Birla Institute of Technology & Science, Pilani

Pilani Campus

PHY F244 : Modern Physics Laboratory

Specific charge of electron e/m_e

1. **Aim and Objectives** : This experiment aims to determine the electron's specific charge, i.e., charge to mass ratio e/m_e.

2. **Keywords** : Lorentz Force, Helmhotz Coil, Biot Savart Law.

3. **Theory** :

   (a) **Basic Principle:**
   The motion of the charged particle q under the influence of the magnetic field $\mathbf{B}$ is governed by the Lorentz force law and is given as:

   $$\mathbf{F} = q\mathbf{v} \times \mathbf{B} \tag{1}$$

   where $\mathbf{v}$ is the velocity vector of the charged particle. In the special case when $\mathbf{v} \perp \mathbf{B}$ then the above equations simplifies to:

   $$F = q\upsilon B \tag{2}$$

   where $\upsilon = |\mathbf{v}|$ and $B = |\mathbf{B}|$. In this configuration $\mathbf{v} \perp \mathbf{B}$, the charged particle will trace out a circular path, as we know the required centripetal force for the circular path of radius r should be the same as the Lorentz force. Here, anyway charge particles are electrons so $q = e = |e|$ and the desired equation is obtained as:

   $$\frac{m_e \upsilon^2}{r} = e\upsilon B \implies \boxed{\frac{e}{m_e} = \frac{\upsilon}{rB}} \tag{3}$$

   If the electron is accelerated by the accelerating potential of U volts, then its kinetic energy can be given as:

   $$\frac{1}{2} m_e \upsilon^2 = eU \implies \upsilon = \sqrt{\frac{2eU}{m_e}} \tag{4}$$

   From Eq. 3 and 4, one can easily obtain the following relation equating the e/m_e with the applied potential and the magnetic field.

   $$\boxed{\frac{e}{m_e} = \frac{2U}{(rB)^2}} \tag{5}$$

   (b) **Magnetic field of Helmhotz Coil:**
   The magnetic field generated by the steady currents can be estimated by the Biot-Savart law. Consider the geometry of the problem as shown in Figure 1. The magnetic field along the axial direction i.e., along the x axis from the current carrying current loop of radius a, is given by,

   $$\mathbf{B}(x) = \frac{\mu_0 I}{4\pi} \int \frac{\vec{d\ell} \times \mathbf{r}}{r^3} \tag{6}$$

   here, $\mathbf{r}$ is the position vector from the source of the current to the location where the field is calculated, and $|\mathbf{r}| = r = \sqrt{x^2 + a^2}$. of the point at which one needs to calculate the magnetic field and $\mathbf{r_0}$ is the position vector of the current source itself with respect to some coordinate system. It is clear that $\vec{d\ell} \perp \mathbf{r}$ and hence the above equation for a small current segment can be written as:

   $$d\mathbf{B} = \frac{\mu_0 I}{4\pi} \frac{d\ell}{r^2} \hat{k} \tag{7}$$

   where $\hat{k} \equiv \hat{dl} \times \hat{r}$ is the unit vector along which the magnetic field will be pointed. As can be seen from the symmetry in Figure 1, the y component of the magnetic field will be canceled out, and only the x component will survive. So, using Eq. 7 and Eq. 6 the differential axial component of the magnetic field is then given by

$$dB_x = |d\mathbf{B}|\cos\theta = \frac{\mu_0 I}{4\pi}\frac{d\ell}{r^2}\cos\theta \tag{8}$$

Moreover, the net magnetic field can be found by integrating the last equation:

$$B_x(x) = \frac{\mu_0 I}{4\pi}\frac{2\pi a}{r^2}\cos\theta = \frac{\mu_0 I}{4\pi}\frac{2\pi a}{r^2}\frac{a}{r} \tag{9}$$

