Another Example Proof From Class

Show : $\neg(p \wedge q) \to \neg r \equiv (\neg p \to \neg r) \wedge (\neg q \to \neg r)$

p	q	r	$(p \wedge q)$	$\neg(p \wedge q)$	$\neg r$	$\neg(p \wedge q) \to \neg r$
T	T	T	T	F	F	T
T	T	F	T	F	T	T
T	F	T	F	T	F	F
T	F	F	F	T	T	T
F	T	T	F	T	F	F
F	T	F	F	T	T	T
F	F	T	F	T	F	F
F	F	F	F	T	T	T

p	q	r	$\neg p$	$\neg q$	$\neg r$	$(\neg p \to \neg r)$	$(\neg q \to \neg r)$	$(\neg p \to \neg r) \wedge (\neg q \to \neg r)$
T	T	T	F	F	F	T	T	T
T	T	F	F	F	T	T	T	T
T	F	T	F	T	F	T	F	F
T	F	F	F	T	T	T	T	T
F	T	T	T	F	F	F	T	F
F	T	F	T	F	T	T	T	T
F	F	T	T	T	F	F	F	F
F	F	F	T	T	T	T	T	T

Using a truth table we see that these two compound propositions are logically equivalent.

Using a replacement style proof we get

LHS: $\neg(p \wedge q) \to \neg r$

$\equiv \neg\neg(p \wedge q) \vee \neg r$ (by definition of implication)

$\equiv (p \wedge q) \vee \neg r$ (by double negation)

$\equiv \neg r \vee (p \wedge q)$ (by commutative)

$\equiv (\neg r \vee p) \wedge (\neg r \vee q)$ (by distributive)

$\equiv (p \vee \neg r) \wedge (q \vee \neg r)$ (by commutative)

$\equiv (\neg\neg p \vee \neg r) \wedge (\neg\neg q \vee \neg r)$ (by double Negation)

RHS: $\equiv (p \rightarrow \neg r) \wedge (q \rightarrow \neg r)$ (by def of implication)