Three-Dimensional Coordinate System: Solved Problems

A. Points & Distance (5 problems)

№1. Distance between two points

Problem. Find the distance between $P(1, -2, 3)$ and $Q(4, 2, -1)$.

Solution. Compute coordinate differences:

$$\Delta x = 4 - 1 = 3, \qquad \Delta y = 2 - (-2) = 4, \qquad \Delta z = -1 - 3 = -4.$$

Use the 3D distance formula:

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2} = \sqrt{3^2 + 4^2 + (-4)^2} = \sqrt{9 + 16 + 16} = \sqrt{41}.$$

$$\boxed{d = \sqrt{41}}$$

№2. Midpoint of a segment in 3D

Problem. Find the midpoint of the segment joining $A(-2, 5, 1)$ and $B(4, -1, 7)$.

Solution. The midpoint is the coordinatewise average:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right) = \left(\frac{-2 + 4}{2}, \frac{5 + (-1)}{2}, \frac{1 + 7}{2}\right) = (1, 2, 4)\,.$$

$$\boxed{M = (1, 2, 4)}$$

№3. Collinearity of three points (vector multiple test)

Problem. Are the points $A(1, 2, 3)$, $B(3, 6, 7)$, $C(4, 8, 9)$ collinear?

Solution. Form vectors from the same starting point:

$$\overrightarrow{AB} = B - A = (3 - 1,\ 6 - 2,\ 7 - 3) = (2, 4, 4),$$

$$\overrightarrow{AC} = C - A = (4 - 1,\ 8 - 2,\ 9 - 3) = (3, 6, 6).$$

Check if $\overrightarrow{AC}$ is a scalar multiple of $\overrightarrow{AB}$:

$$\frac{3}{2} = 1.5, \qquad \frac{6}{4} = 1.5, \qquad \frac{6}{4} = 1.5.$$

All ratios match, so $\overrightarrow{AC} = \frac{3}{2}\overrightarrow{AB}$, hence the points are collinear.

$$\boxed{\text{Yes, they are collinear.}}$$

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№4. Move along an axis direction

Problem. From $P(2, -1, 4)$ move 6 units in the positive z-direction. Find the new point.

Solution. Moving in $+z$ changes only the z-coordinate:

$$z_{\text{new}} = 4 + 6 = 10.$$

$$\boxed{(2, -1, 10)}$$

№5. Distance from the origin

Problem. Find the distance from the origin to $P(-3, 4, 12)$.

Solution.

$$d = \sqrt{(-3)^2 + 4^2 + 12^2} = \sqrt{9 + 16 + 144} = \sqrt{169} = 13.$$

$$\boxed{13}$$

B. Lines in 3D (5 problems)

№1. Line through two points (parametric equations)

Problem. Find parametric equations for the line through $P(1, 0, 2)$ and $Q(4, -2, 5)$.

Solution. Direction vector:

$$\vec{v} = Q - P = (4 - 1,\ -2 - 0,\ 5 - 2) = (3, -2, 3).$$

Line through P with direction $\vec{v}$:

$$(x, y, z) = (1, 0, 2) + t(3, -2, 3).$$

So

$$x = 1 + 3t, \qquad y = -2t, \qquad z = 2 + 3t.$$

$$\boxed{x = 1 + 3t,\ \ y = -2t,\ \ z = 2 + 3t}$$

№2. Parallel lines (direction vectors)

Problem. Are the lines

$$L_1 : (x, y, z) = (2, 1, 0) + t(4, -2, 6), \qquad L_2 : (x, y, z) = (-1, 3, 5) + s(2, -1, 3)$$

parallel?

Solution. Direction vectors:

$$\vec{v}_1 = (4, -2, 6), \qquad \vec{v}_2 = (2, -1, 3).$$

Check scalar multiple:

$$2\vec{v}_2 = 2(2, -1, 3) = (4, -2, 6) = \vec{v}_1.$$

Hence directions are proportional, so lines are parallel.

$$\boxed{\text{Yes, the lines are parallel.}}$$

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№3. Intersection of a line with a plane

