Discrete Math Homework 2
Due Wednesday, January 25 at the beginning of class

General instructions:

- Use standard size paper (8.5 by 11).
- Answer each question in order using a single column.
- Be neat. If we cannot read your solution it is wrong.
- Show your work. If you just write an answer, you will get minimal credit even if the answer is correct.

Question A) Consider functions f and g. For small positive values of x, which function is larger? For large positive values of x, which function is larger? Find the value of x where the two functions cross each other. You may solve part 4, by trying integer values with a calculator.

1) $f(x) = x^2$, $g(x) = 100$

2) $f(x) = \log x$, $g(x) = 100$

3) $f(x) = 2^x$, $g(x) = 100$

4) $f(x) = x^x$, $g(x) = 100$

Question B) The Baskin-Robbins chain famously claimed that they had 31 flavors of ice cream (one for each day in the month).

1) How many different pairs of ice-cream flavors can we create? We can only use a flavor once in a pair, e.g. (Vanilla, Vanilla) is not a legal pair. The order of flavors in the pair does not matter, e.g. (Chocolate, Vanilla) is the same as (Vanilla, Chocolate) *Hint: Think about creating a grid that lists each flavor on the side and on the top.*
2) Suppose that the number of flavors can change… Lets call it n. Give an exact formula for the number of pairs of flavors.
   $Pairs(n) =$
3) Express the formula from part 2 using Big-Theta notation.

Question C) Suppose that we have a 3 symbol passcode protecting our computer. If there are n possible symbols, the number of passcodes is n^3 and we can express the time to try all the passcodes as $Time(n) = kn^3$.

1) Using technology from 2015, we could try all passcodes with n=10 in 1 hour. Find the value of k.
2) At the start of 2017 we received new technology that is twice as fast. Find the new value of k.
3) Using the new technology, for how big an n can we try all passcodes in an hour?

Question D) Suppose that the time to complete a problem depends on its size.

$$Time(n) = \Theta(2^n)$$

We know that a problem of size 10 requires 3 minutes to solve. Find an approximate formula for Time(n) and then predict how long it will take to solve a problem of

1) size n=11
2) size n=20
3) size n=30

The following questions are over material from Rosen section 1.1. If you want to try other similar problems when you study, note that the odd problems have answers in the back of the book.

Question E) Which of the following sentences are propositions. If the sentence is a proposition, give its truth value and its negation.

1) The name of the planet you live on is Mars.
2) x + x = 2x
3) Go to the store.
4) y = 2 + x
5) A fork is used when eating food.

Question F) Let p and q be the propositions

p: The time is noon.
q: It is warm outside.

Express each of these propositions as an English sentence.

1) $p \vee q$
2) $p \rightarrow \neg q$
3) $\neg p \wedge \neg q$
4) $q \wedge (\neg p \vee q)$
5) $p \leftrightarrow q$

Question G) Rosen 1.1 Exercise 14 a, c, d, e (p. 14)

Question H) Rosen 1.1 Exercise 18 (p. 14)

Question I) Rosen 1.1 Exercise 22 a, c, d, e (p. 14)

You may choose to solve one (and only one) of the following Extra Credit Problems. If you submit more than one, only the first will be graded.

Extra Credit 1) Suppose that we have a coin that has two sentences. On the front of the coin is the sentence: "The statement on the other side is false". On the back of the coin is the sentence: "The statement on the other side is false". Are these two sentences propositions? Explain why or why not.

Extra Credit 2) Rosen 1.2 Exercise 18 (p. 23)

Extra Credit 3) Rosen 1.2 Exercise 26 (p. 23)