BIOL 196.04 – Ecological Models and Data – TEST #1 – 25 February, 2014

NB: Each subproblem is worth 10 points. There are a total of 120 points.

NB: There is a table of functions and formulas at the end of the test (pg. 4)

NAME: KEY

1. The following data are from a study of Indigo buntings (birds). Researchers found nests in the forest interior and on the forest edge, and recorded whether the eggs in that nest were predated (eaten by other animals) or not.

Year	Habitat type	# predated	# not
2002	Forest	32	3
2002	Edge	18	17
2003	Forest	28	7
2003	Edge	26	9

a. Find the MLE of the proportion of predated nests, using all nests in both years. Report this rate below:

$\hat{p} = 0.743$

b. Find the 95% likelihood profile confidence interval of predation, using all nests in both years. Report this interval below.

0.666, 0.810

c. Find the MLE of the predation rate, using a model with different predation rates on edges and in the forest interior. Report these rates below:

Edge: 0.629

Forest: 0.857

d. Use LRT to compare these two models (one with a single predation rate, and one in which the rate differs among habitat types). Show the χ^2 statistic, degrees of freedom, and P-value. State your conclusion.

$\chi^2 = 9.84$, df = 1, $P = 0.0017$

2-Habitat model is better ($P < 0.05$)

e. Use AIC to compare these two models. Show the AIC for each model, and state your conclusion.

Single rate: AIC = 31.98

Habitat model: AIC = 24.14

Model with lowest AIC "wins" → Habitat model is better

2. Consider a population of orchids with annual survival probability = 0.55, and annual reproduction probability = 0.45. Each orchid that reproduces makes exactly one capsule, that leads to exactly one young plant.
    a. If a population has 4 plants this year, what is the expected number of plants next year?

annual "birth rate" : 0.45 $\quad N p_{births} = 4 \times 0.45 = 1.8$

annual survival = 0.55 $\quad N p_{survive} = 4 \times 0.55 = 2.2$

Expected # = births + survival = 1.8 + 2.2 = 4.0

    b. Use the axioms of probability to calculate the probability that a population of 4 animals goes extinct within one year.

ALL must die AND ALL must not reproduce

$(1-p)^N = 0.45^4 = 0.041 \qquad (1-p)^N = 0.55^4 = 0.091$

$0.041 \times 0.091 = 0.00375$

3. The following data describe fates of 65 plants in a real population of lady's slipper orchids (in Newton, Massachusetts).

Fate in 1986	# of orchids
Flowering	60
Vegetative	4
Dead	1

    a. What is the MLE of the probability of flowering, conditioned on survival?

64 = # trials (living plants)
60 = # events

$\frac{60}{64} = 0.93\cancel{755}$ 0.9375

Flowering probability	Likelihood	Prior Probability	Posterior Probabiity
0.8	0.00156	1/5 = 0.2	0.00486
0.85	0.01873	0.2	0.05837
0.9	0.11418	0.2	0.35582
0.95	0.18295	0.2	0.57013
0.99	0.00348	0.2	0.01083

    b. Consider five possible values for this flowering probability: 0.8, 0.85, 0.9, 0.95, 0.99. Calculate the likelihood of the data for each of these "models", and enter the numbers into the table at left.

5 pts for log-lik

    c. Use Bayes' theorem to calculate the probability of each of these "models", assuming a uniform prior probability (i.e., all 5 values are equally likely, and, for this calculation, no other values are possible). Enter the prior probabilities and posterior probabilities into the table at left.

3 pts priors
7 pts answers
(3 pts if sum to 1, even w/ mistakes)

4. The following data describe whether islands in an archipelago (off the coast of Finland) were occupied by field voles, during a monitoring study from 1972-1977. Islands were divided into 3 size classes: small (< 1 Ha), medium (1-5 Ha) and large (> 5 Ha).

		# of islands	
year	sizeclass	with.voles	no.voles
1972	small	16	24
1972	medium	11	7
1972	large	11	2
1973	small	17	23
1973	medium	14	4
1973	large	11	2
1974	small	28	12
1974	medium	14	4
1974	large	10	3
1975	small	21	19
1975	medium	12	6
1975	large	7	6
1976	small	4	36
1976	medium	14	4
1976	large	10	3
1977	small	19	21
1977	medium	16	2
1977	large	12	1

a. Using data for large islands only, does variation in the proportion of occupied islands seem to be due to binomial sampling variance only? Explain the reasoning for your conclusion.

$\hat{p} = 0.782$

$E(\sigma^2) = N\hat{p}(1-\hat{p}) = 13(0.782)(0.218) = 2.22$

$\sigma_k^2 = \text{var(with.voles)} = 2.97$

I think these are fairly close, but to really know, I suppose you'd need a statistical test...

b. What model would you use to describe island occupancy? Justify your answer using statistics.

Nearly any answer was acceptable, if it involved fitting >1 model to data and comparing them with AIC or LRT.

FOR POSTERITY, Consider the following models:

	# parameters	AIC
"Null": One rate for all islands/years	1	152.7
3 size class (S, M, L)	3	109.2
years differ	6	143.0
2 size class (S/M, L)	2	107.5
every row differs	18	93.2

... The model with the lowest AIC "wins" though there may be even better ones than these 5

Possibly-useful functions and formulas:

$P(x = k \mid N, p) = \binom{N}{k} p^k (1-p)^{N-k}$	$E[x] = Np$
$\hat{p} = \frac{k}{N}$	dbinom(k,N,p, log = TRUE/FALSE)
	logLik(model.name)
$\widehat{\sigma^2} = Np(1-p)$	glm(Y ~ 1, family = binomial(link = "identity"))
$P_A \approx \frac{N_A}{N}$	glm(Y ~ -1 + treatment, family = binomial(link = "identity"))
$P(model \mid data) \frac{P(data \mid model) P(model)}{P(data)}$	confint(model.name)
	cbind(successes, failures)
$AIC_i = -2\log(L_i) + 2k_1$	AIC(model.name)