

YAKEEN NEET 2.0

2026

Basic Maths and Calculus (Mathematical Tools)

Physics

Lecture - 4

By- Manish Raj (MR Sir)

8 3 1/2 month

→ 30 chapters





Topics to be covered

1 Phase difference & Phasor diagram.

2

Vector

3

A/C \rightarrow 70%

4

S.H.M \rightarrow 30%

Sound wave \rightarrow 20%

Wave optics \rightarrow 30%

EM wave \rightarrow 20%

\rightarrow (a) Yes

(b) NO



Recap of previous lecture

1

2

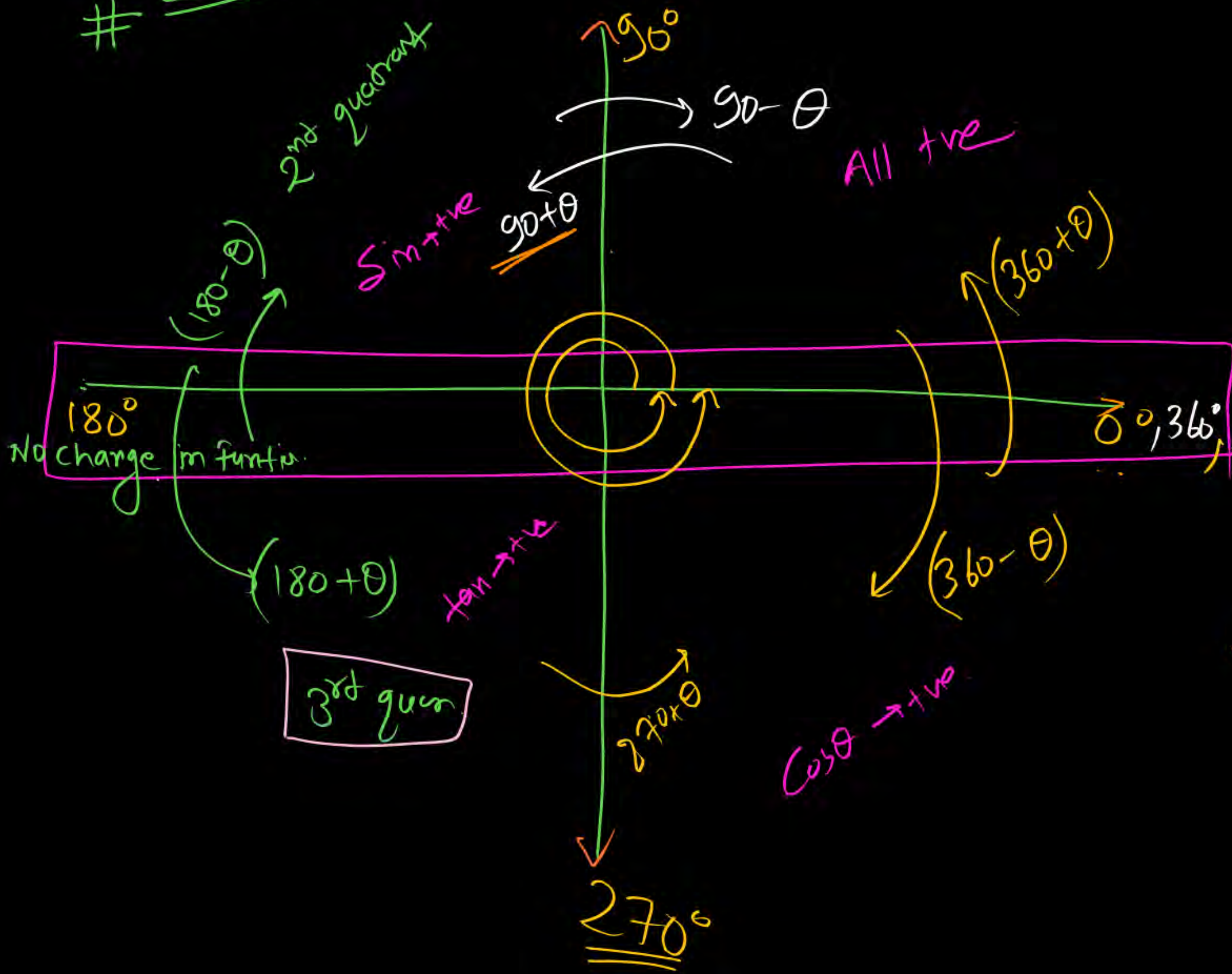
3

4

→ function.

Class full Temprament focus se
Karo , class ka H/w rough me ya
Kuch Question Jo na bane , badhiya ho
H/w usko notes me likh lo.

Revision



$$\sin 0^\circ = 0 = \sin 360^\circ$$

$$\cos 0^\circ = 1 = \cos 360^\circ$$

$$\cos 180^\circ = -1$$

$$\sin 90^\circ = 1$$

$$\sin 270^\circ = -1$$

Step-1 → function kis quadrant me hai uske acc to sign likh lo

$$\sin(180^\circ + \theta) = -\sin \theta$$

$$\begin{pmatrix} 180^\circ + \theta \\ 180^\circ - \theta \\ 360^\circ + \theta \\ 360^\circ - \theta \end{pmatrix} \rightarrow \text{No change}$$

$$\sin(90-\theta) = +\cos\theta$$

$$\cos(90+\theta) = -\sin\theta$$

$$\sin(90+\theta) = +\cos\theta$$

$$\cos(90-\theta) = +\sin\theta$$

$$\sin(180+\theta) = -\sin\theta$$

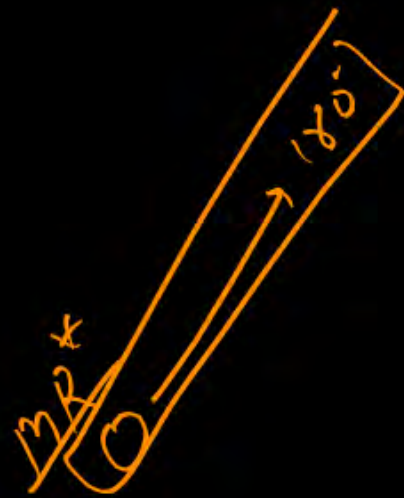
$$\cos(180+\theta) = -\cos\theta$$

$$\cos(180-\theta) = -\cos\theta$$

$$\sin(180+\theta) = -\sin\theta$$

$$\begin{aligned} \sin\theta &\longleftrightarrow \cos\theta \\ \tan\theta &\longleftrightarrow \cot\theta \\ \sec\theta &\longleftrightarrow \csc\theta \end{aligned}$$

Note: θ और θ लेना



$$\sin(270+\theta) = -\cos\theta$$

$$\sin(270-\theta) = -\cos\theta$$

$$\cos(270+\theta) = +\sin\theta$$

$$\sin(360+\theta) = +\sin\theta$$

$$\sin(360-\theta) = -\sin\theta$$

$$\cos(360-\theta) = +\cos\theta$$

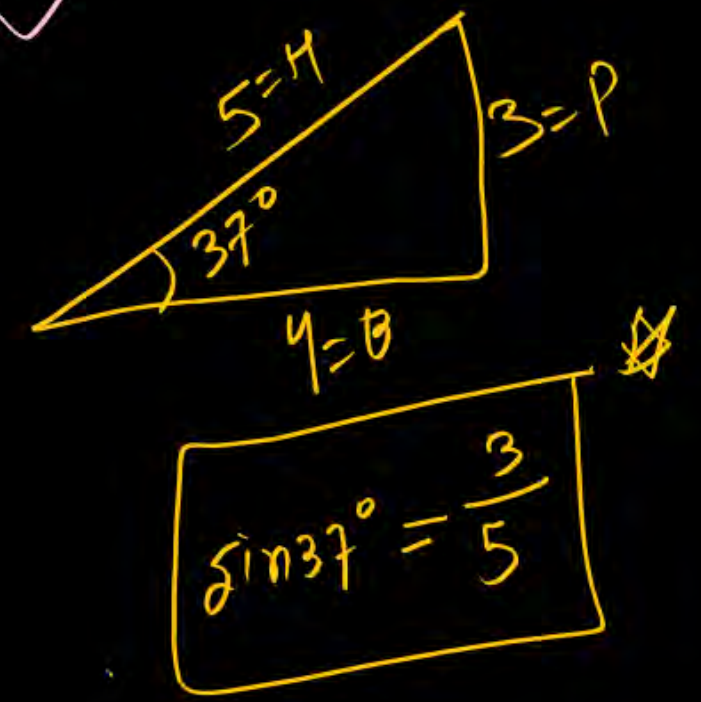
step-1 \rightarrow sign diagram know

$$\# \sin(127^\circ) = \sin(180-53) = +\sin 53^\circ = \frac{4}{5} \quad \checkmark$$

$$\begin{aligned} \# \cos(143^\circ) &= \cos(180-37) = -\cos 37^\circ = -\frac{4}{5} \quad \checkmark \\ &= \cos(90+53) = -\sin 53^\circ = -\frac{4}{5} \quad \checkmark \end{aligned}$$

$$\# \tan(143^\circ) = \tan(180-37) = -\tan 37^\circ = -\frac{3}{4} \quad \checkmark$$

$$\# \sin(750^\circ) = \sin(\underbrace{720^\circ + 30^\circ}_{1^{st} \text{ quarter}}) = +\sin 30^\circ = +\frac{1}{2}$$



Phase of Trigonometric function :-

→ phase is angle Inside Trigonometric function.

$$y = \sin(\theta)$$

$\theta = \text{phase.}$

$$y = 4 \cos(\underbrace{\theta + \phi})$$

phase = $\theta + \phi$

$$y = 3 \sin(\omega t + \phi + \frac{\pi}{4})$$

phase at time 't' = $\omega t + \phi + \frac{\pi}{4}$

$$y = \sin(\underbrace{\omega t + \phi}_{\text{phase}})$$

Phase difference :- Angle difference b/w two same trigonometric function.

$$\begin{cases} y_1 = A \sin(\omega t + \pi/2) \\ y_2 = A \sin(\omega t) \end{cases}$$

