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Hand-in Week 11

Exercise 1

We put the precondition for our factorial as $n \geq 1$. Then create a base case which we can proof is correct, by testing it with $n = 1$. The fact(n =1) will return 1, as n is not larger than 1, so $fact(1) = 1$. As the factorial of 1 is 1 we confirm that the base case is correct.

We also need to test a case for $n > 1$, so let's test for $n = 2$, then we can test our recursive step and see if we can prove that the factorial for $n = 2$ is correct:
Fact(2) will return:
$2 * fact(2 - 1)$
$= \quad 2 * fact(1)$
As the base case is $fact(1) = 1$ we can write that:
$2 * fact(1) = 2 * 1$

As the factorial of 2 is indeed $1 * 2$, we have proven that the factorial for $n = 2$ is correct
We can continue to prove this for n=3, n=4, n=5 and so on the same way.

To prove that it works with any positive integer above 1, we create a case for an integer k > 1 as an input. This integer k could be any positive integer above 1. For this case we know that fact(k) will return:
$fact(k) = k * fact(k - 1)$ for k > 1

Then fact(k) is correct if fact(k-1) is correct, which is the same as saying, if fact(k-1) is correct then:

$$fact(k) = k * [the\ factorial\ of\ (k - 1)]$$, for k > 1

And this is the definition of the factorial of k. ($k!$)

We have proven that, fact(1) is correct and fact(k) is correct IF fact(k-1) is correct for k > 1. We then test for an integer k = 6.

Then fact(6) is correct if fact(5) is correct -> fact(5) is correct if fact(4) is correct ->
fact(4) is correct if fact(3) is correct -> … -> fact(2) is correct if fact(1) is correct.

As we have proven that fact(1) is correct, then fact(2) is correct, which would eventually make fact(6) correct. No matter which integer is chosen for k, it will create a sequence like this:

IF $fact(i - 1)$ is correct then $fact(i)$ is correct, for $1 < i \leq k$

As the sequence use (i-1) to prove the case for (i) then it can be said:
IF fact(1) is correct then fact(2) is correct. (our inductive step is then if fact(i-1) is correct then fact(i) is correct)

So the bases case fact(1) proves the case for fact(2), which proves fact(3), and so on all the way up to fact(k), which proves the function is correct for all positive integers, n.