Discrete Math Homework 11
Due Wednesday, April 5 at the beginning of class

General instructions:

- Use standard size paper (8.5 by 11).
- Answer each question in order using a single column.
- Be neat. If we cannot read your solution it is wrong.
- Show your work. If you just write an answer, you will get minimal credit even if the answer is correct.

Rosen section 5.4

Question A) Rosen 5.4 Exercise 12 (p. 370)

Devise a recursive algorithm for finding x^n **mod** m whenever n, x, and m are positive integers based on the fact that x^n **mod** m = (x^{n-1} **mod**m · x**mod**m) **mod**m.

```
powerMod(x, n, m)
        if  n is 0 return 1
        else return ( powerMod(x, n-1,m) modm * xmodm) modm
```

Question B) Rosen 5.4 Exercise 46 (p. 371)

How many comparisons are required to merge these pairs of lists using Algorithm 10?

In this algorithm, we compare the smallest values until one of the lists is exhausted. Every one of those values required a comparison. The remaining values are then placed and they do not require a comparison. I will mark in green the remaining values.

a) 1,3,5,7,9; 2,4,6,8,10

1, 2, 3, 4, 5, 6, 7, 8, 9, 10 We needed 9 comparisons

b) 1,2,3,4,5; 6,7,8,9,10

1, 2, 3, 4, 5, 6, 7, 8, 9, 10 We needed 5 comparisons

c) 1,5,6,7,8; 2,3,4,9,10

1, 2, 3, 4, 5, 6, 7, 8, 9, 10 We needed 8 comparisons

The interesting thing is to combine two lists of size n/2 requires a minimum of n/2 and a max of n-1 comparisons. In both cases, the work is O(n).

Rosen section 6.1

Question C) Rosen 6.1 Exercise 4 (p. 396)
A particular brand of shirt comes in 12 colors, has a male version and a female version, and comes in three sizes for each sex. How many different types of this shirt are made?

Task1: Select color	12
Task2: Select gender	2
Task3: Select size	3

Total types is 12x2x3 = 72

Question D) Rosen 6.1 Exercise 8 (p. 396)
How many different three-letter initials with none of the letters repeated can people have?

Task1: Select first initial	26
Task2: Select second initial	25
Task3: Select third initial	24

Total is 26x25x24 (Acceptable on the exam)

Which is 15,600.

This problem can also be solved using permutations and is P(26,3)

Question E) Rosen 6.1 Exercise 12 (p. 396) Hint: You will need to use the sum rule.
How many bit strings are there of length six or less, not counting the empty string?

Count the number for each of the lengths n=1, 2, 3, 4, 5, and 6. Each of these are disjoint so we can just add the results together.

N=6 $\underset{2}{\underline{\quad}}\ \underset{2}{\underline{\quad}}\ \underset{2}{\underline{\quad}}\ \underset{2}{\underline{\quad}}\ \underset{2}{\underline{\quad}}\ \underset{2}{\underline{\quad}}$ gives 2^6

N=5 $\underset{2}{\underline{\quad}}\ \underset{2}{\underline{\quad}}\ \underset{2}{\underline{\quad}}\ \underset{2}{\underline{\quad}}\ \underset{2}{\underline{\quad}}$ gives 2^5

N=4 $\underset{2}{\underline{\quad}}\ \underset{2}{\underline{\quad}}\ \underset{2}{\underline{\quad}}\ \underset{2}{\underline{\quad}}$ gives 2^4

N=3 $\underset{2}{\underline{\quad}}\ \underset{2}{\underline{\quad}}\ \underset{2}{\underline{\quad}}$ gives 2^3

N=2 $\underset{2}{\underline{\quad}}\ \underset{2}{\underline{\quad}}$ gives 2^2

N=1 $\underset{2}{\underline{\quad}}$ gives 2^1

Total is 2 + 4 + 8 + 16 + 32 + 64 =126

Question F) Rosen 6.1 Exercise 16 (p. 396) Hint: You will need to use the sum rule. How many strings are there of four lowercase letters that have the letter x in them?

We will list out the possible ways that we can have an x in a string of length 4. We use ? to indicate one of the remaining 25 letters.

x??? ?x?? ??x? ???x There are 4 of these which is also C(4,1)

For each of these we have three tasks

1) Select first non-x 25
2) Select second non-x 25
3) Select third non-x 25

Total is 4x25x25x25

xx?? x?x? x??x ?xx? ?x?x? ??xx There are 6 of these which is also C(4,2)

For each of these we have two tasks

1) Select first non-x 25
2) Select second non-x 25

Total is 6x25x25

xxx? xx?x x?xx ?xxx There are 4 of these which is also C(4,3)

For each of these we have one task

1) Select first non-x 25

Total is 4x25

xxxx There is 1 of these which is also C(4,4)

Total is 1

Adding these results in 4x25x25x25 + 6x25x25 + 4x25 + 1
Is 2500x25 + 150x25 + 101 (Exam answer)
Is 62500 + 3750 + 101
Is 66351

Question G) Rosen 6.1 Exercise 18 (p. 396)
How many 5-element DNA sequences

a) end with A?

