

YAKEEN NEET 2.0

2026

Basic Maths and Calculus (Mathematical Tools)

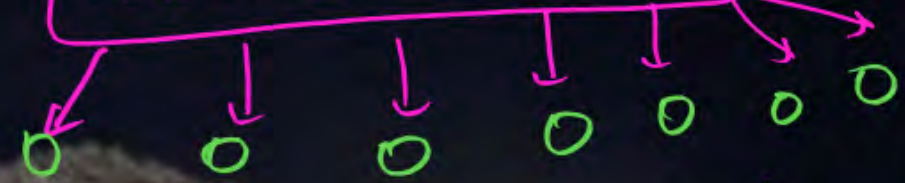
Physics

Lecture - 12

By- Manish Raj (MR Sir)



Lecture 01 to 11



$\frac{1}{2}$ hr (30 min)
Not to know

↓
rough page

~~No of class~~ Yakeen 2.0

4:30 pm

Play with physics

1 hr only

8 lecture - 1



Topics to be covered

1

~~chain Rule~~

outside - Inside Rule ✓✓

2

Partial differentiation ✓✓ (very useful)

3

Maxima and Minima ✓✓

4

Play with Physics \rightarrow In MR^* style

\rightarrow Live
 \rightarrow Not daily (one or two lecture in week)
Topic \rightarrow random from any - chapter

\rightarrow Notes अमरों से बनाता है ||

\rightarrow duration 1 hr

\rightarrow Today's time \rightarrow 4:30 PM

$$\left[\begin{array}{l} \frac{dy}{dx} = \text{The rate of change of } \boxed{y} \text{ w.r.t } x \\ \frac{dx}{dy} = \text{The rate of change in } \boxed{x} \text{ w.r.t } y \end{array} \right.$$

$$\frac{d^2y}{dx^2} = \text{Double deriv}^n \text{ of } y \\ = \text{Rate of change in } \left(\frac{dy}{dx} \right) \text{ with respect to } x$$

$$\# \left| \frac{dy}{dx} \right| = \text{magnitude of Rate of change in } y \text{ w.r.t } x$$

$$\frac{dA}{dt} = \text{Rate of change in Area w.r.t time.}$$

$$\# \frac{d|y|}{dx} = \text{Rate of change in magnitude of } y \text{ w.r.t } x.$$

$$\frac{dx}{dt} = \text{Rate of change in } x \text{ w.r.t } t.$$

$$\# \frac{dy^2}{dx} = \text{The rate of change in } y^2 \text{ w.r.t } x$$

$$\frac{dR}{dt} = \text{Rate of change in } R \text{ w.r.t time}$$

Inst. acceleration \vec{a} = The rate of change in velocity \vec{v} wrt time

$$\vec{a} = \frac{d\vec{v}}{dt}$$

magnitude of accⁿ $= |\vec{a}| = \left| \frac{d\vec{v}}{dt} \right|$

Tangential accⁿ \vec{a}_T = The rate of change in (speed) wrt. time

$$\vec{a}_T = \frac{d|\vec{v}|}{dt}$$

#

magnitude of velocity
 $|\vec{v}|$



Function of a Function

अजीब function \Rightarrow $\begin{matrix} 3, 4, 7, 8 \\ 9, 10 \end{matrix}$
simple differentiable
होना चाहिए



① $y = \sin x \cdot e^x$ ✓ (Product Rule)

⑤ $y = \log x \cdot (x^5)$ ✓
Product Rule

② $y = \sin x + \cos(x)$ ✓ (Addition Rule)

⑥ $y = x^5 - \tan x$ ✓

⑨ $y = \log(\sin x)$ ✗

③ $y = \sin(e^x)$ ✗ अजीब है ✗

⑦ $y = e^{(4x)}$ ✗

⑩ $y = \sin(\log x)$ ✗

④ $y = \sin(x^2 + 2x)$ ✗ अजीब है ✗

⑧ $y = (x^2 + 4)^4$ ✗

⑪ $y = e^x \times \sin x$ ✓
Product Rule

$y = \sin x \times (x^2 + 2x)$ ✓

Note

(Q) If $y = \sin(x^2 + 3x)$ then find $\frac{dy}{dx}$??

