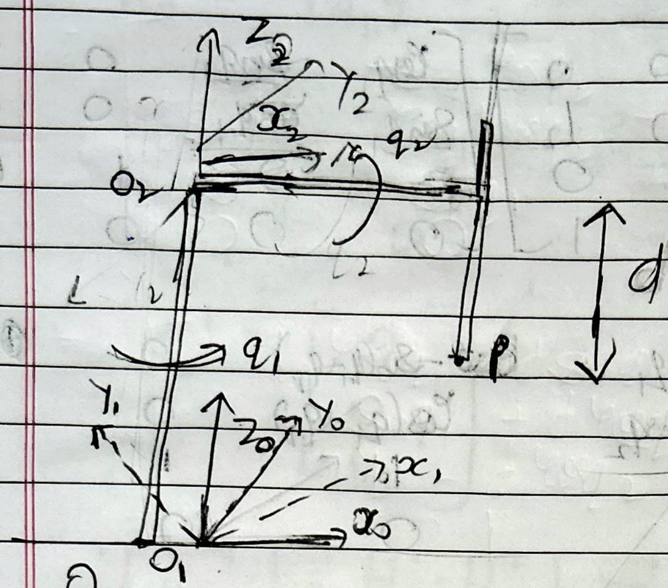
$$\therefore \begin{bmatrix} \vec{p}_0 \\ 1 \end{bmatrix} = H_0^2 \begin{bmatrix} \vec{p}_2 \\ 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} \vec{p}_3 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(q_1+q_2) & -\sin(q_1+q_2) & 0 & -l_2 \sin q_1 \\ \sin(q_1+q_2) & \cos(q_1+q_2) & 0 & l_2 \cos q_1 \\ 0 & 0 & 1 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ L_3 \\ -Z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \vec{p}_3 \\ 1 \end{bmatrix} = \begin{bmatrix} -l_3 \sin(q_1+q_2) - l_2 \sin q_1 \\ l_3 \cos(q_1+q_2) + l_2 \cos q_1 \\ L_1 - Z \\ 1 \end{bmatrix}$$

$$\therefore \vec{p}_3 = \begin{bmatrix} -l_3 \sin(q_1+q_2) - l_2 \sin q_1 \\ l_3 \cos(q_1+q_2) + l_2 \cos q_1 \\ L_1 - Z \end{bmatrix}$$

9.



$R_0^1 = R_{z,q_1}, \; d_0^1 = 0$

$\therefore R_0^1 = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 \\ \sin q_1 & \cos q_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore H_0^1 = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 & 0 \\ \sin q_1 & \cos q_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$R_1^2 = R_{x,q_2}$

$R_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos q_2 & -\sin q_2 \\ 0 & \sin q_2 & \cos q_2 \end{bmatrix}$

$d_1^2 = L_1 \hat{k}$

$$\therefore H_1^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos q_2 & -\sin q_2 & 0 \\ 0 & \sin q_2 & \cos q_2 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \vec{P_2} = \begin{bmatrix} L_2 \\ 0 \\ -d \end{bmatrix}$$

$$\therefore \left[\vec{P_0}\right]_1 = H_0^1 \; H_1^2 \cdot \left[\vec{P_2}\right]_1$$

$$\cancel{\therefore \left[\vec{P_0}\right]_1 =}$$

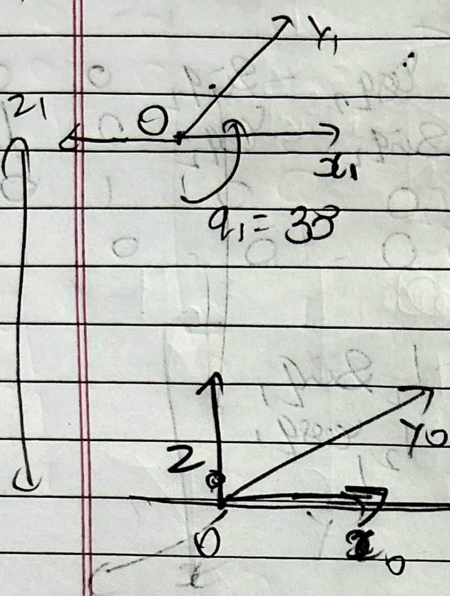
$$\left[\vec{P_0}\right]_1 = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 & 0 \\ \sin q_1 & \cos q_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos q_2 & \sin q_2 & 0 \\ 0 & \sin q_1 & \cos q_2 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \left[\vec{P}\right]_1$$