___	___	___	___	A	
4	4	4	4		Total = 4^4 = 256

b) start with T and end with G?

T	___	___	___	G	
	4	4	4		Total = 4^3 = 64

c) contain only A and T?

___	___	___	___	___	
2	2	2	2	2	Total = 2^5 = 32

d) do not contain C?

___	___	___	___	___	
3	3	3	3	3	Total = 3^5 = 243

Question H) Rosen 6.1 Exercise 32 (p. 397)
How many strings of eight uppercase English letters are there

a) if letters can be repeated?

We have 8 slots to fill and 26 choices for each is 26^8= 208,827,064,576

b) if no letter can be repeated?

We have 8 slots to fill and one fewer choice for each is
26x25x24x23x22x21x20x19 = 62,990,928,000

c) that start with X, if letters can be repeated?

We have 7 slots to fill and 26 choices for each is 26^7= 8,031,810,176

d) that start with X, if no letter can be repeated?

We have 7 slots to fill. Since we can not repeat, the letter after X can not be an X and we only have 25 choices. Each one after has one fewer choice
25x24x23x22x21x20x19 = 2,422,728,000

e) that start and end with X, if letters can be repeated?

We have 6 slots to fill and 26 choices for each is 26^6= 308,915,776

f) that start with the letters BO (in that order), if letters can be repeated?

We have 6 slots to fill and 26 choices for each is 26^6= 308,915,776

g) that start and end with the letters BO (in that order), if letters can be repeated?

We have 4 slots to fill and 26 choices for each is 26^4= 456,976

h) that start or end with the letters BO (in that order), if letters can be repeated?

We need to use inclusion/exclusion for this.

#Start BO + #End BO - #(Start and end BO)

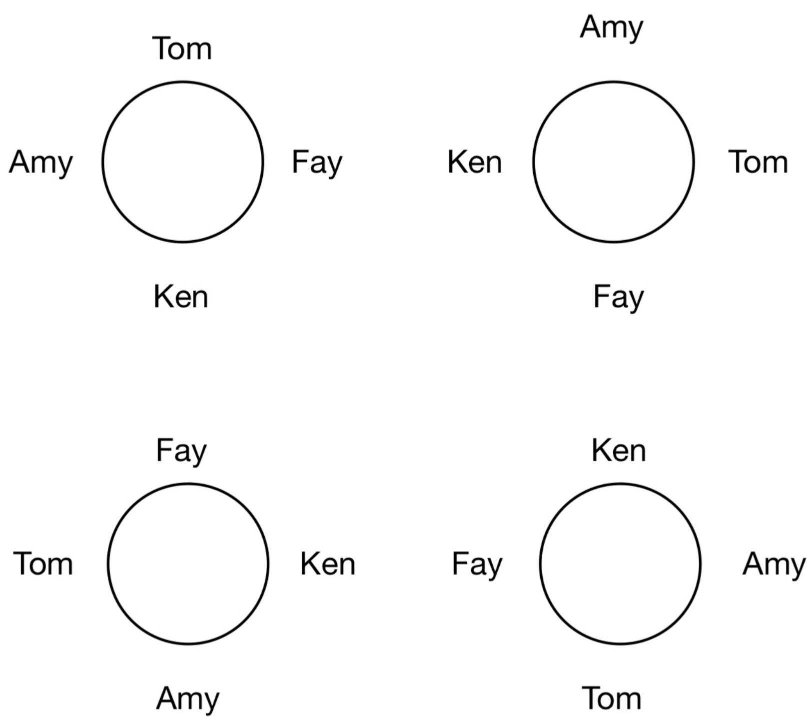
308,915,776 + 308,915,776 – 456,976

= 617,374,576

Question I) Rosen 6.1 Exercise 44 (p. 397)
How many ways are there to seat four of a group of ten people around a circular table where two seatings are considered the same when everyone has the same immediate left and immediate right neighbor?

We label the chairs 1, 2, 3, 4. We select the person for each chair in 10x9x8x7 = 5040.

We need to divide by the number of seatings that are considered the same. Lets consider the seating with Tom, Fay, Ken, and Amy. There are three other seatings that would be the same (see the image.) We need to divided the number of seatings we found by 4 to get 5040 / 4 = 1260.



Question J) Rosen 6.1 Exercise 46 (p. 397)
In how many ways can a photographer at a wedding arrange 6 people in a row from a group of 10 people, where the bride and the groom are among these 10 people, if

a) the bride must be in the picture?