Soln

$$y = \sin(x^2 + 3x)$$

algebraic

Trigonometric
const fun

Chain Rule

$$x^2 + 3x = t \quad (\text{Let})$$

differential w.r.t x

$$\frac{dx^2}{dx} + 3 \frac{dx}{dx} = \frac{dt}{dx}$$

$$2x + 3 = \frac{dt}{dx} \quad \text{--- (1)}$$

Putting value of 't' in main eqn

$$y = \sin(t)$$

diffn of y w.r.t 't'

$$\frac{dy}{dt} = \frac{d(\sin(t))}{dt}$$

$$\frac{dy}{dt} = \cos t$$

Putting value of 't' & 'dt'

$$\begin{aligned} dy &= \cos(x^2 + 3x) \times dt \\ &= \cos(x^2 + 3x) \times (2x + 3) dx \end{aligned}$$

$$\left(\frac{dy}{dx} \right) = \cos(x^2 + 3x) \times (2x + 3)$$



1/5/12

Chain Rule



✓ (Outside) - (Inside) Rule (MR*)

$$\frac{dy}{dx} = \left(\text{differentiation of } \underline{\text{outer}} \text{ function keep Inside as it is} \right) \times \left(\text{diff'n of inner function wrt } x \right)$$

NOTE

$$y = A \cos(kx)$$

Constⁿ

outer function

inner

$$\frac{dy}{dx} = A (-\sin(kx)) \times \frac{d(kx)}{dx}$$

$$= -A \sin(kx) \cdot k$$

$$\# \frac{dy}{dx} = -Ak \sin(kx)$$

$$\frac{d \cos(\text{outer})}{dx} = -\sin(\text{outer}) \times \frac{d(\text{outer})}{dx}$$

note

$$y = A e^{(5x)}$$

$$\frac{dy}{dx} = A \frac{d e^{5x}}{dx}$$

$$= A \times e^{(5x)} \times \left(\frac{d 5x}{dx} \right)$$

$$= A e^{5x} \times 5$$

$$\# \boxed{\frac{dy}{dx} = 5A e^{5x}}$$

$$y = e^{-x}$$

$$\frac{dy}{dx} = \frac{d e^{-x}}{dx}$$

$$= e^{-x} \left(\frac{d(-x)}{dx} \right)$$

$$= e^{-x} \left(- \frac{dx}{dx} \right)$$

$$\frac{dy}{dx} = - e^{-x}$$

$$y = 3 \sin(x^2 + 2)$$

$$y = 3 \sin(x^2 + 2)$$

$$\frac{dy}{dx} = 3 \cos(x^2 + 2) \times (2x + 0)$$

$$\boxed{\frac{dy}{dx} = 6x \cos(x^2 + 2)}$$

Note

$$(1) \quad y = e^{2x^2}$$

inside function.

Soln

$$y = e^{(2x^2)}$$

$$\frac{dy}{dx} = e^{(2x^2)} \times \frac{d(2x^2)}{dx}$$

$$= e^{2x^2} \times 2(2x)$$

$$= 2x \times 2 e^{2x^2}$$

Note लिखो

ω, ϕ are constant
then find $\frac{dx}{dt}$, $\frac{d^2x}{dt^2}$

Q) Position of object given as

$$x = A \sin(\underbrace{\omega t + \phi}_{\text{Inside}})$$

Solⁿ

$$\frac{dx}{dt} = A \cos(\omega t + \phi) \times \frac{d(\omega t + \phi)}{dt}$$

$$= A \cos(\omega t + \phi) \times \left(\omega \left(\frac{dt}{dt} \right) + \frac{d\phi}{dt} \right)$$

$$\frac{dx}{dt} = A\omega \cos(\omega t + \phi)$$

$$\frac{d^2x}{dt^2} = A\omega \left[-\sin(\omega t + \phi) \times (\omega) \right]$$
$$\frac{d^2x}{dt^2} = -\omega^2 A \sin(\omega t + \phi)$$

$$\left(\frac{d^2x}{dt^2} \right) = -\omega^2 A \sin(\omega t + \phi)$$

putting value of (x)

