MATH1231/1241 Calculus Revision Additional Solutions to Part 1

August 10, 2019

These solutions were written and typed by Yasin Khan. Please be ethical with this resource. It is for the use of MathSoc members - do not repost it on other forums or groups without asking for permission. If you appreciate our resources, please consider supporting us by coming to our events! Also, happy studying :)

We cannot guarantee that our answers are correct - please notify us of any errors or typos at unswmathsoc@gmail.com, or on our Facebook page. There are sometimes multiple methods of solving the same question. Remember that in the real class test, you will be expected to explain your steps and working out.

Question 1 $\int x^n e^{x^2} dx$

$$\begin{aligned}&\int x^n e^{x^2} dx \\ &= \int x^{n-1} x e^{x^2} dx\end{aligned}$$

By considering integration by parts, we have:

$$\begin{aligned} u &= x^{n-1} & dv &= xe^{x^2}\,dx \\ du &= (n-1)x^{n-2}\,dx & v &= \frac{1}{2}e^{x^2}\end{aligned}$$

Hence we get

$$\begin{aligned} I_n &= uv - \int v du \\ &= \frac{1}{2}e^{x^2}x^{n-1} - \frac{n-1}{2}x^{n-2}e^{x^2} \\ &= \frac{1}{2}e^{x^2}x^{n-1} - \frac{n-1}{2}I_{n-2}\end{aligned}$$

Therefore our reduction formula is:

$$I_n = \frac{1}{2}e^{x^2}x^{n-1} - \frac{n-1}{2}I_{n-2}$$

Question 2 $\int \sqrt{1-x^2}dx$

Using the substitution $x = \sin\theta$

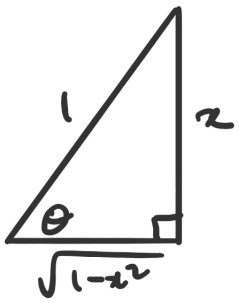
$$dx = \cos\theta d\theta$$

Hence,

$$\begin{aligned}
\int \sqrt{1-x^2}dx &= \int \sqrt{1-\sin^2\theta}\cos\theta d\theta \\
&= \int \cos^2\theta d\theta \\
&= \frac{1}{2}\int 1+\cos 2\theta d\theta \\
&= \frac{1}{2}\left(\theta + \frac{1}{2}\sin 2\theta\right) + C \\
&= \frac{1}{2}\left(\theta + \sin\theta\cos\theta\right) + C
\end{aligned}$$

Now, bring back into terms of x, we consider trig ratios,

$$\sin\theta = x \implies x = \sin^{-1}\theta$$



From the diagram we compute,

$$\cos\theta = \sqrt{1-x^2}$$

Therefore we have,

$$\int \sqrt{1-x^2}dx = \frac{1}{2}(\sin^{-1} x + x\sqrt{1-x^2}) + C$$

Question 3 $\int \sin^2 x \cos^2 x dx$

Using the identities we learned, we can rewrite the integral in linear terms,

$$\begin{aligned}
\int \sin^2 x \cos^2 x dx &= \int \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right) dx \\
&= \frac{1}{4}\int (1-\cos 2x)(1+\cos 2x) dx \\
&= \frac{1}{4}\int 1-\cos^2 2x dx \\
&= \frac{1}{4}\int \sin^2 2x dx \\
&= \frac{1}{8}\int 1-\cos 4x dx \\
&= \frac{1}{8}\left(x - \frac{\sin 4x}{4}\right) + C
\end{aligned}$$