So $\phi = \phi_1 - \phi_2$

$$= \omega t + \pi/2 - \omega t$$

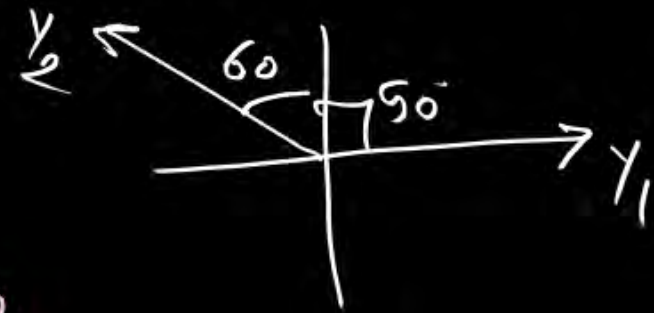
$$\boxed{\phi = \pi/2}$$

Ans

$$\begin{aligned} y_1 &= A \sin(\omega t + \pi/3) \\ y_2 &= A \sin(\omega t - \pi/3) \end{aligned}$$

$$\begin{aligned} \phi (\text{phase diff}) &= \phi_1 - \phi_2 \\ &= \omega t + \pi/3 - (\omega t - \pi/3) \\ &= \omega t + \pi/3 - \omega t + \pi/3 \\ &= \frac{2\pi}{3} \end{aligned}$$

$$\left\{ \begin{array}{l} Y_1 = A \cos(\omega t - \pi/2) \text{ पीछे} \\ Y_2 = A \cos(\omega t + \pi/3) \text{ आगे} \end{array} \right.$$



$$\phi = \phi_1 - \phi_2$$

$$= (\omega t - \pi/2) - (\omega t + \pi/3)$$

$$= \cancel{\omega t} - \pi/2 - \cancel{\omega t} - \pi/3$$

$$\phi_2 - \phi_1 = +\frac{5\pi}{6} = 150^\circ$$

$$Y_1 = A \sin(\omega t)$$

$$Y_2 = A \sin(\omega t + \pi/6)$$

$$\phi = \phi_1 - \phi_2 = -\pi/6$$

~~$$\# \begin{array}{l} Y_1 = A \sin(\omega t) \\ Y_2 = A \sin(\omega t + \pi/2) \end{array}$$~~

~~$$\phi = \phi_1 - \phi_2 =$$~~

next p.

#

$$y_1 = A \sin(\omega t)$$

#

$$y_2 = A \sin(\omega t + \pi/2) = A \cos(\omega t) \quad \checkmark \text{ आगे}$$

$$\phi = \phi_1 - \phi_2 \\ = -\pi/2$$

$$y_1 = A \sin(\omega t)$$

$$y_2 = A \cos(\omega t) \leftarrow \text{आगे } (\pi/2)$$

$$\phi = \pi/2 = \phi_2 - \phi_1 \quad (y_2 \text{ आगे})$$

MR* Box

$$\sin(90^\circ + \omega t) = \cos(\omega t) \quad \checkmark$$

find phase difference b/w γ_1 & γ_2

$$\begin{cases} \gamma_1 = 4 \sin(\theta + \pi/4)_{\text{अगे}} \\ \gamma_2 = 3 \sin(\theta - \pi/2)_{\text{पीछे}} \end{cases}$$

$$\begin{aligned} \phi &= \phi_1 - \phi_2 \\ &= (\theta + \pi/4) - (\theta - \pi/2) \\ &= \pi/4 + \pi/2 \end{aligned}$$

$$\boxed{\phi = \frac{3\pi}{4} \text{ rad} = \underline{\underline{135^\circ}}}$$

$$\gamma_1 = 4 \sin(\theta + \pi/3)_{\text{अगे}}$$

$$\gamma_2 = 3 \sin(\theta + \pi/6)_{\text{अगे}}$$

$$\begin{aligned} \text{Soln} \quad \phi &= \phi_1 - \phi_2 \\ &= \theta + \pi/3 - (\theta + \pi/6) \\ &= \pi/3 - \pi/6 \end{aligned}$$

$$= \frac{2\pi - \pi}{6} = \left(\frac{\pi}{6} \right)$$

30°

$$\gamma_1 = 3 \cos(\omega t + \pi/2)$$

$$\gamma_2 = 3 \sin(\omega t)$$

$$\begin{aligned} \text{Soln} \quad \phi &= \phi_1 - \phi_2 \\ &= \omega t + \pi/2 + \pi/2 - \omega t \\ &\quad \boxed{\phi = \pi} \text{ rad} \end{aligned}$$

$$\gamma_1 = 3 \sin(\omega t)$$

$$\gamma_2 = 3 \cos(\omega t - \pi/2)$$

$$\text{Soln} \quad \phi = \underline{\underline{\text{zero}}}$$

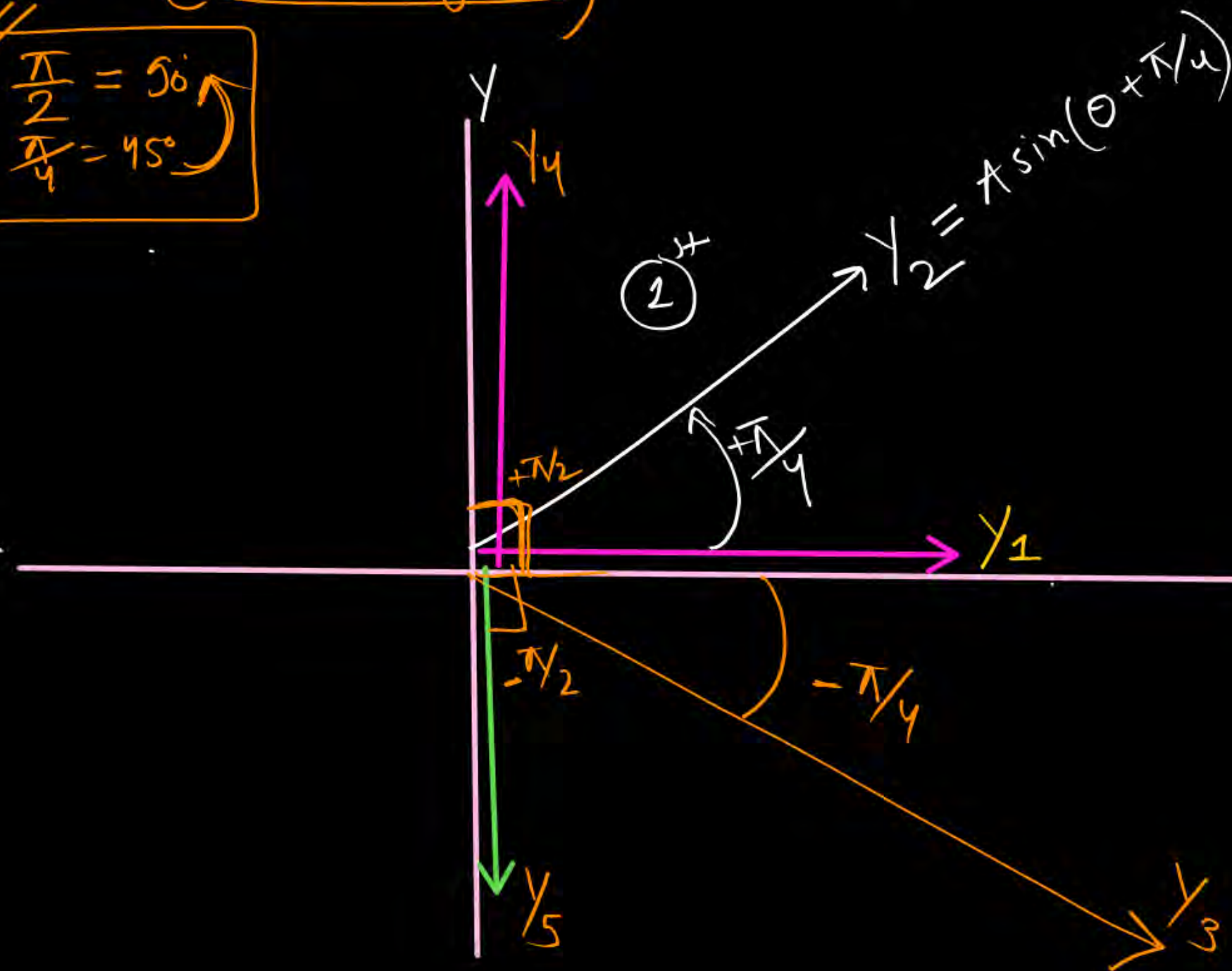
Phase diagram

(This is not graph)

Representation of Trigonometric function with Phase (Angle)