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

*

$$y = \sin(x^2)$$

\uparrow out 2
 \uparrow inn 2

$$\frac{dy}{dx} = \cos(x^2) \times (2x)$$

$$\textcircled{\#} \quad \frac{dy}{dx} = \cos(x^2) \times 2x$$

$$y = \sin^2 x = (\sin x)^2$$

$$y = (\sin x)^2$$

\uparrow inn 2 func.

$$y = 2(\sin x)^{2-1} \times \cos x$$

$$y = 2 \sin x \times \cos x$$

$$y = \sin(2x)$$

$$\frac{dx^2}{dx} = 2x$$

$$\frac{dt^2}{dx} = 2t$$

$$y = \sin(x^3)$$

↑
Inner

$$\frac{dy}{dx} = \cos(x^3) \times 3x^2$$

$$y = \sin^3 x = (\sin x)^3$$

↑
Inner

$$\frac{dy}{dx} = 3(\sin x)^2 \times \cos x$$

$$\frac{dy}{dx} = 3 \sin^2 x \cos x$$

$$y = \log_e(\underbrace{x^3 - 2x^2}_{\text{Inside function}})$$

$$\frac{dy}{dx} = \frac{1}{(x^3 - 2x^2)} \times \frac{d(x^3 - 2x^2)}{dx}$$

$$\frac{dy}{dx} = \frac{1}{(x^3 - 2x^2)} \times (3x^2 - 4x)$$

~~MPX BOX~~

$$\sin(2\theta) = 2 \sin\theta \cdot \cos\theta$$



$$y = \log_e(x)$$

$$\frac{dy}{dx} = \frac{1}{x}$$

$$y = \cos^2(x^3 + 2x)$$

do not write
no use in
physics

$$y = \left(\underbrace{\cos(x^3 + 2x)}_{\text{inside}} \right)^2$$


$$\frac{dy}{dx} = 2 \left[\cos(x^3 + 2x) \right]^{2-1} \times (-\sin(x^3 + 2x)) \times (3x^2 + 2)$$

$$\frac{dy}{dx} = -2 \cos(x^3 + 2x) \sin(x^3 + 2x) (3x^2 + 2) \checkmark$$

$y = (\underbrace{x^2+4}_{\text{Inner funct.}})^3$


$$\begin{aligned}\frac{dy}{dx} &= 3(x^2+4)^{3-1} \times \frac{d(x^2+4)}{dx} \\ &= 3(x^2+4)^2 \times (2x+0)\end{aligned}$$

$$\frac{dy}{dx} = 6x(x^2+4)^2$$



$$y = \alpha \sin(\beta t)$$

where α & β are constant


$$\frac{dy}{dt} = \alpha \cos(\beta t) \times \beta$$

$$\# \boxed{\frac{dy}{dt} = \alpha \beta \cos(\beta t)}$$

H/W

$$(2) y = e^{\sin x}$$

$$(2) y = \cos(x^2 + 4x)$$

$$(3) y = (x^2 + 4x)^2$$

$$(4) y = 4e^{3t}$$

$$(5) y = 5 \sin(4 + 3t)$$

$$(6) y = \log(3x + 4)$$

$$(7) y = e^{-4x + 3}$$

$$(8) y = (x^4 - 1)^{50}$$

Home work

(*) (*)



Partial Differentiation



gravitation, work
energy
electrostatic

if v is a function of (x, y, z)

Write down

$$\left[\left(\frac{\partial v}{\partial x} \right)_{y, z \text{ const}} + \left(\frac{\partial v}{\partial y} \right)_{x, z \text{ const}} + \left(\frac{\partial v}{\partial z} \right)_{x, y \text{ const}} \right]$$

$$\vec{E} = - \left[\left(\frac{\partial v}{\partial x} \right)_{y, z \text{ const}} \hat{i} + \left(\frac{\partial v}{\partial y} \right)_{x, z \text{ const}} \hat{j} + \left(\frac{\partial v}{\partial z} \right)_{x, y \text{ const}} \hat{k} \right]$$

Q1) If electric potential $V = 3x^2yz$ then find electric field.