$$\boxed{B_x(x) = \frac{\mu_0 I}{2}\frac{a^2}{(x^2+a^2)^{3/2}}} \tag{10}$$

Now, the Helmholtz coil is the assembly of two identical circular coils separated by a distance equal to the radius of the coils. The direction of the current is also the same for both coils. The advantage of using a Helmholtz coil is that it can generate a uniform magnetic field at the center of this assembly. The typical field profile and the schematic of the Helmholtz coil are shown in Figure 2. We obtained the magnetic field by the single current-carrying loop in Eq. 10. We can extend this concept to two loops. Let us now say $a = R$ and $B_1 \equiv= B_x$ as the net magnetic field of a single loop, the Eq. 10 is written as:

$$B_1(x) = \frac{\mu_0 I}{2}\frac{R^2}{(x^2+R^2)^{3/2}} \tag{11}$$

The net magnetic field by two coils is then given by:

$$B(x) = 2B_1(x) = \mu_0 I\frac{R^2}{(x^2+R^2)^{3/2}} \tag{12}$$

For $x = R/2$ where our main setup is placed, the net magnetic field of a single current loop is reduced to,

$$B = \left(\frac{4}{5}\right)^{3/2}\frac{\mu_0 I}{R} \tag{13}$$

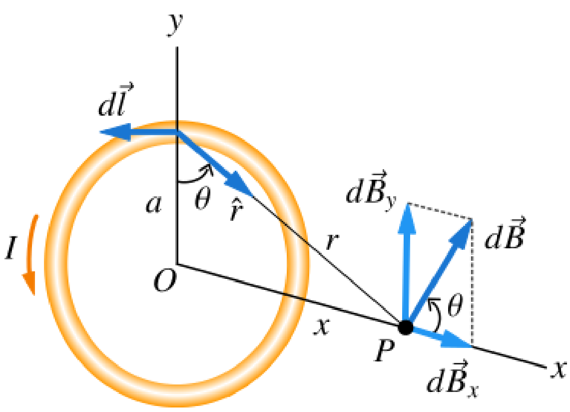

Figure 1: Magnetic field by a current carrying loop.

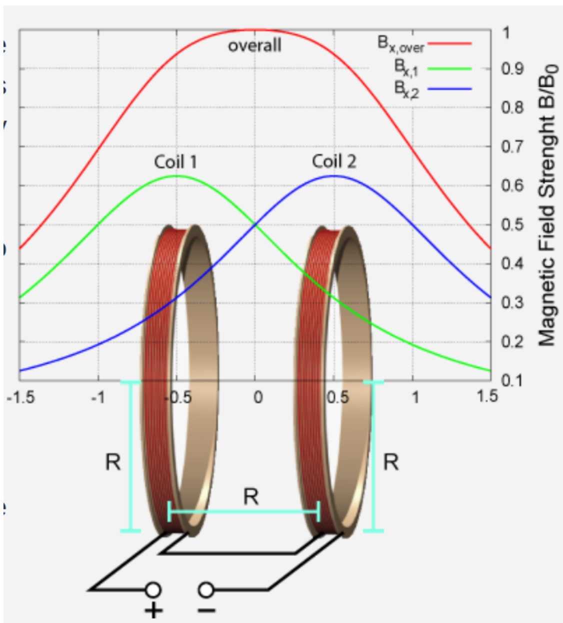

Figure 2: Helmhotz coil setup.[Image Courtsey: Click Here.]

and if the coils are having N turns, the magnetic field at the center is then given by,

$$\boxed{B = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 N I}{R}} \tag{14}$$

(c) Using the Eq. 5 and Eq. 14 the specific charge can be calculated as:

$$\frac{e}{m_e} = \frac{2U}{r^2}\frac{125}{64}\frac{R^2}{(\mu_0 N I)^2} \implies \boxed{\frac{e}{m_e} = \left[\frac{125R^2}{32(\mu_0 N)^2}\right]\frac{1}{r^2}\frac{U}{I^2}} \tag{15}$$

- $R = 20$ cm is the Radius of the Coils
- $N = 154$ number of turns of the coils
- I is the current passing through the coils.
- r is the radius of the electron circular path.
- U is the applied potential to accelerate electrons.

4. **Experimental Procedure** :

(a) Initially, keep the current through the coils zero.

(b) Increase the potential of the electron gun till you see the orange glow (electrons collide with the gas atoms in the bulb, and the excitation and de-excitation of atoms emit the photon in the visible range, and hence the glow is seen).