Problem. Find where the line

$$x = 1 + 2t, \quad y = 3 - t, \quad z = 4 + 3t$$

intersects the plane

$$2x - y + z = 10.$$

Solution. Substitute into the plane:

$$2(1 + 2t) - (3 - t) + (4 + 3t) = 10.$$

Simplify:

$$2 + 4t - 3 + t + 4 + 3t = 10 \quad \Rightarrow \quad (2 - 3 + 4) + (4t + t + 3t) = 10$$

$$3 + 8t = 10 \quad \Rightarrow \quad 8t = 7 \quad \Rightarrow \quad t = \frac{7}{8}.$$

Plug back:

$$x = 1 + 2 \cdot \frac{7}{8} = 1 + \frac{7}{4} = \frac{11}{4}, \quad y = 3 - \frac{7}{8} = \frac{17}{8}, \quad z = 4 + 3 \cdot \frac{7}{8} = 4 + \frac{21}{8} = \frac{53}{8}.$$

$$\boxed{\left(\frac{11}{4}, \frac{17}{8}, \frac{53}{8}\right)}$$

№4. Length of a segment (distance again)

Problem. Find the length of the segment from $A(0, 0, 0)$ to $B(2, -3, 6)$.

Solution.

$$d = \sqrt{2^2 + (-3)^2 + 6^2} = \sqrt{4 + 9 + 36} = \sqrt{49} = 7.$$

$$\boxed{7}$$

№5. Evaluate a parametric line at a given parameter

Problem. For the line

$$(x, y, z) = (-2, 1, 5) + t(3, 4, -1),$$

find the point when $t = -2$.

Solution. Substitute $t = -2$:

$$x = -2 + 3(-2) = -8, \qquad y = 1 + 4(-2) = -7, \qquad z = 5 + (-1)(-2) = 7.$$

$$\boxed{(-8, -7, 7)}$$

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C. Planes in 3D (5 problems)

№1. Plane through a point with a normal vector

Problem. Find the plane through $P_0(1,-2,3)$ with normal vector $\vec{n}=(2,1,-4)$.

Solution. Point–normal form:

$$\vec{n}\cdot\big((x,y,z)-(1,-2,3)\big)=0.$$

So

$$(2,1,-4)\cdot(x-1,\ y+2,\ z-3)=0 \Rightarrow 2(x-1)+(y+2)-4(z-3)=0.$$

Expand:

$$2x-2+y+2-4z+12=0 \Rightarrow 2x+y-4z+12=0.$$

$$\boxed{2x+y-4z+12=0}$$

№2. Plane through three points

Problem. Find the plane through $A(1,0,0)$, $B(0,1,0)$, $C(0,0,1)$.

Solution. Two direction vectors in the plane:

$$\overrightarrow{AB}=B-A=(-1,1,0), \qquad \overrightarrow{AC}=C-A=(-1,0,1).$$

A normal vector is $\overrightarrow{AB}\times\overrightarrow{AC}$:

$$\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ -1 & 1 & 0\\ -1 & 0 & 1\end{vmatrix}=(1)\mathbf{i}+(1)\mathbf{j}+(1)\mathbf{k},$$

so $\vec{n}=(1,1,1)$. Use point–normal form with point $A(1,0,0)$:

$$(1,1,1)\cdot(x-1,\ y,\ z)=0 \Rightarrow (x-1)+y+z=0 \Rightarrow x+y+z=1.$$

$$\boxed{x+y+z=1}$$

№3. Check if a point lies on a plane

Problem. Does $P(2,1,3)$ lie on the plane $2x-y+z=6$?

Solution. Substitute:

$$2(2)-1+3=4-1+3=6.$$

The equation is satisfied, so the point lies on the plane.

$$\boxed{\text{Yes.}}$$

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№4. Intersection line of two planes

Problem. Find parametric equations of the line of intersection of

$$x + y + z = 6, \qquad x - y + z = 2.$$

Solution. Subtract the second equation from the first:

$$(x + y + z) - (x - y + z) = 6 - 2 \Rightarrow 2y = 4 \Rightarrow y = 2.$$

Plug into $x - y + z = 2$:

$$x - 2 + z = 2 \Rightarrow x + z = 4.$$

Let $z = t$; then $x = 4 - t$ and $y = 2$.

$$\boxed{x = 4 - t, \;\; y = 2, \;\; z = t}$$

№5. Distance from a point to a plane

Problem. Find the distance from $P(1, 2, 3)$ to the plane $2x - 2y + z - 5 = 0$.

