Birla Institute of Technology & Science, Pilani
Pilani Campus

PHY F244 : Modern Physics Laboratory

Rydberg's constant using Balmer Series

1. **Aim and Objectives** : This experiment aims to determine the Rydberg's constant using the Balmer series spectroscopic lines.

2. **Keywords** : Hydrogen atom, Bohr Atomic model, Balmer Seris, Diffraction, Bragg's law.

3. **Theory** :

   (a) **Hydrogen Atom:** The hydrogen atom, as the simplest atom consisting of a single proton and a single electron, plays a crucial role in understanding atomic structure and quantum mechanics. Its energy levels are determined by the solutions to the Schrödinger equation for an electron in the Coulomb potential created by the proton. These energy levels are quantized, meaning the electron can only occupy specific discrete energy states.

   A simple derivation of the energy levels in Bohr's atomic model can be achieved by combining basic physics principles: centripetal force, quantization of angular momentum, and the total energy of an electron in a circular orbit.

   - The electron (mass m_e and charge e) moves in a circular orbit around the nucleus (radius r), where the electrostatic force provides the necessary centripetal force:

   $$\frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r^2} = \frac{m_e v^2}{r}$$

   - Bohr proposed that the angular momentum of the electron is quantized:

   $$m_e v r = n\hbar$$

   Where: n: Principal quantum number ($n = 1, 2, 3, \ldots$).
   $\hbar$: Reduced Planck's constant ($\hbar = \frac{h}{2\pi}$).
   Rearranging for v:

   $$v = \frac{n\hbar}{m_e r}$$

   - Substitute $v = \frac{n\hbar}{m_e r}$ into the centripetal force equation:

   $$\frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r^2} = \frac{m_e \left(\frac{n\hbar}{m_e r}\right)^2}{r}$$

   Simplify:

   $$\frac{1}{4\pi\varepsilon_0} \cdot e^2 = \frac{n^2\hbar^2}{m_e r}$$

   Solve for r:

   $$r_n = \frac{4\pi\varepsilon_0 n^2 \hbar^2}{m_e e^2}$$

   This is the radius of the n-th orbit. For $n = 1$, r_1 is called the **Bohr radius**:

   $$r_1 \approx 0.529\,\text{Å}\,(10^{-10}\,\text{m})$$

   - The total energy E is the sum of kinetic energy (K) and potential energy (U):
   - **Kinetic Energy**:

   $$K = \frac{1}{2} m_e v^2$$

From the centripetal force equation:

$$m_e v^2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r}$$

Substitute v^2:

$$K = \frac{1}{2} \cdot \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r}$$

Potential Energy

$$U = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r}$$

Total Energy

$$E = K + U = \frac{1}{2} \cdot \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r} - \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r}$$

- Simplify:

$$E = -\frac{1}{2} \cdot \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r}$$

- Substitute $r_n = \frac{4\pi\varepsilon_0 n^2 \hbar^2}{m_e e^2}$ into E:

$$E_n = -\frac{1}{2} \cdot \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{\frac{4\pi\varepsilon_0 n^2 \hbar^2}{m_e e^2}}$$

Simplify:

$$\boxed{E_n = -\frac{m_e e^4}{8\varepsilon_0^2 h^2} \cdot \frac{1}{n^2} \approx \frac{-13.6}{n^2} \quad \text{eV}}$$

- **The Rydberg constant** R_∞ is derived from fundamental constants and is given by the following expression:

$$R_\infty = \frac{m_e e^4}{8\varepsilon_0^2 h^3 c}$$

For hydrogen (R_H), the reduced mass of the electron-proton system is used instead of the electron mass:

$$R_H = R_\infty \cdot \frac{\mu}{m_e}$$

where:

$$\mu = \frac{m_e m_p}{m_e + m_p}$$

is the reduced mass of the electron-proton system, and m_p is the mass of the proton. This correction accounts for the finite mass of the proton. As we know $\mu \approx m_e \implies R_H \approx R_\infty$.