$$\frac{\pi}{2} = 90^\circ$$

$$\frac{\pi}{4} = 45^\circ$$



$$y_1 = A \sin \theta$$

$$y_2 = A \sin(\theta + \pi/4) = A \sin(\theta + 45^\circ)$$

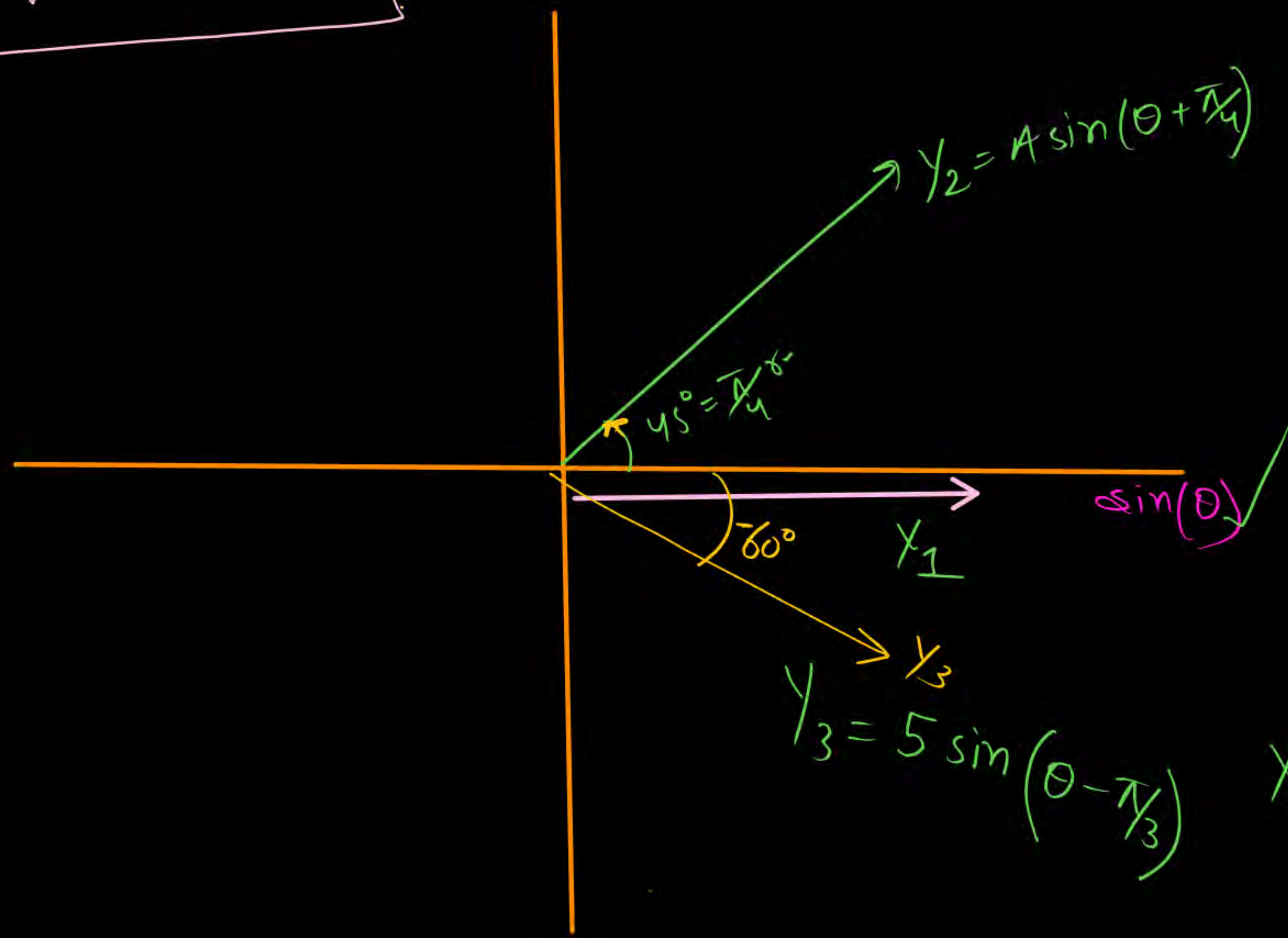
$$y_3 = A \sin(\theta - \pi/4) = A \sin(\theta - 45^\circ)$$

$$y_4 = A \sin(\theta + \pi/2)$$

$$y_5 = A \sin(\theta - \pi/2)$$

+ve \rightarrow Anticlock
 -ve \rightarrow clockwise

\leftarrow map box



map box

$$\frac{\pi}{3} \text{ rad} = \frac{180^\circ}{3} = 60^\circ$$

$$\frac{\pi}{6} \text{ rad} = \frac{180^\circ}{6} = 30^\circ$$

$$\frac{\pi}{2} \text{ rad} = 90^\circ$$

$$\frac{\pi}{4} \text{ rad} = 45^\circ$$

$$y_1 = 3 \sin \theta$$

$$y_2 = 4 \sin(\theta + \pi/4)$$

$$y_3 = 5 \sin(\theta - \pi/3)$$

Draw this in phasor diagram.

#

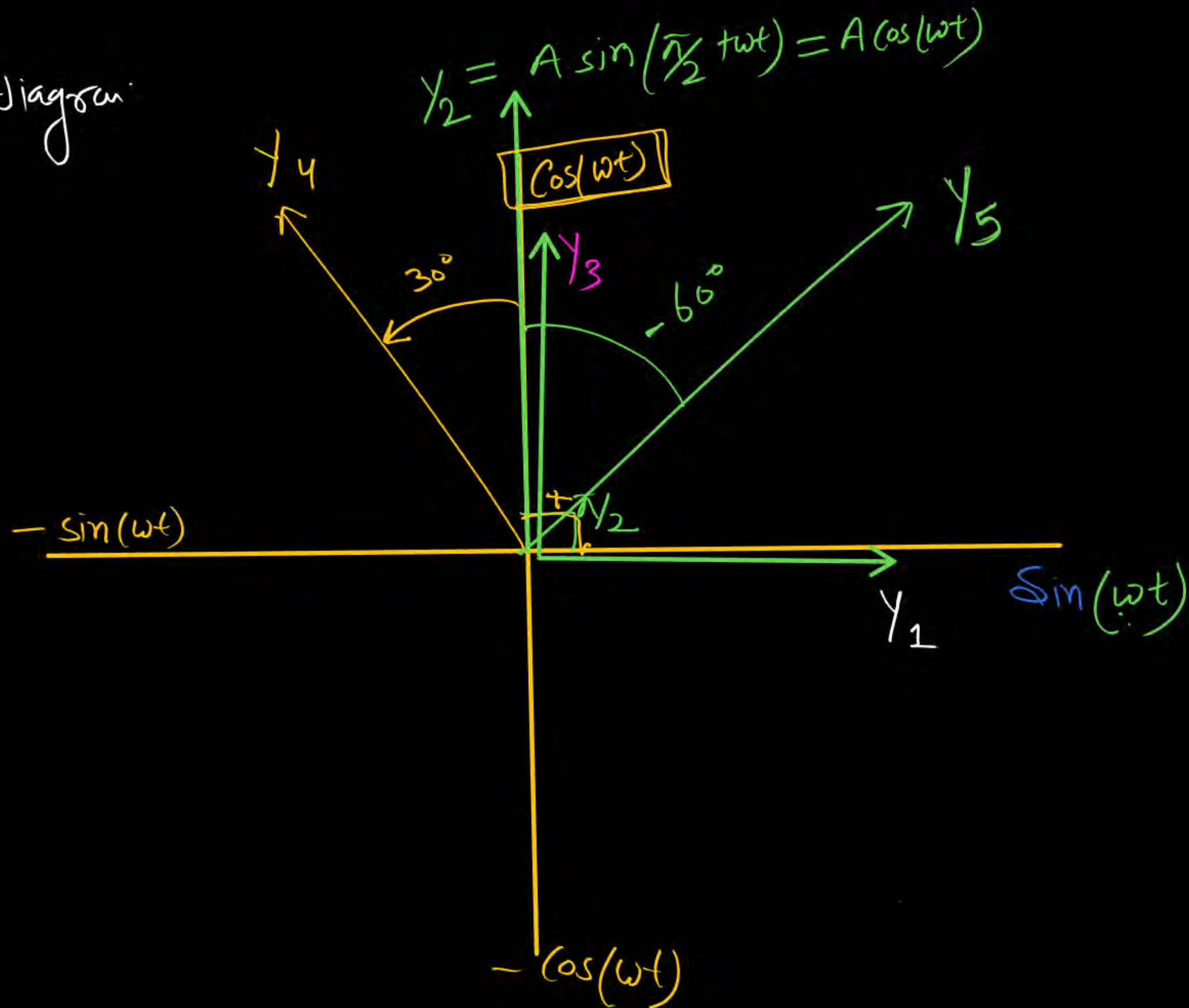
$$Y_1 = A \sin(\omega t)$$

$$Y_2 = A \sin(\omega t + \pi/2)$$

$$Y_3 = A \cos(\omega t)$$

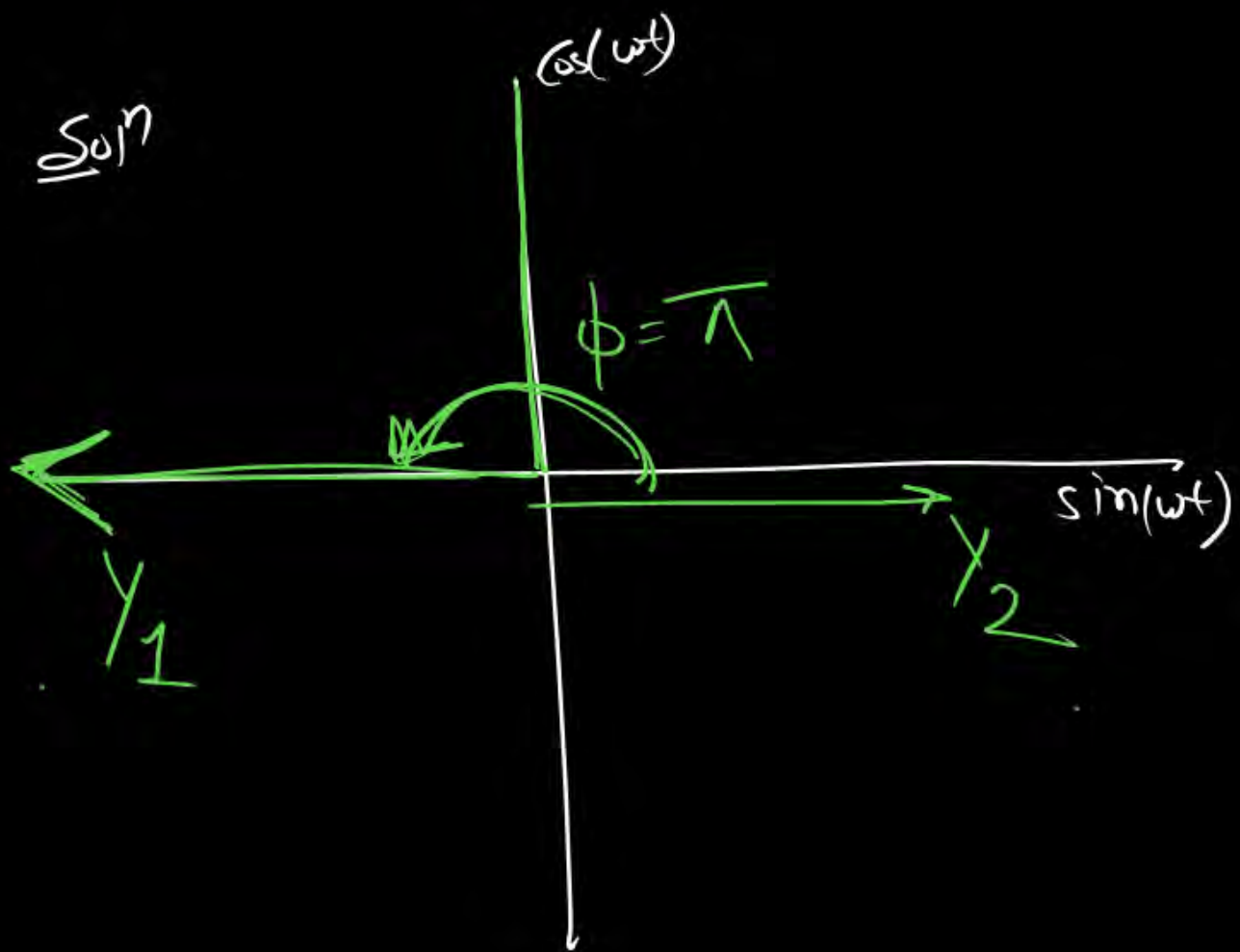
$$\# Y_4 = A \cos(\omega t + 30^\circ)$$

