

Note on proposition.

Is the following a proposition?

$$3 + x = 5$$

Our answer is going to be no. The reason is that we are going to require that any variable be bound. We can make this a proposition by adding in a specification for x.

$$3 + x = 5 \text{ when } x \text{ is } 2$$

$$3 + x = 5 \text{ when } x \text{ is } 4$$

What about

$$x = x$$

We will still answer no. If x is not bound, it isn't a proposition. Part of the problem here is that when you see that statement, you automatically bind the x and get a true proposition

$$x = x \text{ when } x \text{ is an integer value.}$$

We also have a meaning for = so ingrained in us, that it is hard to think about anything else. Is the following true?

$$x = x \text{ when } x \text{ is my dog}$$

Notes on implication wording in translation from English to Logic.

P is sufficient for Q	P is necessary for Q
If you can show P, then Q is true	If you can show Q, then you can show P
Q if P	Q only if P
$P \rightarrow Q$	$Q \rightarrow P$

Look for the following tags to help determine which side of the implication the proposition is on.

Left side of the implication (premise)

Source of the implications

$$X \rightarrow$$

If X

When X

X is sufficient

Right side of the implication (conclusion)

Target of the implication

$$\rightarrow X$$

only if X

X follows

X is necessary

Another way of stating the implication $P \rightarrow Q$ in English is

$$S = \text{"Q Unless NOT P"}$$

We can explain why this is the case by the following reasoning. If want to know the truth value of S, respond with the truth value of Q, unless P is not true (in which case the response will be true).