Soln

write down

$$\textcircled{\#} \left(\frac{\partial V}{\partial x} \right)_{y,z} = \frac{\partial 3x^2yz}{\partial x} = 3yz \frac{\partial x^2}{\partial x} = 3yz(2x) = 6xyz \hat{i} \quad \text{--- (1)}$$

$$\textcircled{\#} \left(\frac{\partial V}{\partial y} \right)_{x,z} = 3x^2z \frac{\partial y}{\partial y} \hat{j} = 3x^2z \hat{j}$$

$$\textcircled{\#} \left(\frac{\partial V}{\partial z} \right) = 3x^2y \frac{\partial z}{\partial z} \hat{k} = 3x^2y \hat{k}$$

$$\vec{E} = - \left[6xyz \hat{i} + 3x^2z \hat{j} + 3x^2y \hat{k} \right]$$

9f $V = x^2y + y^2z + z^2x$ find $|\vec{E}| = ??$

given in
Quelt.

$$\vec{E} = - \left[\frac{\partial V}{\partial x} \hat{i} + \frac{\partial V}{\partial y} \hat{j} + \frac{\partial V}{\partial z} \hat{k} \right]$$

H/W

#

$$y = x^2$$

find $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{d x^2}{dx}$$

#

$$\frac{dy}{dx} = 2x$$

$$y = x^2$$

find $\frac{dy}{dt} = ??$

diffⁿ w.r.t. time

$$\frac{dy}{dt} = \frac{dx^2}{dt} \times \frac{dx}{dx}$$

$$\frac{dy}{dt} = \frac{dx^2}{dx} \times \frac{dx}{dt}$$

$$\frac{dy}{dt} = 2x \times \frac{dx}{dt}$$

$$\frac{dy}{dx} = 2x$$

$$\left(\frac{dy}{dt} \right) = 2x \frac{dx}{dt}$$

Q. If radius of circle is increasing at rate $\frac{1}{\pi}$ m/s then find rate of change in Area with respect to time.

$$\# \quad \frac{dR}{dt} = \frac{1}{\pi} \text{ m/s}$$

$$\# \quad \frac{dA}{dt} = ??$$

$$A = \pi R^2$$

diffⁿ w.r.t R

$$\frac{dA}{dR} = \pi \frac{dR^2}{dR}$$

$$\frac{dA}{dR} = \pi 2R$$

$$\frac{dA}{dt} = 2\pi R \frac{dR}{dt}$$

$$= 2\cancel{\pi} R \left(\frac{1}{\cancel{\pi}} \right) = 2R \text{ (m}^2\text{/s)} \quad \text{unit}$$

Hint
Likhnah

① Radius of solid sphere Increases $\frac{1}{\pi}$ m/s then find
Rate of change in Volume of sphere w.r.t time
When Radius is 2m.

Solⁿ

$$\frac{dR}{dt} = \frac{1}{\pi} \text{ m/s}$$

$$\frac{dV}{dt} = ?$$

$$V = \frac{4}{3} \pi R^3$$

diffⁿ w.r.t 'R'

$$\frac{dV}{dR} = \frac{4}{3} \pi \frac{dR^3}{dR}$$

$$\frac{dV}{dR} = \frac{4}{3} \pi \cancel{3} R^2$$

$$\left(\frac{dV}{dt} \right) = 4\pi R^2 \frac{dR}{dt}$$
$$= 4\pi (2)^2 \times \frac{1}{\pi}$$

$$= 4 \times 4$$
$$\left(\frac{dV}{dt} \right) = 16 \text{ } \left(\frac{\text{m}^3}{\text{sec}} \right)$$

Maxima/minima

likhna hai

Happy



Minima

* $\text{at } x_2 \left(\frac{dy}{dx} \right) = m = 0$
minima

Slope \rightarrow Increased ☺

$\frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d^2y}{dx^2} = +ve$
↑ slope

