Discrete Math Homework 5
Due Wednesday, February 15 at the beginning of class

General instructions:

- Use standard size paper (8.5 by 11).
- Answer each question in order using a single column.
- Be neat. If we cannot read your solution it is wrong.
- Show your work. If you just write an answer, you will get minimal credit even if the answer is correct.

Rosen section 1.6.

Question A) Rosen 1.6 Exercise 4 (p. 78)

Question B) Rosen 1.6 Exercise 10 a, c, e (p. 79)

Question C) Rosen 1.6 Exercise 16 (p. 79)

Question D) Rosen 1.6 Exercise 30 (p. 80)

Rosen section 1.7.

Question E) Rosen 1.7 Exercise 6 (p. 91)

Question F) Rosen 1.7 Exercise 12 (p. 91)

Question G) Show that if n is an integer and $n^3 - 7$ is odd, then n is even using a proof by contradiction.

Question H) Rosen 1.7 Exercise 26 (p. 91). This is a bi-conditional, so you must prove both directions.

Question I) Consider our proof that $\sqrt{2}$ is irrational.
Proof:
Any rational number can be written as p/q, where p and q are integers where $q \neq 0$ and p and q share no common factors.

Proof by contradiction:
Assume that $\sqrt{2}$ is rational. This means that we can write

$$\sqrt{2} = \frac{p}{q}$$

where p and q are integers that share no common factors and $q \neq 0$.
Square both sides to get

$$2 = \frac{p^2}{q^2}$$

We can then multiply both sides by q squared

$$2q^2 = p^2$$

Both sides of this equation are integers. If we list out the factors of p squared, 2 must be there. Therefore, we can conclude that 2 is also a factor of p. Thus we can write p as p=2k for some integer k. Replacing p, we get

$$2q^2 = (2k)^2$$

Dividing both side by 2 results in

$$q^2 = 2k^2$$

Again, both sides are integers and we know that 2 must be a factor of q squared. And we can then conclude that 2 is a factor of q.

We have a contradiction because p and q share 2 as a common factor.
Therefore, $\sqrt{2}$ must be irrational.
QED.

We could also prove that $\sqrt{3}$ is irrational by replacing 2 by 3 in our argument above (except in the exponents).

The argument must fail if we try to show that $\sqrt{9}$ is irrational by replacing 2 by 9. At which step does the proof fail?

You may choose to solve one (and only one) of the following Extra Credit Problems. If you submit more than one, only the first will be graded.

Extra Credit 1) Rosen 1.7 Exercise 22 (p. 91)

Extra Credit 2) Consider the following proof that 1=0: Let S be the infinite sum of terms where 1 and -1 alternate

S = 1 + (-1) + 1 + (-1) + 1 + (-1) + …

We can compute this sum by pairing each of the 1's with a -1.

S = **[**1 + (-1) **]** + **[**1 + (-1) **]** + **[**1 + (-1) **]** + …

Which is just

S = 0 + 0 + 0 + …
S = 0

But we could have paired up the values as follows:

S = 1 + **[** (-1) + 1 **]** + **[** (-1) + 1 **]** + **[** (-1) + 1 **]**+ …

Which is just

S = 1 + 0 + 0 + 0 + …

$S = 1$

We can therefore conclude that $1 = 0$.

If this argument is correct, show that we can prove every number x would be equal to every other number y.