Birla Institute of Technology & Science, Pilani
Pilani Campus

PHY F244 : Modern Physics Laboratory

Determination of Verdet Constant - Faraday Effect

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1. **Aim and Objectives** : The objective of this experiment to determine the Verdet constant using the magneto-optic effect or Farady effect.

2. **Keywords** : Magneto-optic Effect, Farady Effect, Verdet constant.

3. **Theory** :

   The Verdet constant quantifies the rotation of the plane of polarization of linearly polarized light as it propagates through a material in the presence of a magnetic field. This phenomenon, known as the Faraday effect, arises due to the differential interaction of left- and right-circularly polarized light with the medium.

   (a) **Decomposition of Linearly Polarized Light into LCP and RCP Components:** A linearly polarized (LP) light wave propagating along the $+z$ direction with a phase factor $\omega t - kz$ can be decomposed into left-circularly polarized (LCP) and right-circularly polarized (RCP) components. The electric field of LP light polarized along the x-axis can be written as:

   $$\mathbf{E}_{LP}(z,t) = E_0 \cos(\omega t - kz)\,\hat{x} \tag{1}$$

   This can be decomposed as a sum of LCP and RCP components:

   $$\mathbf{E}_{LP}(z,t) = \frac{1}{2}\left(\mathbf{E}_{LCP}(z,t) + \mathbf{E}_{RCP}(z,t)\right) \tag{2}$$

   where the electric fields of LCP and RCP light are given by:

   $$\mathbf{E}_{LCP}(z,t) = E_0\left(\cos(\omega t - kz)\,\hat{x} + \sin(\omega t - kz)\,\hat{y}\right) \tag{3}$$

   $$\mathbf{E}_{RCP}(z,t) = E_0\left(\cos(\omega t - kz)\,\hat{x} - \sin(\omega t - kz)\,\hat{y}\right) \tag{4}$$

   (b) **Relation Between Displacement r, Dipole Moment, and Dielectric Constant:** The displacement $\mathbf{r}(t) = x(t)\,\hat{x} + y(t)\,\hat{y}$ of a charged particle with charge q relative to its equilibrium position induces a dipole moment:

   $$\mathbf{p} = q\mathbf{r} \tag{5}$$

   If there are N dipoles per unit volume, the total polarization $\mathbf{P}$ is:

   $$\mathbf{P} = N\mathbf{p} = Nq\mathbf{r} \tag{6}$$

   The total electric displacement $\mathbf{D}$ in the material is related to the electric field $\mathbf{E}$ and the polarization $\mathbf{P}$ as:

   $$\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P} \tag{7}$$

   By definition, the displacement $\mathbf{D}$ is related to the electric field $\mathbf{E}$ through the dielectric constant ε:

   $$\mathbf{D} = \varepsilon\mathbf{E} \tag{8}$$

   Equating the two expressions for $\mathbf{D}$, we have:

   $$\varepsilon\mathbf{E} = \varepsilon_0\mathbf{E} + Nq\mathbf{r} \tag{9}$$

   Solving for ε, we obtain:

   $$\varepsilon = \varepsilon_0\left(1 + \frac{Nq\mathbf{r}}{\varepsilon_0\mathbf{E}}\right) \tag{10}$$

(c) **Equation of Motion for the Dipole:** We now derive the equation of motion for a dipole under the influence of electric and magnetic fields. Consider a particle with mass m and charge q subjected to the following forces:

Restoring Force: The restoring force due to the harmonic binding potential is proportional to the displacement $\mathbf{r}$ from the equilibrium position. It is given by:

$$\mathbf{F}_{\text{restoring}} = -\kappa \mathbf{r} \tag{11}$$

where κ is the effective spring constant for the dipole.

Electric Force: The force on the charged particle due to the electric field $\mathbf{E}$ of the light wave is:

$$\mathbf{F}_{\text{electric}} = q\mathbf{E} \tag{12}$$

Lorentz Force: When the magnetic field $\mathbf{B}_0$ is applied along the z-axis, and the particle moves with velocity $\mathbf{v}$, the Lorentz force is:

$$\mathbf{F}_{\text{magnetic}} = q\,\mathbf{v} \times \mathbf{B}_0 \tag{13}$$

Assuming $\mathbf{B}_0 = B_0\hat{z}$, the velocity $\mathbf{v} = \dot{\mathbf{r}}$, the Lorentz force can be written as:

$$\mathbf{F}_{\text{magnetic}} = q\,\dot{\mathbf{r}} \times B_0\hat{z} \tag{14}$$

The total force $\mathbf{F}$ acting on the particle is the sum of these three forces:

$$\mathbf{F} = \mathbf{F}_{\text{restoring}} + \mathbf{F}_{\text{electric}} + \mathbf{F}_{\text{magnetic}} \tag{15}$$

By Newton's second law, the equation of motion becomes:

$$m\ddot{\mathbf{r}} = -\kappa\mathbf{r} + q\mathbf{E} + q\,\dot{\mathbf{r}} \times B_0\hat{z} \tag{16}$$

(d) **Complex Form of the Equation of Motion:** We separate the components of $\mathbf{r} = x(t)\,\hat{x} + y(t)\,\hat{y}$. The electric field is assumed to lie in the x-y plane, and the magnetic field is along the z-axis. We decompose the equation of motion for the x and y components.