$$\# Y_5 = A \cos(\omega t - 60^\circ)$$



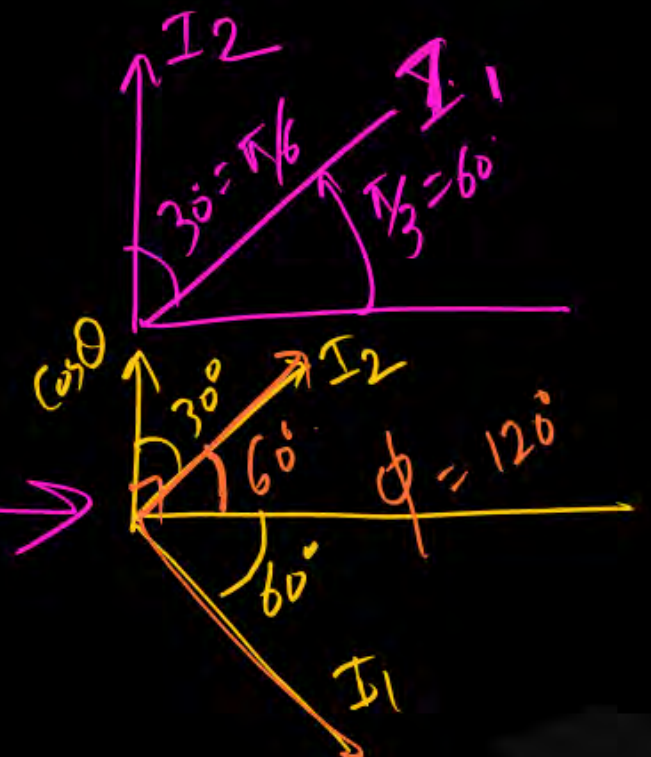
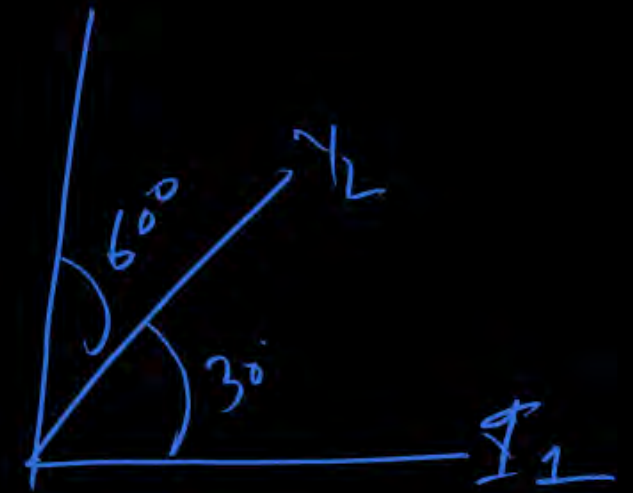
(Q) $y_1 = 3 \cos(\omega t + \pi/2)$

$$y_2 = 3 \sin(\omega t)$$



VECTOR/SHM/Wave/Wave Optics/EM Wave/ A/C

Equation - 1	Equation - 2	Phase difference
$I_1 = I_0 \sin(\theta + \underbrace{\pi/3}_{60^\circ})$	$I_2 = I_0 \sin(\theta - \underbrace{\pi/6}_{30^\circ})$	$\phi = 90^\circ = \pi/2$
$I_1 = I_0 \sin(\theta + \underbrace{\pi/3}_{60^\circ})$	$I_2 = I_0 \cos(\theta + \underbrace{\pi/3}_{60^\circ})$	$\phi = \pi/2$
$I_1 = 4 \sin(\omega t)$	$I_2 = I_0 4 \cos(\omega t - \underbrace{\pi/3}_{60^\circ})$	$\phi = 30^\circ = \pi/6$
$I_1 = 3 \sin(\omega t + \pi/2)$	$I_2 = 5 \cos(\omega t)$	$\phi = 0$
$I_1 = 4 \cos(\omega t - \pi/2)$	$I_2 = 4 \sin(\omega t + \pi/2)$	$\phi = \pi/2$ (pt)
$I_1 = 3 \sin(\omega t + \pi/3)$	$I_2 = 3 \cos(\omega t)$	$\pi/6$
$I_1 = 4 \sin(\theta - 60^\circ)$	$I_2 = 4 \cos(\theta - 30^\circ)$	$120^\circ = \frac{2\pi}{3} \text{ rad.}$



Question

Equation of a.c. current $I = -I_0 \cos(\omega t)$ and a/c voltage is $V = V_0 \sin(\omega t - \pi/6)$ then find phase difference between current and voltage:

WEET

1 $\frac{2\pi}{3}$

2 $\frac{\pi}{3}$ 67°/0

3 $\frac{\pi}{6}$

4 $\frac{5\pi}{6}$



Trigonometric function

maximum / minima.

$$y = 3 \sin \theta$$

$$\longrightarrow y_{\max} = 3$$

$$y_{\min} = -3$$

$$y = 3 \cos \theta + 4 \cos \theta$$

$$\longrightarrow y_{\max} = +7$$

$$y_{\min} = -7$$

$$y = (3+4) \cos \theta = 7 \cos \theta$$

MP box

$$y = a \sin \theta + b \cos \theta$$
$$y_{\max} = +\sqrt{a^2 + b^2}$$
$$y_{\min} = -\sqrt{a^2 + b^2}$$

$$y = 3 + 4 \sin \theta$$



$$y_{\max} = 3 + 4(\sin \theta)_{\max}$$

$$y_{\max} = 7$$

$$y_{\min} = 3 - 4$$
$$= -1$$

we will learn it
in max/min
3 min

$$y = 3 \sin \theta + 4 \cos \theta$$

$$\longrightarrow y_{\max} = \sqrt{3^2 + 4^2}$$
$$= \sqrt{25} = +5$$

$$y_{\min} = -5$$

$$\# y = 3 \sin \theta + 4 \sin \theta \longrightarrow y_{\max} = 7 \quad y_{\min} = -7$$

Question



If $y = A \sin \theta + B \cos \theta$ then find maximum value of y .

$$y = \sqrt{A^2 + B^2} \quad \text{Ans}$$

Question



If $y = \sin \theta + 2 \cos \theta$ then find maximum value of y .

Soln

$$y = \sin \theta + 2 \cos \theta$$

$$y_{\max} = \sqrt{1^2 + 2^2}$$
$$= \sqrt{5} \quad \checkmark \checkmark$$

$$y = \frac{4}{3\sin\theta + 2\cos\theta}$$

H/w

find minimum value of 'y'.

$$y_{\min} = \frac{4}{(3\sin\theta + 2\cos\theta)_{\max}}$$

Question



Current in A/C circuit is $I_1 = I_0 \sin (\omega t - 30^\circ)$ and voltage across it $V = V_0 \cos (\omega t)$

Find phase difference?

n/w

Do it right now



1. Equation of current and voltage

$$I = 10 \sin \left(\theta + \frac{\pi}{3} \right) \text{ and } V = 10 \cos \left(\theta - \frac{\pi}{6} \right)$$

then phase difference between current and voltage.