$$= \begin{bmatrix} \cos q_1 & -\sin q_1 \cos q_2 & \sin q_1 \sin q_2 & 0 \\ \sin q_1 & \cos q_1 \cos q_2 & -\cos q_1 \sin q_2 & 0 \\ 0 & \sin q_1 & \cos q_2 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_2 \\ 0 \\ -d \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} l_2 \cos q_1 + d \sin q_1 \cos q_2 \\ l_2 \sin q_1 + d \cos q_1 \sin q_2 \\ L_1 - d \cos q_2 \\ 1 \end{bmatrix}$$

$$\therefore \vec{P_0} = \begin{bmatrix} l_2 \cos q_1 + d \sin q_1 \cos q_2 \\ l_2 \sin q_1 + d \cos q_1 \sin q_2 \\ L_1 - d \cos q_2 \end{bmatrix}$$

10.



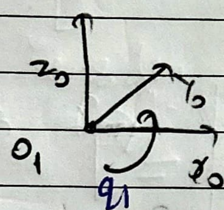
$$R^0_1 = R_{x,q_1} \cdot R_{z,q_2}$$

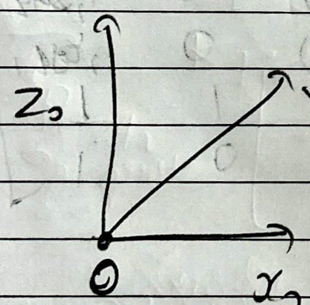
$$d^1_0 = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$$

$$\vec{p}_1 = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$$

Consider the following

(i)

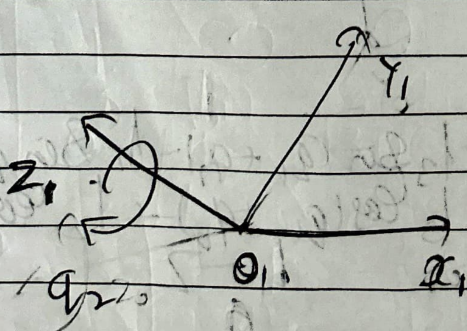




☆ Here, the drone moves up by 10m.

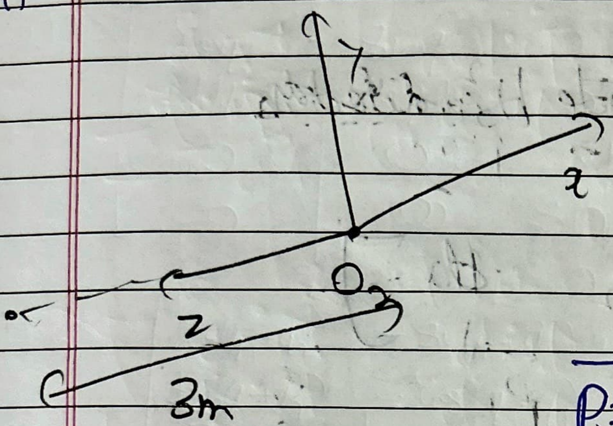
☆ It rotates about the x axis by $q_1 = 30°$

(ii)



☆ It rotates about the new z axis by $q_2 = 65$

(iii)



$$\vec{P_3} = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$$

☆ Here, $R_0^1 = R_{xq_1}$

$$d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$$

Also, $R_1^2 = R_{zq_2}$, $d_1^2 = 0$

∴ We can now write ~~directly~~

$$H_0^2 = H_0^1 \cdot H_1^2 \cdot \vec{P_2}$$

where $H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix}$, $H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix}$

$\therefore \cancel{H_0^1} = \begin{bmatrix} R_{xq_1} & d_1 \\ 0 & 1 \end{bmatrix}$

