BIO 133 Homework 2 Solutions

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General feedback

rounding

- don't round until the end
- should feel comfortable using R as a calculator

likelihood

- P(data|model) = L(model|data)
- For a binomial process, this is the output of `dbinom()` OR `logLik(glm(cbind(success, failures) ~ 1, family = binomial(link = "identity")))`
- Bernouli probablities are not the same as binomial probabilities because Bernouli is events in a particular order! Remember that $\binom{N}{k}$ is in the binomial probability equation!
- Likelihood ratio and posterior probability ratio should be the same when priors are uninformative.

1 Probability for two models

```
p <- c("fair" = 0.5, "weighted" = 0.55)
k <- 110
N <- 200

binProb <- dbinom(k, N, p)
binProb
```

```
##       fair   weighted
## 0.02079869 0.05663140
```

2 Calculate posteriors with flat priors

```
priors <- c(0.5, 0.5)
pdata <- sum(binProb * priors)
pmodels <- binProb * priors / pdata
pmodels
```

```
##      fair  weighted
## 0.2686125 0.7313875
```

3 Likelihood ratio

```
binProb[2] / binProb[1]
```

```
## weighted
## 2.722834
```

```
pmodels[2] / pmodels[1]
```

```
## weighted
## 2.722834
```

Model 2 is about 2.7x more likely than model 1

OR

```
binProb[1] / binProb[2]
```

```
##      fair
## 0.3672644
```

LR and probability ratio are the same.

4 What would Nate's prior need to be to make model 1 more likely?

$$P(model_1|data) = \frac{P(data|model_1)P(model_1)}{P(data)}$$

$$P(model_1|data) = \frac{P(data|model_1)P(model_1)}{P(data|model_1)P(model_1) + P(data|model_2)P(model_2)}$$

1. Set P(model1) to x and P(model2) to 1-x
2. Set P(model1 | data) to 0.5
3. Remember that $P(data) = P(data|model_1)x + P(data|model_2)(1 - x)$
4. Solve for x

Nate believes the P(heads) = 0.5

set Nate's prior belief to x

P(model1) = x

P(model2) = 1-x because we are assuming there are only 2 possible models

P(data|model1) = 0.02079869 P(data|model2) = 0.05663140

P(data) = 0.02079869x + 0.05663140(1-x)

0.5 = 0.02079869x/(0.02079869x + 0.05663140(1-x))

0.5 = 0.02079869x/(0.02079869x + 0.05663140 - 0.05663140x)

0.5 = 0.02079869x/0.05663140 - 0.03583271x

$0.5(0.05663140 - 0.03583271x) = 0.02079869x$

$0.0283157 - 0.01791635x = 0.02079869x$

$0.0283157 = 0.03871504x$

$x = 0.7313876$

A numerical solution using a for-loop (for example) would also have been OK