(c) Now, increase the current, and you will see the electron beam is bent according to the Lorentz force law. Rotate the tube till you get the circular path (typically if $\mathbf{v}$ is not perpendicular to $\mathbf{B}$, the electron path would be helix).

(d) There is a ladder-type metal frame placed inside the bulb. We can see when the electron will hit the equidistant markers of that ladder.

(e) For a given applied potential U, you vary the current in coils (change the magnetic field strength) so that the electron hits the first marker of the ladder. This would give the diameter of the electron's circular path.

(f) One can vary the U and obtain the I required such that desired r is reached.

5. **Tasks** :

(a) Create an appropriate observation table to take your observations.

(b) Vary the applied potential U (Volts) and note down the current I (Amps) needed, such that the electron beam hit the first marker such that the radius of the electron's circular path is $r = 2$ cm.

(c) Change the U and repeat step (b).

(d) Now repeat step (b) and step (c) for another path radius $r = 3, 4, 5$ cm.

(e) Plot the graph between U and I^2 for a particular value of r and find the slope of the graph.

(f) Use the slope of the graph and Eq. 15 to obtain the value of e/m_e for each r values.

(g) Obtain the average value of e/m_e and also estimate the percentage error.

6. **Observations and Results** :

S.N.	U (V)	I (A) [$r = 2$ cm]	I (A) [$r = 3$ cm]	I (A) [$r = 4$ cm]	I (A) [$r = 5$ cm]
1					
2					
3					

The instruction manual for the experimental setup is also appended.

Related topics

Cathode rays, Lorentz force, electron in crossed fields, electron mass, electron charge.

Principle

Electrons are accelerated in an electric field and enter a magnetic field at right angles to the direction of motion. The specific charge of the electron is determined from the accelerating voltage, the magnetic field strength and the radius of the electron orbit.

Equipment

Narrow beam tube	06959.00	1
Pair of Helmholtz coils	06960.00	1
Power supply, 0...600 VDC	13672.93	1
Power supply, universal	13500.93	1
Digital multimeter	07134.00	2
Connecting cord, l = 100 mm, red	07359.01	1
Connecting cord, l = 100 mm, blue	07359.04	1
Connecting cord, l = 750 mm, red	07362.01	5
Connecting cord, l = 750 mm, blue	07362.04	3
Connecting cord, l = 750 mm, yellow	07362.02	3

Tasks

Determination of the specific charge of the electron (e/m_0) from the path of an electron beam in crossed electric and magnetic fields of variable strength.

Set-up and procedure

The experimental set up is as shown in Fig. 1. The electrical connection is shown in the wiring diagram in Fig. 2. The two coils are turned towards each other in the Helmholtz arrangement. Since the current must be the same in both coils, connection in series is preferable to connection in parallel. The maximum permissible continuous current of 5 A should not be exceeded.

If the polarity of the magnetic field is correct, a curved luminous trajectory is visible in the darkened room. By varying the magnetic field (current) and the velocity of the electrons (acceleration and focussing voltage) the radius of the orbit can be adjusted, such that it coincides with the radius defined by the luninous traces. When the electron beam coincides with the luminous traces, only half of the circle is observable. The radius of the circle is then 2, 3, 4 or 5 cm.

For detailed description of the narrow beam tube, please refer to the operating instructions.

If the trace has the form of a helix this must be eliminated by rotating the narrow beam tube around its longitudinal axis.

Theory and evaluation

If an electron of mass m_0 and charge e is accelerated by a potential difference U it attains the kinetic energy:

$$e \cdot U = \frac{1}{2} \cdot m_0 \cdot v^2 \qquad (1)$$

where v is the velocity of the electron.

In a magnetic field of strength $\vec{B}$ the Lorentz force acting on an electron with velocity $\vec{v}$ is:

$$\vec{F} = e \cdot \vec{v} \times \vec{B}$$

If the magnetic field is uniform, as it is in the Helmholtz arrangement the electron therefore follows a spiral path along the magnetic lines of force, which becomes a circle of radius r if $\vec{v}$ is perpendicular to $\vec{B}$.

Fig.1: Experimental set-up for determining the specific charge of the electron.