# Errata for:
# Data-driven Modeling of Structured Populations: A Practical Guide to the Integral Projection Model

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**Section 2.3:** We think that the statement about "piecewise continuous" in the footnote is true, but at least one proof in the book isn't valid regardless of what the curves are that divide $\mathbf{Z}^2$ into subregions. A slightly less general definition, which should be sufficient for any applications, is as follows. For the basic model where the individual-level state space $\mathbf{Z}$ is a bounded interval $[L, U]$, a partition of $\mathbf{Z}$ is a set of intervals

$$\mathbf{Z}_1 = (z_0, z_1), \mathbf{Z}_2 = (z_1, z_2), \cdots, \mathbf{Z}_m = (z_{m-1}, z_m) \tag{1}$$

where

$$L = z_0 < z_1 < z_2 < \cdots < z_M = U. \tag{2}$$

A partition breaks $\mathbf{Z}^2$ into a set of open rectangles

$$\mathbf{Z}_{ij} = \mathbf{Z}_i \times \mathbf{Z}_j = \{(z', z) : z' \in \mathbf{Z}_i, z \in \mathbf{Z}_j\}.$$

Define the kernels $K_{ij}$ to be $K$ restricted to $\mathbf{Z}_{ij}$.

We say that $K$ is *piecewise continuous* if there exists a partition such that each of the kernels $K_{ij}$ is continuous on $\mathbf{Z}_{ij}$, and can be defined on the boundary of $\mathbf{Z}_{ij}$ so that it is continuous on the closed rectangle $\bar{\mathbf{Z}}_{ij}$ consisting of $\mathbf{Z}_{ij}$ and its boundary.

The reason this definition works is the general theory in Chapter 6 (originally in the Appendices to **?**) applied to the closed intervals $\bar{\mathbf{Z}}_i = [z_0, z_1]$ as a set of continuous components, with continuous component-to-component kernels $K_{ij}$. There are then state distribution functions $n_i(z, t)$ defined on each $\bar{\mathbf{Z}}_i$, and in terms of the general theory, this is an IPM with continuous kernels. But we can also think of it as defining a single distribution function $n(z, t)$ on all of $[L, U]$, consisting of $n_1, n_2, \cdots n_M$ side-by-side.

Each of the points $z_i$ is in two adjacent components, but this doesn't matter because single points contribute nothing to the As an example consider $\mathbf{Z} = [0, 2]$ and the (nonsensical) kernel $K(z', z) = 1$ if $z' > 1$, and 0 otherwise. The partition is $\mathbf{Z}_1 = (0, 1), \mathbf{Z}_2 = (1, 2)$ and the kernels are

$$K_{11} = K_{12} \equiv 0, K_{21} = K_{22} \equiv 1. \tag{3}$$

The population dynamics are

$$\begin{aligned} n_1(z', t+1) &= \int_0^1 K_{11}(z', z) n_1(z, t) dz + \int_1^2 K_{12}(z', z) n_2(z, t) dz = 0 \\ n_2(z', t+1) &= \int_0^1 K_{21}(z', z) n_1(z, t) dz + \int_1^2 K_{22}(z', z) n_2(z, t) dz \\ &= \int_0^1 n_1(z, t) dz + \int_1^2 n_2(z, t) dz. \end{aligned} \tag{4}$$

So $n_1(1, t+1) = 0$, while $n_2(1, t+1) > 0$ unless there were no individuals at time $t$. However, the values of $n_1$ and $n_2$ at the one point $z = 1$ have no effect on the integrals in the population dynamics, so we can regard $n(1, t+1)$ as being undefined, or give it an arbitrary value such the average $n_1(1, t+1)$ and $n_2(1, t+1)$.

The contrived kernel is (4) a counter-example to the claim in section 6.9 that a piecewise continuous kernel, as defined in that section, maps $L_1(\mathbf{Z})$ into $C(\mathbf{Z})$. The gap in the proof is the assertion that the functions $f_n$ converge almost everywhere, which is not necessarily true regardless of how the partitioning into sets $\mathcal{U}_k$ is done. But on the set of domains $\bar{\mathbf{Z}}_i$ the component kernels are all continuous and the $f_n$ converge pointwise, which is sufficient for the rest of the proof. In example (4), $n_1$ is continuous on $\bar{\mathbf{Z}}_1$ and $n_2$ is continuous on $\bar{\mathbf{Z}}_2$, and that is exactly what it means to be continuous on the state space with domains $\bar{\mathbf{Z}}_1$ and $\bar{\mathbf{Z}}_2$.