Task 1: Place the Bride	6	(6 places to select from)
Task 2: Select person free slot 1	9	
Task 3: Select person free slot 2	8	
Task 4: Select person free slot 3	7	
Task 5: Select person free slot 4	6	
Task 6: Select person free slot 5	5	

Total is 6x9x8x7x6x5 = 90,720

b) both the bride and groom must be in the picture?

Task 1: Place the Bride	6	
Task 2: Place the Groom	5	(5 places after bride)
Task 3: Select person free slot 1	8	
Task 4: Select person free slot 2	7	
Task 5: Select person free slot 3	6	
Task 6: Select person free slot 4	5	

Total is 6x5x8x7x6x5 = 50,400

c) exactly one of the bride and the groom is in the picture?

Task 1: Decide Bride/Groom	2	
Task 2: Place them	6	
Task 3: Select person free slot 1	8	(8 people not bride/groom)
Task 4: Select person free slot 2	7	
Task 5: Select person free slot 3	6	
Task 6: Select person free slot 4	5	
Task 6: Select person free slot 5	4	

Total is 2x6x5x8x7x6x5x4 = 80,640

We can also compute this by computing the number of arrangements that have the bride and not the groom:

Part A - Part B = 90,720 - 50,400 = 40,320

The number of arrangements that have the groom and not the bride is similarly

90,720 - 50,400 = 40,320

We add these to get 80,640

Question K) Rosen 6.1 Exercise 48 (p. 398)

How many bit strings of length seven either begin with two 0s or end with three 1s?

Begin with two 0s:

```
        0    0    ____   ____   ____   ____   ____

                   2      2      2      2      2     Total = 2^5 = 32
```

End with three 1s:

```
        ____   ____   ____   ____   1      1      1

         2      2      2      2                      Total = 2^4 = 16
```

Begin with two 0s and End with three 1s:

```
        0      0    ____   ____   1      1      1

                     2      2                        Total = 2^2 = 4
```

Use inclusion/exclusion to find the number of bit string that begin with 00 or end with 111:

= 32 + 16 – 4 = 44

Question L) Rosen 6.1 Exercise 55 (p. 398) Suppose that a password for a computer system must have at least 8, but no more than 12, characters, where each character in the password is a lowercase English letter, an uppercase English letter, a digit, or one of the six special characters *, >, <, !, +, and =.

a) How many different passwords are available for this computer system?

Total choices for a character are 26 lower case + 26 upper case + 10 digits + 6 special characters is 68.

Compute the number for each length and then add

N=8 68^8= 457,163,239,653,376

N=9 68^9= 3.108710029642957e16

N=10 68^{10}= 2.113922820157211e18

N=11 68^{11}= 1.437467517706903e20

N=12 68^{12}= 9.774779120406942e21

Total is 9.920671339261326e21

b) How many of these passwords contain at least one occurrence of at least one of the six special characters?

The trick here is not to try to count the passwords that contain special characters, but to count the ones that don't and subtract from the total.

Total choices for a character are 26 lower case + 26 upper case + 10 digits is 62.
Compute the number for each length and then add
N=8 62^8= 218,340,105,584,896
N=9 62^9= 1.353708654626355e16
N=10 62^{10}= 8.392993658683402e17
N=11 62^{11}= 5.20365606838371e19
N=12 62^{12}= 3.2262667623979e21

Total is 9.920671339261326e21 - 3.279156377874257e21 = 6.641514961387068e21

c) Using your answer to part (a), determine how long it takes a hacker to try every possible password, assuming that it takes one nanosecond for a hacker to check each possible password.

There are 10^9 nanoseconds in a second
$9.920671339261326 \times 10^{21} / 10^9$ is $9.920671339261326 \times 10^{12}$ seconds which is approximately 314, 500 years.

You may choose to solve one (and only one) of the following Extra Credit Problems. If you submit more than one, only the first will be graded.

Extra Credit 1) Rosen 5.4 Exercise 48 (p. 371)
What is the least number of comparisons needed to merge any two lists in increasing order into one list in increasing order when the number of elements in the two lists are:

The fastest merge is the one that exhausts the shortest list without using any value from the larger list. That requires one comparison for each value in the shorter list. The slowest merge is one that exhausts a list with just on value remaining in the other list. That requires one comparison less than the total number of values combined.

a) 1, 4? Least is 1 Most is 4

b) 2, 4? Least is 2 Most is 5

c) 3, 4? Least is 3 Most is 6

d) 4, 4? Least is 4 Most is 7

Extra Credit 2) Rosen 6.1 Exercise 66 (p. 398)

Use a tree diagram to find the number of ways that the World Series can occur, where the first team that wins four games out of seven wins the series.