- The energy level of the Hydrogen atom can be re-written in terms of Rydberg's constant as:

$$E_n = -hcR_\infty \frac{1}{n^2} \tag{1}$$

- **The hydrogen spectroscopic lines** refer to the various wavelengths of light emitted or absorbed by hydrogen atoms as their electrons transition between discrete energy levels. These transitions produce distinct spectral lines, which can be categorized into different series based on the final energy level of the electron. The wavelength of the spectral lines is obtained by

$$\Delta E = E_{n+1} - E_n = h\nu = \frac{hc}{\lambda} == hcR_\infty \left[\frac{1}{n^2} - \frac{1}{(n+1)^2}\right]$$

as a result,

$$\frac{1}{\lambda} == R_{\infty}\left[\frac{1}{n^2} - \frac{1}{(n+1)^2}\right] \tag{2}$$

- **Summary of Key Spectral Series:**

Series	Transitions	Wavelength Range	Region
Lyman	$n \geq 2 \rightarrow n = 1$	91.2 nm to 121.6 nm	Ultraviolet
Balmer	$n \geq 3 \rightarrow n = 2$	410.2 nm to 656.3 nm	Visible
Paschen	$n \geq 4 \rightarrow n = 3$	820.0 nm to 1875 nm	Infrared
Brackett	$n \geq 5 \rightarrow n = 4$	1465 nm to 4050 nm	Infrared
Pfund	$n \geq 6 \rightarrow n = 5$	2270 nm to 7460 nm	Infrared
Humphreys	$n \geq 7 \rightarrow n = 6$	Infrared	Infrared

Table 1: Hydrogen Spectroscopic Lines

- The Balmer series is only in the visible region and hence in this experiment we use the spectroscopic lines of Balmer series (i.e. having the transtion to $n = 2$ from higher energy levels.) to measure the value of Rydberg's constant. So the above equation for the Balmer series would be:

$$\frac{1}{\lambda} == R_{\infty}\left[\frac{1}{2^2} - \frac{1}{n^2}\right] \quad ; \quad n \geq 3 \tag{3}$$

- **Balmer Series Spectra with Wavelength and Color Information** is tabulated below.

Transition	Wavelength (λ)	Color	Line Name
$n = 3 \rightarrow n = 2$	656.3 nm	Red	H_α
$n = 4 \rightarrow n = 2$	486.1 nm	Blue-Green	H_β
$n = 5 \rightarrow n = 2$	434.0 nm	Blue	H_γ
$n = 6 \rightarrow n = 2$	410.2 nm	Violet	H_δ

Table 2: Balmer Series Spectra.

4. **Tasks** :

   (a) Prepare appropriate Observation table.

   (b) Calculate the least count of the spectrometer.

   (c) Calculate the Grating constant d from the lines/mm given on the Grating.

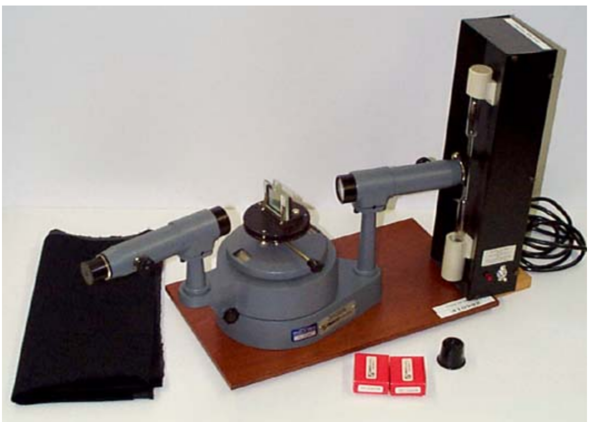

Figure 1: Typical experimental setup.

(d) Use the Bragg's diffraction formula $d\sin\theta = m\lambda$ to estimate the wavelength of Red, Blue-Green, Blue and Violet color lines, here m is the order of the diffraction.

(e) The diffraction angle θ should be calculated as $\theta = 0.5 \times (\theta_{\text{left}} + \theta_{\text{right}})$.

(f) Now use Table 2 (for n values of specific color) and Eq. 3 to collect the data for plotting $1/\lambda$ and $[1/4 - 1/n^2]$.

(g) The slope of the graph will give the desired Rydberg's constant.

(h) Estimate the percentage error from the theoretical value.

5. **Observations and Results** :

Sl. No.	**Color**	n	m	$\theta_R(^\circ)$	$\theta_L(^\circ)$	$\lambda\,(\text{nm})$	$1/4 - 1/n^2$
1	Violet (v)	6	1				
2	Blue-Violet (bv)	5	1				
3	Blue-Green (bg)	4	1				
4	Red (r)	3	1				
5	Violet (v)	6	2				
6	Blue-Violet (bv)	5	2				
7	Blue-Green (bg)	4	2				
8	Red (r)	3	2				