For the x-component:

$$m\ddot{x} + \kappa x = qE_x + qB_0\dot{y} \tag{17}$$

For the y-component:

$$m\ddot{y} + \kappa y = qE_y - qB_0\dot{x} \tag{18}$$

To solve these coupled differential equations, we introduce the complex form of the displacement:

$$r(t) = x(t) + iy(t) \tag{19}$$

The first and second time derivatives are:

$$\dot{r}(t) = \dot{x}(t) + i\dot{y}(t), \quad \ddot{r}(t) = \ddot{x}(t) + i\ddot{y}(t) \tag{20}$$

The two-component system can be combined into a single complex equation:

$$m\ddot{r} + \kappa r = qE(t) + i\Omega_c m\dot{r} \tag{21}$$

where $\Omega_c = \frac{qB_0}{m}$ is the cyclotron frequency.

We assume a solution of the form $r(t) = r_0 e^{i\omega t}$. The first and second derivatives of $r(t)$ are:

$$\dot{r}(t) = i\omega r_0 e^{i\omega t}, \quad \ddot{r}(t) = -\omega^2 r_0 e^{i\omega t} \tag{22}$$

Substituting these into the equation of motion, we get:

$$(-m\omega^2 + \kappa)r_0 e^{i\omega t} = qE_0 e^{i\omega t} - \Omega_c m\,\omega\, r_0 e^{i\omega t} \tag{23}$$

Reorganizing, we have:

$$r_0\left(\kappa - m\omega^2 - i\Omega_c m\omega\right) = qE_0 \tag{24}$$

The displacement r_0 is given by:

$$r_0 = \frac{qE_0}{\kappa - m\omega^2 - i\Omega_c m\omega} \tag{25}$$

(e) **Calculation of Refractive Indices n_L and n_R:** Using the expression for r_0, the dielectric constants ε_L and ε_R for LCP and RCP light are:

$$\varepsilon_L = \varepsilon_0\left(1 + \frac{Nq^2}{\varepsilon_0(\kappa - m\omega^2 + i\Omega_c m\omega)}\right) \tag{26}$$

$$\varepsilon_R = \varepsilon_0\left(1 + \frac{Nq^2}{\varepsilon_0(\kappa - m\omega^2 - i\Omega_c m\omega)}\right) \tag{27}$$

The refractive indices for LCP and RCP light are related to the dielectric constants as:

$$n_L = \sqrt{\frac{\varepsilon_L}{\varepsilon_0}}, \quad n_R = \sqrt{\frac{\varepsilon_R}{\varepsilon_0}} \tag{28}$$

Using the expressions for ε_L and ε_R, the refractive indices are:

$$n_L = \sqrt{1 + \frac{Nq^2}{\varepsilon_0(\kappa - m\omega^2 + i\Omega_c m\omega)}} \tag{29}$$

$$n_R = \sqrt{1 + \frac{Nq^2}{\varepsilon_0(\kappa - m\omega^2 - i\Omega_c m\omega)}} \tag{30}$$

Using the small perturbation approximation $\sqrt{1+x} \approx 1 + \frac{x}{2}$, we obtain:

$$n_L \approx 1 + \frac{1}{2} \cdot \frac{Nq^2}{\varepsilon_0(\kappa - m\omega^2 + i\Omega_c m\omega)} \tag{31}$$

$$n_R \approx 1 + \frac{1}{2} \cdot \frac{Nq^2}{\varepsilon_0(\kappa - m\omega^2 - i\Omega_c m\omega)} \tag{32}$$

To compute the difference in refractive indices, we use the identity:

$$\frac{1}{a+ib} - \frac{1}{a-ib} = \frac{2ib}{a^2+b^2} \tag{33}$$

We identify $a = \kappa - m\omega^2$ and $b = \Omega_c m\omega$. Applying this identity, we get:

$$n_L - n_R = \frac{Nq^2}{2\varepsilon_0} \cdot \frac{2i\Omega_c m\omega}{(\kappa - m\omega^2)^2 + (\Omega_c m\omega)^2} \tag{34}$$

$$n_L - n_R = \frac{iNq^2\,\Omega_c\, m\,\omega}{\varepsilon_0\left((\kappa - m\omega^2)^2 + (\Omega_c m\omega)^2\right)} \tag{35}$$