A/c chapt 2

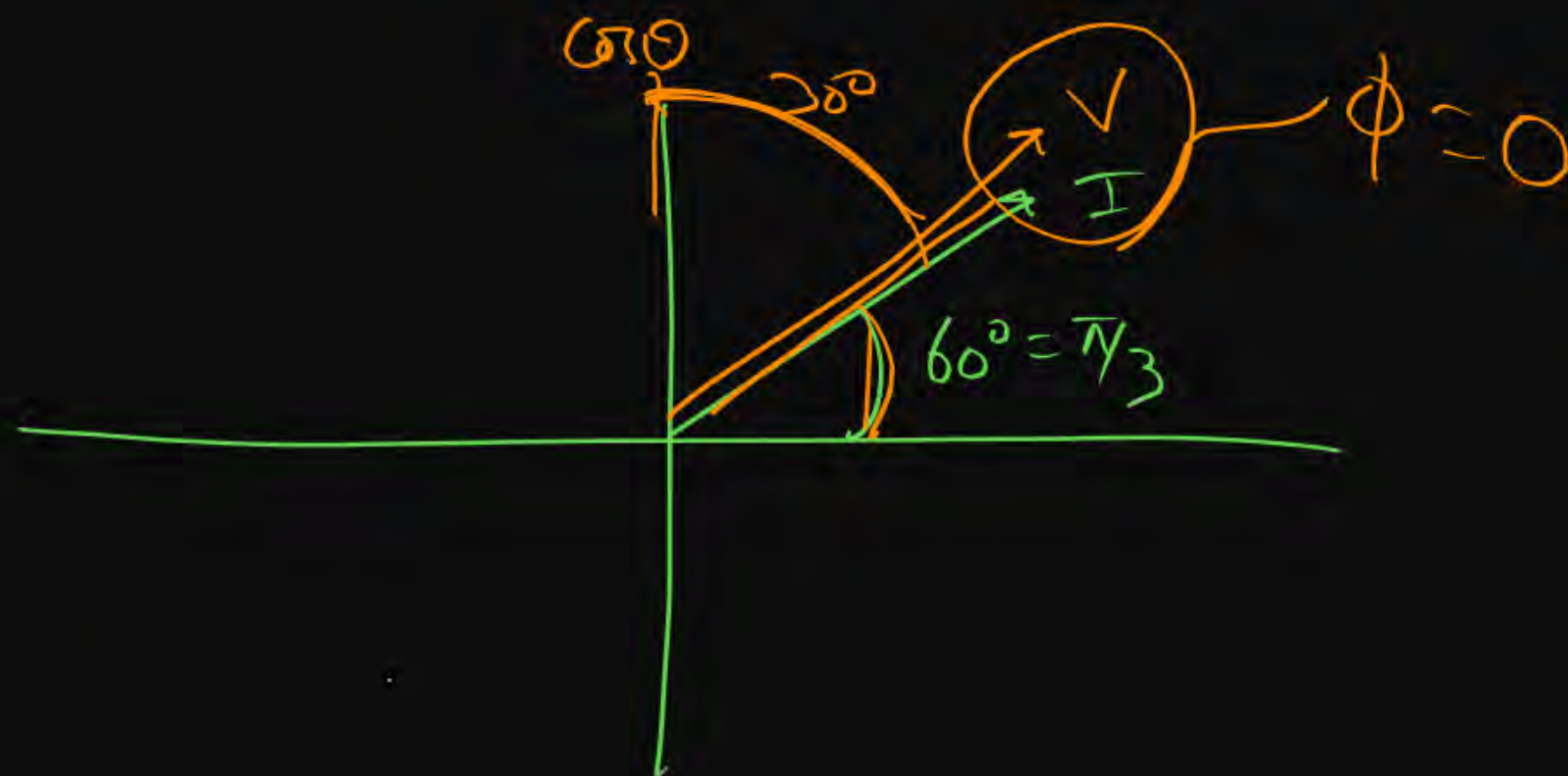
(a) zero

(b) $\pi/2$

(c) $\pi/6$

(d) $\pi/3$

$\phi = ??$



If $y_1 = 2 \sin (5\pi t)$ and $y_2 = 2 \cos (5\pi t - \pi/3)$, what is the phase difference between the two waveforms?

If $y_1 = 4 \sin (\omega t - \pi/6)$ and $y_2 = 4 \sin (\omega t + \pi/6)$, what is the phase difference between the two waveforms?

- 1 $\pi/6$
- 2 $\pi/3$
- 3 $\pi/2$
- 4 3π

Two waves are represented by the equations $y_1 = 4 \sin (3t)$ and $y_2 = 4 \sin (3t + \pi/2)$. Determine the phase difference between the two waves.

Question

H/W

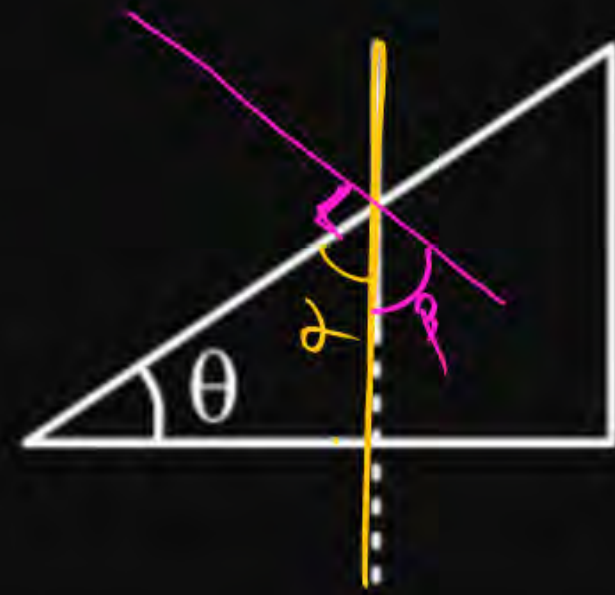


The equation of two waves are given as $y_1 = 3 \sin (4\pi t)$ and $y_2 = 3 \cos (4\pi t + \pi/3)$. Determine the phase difference between the two waves.

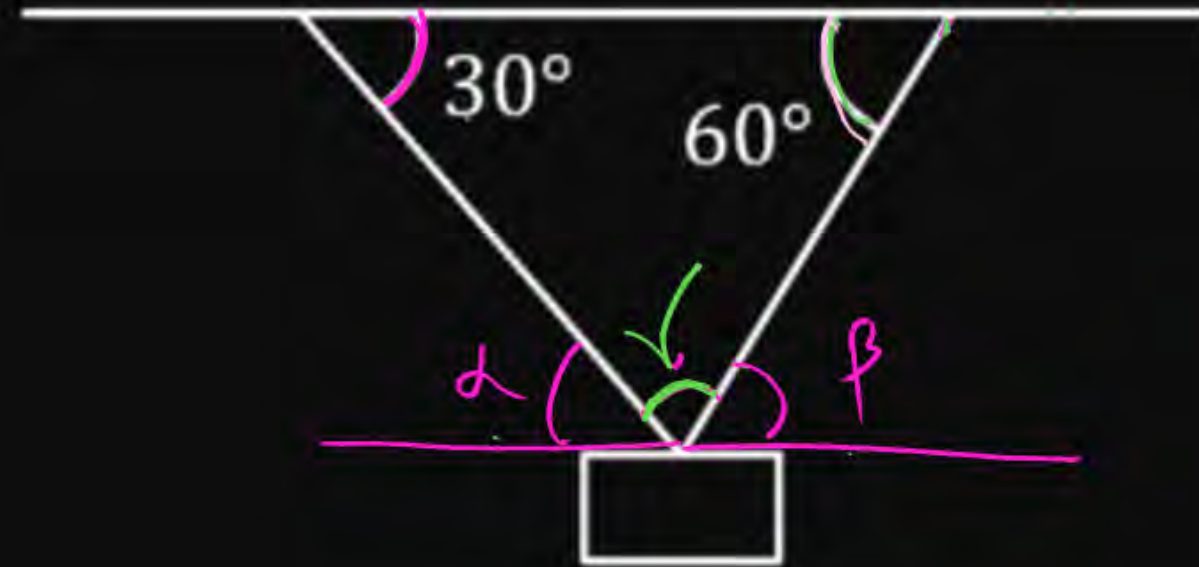
More Concept of Trigonometry required in physics

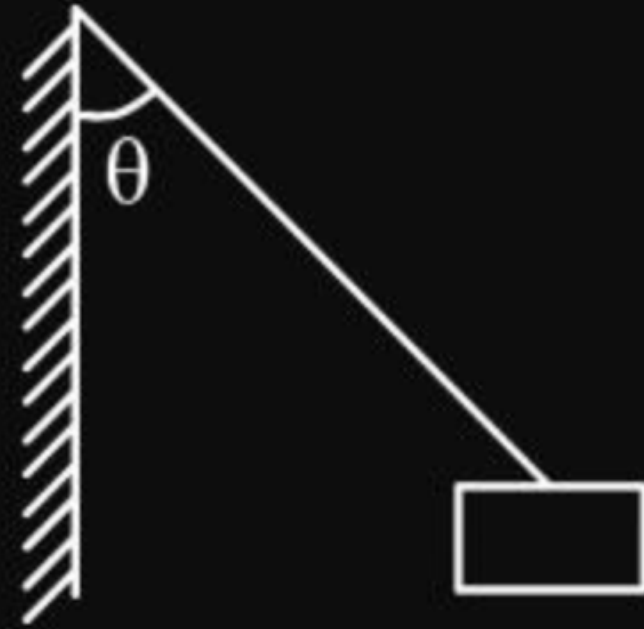
H/W

find $\alpha = ?$
 $\beta = ?$



find α , β & $\gamma = ?$





A/P series : — Sequence of number where the difference between any two consecutive number is constant → common diff.

1, 2, 3, 4, 5, 6, 7, 8, 9, ... → A/P series ✓
 $d = 1$

2, 4, 6, 8, 10, 12, 14, ... → A/P

3, 5, 7, 9, 12, 15, 17

$d = 2$

$$d (\text{Common diff}) = n^{\text{th}} - (n-1)^{\text{th}}$$

AP → No A/P series

✓ 1, 3, 5, 7, 9, 11, 13, 15, 17, ... → A/P ✓ $d = 2$

Question



Which of the following series is not arithmetic progression.

1 2, 8, 15, 15, 27, _____

2 3, 6, 12, 24, _____

3 4, 1, -2, -5, -8, _____

4 -5, -3, -1, 1, _____

HW

Question

H/W \rightarrow ***
PhD. on.

