Building Blocks Homework 1 Solution

Question A) For each of the following numbers, determine if it is a member of the following sets: **Z**, **N**, $\mathbf{Z^+}$, **R**, **Q, (0, 3]**

Values: 0, 3, $\sqrt{5}$, -3/5, $\sqrt{\frac{1}{9}}$, π

0 is an integer, but it is not positive. Every integer is rational, every rational is real. It is a member of **Z**, **N**, **R**, **Q**
It is not a member of **(0, 3]** since it is open on the left endpoint.

3 is a positive integer. Every integer is rational, every rational is real. It is a member of **Z**, **N**, $\mathbf{Z^+}$, **R**, **Q**
It is a member of **(0, 3]** since it is closed on the right endpoint.

$\sqrt{5}$ is not an integer and it is not rational. It is a member **R**
It is a member of **(0, 3]** since it is between the endpoints.

-3/5 is not an integer. It is rational because it is the ratio of two integer values.
It is a member of **R**, **Q**
It is not a member of **(0, 3]**

$\sqrt{\frac{1}{9}}$ is not an integer. It is rational because it is equivalent to $\frac{\sqrt{1}}{\sqrt{9}}$ which is the ratio of two integer values. It is a member of **R**, **Q.** It is a member of **(0, 3]**

π is not an integer and it is not rational. It is a member **R**
It is not a member of **(0, 3]** .

Question B) Evaluate each of the following expressions in decimal form.

Expressions: 2×3^3, $5 \times 5^4 \times 7^{-2}$,$(2^3)^5$,$9^{\left(\frac{7}{2}\right)}$,$\left(\frac{2}{3}\right)^{-3}$

$2 \times 3^3 = 2 \times 3 \times 3 \times 3 = 54$

$5 \times 5^4 \times 7^{-2} = \frac{5^{1+4}}{7^2} = \frac{5^5}{49}$ (Good enough for an exam.)

By calculator it evaluates to approximately 63.77

$(2^3)^5 = 2^{3\times 5} = 2^{15}$

By calculator it evaluates to 32,768.

$9^{\left(\frac{7}{2}\right)} = 9^{\left(\frac{1}{2}\right)^7} = 3^7$

By calculator it evaluates to 2,187.

$$\left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^3 = \frac{3^3}{2^3} = \frac{27}{8}$$

By calculator it evaluates to 3.375.

Question C) Evaluate each of the following expressions in decimal form.

Expressions: $log_2(2) + log_2(4)$, $log(25)$, $log_5\left(5^{(1+\pi)}\right)$, $\ln(10)$, $\frac{log(5)}{log(10)}$

$log_2(2) + log_2(4) = log_2(2 \times 4) = log_2(2^3) = 3$

Or

$log_2(2) + log_2(4) = log_2(2^1) + log_2(2^2) = 1 + 2 = 3$

Note: Our goal for evaluations with integer arguments is to factor them and look for the base of the log we are using.

$log(25) = log(5^2) = 2log(5)$

By calculator it evaluates to approximately 1.40

$log_5\left(5^{(1+\pi)}\right) = 1 + \pi$

By calculator it evaluates to approximately 4.141

$\ln(10) = \ln(2 \times 5) = \ln(2) + \ln(5)$

By calculator it evaluates to approximately 2.30.

Note: This is one of those cases where leaving it as $\ln(10)$ is about as informative as the equivalent expression.

$\frac{log(5)}{log(10)} = \frac{log(5)}{log(10^1)} = \frac{log(5)}{1} = \log(5)$

By calculator it evaluates to approximately 0.699.

Note: You can not combine into a single log. We need to evaluate each and then divide.

Question D) Evaluate each of the following expressions in decimal form.
Expressions: $\frac{7!}{4!\times 3!}$, $\lfloor ^{1127}/_{10} \rfloor$, $\lfloor ^{-8}/_{3} \rfloor$, $\lfloor -1.2 \rfloor + \lceil -2.2 \rceil$, $\lfloor (n-1)(2n+1) - e \rfloor$
where n is an integer and e is a real value in the range (0, 1).

$\frac{7!}{4!\times 3!} = \frac{7\times6\times5\times4\times3\times2\times1}{4\times3\times2\times1\times3\times2\times1} = \frac{7\times6\times5\times4\times3\times2\times1}{4\times3\times2\times1\times3\times2\times1} = \frac{7\times6\times5}{3\times2\times1} = 7\times5 = 35$

$\lfloor ^{1127}/_{10} \rfloor = \lfloor 112.7 \rfloor = 112$

$\lfloor ^{-8}/_{3} \rfloor = \lfloor ^{-9}/_{3} + {}^{1}/_{3} \rfloor = \lfloor -3 + {}^{1}/_{3} \rfloor = -3 + \lfloor {}^{1}/_{3} \rfloor = -3 + 0 = -3$

$\lfloor -1.2 \rfloor + \lceil -2.2 \rceil = -2 + -2 = -4$

$\lfloor (n-1)(2n+1) - e \rfloor$ where n is an integer and e is a real value in the range (0, 1).