Instead, we can write H_0^2 directly as

$$H_0^2 = \begin{bmatrix} R_{xq_1} R_{zq_2} & d_1 \\ 0 & 1 \end{bmatrix}$$

$$R_{xq_1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 \\ 0 & 1/2 & \sqrt{3}/2 \end{bmatrix}$$

$$R_{zq_2} = \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 \\ \sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore R_0^2 = R_{xq_1} \cdot R_{zq_2}$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 \\ 0 & 1/2 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 \\ \sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_0^2 = \begin{bmatrix} 1 & -\sqrt{3}/2 & 0 \\ 3/4 & \sqrt{3}/4 & -1/2 \\ \sqrt{3}/4 & 1/4 & \sqrt{3}/2 \end{bmatrix}$$

$$\therefore H_0^2 = \begin{bmatrix} 1 & -\sqrt{3}/2 & 0 & 0 \\ 3/4 & \sqrt{3}/4 & -1/2 & 0 \\ \sqrt{3}/4 & 1/4 & \sqrt{3}/2 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \begin{pmatrix} \vec{p_0} \\ 1 \end{pmatrix} = H_0^2 \cdot \begin{pmatrix} \vec{p_2} \\ 1 \end{pmatrix}$$

$$= \begin{bmatrix} 1 & -\sqrt{3}/2 & 0 & 0 \\ 3/4 & \sqrt{3}/4 & -1/2 & 0 \\ \sqrt{3}/4 & 1/4 & \sqrt{3}/2 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$\therefore p_0 = \begin{bmatrix} 0 \\ -3/2 \\ 3\sqrt{3}/2 + 10 \end{bmatrix}$$

2. Consider R_0^1 and R_1^0

We know that $R_0^1 = (R_1^0)^{-1} = (R_1^0)^T$

and $R_1^0 = (R_0^1)^{-1} = (R_0^1)^T$

Now, $R_0^1 \cdot R_1^0 = (R_0^1) \cdot (R_0^1)^{-1} = I$

but $R_0^1 = (R_0^1)^T$

$\therefore R_0^1 \cdot (R_0^1)^T = I$

Consider $R_0^1 = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$

$\therefore R_0^1 \cdot (R_0^1)^T = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} r_{11} & r_{21} & r_{31} \\ r_{12} & r_{22} & r_{32} \\ r_{13} & r_{23} & r_{33} \end{bmatrix}$

$\therefore R_0^1 \cdot (R_0^1)^T = I$

$\therefore (r_{11})^2 + (r_{12})^2 + (r_{13})^2 = 1$

and

$(r_{11} \cdot r_{21}) + (r_{12} \cdot r_{22}) + (r_{13} \cdot r_{23}) = 0$

~~So we can say that~~

$r_i \cdot r_j$

Now, consider

$$R_o' = \left[\vec{r_1} \quad \vec{r_2} \quad \vec{r_3} \right]$$

$$\therefore R_o' \cdot (R_o')^T = \left[\vec{r_1} \quad \vec{r_2} \quad \vec{r_3} \right] \begin{bmatrix} \vec{r_1} \\ \vec{r_2} \\ \vec{r_3} \end{bmatrix}$$

$$(\vec{r_i})^T \cdot \vec{r_i} = 1$$

$$\text{and} \quad (\vec{r_i})^T \cdot \vec{r_j} = 0 \; (i \neq j)$$

Hence, $\vec{r_i}$ and $\vec{r_j}$ are orthogonal to each other

3

To prove: $|R_\theta'| = 1$

We know that $(R_\theta')^T \cdot R_\theta' = I$

$\det(A \cdot B) = \det(A) \cdot \det(B)$

$\therefore (|R_\theta'|)^2 = 1$

$\det(R) = \det(I) = 1$

6. $R S(\vec{a}) R^T \vec{b}$

$= R \cdot (S(\vec{a}) \cdot (R^T \vec{b}))$

$= R \cdot (\vec{a} \times (R^T \vec{b}))$

$= R(\vec{a}) \times R(R^T \vec{b})$

$= R(\vec{a}) \times \vec{b}$

$= (R\vec{a}) \times \vec{b}$

$= S(R\vec{a}) \vec{b}$

$\therefore (R S(\vec{a}) \cdot R^T) = S(R\vec{a})$