Team 20 Statement of Work
Wind Park Design Problem

2.1. Problem Characterization

Wind park layout optimization is considered an important topic in the energy reuse area. Given an area of wind park, different factors such as wind speed, positioning of the individual wind turbines and total number of wind turbines produce different output.

This optimization problem has been addressed in a lot of published literature, including one which uses genetic algorithm to minimize a weighted sum of wind energy and turbine costs, with the wind farm divided into a square grid to facilitate the encoding of a 0–1 type solution. Another paper also proposed using genetic algorithm to maximize an economic function but an important factor, wake loss, is not taking into consideration. Another paper presents a general framework for wind park optimization by providing specific mathematical models to considering wake loss and wind direction.

For this report, we obtain the wind speed and a square grid map of the waterloo region.

2.2 Problem Formulation

This report will research on the problem of optimizing a wind park layout in a given area in Waterloo, Ontario. Internal factors are number of wind turbines being used, type of turbine being used, and the positions of each turbine. External factors include wind direction, wind speed, landform of the wind park location and wake loss.

Assumption 1: Wind speed is assumed to be the same in the area of wind park, and landform is assumed to be a relatively flat ground so that every wind turbine would have the same height.
Assumption 2: The type of turbine used is assumed to be the same for all turbines and each of them has a 20m rotor diameter.
Assumption 3: Wind turbine location is characterized by its two dimensional Cartesian coordinate (x,y). And the number of wind turbines being used is N.
Assumption 4: Any two turbines in a wind farm are separated from each other at least 4 rotor diameters. This is ensuring to reduce the interactions between two adjacent turbines. Therefore, in our case, the distance between two turbines is 80m.
Assumption 5: Assuming the wind turbines are all built perpendicular to the wind direction. In this report, we will assume the wind is from west to east.

The cost function used for calculating the cost per year as a function of number of turbines N (from paper) is:

$$\text{costs} = N\left(\frac{2}{3} + \frac{1}{3} exp(-0.00174N^2)\right)$$

and the total power extracted by all wind turbines is:

$$P = NPtp$$

where P is the total power, N is the number of turbine and Ptp is single wind turbine power production.

The single wind turbine power production can be calculated by:

$$Ptp = t*P$$

where t is number of seconds in a year and P is wind power production of a single turbine per second which can be calculated as

$$P = 0.5 \times \rho \times A \times C_p \times V^3 \times N_g \times N_b$$

where ρ is air density, A is rotor swept area, C_p is performance coefficient, V is wind speed, N_g is generator efficiency , N_b is gear box bearing efficiency.

Wake loss is an important factor as it deals with the situation when wind encounters a wind turbine, and a linearly wake behind the turbine occurs.s For the wake loss model, we will use the following formula from reference paper:

$$V_{def} = (1 - \sqrt{1 - C_\tau})/(1 + \kappa d/R)^2$$

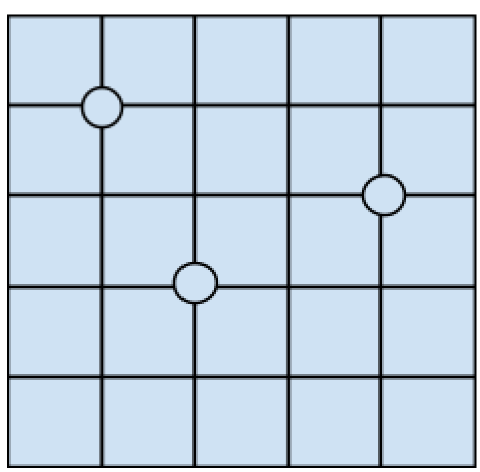
where C_τ is the thrust coefficient, κ is the wake spreading constant, and d is distance behind the turbine following the wind direction.

The optimization function being used here is a widely accepted one for minimum cost per unit energy produced as:

$$\min(\frac{costs}{P})$$

The objective is to find the number of turbine used, N, and all turbine locations (xi, yi) for i = 1...N that minimize $\frac{costs}{P}$.

2.3. Problem Modeling



In our problem, we will divide a given area into LxL grid with each little square representing 80m to satisfy **Assumption 4** in 2.2. The wind turbines are being placed on the intersection of grids. The figure above shows one possible solution for the problem of 6x6 grid and 3 wind turbines. The locations of wind turbine are just being represented by x-y location in the grid graph. In the graph above, the locations of three turbines from left to right are (1,1), (2,3), (4,2), since each box is representing 80mx80m in dimension.

2.4. Reduced Size Problem

A reduced size problem can be carried out in a square grid of 3x3 as the wind farm size. Each 1x1 grid has the dimension of 80m x 80m.

2.5. Real Problem

The real size problem is set to be carried out in a square grid of 100x100 as the wind farm size. Each 1x1 grid has the dimension of 80m x 80m.