H/W भेजना है



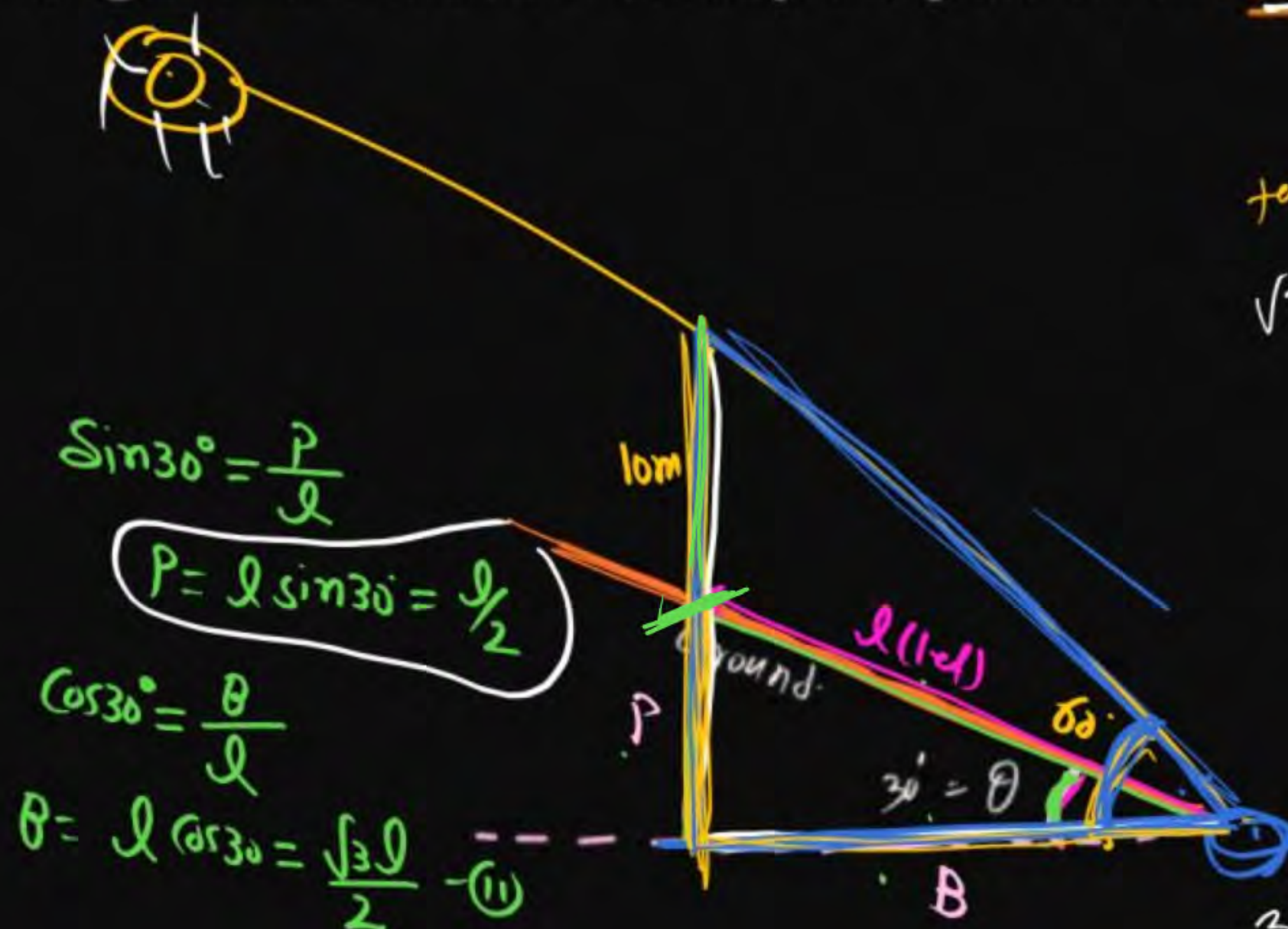
A vertical pole of height $h = 10$ m stands on ground that slopes upwards at a constant angle $\alpha = 30^\circ$ with the horizontal. If the sun's angle of elevation above the horizontal is $\theta = 60^\circ$, what is the length of the shadow cast by the pole on the sloping ground?

1 5 m

2 10 m ✓

3 $10\sqrt{3}$ m

4 $\frac{10}{\sqrt{3}-1}$ m



$$\sin 30^\circ = \frac{P}{l}$$

$$P = l \sin 30^\circ = \frac{l}{2}$$

$$\cos 30^\circ = \frac{B}{l}$$

$$B = l \cos 30^\circ = \frac{\sqrt{3}l}{2} \quad \text{--- (1)}$$

Inclined plane

$$\tan 60^\circ = \frac{10+P}{B}$$

$$\sqrt{3} = \frac{10 + \frac{l}{2}}{\frac{\sqrt{3}l}{2}}$$

आगे solve

$$\sqrt{3} \times \frac{\sqrt{3}l}{2} = 10 + \frac{l}{2}$$

$$\frac{3l}{2} = 10 + \frac{l}{2}$$

$$\frac{3l-l}{2} = 10 \quad \frac{2l}{2} = 10 \quad l = 10$$

Question

challenge question (H/W)

$$\cos(120) = -\frac{\sqrt{3}}{2} = -\frac{1.71}{2}$$



Suggest suitable match between function given in the first column and its description given in the second column.

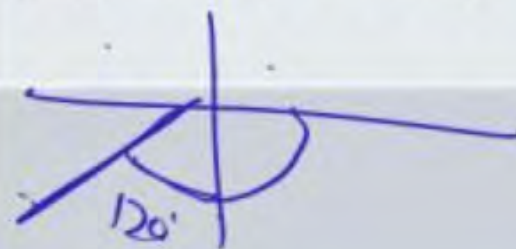
1 A → PT, B → QT, C → QT, D → PS

2 A → PT, B → QS, C → QT, D → PS

3 A → QT, B → QS, C → PT, D → PS

4 A → QS, B → PT, C → QT, D → PS

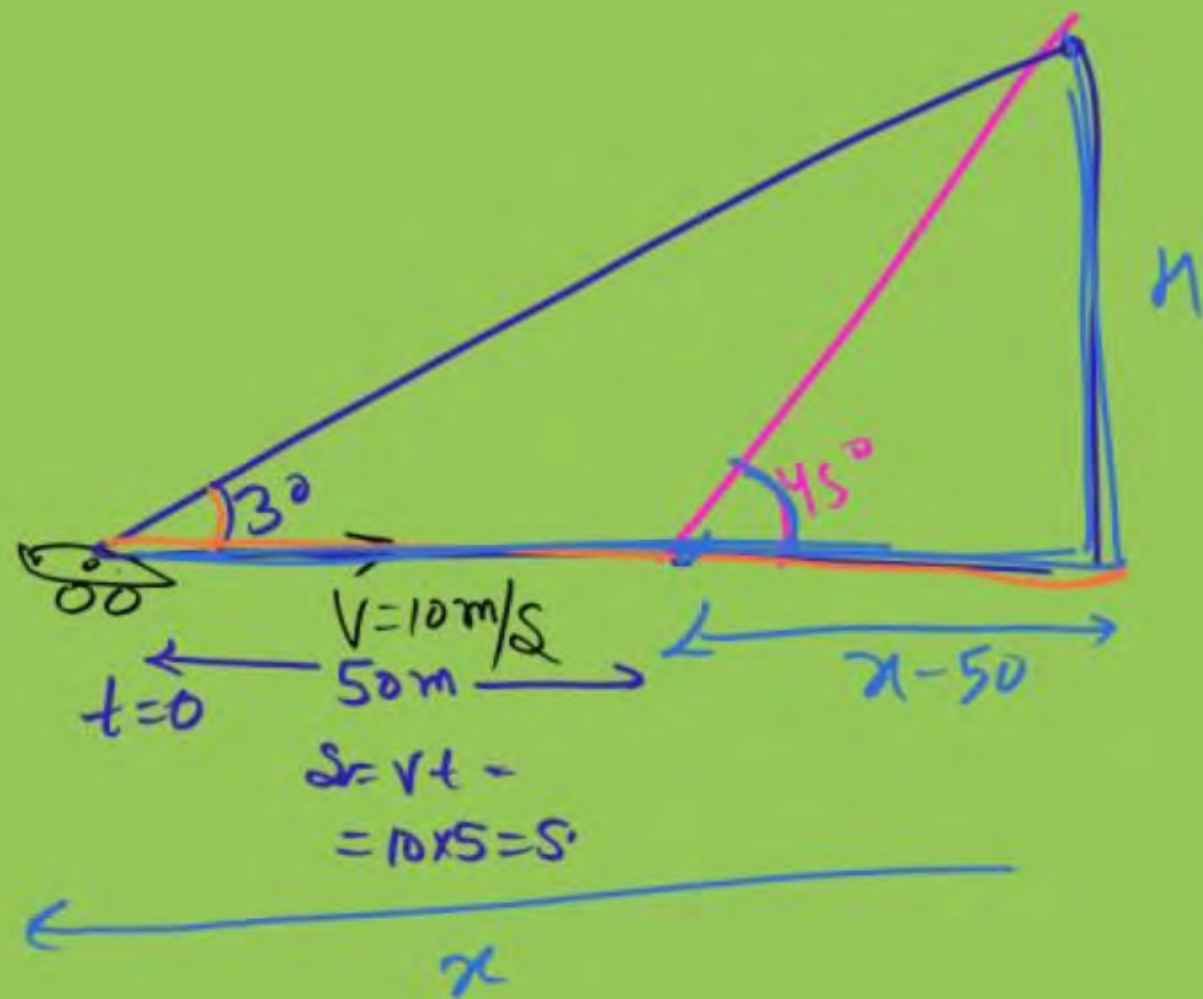
Column-I	Column-II
(A) $\sin(390^\circ)$	(P) Positive
(B) $\sin(-30^\circ)$	(Q) Negative
(C) $\cos 120^\circ$	(R) Zero
(D) $\tan(-120^\circ)$	(S) Modulus is greater than one
	(T) Modulus is less than one



$$\sin(390) = \sin(\underbrace{360}_A + \underbrace{30}_B) \Rightarrow \sin 30 = \left| \frac{1}{2} \right|$$

$$\sin(-30) = -\frac{1}{2}$$





$$\tan 30^\circ = \frac{H}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{H}{x} \quad \text{--- (1)}$$

$$x = \sqrt{3}H$$

$$\tan 45^\circ = \frac{H}{x - 50} = 1$$

$$H = x - 50$$

$$H = \sqrt{3}H - 50$$

$$50 = \sqrt{3}H - H$$

$$50 = H(\sqrt{3} - 1) \quad H = \frac{50}{\sqrt{3} - 1}$$

$$\tan 3^\circ = 3^\circ \quad \text{X}$$

$$\tan(3^\circ) = 3^\circ = 3 \left(\frac{\pi \text{ rad}}{180} \right) \quad \text{cor.}$$

$$(\tan 3^\circ) \neq (\sin 3^\circ) \quad \text{correct}$$

b/c AS

Question

Find value:

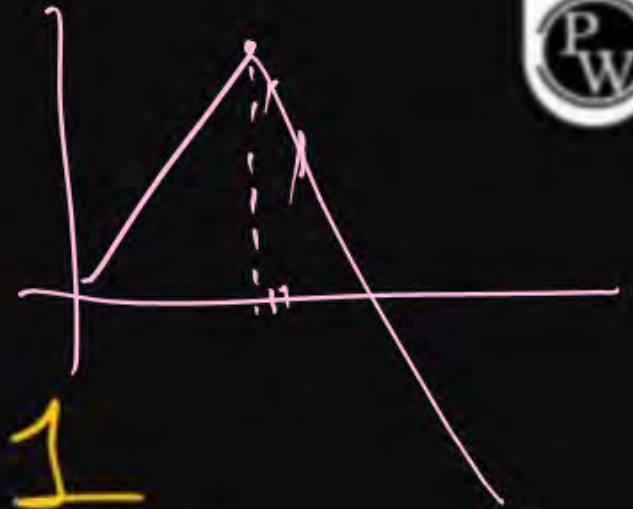
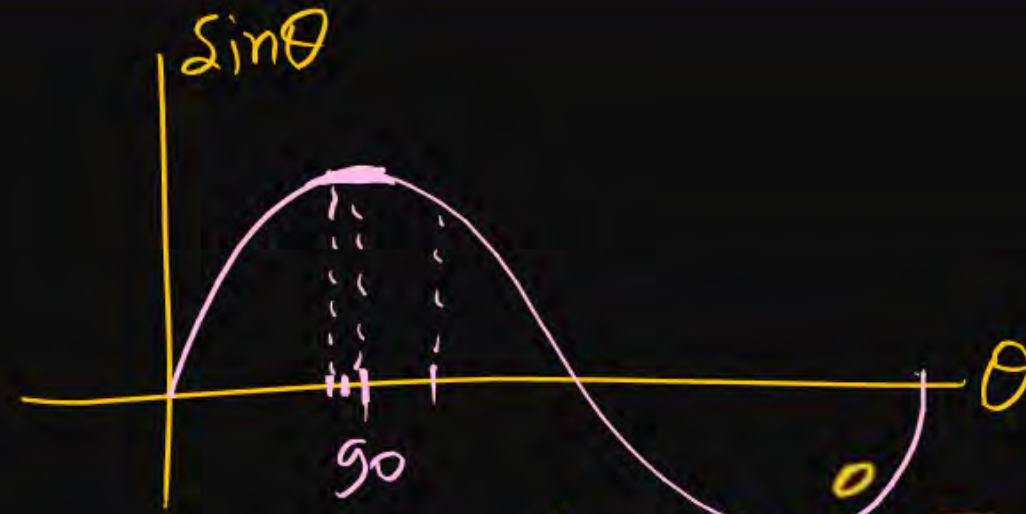
(i) $\sin 2^\circ = 2^\circ \times \frac{\pi \text{ rad}}{180}$

(ii) $\tan 3^\circ = 3 \left(\frac{\pi \text{ rad}}{180} \right)$

(iii) $\cos 3^\circ = 1$

(iv) $\sin (88.5^\circ) \approx 1$

Answer

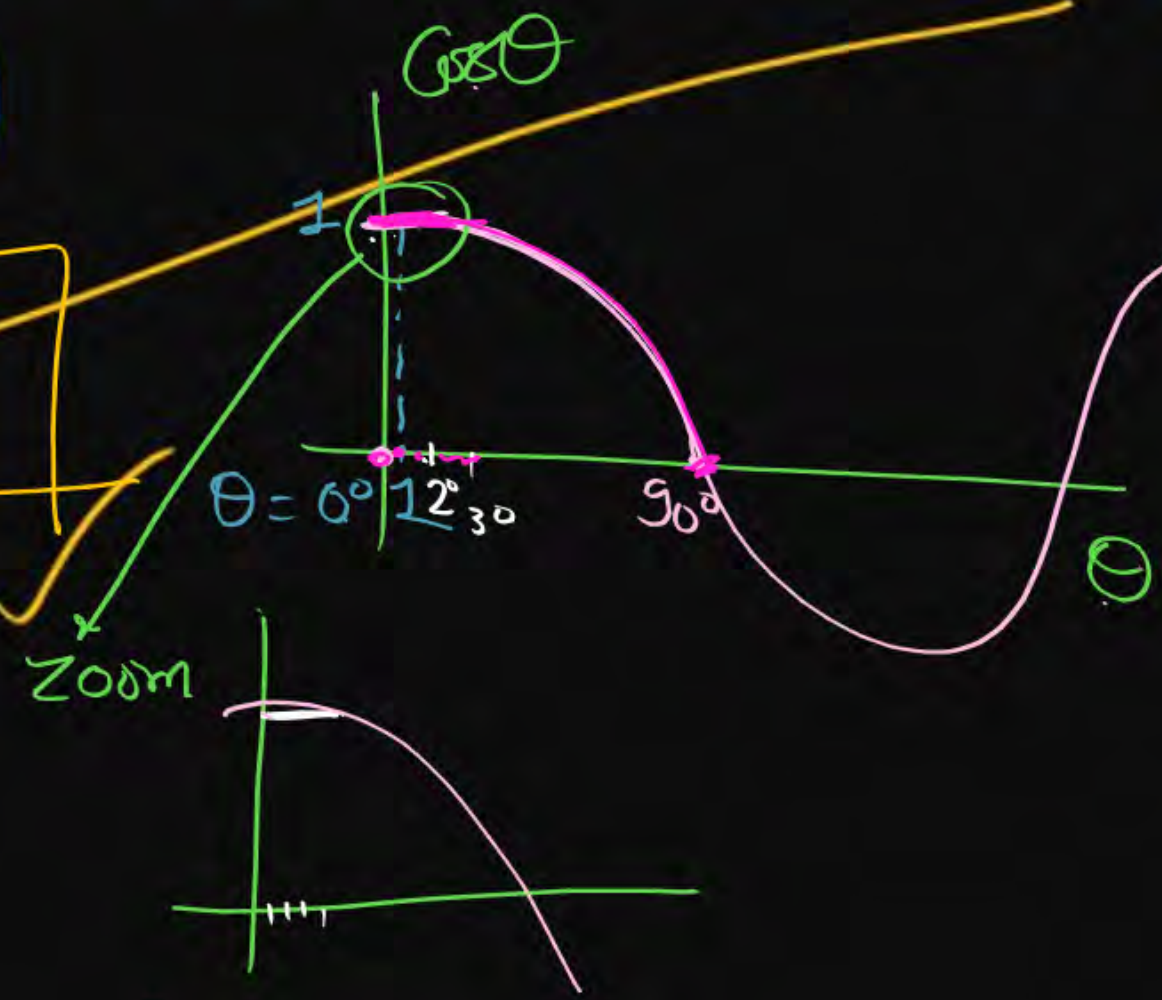
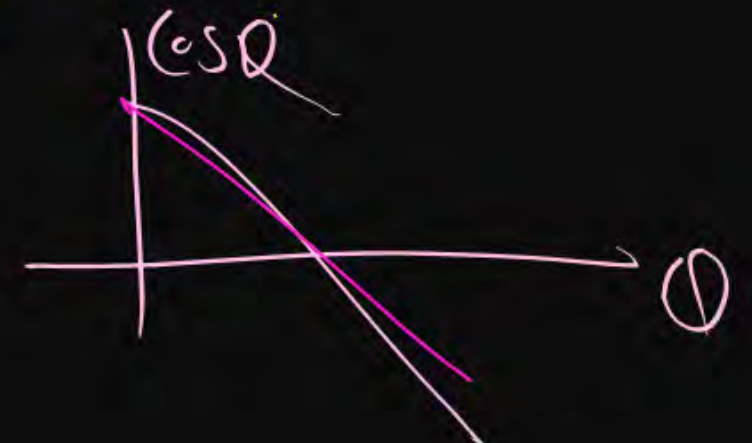