Sad



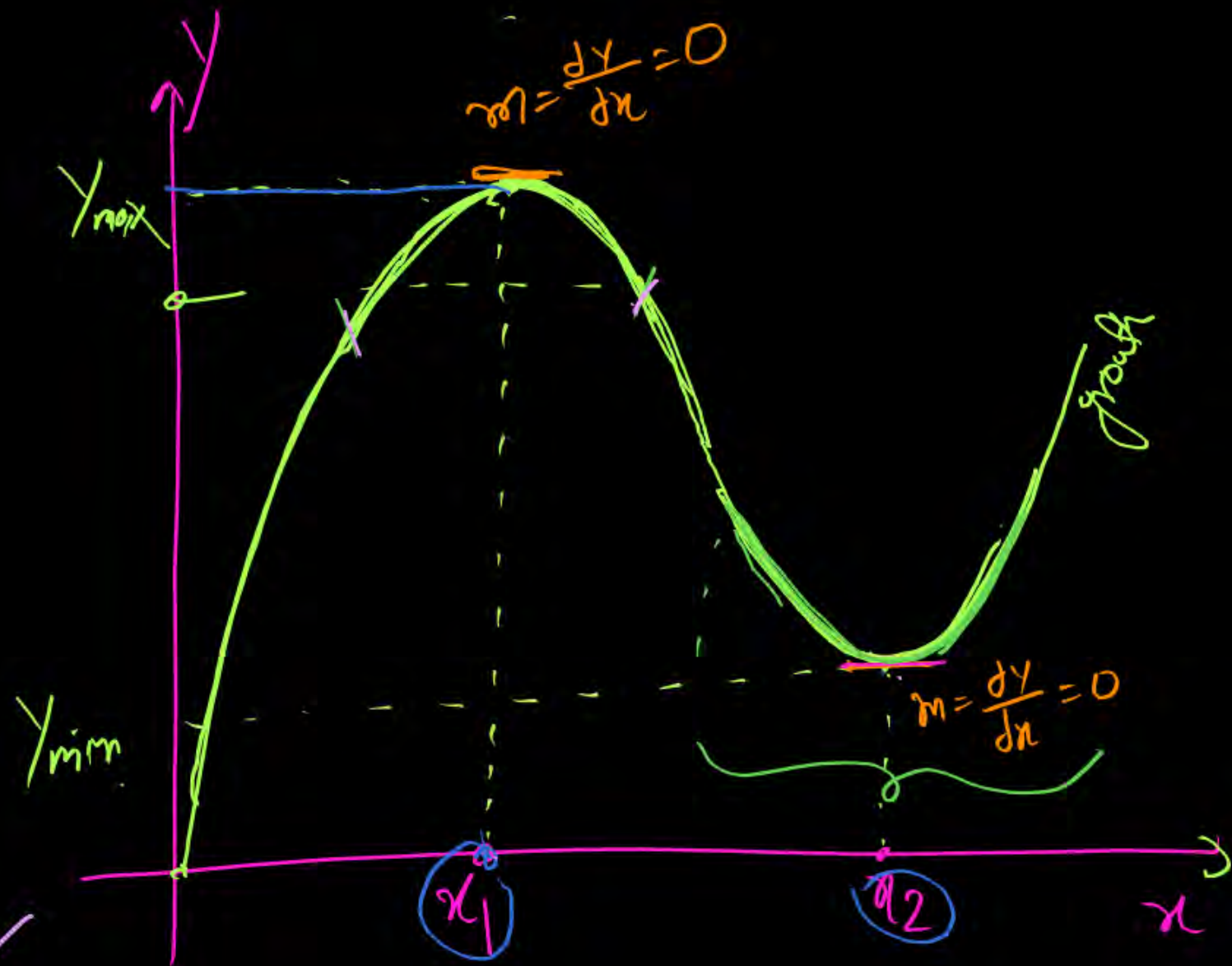
Maxima

$\text{at } x_1 \left(\frac{dy}{dx} \right) = 0$
Maxima

real \rightarrow Maxima

Slope decreasing ☹

$\frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d^2y}{dx^2} = -ve$
↑ slope



THANK
YOU