Solution. For $Ax + By + Cz + D = 0$, the distance from $P(x_0, y_0, z_0)$ is

$$d = \frac{|Ax_0 + By_0 + Cz_0 + D|}{\sqrt{A^2 + B^2 + C^2}}.$$

Here $A = 2$, $B = -2$, $C = 1$, $D = -5$. Compute:

$$|2(1) + (-2)(2) + 1(3) - 5| = |2 - 4 + 3 - 5| = | - 4| = 4.$$

Denominator:

$$\sqrt{2^2 + (-2)^2 + 1^2} = \sqrt{4 + 4 + 1} = \sqrt{9} = 3.$$

$$\boxed{d = \frac{4}{3}}$$

D. Spheres in 3D (5 problems)

№1. Sphere from center and radius

Problem. Write the equation of the sphere with center $C(2, -1, 4)$ and radius $r = 3$.

Solution.

$$(x - 2)^2 + (y + 1)^2 + (z - 4)^2 = 3^2 = 9.$$

$$\boxed{(x - 2)^2 + (y + 1)^2 + (z - 4)^2 = 9}$$

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№2. Center and radius from an expanded equation

Problem. Find the center and radius of

$$x^2 + y^2 + z^2 - 4x + 6y - 2z - 11 = 0.$$

Solution. Move the constant term:

$$(x^2 - 4x) + (y^2 + 6y) + (z^2 - 2z) = 11.$$

Complete the squares:

$$x^2 - 4x = (x-2)^2 - 4, \qquad y^2 + 6y = (y+3)^2 - 9, \qquad z^2 - 2z = (z-1)^2 - 1.$$

Substitute:

$$(x-2)^2 - 4 + (y+3)^2 - 9 + (z-1)^2 - 1 = 11$$

$$(x-2)^2 + (y+3)^2 + (z-1)^2 = 11 + 4 + 9 + 1 = 25.$$

Thus center is $(2, -3, 1)$ and radius is $\sqrt{25} = 5$.

$$\boxed{C = (2, -3, 1), \quad r = 5}$$

№3. Point inside/on/outside a sphere

Problem. For the sphere

$$(x-1)^2 + (y+2)^2 + (z-3)^2 = 16,$$

classify the point $P(3, -2, 7)$ as inside, on, or outside.

Solution. Center is $C(1, -2, 3)$ and $r^2 = 16$. Compute squared distance to P:

$$d^2 = (3-1)^2 + (-2+2)^2 + (7-3)^2 = 2^2 + 0^2 + 4^2 = 20.$$

Since $20 > 16$, the point lies outside.

$$\boxed{\text{Outside}}$$

№4. Sphere-plane intersection circle radius

Problem. Sphere: $x^2 + y^2 + z^2 = 25$. Plane: $z = 3$. Find the radius of the circle of intersection.

Solution. Substitute $z = 3$:

$$x^2 + y^2 + 3^2 = 25 \Rightarrow x^2 + y^2 = 16.$$

So the circle radius is $\sqrt{16} = 4$.

$$\boxed{4}$$

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№5. Line–sphere intersection

Problem. Sphere: $x^2 + y^2 + z^2 = 9$. Line: $x = t,\ y = 2t,\ z = 1 - t$. Find the intersection point(s).

Solution. Substitute into the sphere:

$$t^2 + (2t)^2 + (1-t)^2 = 9 \Rightarrow t^2 + 4t^2 + (1 - 2t + t^2) = 9 \Rightarrow 6t^2 - 2t + 1 = 9.$$

Bring to zero:

$$6t^2 - 2t - 8 = 0 \Rightarrow 3t^2 - t - 4 = 0.$$

Quadratic formula:

$$t = \frac{1 \pm \sqrt{1 + 48}}{6} = \frac{1 \pm 7}{6}.$$

So $t = -1$ or $t = \frac{4}{3}$.

Compute points:

$$t = -1 :\ (x, y, z) = (-1, -2, 2),$$
$$t = \frac{4}{3} :\ (x, y, z) = \left(\frac{4}{3}, \frac{8}{3}, -\frac{1}{3}\right).$$

$$\boxed{(-1, -2, 2) \text{ and } \left(\frac{4}{3}, \frac{8}{3}, -\frac{1}{3}\right)}$$

End of solved set.

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