(f) **Phase Difference $\Delta\phi$:** The phase difference between LCP and RCP light after propagating a distance d is:

$$\Delta\phi = k|n_L - n_R| \cdot d = \frac{2\pi}{\lambda} \cdot |n_L - n_R| \cdot d \tag{36}$$

Substituting $n_L - n_R$, we have:

$$\Delta\phi = \frac{2\pi}{\lambda} \cdot \frac{N q^2 \Omega_c m \omega d}{\varepsilon_0 ((\kappa - m\omega^2)^2 + (\Omega_c m\omega)^2)} \tag{37}$$

To compute the physical phase difference, we extract the real part of the phase difference. The final expression for the phase difference $\Delta\phi$ is:

$$\Delta\phi = \frac{4\pi^2 N q^3 B c d}{\lambda^2 \varepsilon_0 (\kappa - m\omega^2)^2} \tag{38}$$

(g) **Rotation of Polarization and Verdet Constant:**
The rotation of the polarization angle θ after traveling a distance d is:

$$\theta = \frac{\Delta\phi}{2} = \frac{1}{2} \frac{4\pi^2 N q^3 B c d}{\lambda^2 \varepsilon_0 (\kappa - m\omega^2)^2} \tag{39}$$

As $m = 9.1 \times 10^{-31}$ kg and hence we can ignore the $m\omega^2$ term to the good approximation.

$$\theta \approx \left[\frac{2\pi^2 N q^3 c}{\lambda^2 \varepsilon_0 \kappa^2}\right] B d \approx VBd \tag{40}$$

$$\boxed{\theta = VBd} \tag{41}$$

The Verdet constant V relates the rotation of polarization θ to the magnetic field strength B and the propagation distance d. It is given by:

$$V = \frac{2\pi^2 N q^3 c}{\lambda^2 \varepsilon_0 \kappa^2} \propto \frac{1}{\lambda^2} \tag{42}$$

(h) **Given Data:**
In the experimental setup given $d = 10$ cm is the length of the Glass rod inside the solenoid, and $N_{\text{coils}} = 2508$ is the number of turns in Solenoid. Wavelength of Red Laser is 650 nm and Green Laser is 532 nm.

4. **Experimental Procedure** :

(a) Mount the laser source on one end of the optical rail. Place the polarizer, sample (inside the solenoid), analyzer, and photodetector sequentially along the rail. Ensure the laser beam passes through the center of all components.

(b) Set the polarizer and analyzer to initially aligned positions (zero rotation). With no current in the solenoid, adjust the analyzer such that maximum intensity is detected on the photodetector.

(c) Gradually increase the current I through the solenoid using the power supply. For each current value, note the corresponding angle of rotation θ by adjusting the analyzer to restore maximum intensity. Record the current I, the corresponding magnetic field B (calculated as $B = \mu_0 NI$), and the rotation θ.

(d) Repeat the measurements for a range of current values (e.g., from 0 to the maximum allowed by the setup). Record at least 5-10 data points for better accuracy.

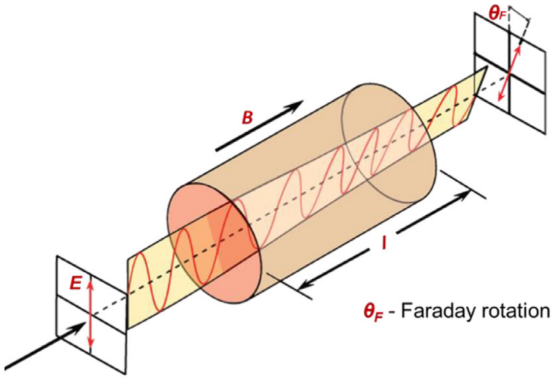

Figure 1: Farady rotation of the polarization.

Figure 2: Experimental setup for the measurement of the Verdet constant.

(e) Plot θ (in degrees or radians) vs. B (in tesla). Determine the slope of the linear plot θ/B, which corresponds to Vd. Divide the slope by the sample length d to obtain the Verdet constant V.

5. **Tasks** :

   (a) Create an appropriate observation table to take your observations.

   (b) Plot the graph between the θ and B and obtain the Verdet constant by calculating slope of the graph.

   (c) Change the laser from Red to Green and then again measure the Verdet constant.

   (d) Comment on the difference between the Verdet constant for different wavelengths.

6. **Observations and Results** :

S.N.	I (Amp)	B (Gauss)	θ (deg) [Red Laser]	θ (deg) [Green Laser]
0	0	0		
1				
2				
3				