Examples:

$\forall x \exists y . x + y > 10$

is true. You give me any x, and I can give you back a y that makes it true.

$\exists x \forall y . x + y > 10$

is false. Suppose there is an x=D that makes this true. Then

$\forall y . D + y > 10$

and we can find a counter example.

Examples:

$\exists x \exists y . x + y = 0$

The truth value depends on the domain of x and y

$\exists x \exists y . x + y = 0$ where x,y are Real

$\exists x \exists y . x + y = 0$ where x,y are Naturals

$\exists x \exists y . x + y = 0$ where x,y are positive Integers

If all my quantifiers are of the same type, the order does not matter.
If the quantifiers are of different types, then the order matters.

Let Likes(x,y) be "x likes y"

$\forall x \exists y . Likes(x, y)$

Means For every x there is a y where x likes y. = Everybody likes somebody

$\exists x \forall y . Likes(x, y)$

Means There is some x where for all y, x likes y = There is somebody that likes everybody.

$\exists y \forall x . Likes(x, y)$

Means There is some y, that every x likes. = There is somebody that everybody likes

$\forall y \exists x . Likes(x, y)$

Means For all y, there is some x that likes y. = Everybody is liked by somebody.

Example:

Everybody has exactly one best friend.

1) people
2) for all people, there is a best friend

3) B(x,y) is "x is the best friend of y"
4) $\forall y \exists x . B(x, y)$ **but this does not guarantee uniqueness.**

Easy solution use !

$\forall y \exists ! x . B(x, y)$

Harder solution: We need to say that and every other person besides x is not the best friend of y.

$\forall y \exists x . \{B(x, y) \wedge \forall z . [(x \neq z) \rightarrow \neg B(z, y)]\}$

Example:

Fred has at least two best friends.

$\exists x \exists y . \{(x \neq y) \wedge B(x, fred) \wedge B(y, fred)\}$

Example:

The product of two negative real numbers is positive.

1) Real numbers
2) Every x,y
3) Positive write as x>0, negative write as x<0
4) $\forall x . \forall y . [(x<0 \wedge y<0) \rightarrow xy>0]$ where x, y are Real

$\forall x . \forall y . [xy>0]$ where x, y are negative Real

Example:

Determine the truth value of

$\forall x . \forall y . \exists z [z=(x+y)/2]$ where x, y, z are Real

is true. For any x and y, (x+y)/2 is Real

Example:

Determine the truth value of

$\forall x . \forall y . \exists z [z=1/(x+y)]$ where x, y, z are Real

is false. Counter example: x=0, y=0 results in

$\exists z [z=1/(0+0)]$ where z is Real

and there is no such z.