

sorted_disks sort_lawnmower(const disk_state& before) {

```

    sorted_disks BLIN = before // 1
    Left_index_limit = 0 // 1
    Right_index_limit = n - 2; // 1
    j = 0; // used to traverse BLIN's disk vector // 1
    reverse = false; // 1

```

```

    for (i = 0; i < n/2; i++) { // n/2 steps
        while(1) { // MAX_AMOUNT while-contraint
            if (Left_index_limit == Right_index_limit) { // 1 + max(3, 13, 12) = 14
                BLIN.swap(index_of_disks[Left_index_limit], index_of_disks[Right_index_limit]) // 1
                BLIN.swap_counter++ // 1
                break // 1
            } else if (!reverse) { // 1 + max(12, 0) = 13
                if (BLIN.index_of_disks[j] == DISK_DARK && BLIN.index_of_disks[j+1] == DISK_LIGHT) { // 3 + max(2, 0) = 5
                    BLIN.swap(index_of_disks[Left_index_limit], index_of_disks[Right_index_limit]) // 1
                    BLIN.swap_counter++ // 1
                }
                if (j == Right_index_limit) { 1 + max(5, 0) = 6
                    j-- // 1
                    break // 1
                }
                j++ // 1
            }
        }
    }

```

Proof that

$$g(n) = 7n^2 + 7n + 6$$

$$f(n) = n^2$$

$$g(n) \in O(f(n))$$

$$\equiv 7n^2 + 7n + 6 \in O(n^2)$$

using limits (L'Hopital)

$$\lim_{n \rightarrow \infty} \frac{g(n)}{f(n)} = L, \text{ non-constant}$$

$$\lim_{n \rightarrow \infty} \frac{d}{dn} (7n^2 + 7n + 6) = \infty$$

$$\frac{d}{dn} (n^2) = 2n$$

else {

$$\lim_{n \rightarrow \infty} \frac{14n + 7}{2n} = \infty$$

$$\lim_{n \rightarrow \infty} \frac{14}{2} = 7$$

$$\therefore 7n^2 + 7n + 6 \in O(n^2)$$

return BLIN object's sorted disks} // 1

$$5 + (n/2)(n-1)(14) + 1$$

$$5 + (7n^2 + 7n) + 1 = 7n^2 + 7n + 6$$

```
sorted_disks sort_left_to_right(const disk_state & before) {
```

```
    sorted_disks BLIN = before // 1 step
```

```
    for (i = 0; i < n/2; i++) { // n/2 steps
```

```
        for (size j = 0; j < n - 1; j++) { // n-1 steps
```

```
            if (BLIN.index_of_disks[j] == DISK_DARK && BLIN.index_of_disks[j+1] == DISK_LIGHT) { // B + max(2, 0)
```

```
                // DISCLAIMER: This just the idea, not syntactically correct.
```

```
                BLIN.swap(index_of_disks[j], index_of_disks[j+1]) // 1 step
```

```
                BLIN.swap_counter++ // 1 step
```

```
            }
```

```
        }
```

```
    }
```

```
    return BLIN object's sorted disks // 1 step
```

```
}
```

$$3 + 2 = 5$$

$$(n-1)(5) = 5n-5$$

$$\left(\frac{n}{2}\right)(5n-5) = \frac{5}{2}n^2 - \frac{5}{2}n$$

$$1 + \left(\frac{5n^2}{2} - \frac{5n}{2}\right) + 1$$

$$\frac{5}{2}n^2 - \frac{5}{2}n + 2 \quad \left(\begin{smallmatrix} \text{total} \\ \text{steps} \end{smallmatrix}\right)$$

Proving using L'Hopital that if $g(n) = O(f(n))$ the

$$\lim_{n \rightarrow \infty} \frac{g'(n)}{f'(n)} = L$$

where L is a non-negative constant

$$\text{so here } g(n) = \frac{5}{2}n^2 - \frac{5n}{2} + 2 \quad f(n) = n^2$$

$$\lim_{n \rightarrow \infty} \frac{\frac{d}{dn} \left(\frac{5}{2}n^2 - \frac{5n}{2} + 2 \right)}{\frac{d}{dn} (n^2)} = \frac{\infty}{\infty}$$

$$\lim_{n \rightarrow \infty} \frac{\frac{d}{dn} \left(\frac{5}{2}n^2 - \frac{5n}{2} + 2 \right)}{\frac{d}{dn} (n^2)} \Rightarrow \lim_{n \rightarrow \infty} \frac{\frac{5}{2}n - \frac{5}{2}}{2n} = \frac{5}{2}$$

$$\therefore \frac{5}{2}n^2 - \frac{5n}{2} + 2 \in O(n^2)$$