

Errata for:

*Data-driven Modeling of Structured Populations: A  
Practical Guide to the Integral Projection Model*

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**Page 15:** Equation (2.3.6),  $C_1$  should be  $C_0$  as it is in the life-cycle diagram, Figure 2.2.

**Section 2.3:** We think that the statement about “piecewise continuous” in the footnote is true, but at least one proof in the book isn’t valid regardless of what the curves are that divide  $\mathbf{Z}^2$  into subregions. A slightly less general definition, which should be sufficient for any applications, is as follows. For the basic model where the individual-level state space  $\mathbf{Z}$  is a bounded interval  $[L, U]$ , a partition of  $\mathbf{Z}$  is a set of intervals

$$\mathbf{Z}_1 = (z_0, z_1), \mathbf{Z}_2 = (z_1, z_2), \dots, \mathbf{Z}_m = (z_{m-1}, z_m) \quad (1)$$

where

$$L = z_0 < z_1 < z_2 < \dots < z_M = U. \quad (2)$$

A partition breaks  $\mathbf{Z}^2$  into a set of open rectangles

$$\mathbf{Z}_{ij} = \mathbf{Z}_i \times \mathbf{Z}_j = \{(z', z) : z' \in \mathbf{Z}_i, z \in \mathbf{Z}_j\}.$$

Define the kernels  $K_{ij}$  to be  $K$  restricted to  $\mathbf{Z}_{ij}$ .

We say that  $K$  is *piecewise continuous* if there exists a partition such that each of the kernels  $K_{ij}$  is continuous on  $\mathbf{Z}_{ij}$ , and can be defined on the boundary of  $\mathbf{Z}_{ij}$  so that it is continuous on the closed rectangle  $\bar{\mathbf{Z}}_{ij}$  consisting of  $\mathbf{Z}_{ij}$  and its boundary.

The reason this definition works is the general theory in Chapter 6 (originally in the Appendices to ?) applied to the closed intervals  $\bar{\mathbf{Z}}_i = [z_0, z_1]$  as a set of continuous components, with continuous component-to-component kernels  $K_{ij}$ . There are then state distribution functions  $n_i(z, t)$  defined on each  $\bar{\mathbf{Z}}_i$ , and in terms of the general theory, this is an IPM with continuous kernels. But we can also think of it as defining a single distribution function  $n(z, t)$  on all of  $[L, U]$ , consisting of  $n_1, n_2, \dots, n_M$  side-by-side.

Each of the points  $z_i$  is in two adjacent components, but this doesn't matter because single points contribute nothing to the As an example consider  $\mathbf{Z} = [0, 2]$  and the (nonsensical) kernel  $K(z', z) = 1$  if  $z' > 1$ , and 0 otherwise. The partition is  $\mathbf{Z}_1 = (0, 1)$ ,  $\mathbf{Z}_2 = (1, 2)$  and the kernels are

$$K_{11} = K_{12} \equiv 0, K_{21} = K_{22} \equiv 1. \quad (3)$$

The population dynamics are

$$\begin{aligned} n_1(z', t+1) &= \int_0^1 K_{11}(z', z)n_1(z, t)dz + \int_1^2 K_{12}(z', z)n_2(z, t)dz = 0 \\ n_2(z', t+1) &= \int_0^1 K_{21}(z', z)n_1(z, t)dz + \int_1^2 K_{22}(z', z)n_2(z, t)dz \\ &= \int_0^1 n_1(z, t)dz + \int_1^2 n_2(z, t)dz. \end{aligned} \quad (4)$$

So  $n_1(1, t+1) = 0$ , while  $n_2(1, t+1) > 0$  unless there were no individuals at time  $t$ . However, the values of  $n_1$  and  $n_2$  at the one point  $z = 1$  have no effect on the integrals in the population dynamics, so we can regard  $n(1, t+1)$  as being undefined, or give it an arbitrary value such the average  $n_1(1, t+1)$  and  $n_2(1, t+1)$ .

The contrived kernel is (??) a counter-example to the claim in section 6.9 that a piecewise continuous kernel, as defined in that section, maps  $L_1(\mathbf{Z})$  into  $C(\mathbf{Z})$ . The gap in the proof is the assertion that the functions  $f_n$  converge almost everywhere, which is not necessarily true regardless of how the partitioning into sets  $\mathcal{U}_k$  is done. But on the set of domains  $\bar{\mathbf{Z}}_i$  the component kernels are all continuous and the  $f_n$  converge pointwise, which is sufficient for the rest of the proof. In example (??),  $n_1$  is continuous on  $\bar{\mathbf{Z}}_1$  and  $n_2$  is continuous on  $\bar{\mathbf{Z}}_2$ , and that is exactly what it means to be continuous on the state space with domains  $\bar{\mathbf{Z}}_1$  and  $\bar{\mathbf{Z}}_2$ .