$\cos 0^\circ = 1$

$\cos 1^\circ = 1$

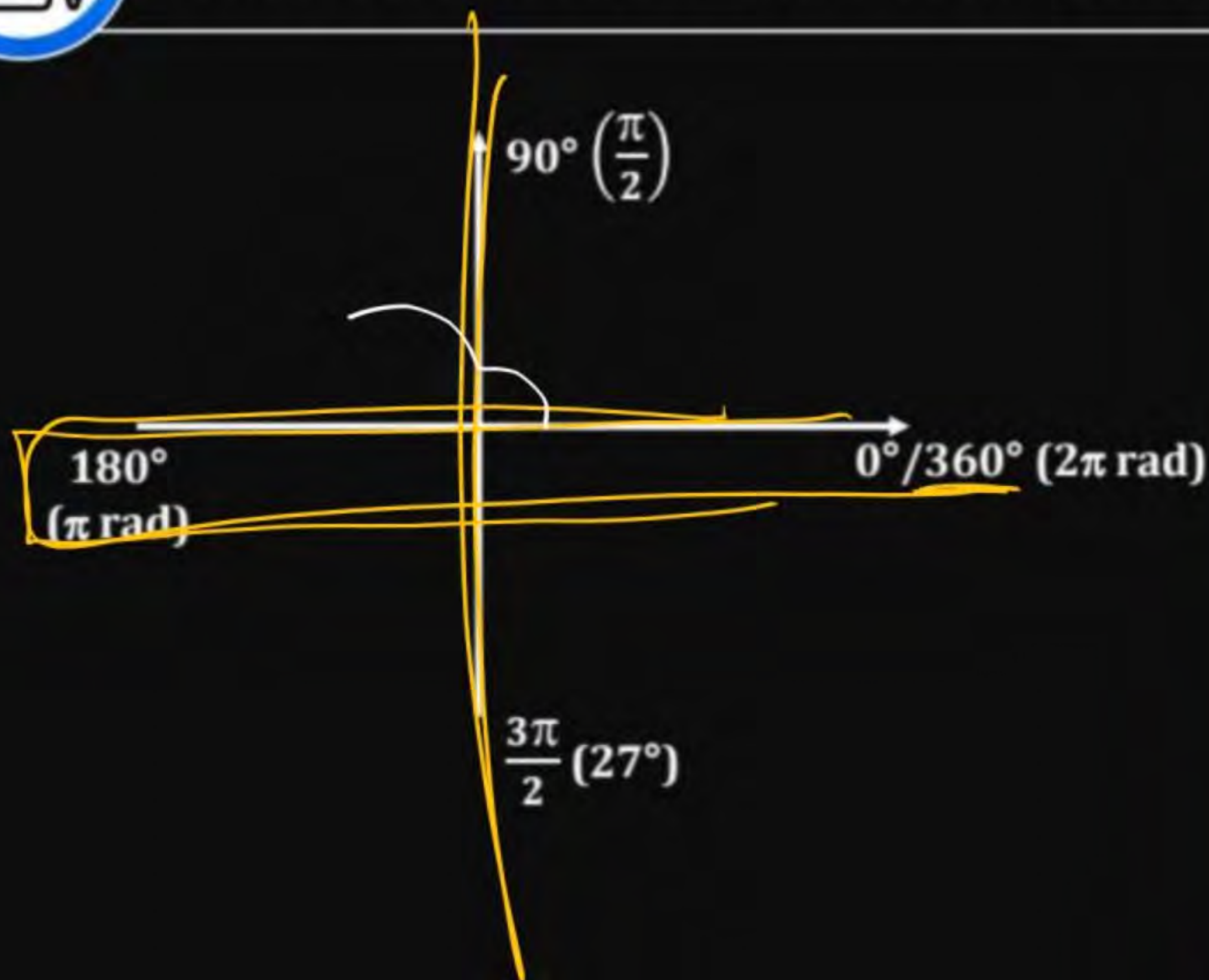
$\cos 2^\circ = 1$

$\cos 3^\circ = 1$





TRIGONOMETRY FUNCTION CHANGE



h/w *ray*

$$\sin (90^\circ - \theta) = + \cos \theta$$

$$\cos (90^\circ - \theta) = + \sin \theta$$

$$\sin (90^\circ + \theta) = + \cos \theta$$

$$\cos (90^\circ + \theta) = - \sin \theta$$

$$\sin (180^\circ - \theta) = \sin \theta$$

$$\cos (180^\circ - \theta) = - \cos \theta$$

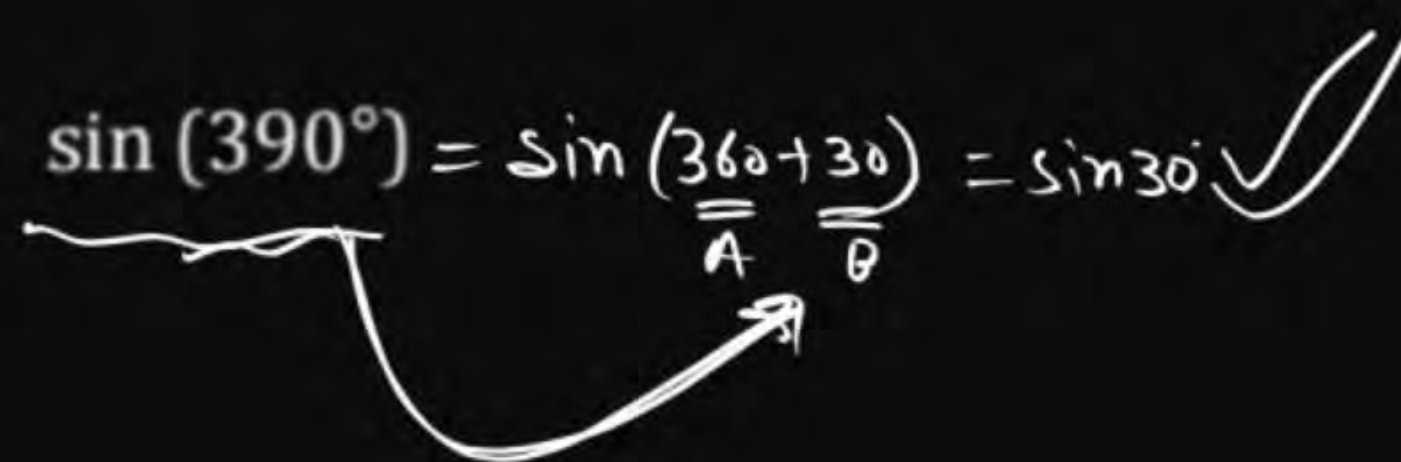
Find value of

(i) $\sin(-30^\circ) = -\frac{1}{2}$

(ii) $\cos(-60^\circ) = +\frac{1}{2}$

(iii) $\sin(120^\circ) = \frac{\sqrt{3}}{2}$

(iv) $\sin(390^\circ) = \sin(\underbrace{360}_{\text{A}} + \underbrace{30}_{\text{B}}) = \sin 30^\circ$ ✓



H/W

$$\sin(360^\circ) = 0$$

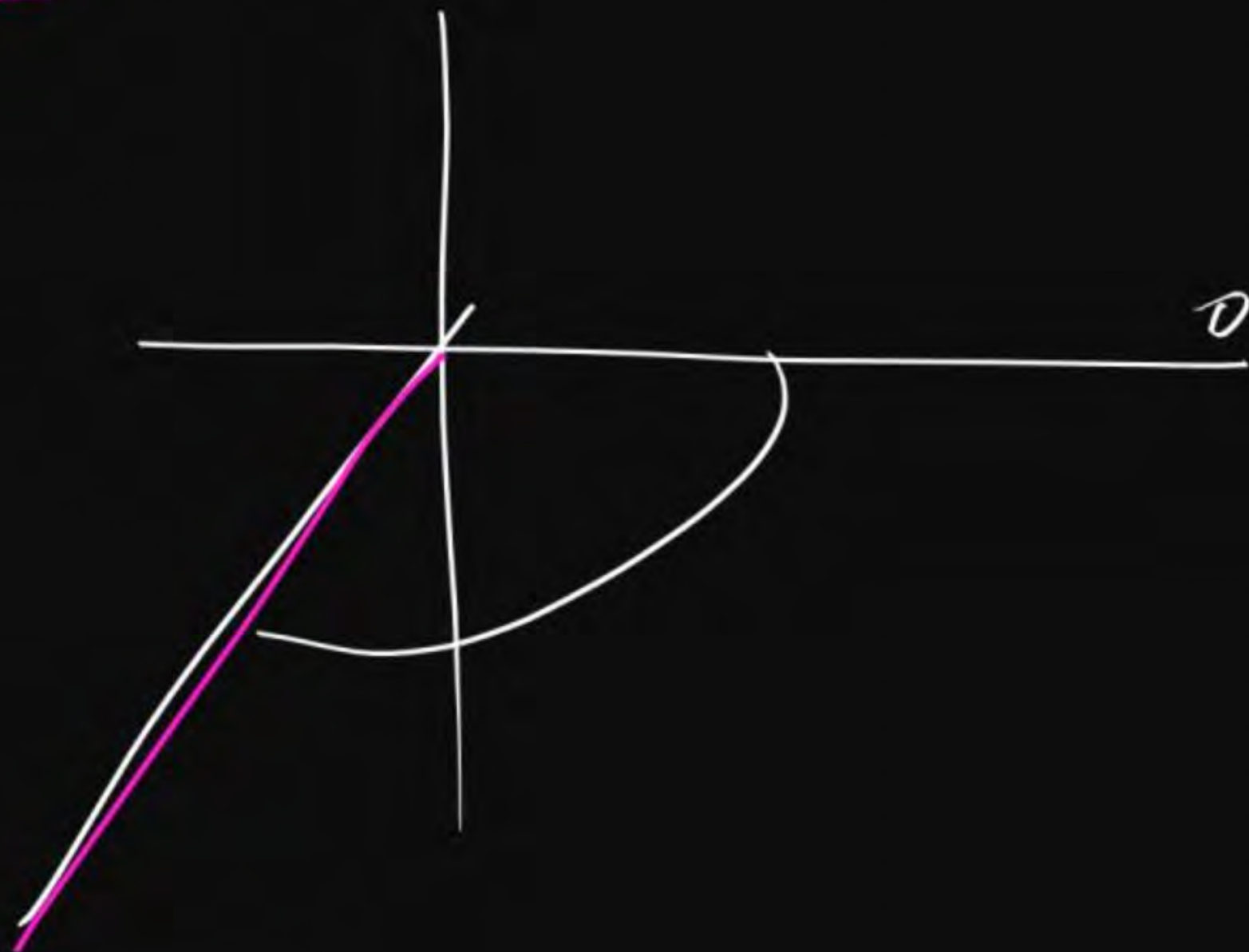
$$\checkmark \sin(450^\circ) = \sin(\underline{360} + 90) = \sin 90 = 1$$

$$\# \sin(-90^\circ) = -1$$

$$\sin(120^\circ) = \sqrt{3}/2$$

$$\checkmark \boxed{\sin(-150^\circ) = -\frac{1}{2}}$$

$$\sin(-\theta) = -\sin \theta$$



H/W

$$\cos(300^\circ) = \cos(360 - 60) = +\cos 60 = +\frac{1}{2}$$

$$\begin{aligned}\cos(330^\circ) &= \cos(360 - 30) = \cos 30 \\ &= \frac{\sqrt{3}}{2}\end{aligned}$$
$$\begin{aligned}&= \cancel{\cos 360}^1 \cos 60 + \cancel{\sin 360}^0 \sin 60 \\ &= 1 \cos 60 + 0 = \frac{1}{2}\end{aligned}$$

$$\tan(240^\circ) = \tan(180 + 60) = \tan 60 = \sqrt{3}$$

$$\cos(-30^\circ) = \cos(30) = \frac{\sqrt{3}}{2}$$

$$\tan(-60^\circ) = -\tan 60 = -\sqrt{3}$$

$$\cot(-45^\circ) = \frac{1}{\tan(-45)} = \frac{1}{-1} = -1$$

Question

M/O



If $y = 3 \cos(3\theta)$, then find angle at which y will be zero.

$$y = 3 \cos(3\theta)$$

$$3\theta = 90$$

$$\theta = \frac{90}{3} = 30^\circ \checkmark$$

THANK
YOU