Building Blocks Homework 7
Solution

For each of the following relations explain if it is

- Reflexive, Irreflexive, or neither
- Symmetric
- Antisymmetric
- Transitive
- Partial Order
- Total Order
- Equivalence Relation
- Describe the equivalence classes if it is an equivalence relation.

A) R is the relation over integers where (m,n) is in the relationship if the product of the digits in m is greater than or equal to the product of the digits in n.

EG. m=1223 and n = 111111 is in the relation, but m=119 and n= 2321 is not in the relation. (1223 has product 12 and 111111 has product of 1)

Reflexive (true) – The product of the digits in m is greater than or equal to the product of the digits in m. (It's the same)
Irreflexive(false)
Symmetric (false)
Antisymmetric (false) – We have cases where there is a single arrow and cases with a double arrow. Ex: m=12 and n=21. We have both (12, 21) and (21, 12) in the relation. Ex: m=13 and n=2. We have (13,21) but not (21, 13)
Transitive: This will be true. Each number will have a product of digits which will be transitive under greater than or equal.
Partial Order (false) – Must be antisymmetric.
Total Order (false) – Must be partial.
Equivalence Relation (false) – Must be symmetric.

B) R is the relation over positive integers where (m,n) is in the relation if m is a multiple of n.

EG. m=12 and n=3 is in the relation, but m=12 and n = 5 is not in the relation.

Reflexive (true) – Every positive integer is a multiple of itself.
Irreflexive(false)
Symmetric (false)
Antisymmetric (true) – If a is a multiple of b then we can write a=bk for some integer k. The only way that b could be a multiple of a is if b=aj for some integer j. Combining we have a=bk=(aj)k and therefore that 1=jk. The only way this can happen for integer j and k is if both are 1. In which case a=b.
Transitive: if a is a multiple of b and b is a multiple of c, we know that b=kc and a=jb. So a=jb=jkc and jk is an integer so, a is a multiple of c.
Partial Order (true) – Is reflexive, anti-symmetric and transitive.
Total Order (false) – There are some values that are not multiples of each other. For example, (12, 5) and (5, 12) are both not in the relation..
Equivalence Relation (false) – Must be symmetric.

C) **R is the relation over non-empty sets where (s,t) is in the relation if set s and set t have an empty intersection.**

EG. s={h, o, t} and t ={w, a, r, m } is in the relation, but s={h, o, t} and t ={c, o, l, d} is not in the relation.

Reflexive (false) – The intersection of a set with itself is just the set. The only way that intersection is empty is if the set was the empty set, but that has been ruled out. There are no loops.
Irreflexive(true)
Symmetric (true)
Antisymmetric (false) – If s and t have a non-empty intersection, then t and s will as well.
Transitive: This will be false. Consider s={a,b}, t={c,d} and v={a}. s and t have an empty intersection and are in the relation. T and v also have an empty intersection and will be in the relations. For transitivity to be true, there would have to be a relation of s with v, but their intersection is not empty.
Partial Order (false) – Must be transitive.
Total Order (false) – Must be partial.
Equivalence Relation (false) – Must be transitive.

D) R is the relation over real numbers where (x,y) is in the relation if x is greater than or equal to y squared.

EG. x=5.2 and y=2.0 is in the relation, but x= -10 and y=-2.3 is not in the relation.

Reflexive (false)
Irreflexive(false) – There are cases where x, x is in the relation (x=1/2) and cases where it is not in the relation (x=2)
Symmetric (false)
Antisymmetric (false) – There are cases where both directions are in the relation. Example: x=2 and y=3. Both (2,3) and (3,2) are in the relation. There are cases where only one direction is in the relation. Example: x=2 and y=-1. (2,-1) is in the relation and (-1,2) is not in the relation.
Transitive: This will be false. It is a bit tricky to find a counter example and it really requires values that are between 0 and 1. Here is one set of values: x=1/2, y=7/10, z=8/10.

  x>y^2 is ½ > 49/100 is true.
  y>z^2 is 7/10 > 64/100 is true
  x>z^2 is ½ > 64/100 is false, so not transitive.

Partial Order (false) – Must be antisymmetric.
Total Order (false) – Must be partial.
Equivalence Relation (false) – Must be symmetric.

E) R is the relation over non-empty strings of bits where (s,t) is in the relationship if the first bit in s is the same as the first bit in t and the last bit in s is the same as the last bit in t.

EG. s=1110 and t = 1110110 is in the relation, but s=1001 and t= 1110 is not in the relation.

Reflexive (true)
Irreflexive(false) – for every non-empty bit string the first and last bits must match
Symmetric (true)
Antisymmetric (false) – For every pair of values, the order does not matter. If s and t match in the first bit t and s must match as well.
Transitive: This will be true. If s and t match on the first bit and t and w match on the first bit then s and w match as well. Similarly for the last bits.
Partial Order (false) – Must be antisymmetric.
Total Order (false) – Must be partial.
Equivalence Relation (True)

We have 4 classes:

- **First bit 0, last bit 0**
- **First bit 0, last bit 1**
- **First bit 1, last bit 0**
- **First bit 1, last bit 1**

You may choose to solve one (and only one) of the following Extra Credit Problems. If you submit more than one, only the first will be graded.

Extra Credit 1) Give an example of a relation over the set {0, 1, 2, 3} that is not reflexive, not symmetric, and not transitive

R = { (0,0) (0,1), (1,2), (2,1) }

Not reflexive – missing (1,1) and others
Not symmetric – has (0,1) but not (1,0)
Not transitive – has (0,1) and (1,2) but not (0,2)

The problem did not specify, but this relation is also
Not Irreflexive – has (0,0)
Not AntiSymmetric - has both (1,2) and (2,1)

Extra Credit 2) Suppose that R is a partial order. In the graph representation of R over a finite set of size n, what is the longest chain possible? (A chain will be a list of unique elements where each has an arrow into the next there is no arrow out to another element.)

A chain cannot be any longer than n elements. Can we find a partial order that has length n? In a partial order we can have two elements that are not related to each other… Unfortunately, if that is the case, then one or the other can not be part of the chain. For the longest chain, we need everything to be related and the partial order must be a total order which will form a chain of length n.