$\lfloor (n-1)(2n+1) - e \rfloor = (n-1)(2n+1) + \lfloor -e \rfloor = (n-1)(2n+1) - 1$

Question E) Compute the value of the following ***arithmetic*** sum ***using Gauss's method*** that we discussed in class.

$$A = 25 + 30 + 35 + 40 + 45 + 50 + 55 + 60 + 65 + 70 + 75$$

Identify first=25, last=75 and # of values=11
Sum is (first+last)(n/2) = (25 +75)(11/2) = 100(11/2) = 50(11) = 550

Question F) Compute the value of the following ***arithmetic*** sum ***using Gauss's method*** that we discussed in class.

$$B = 2\pi + 4\pi + 6\pi + 8\pi + 10\pi + 12\pi + 14\pi + 16\pi$$

Identify first=2π, last=16π and # of values=8
Sum is (first+last)(n/2) = (2π +16π)(8/2) = 16π (4) = 64π
By calculator it evaluates to approximately 201.06

Question G) Compute the value of the following ***geometric*** sum ***using the method*** that we discussed in class (multiply by the common ratio, subtract and solve).

$$G = (5^2) + (5^3) + (5^4) + \cdots + (5^{19})$$

$$5G = (5^3) + (5^4) + (5^5) + \cdots + (5^{20})$$
$$G = (5^2) + (5^3) + (5^4) + \cdots + (5^{19})$$

Subtracting results in all the red terms cancelling

$$5G - G = (5^{20}) - (5^2)$$

$$G = \frac{(5^{20}) - (5^2)}{5\text{-}1}$$

$$G = \frac{(5^{20}) - (5^2)}{4} = \frac{5^2(5^{18}\text{-}1)}{4}$$

By calculator it evaluates to 23,841,857,910,150

Question H) Compute the value of the following ***geometric*** sum ***using the method*** that we discussed in class (multiply by the common ratio, subtract and solve).

$$F = \left(\frac{2}{3}\right) + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \cdots + \left(\frac{2}{3}\right)^{14}$$

$$\left(\frac{2}{3}\right)F = \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \cdots + \left(\frac{2}{3}\right)^{14} + \left(\frac{2}{3}\right)^{15}$$

Subtracting gives

$$\left(\frac{2}{3}\right)F - F = \left(\frac{2}{3}\right)^{15} - \left(\frac{2}{3}\right)$$

$$\left(-\frac{1}{3}\right)F = \left(\frac{2}{3}\right)\left\{\left(\frac{2}{3}\right)^{14} - 1\right\}$$

$$F = \left(\frac{2}{3}\right)\left\{\left(\frac{2}{3}\right)^{14} - 1\right\} \div \left(-\frac{1}{3}\right)$$

Dividing by a fraction is the same as multiplying by the reciprocal

$$F = \left(\frac{2}{3}\right)\left\{\left(\frac{2}{3}\right)^{14} - 1\right\} \times \left(-\frac{3}{1}\right)$$

Canceling gives

$$F = 2\left\{1 - \left(\frac{2}{3}\right)^{14}\right\}$$

By calculator it evaluates to approximately 1.993

You may choose to solve one (and only one) of the following Extra Credit Problems. If you submit more than one, only the first will be graded.

Extra Credit 1) Consider the following summation. While it isn't geometric, we can use the same trick to compute it. Step 1: Multiply by the base and subtract. Step 2: Take the result and solve it.

$$G = 1(3^1) + 2(3^2) + 3(3^3) + \cdots + 15(3^{15})$$

$$3G = 1(3^2) + 2(3^3) + 3(3^4) + \cdots + 15(3^{16})$$

We subtract, but corresponding terms don't cancel exactly, we get

$$G - 3G = 1(3^1) + 1(3^2) + 1(3^3) + \cdots + 1(3^{15}) - 15(3^{16})$$

So we get an arithmetic sequence with an extra term at the end.

$$G - 3G = \frac{3^{16}\text{-}1}{2} - 15(3^{16})$$

$$G = \frac{1\text{-}3^{16}}{4} + \frac{15}{2}(3^{16})$$

Extra Credit 2) Show that the expressions $\lfloor 3x \rfloor$ and $3\lfloor x \rfloor$ are not the same for some values of x.

Consider x=1/2.

$\lfloor 3(1/2) \rfloor = \lfloor (3/2) \rfloor = 1$

$3\lfloor 1/2 \rfloor = 3(0) = 0$

These are different.