Discrete Math Homework 12 Solution

Revised Solution – 4/12/2017

Change in Problem H, part a

General instructions:

- Use standard size paper (8.5 by 11).
- Answer each question in order using a single column.
- Be neat. If we cannot read your solution it is wrong.
- Show your work. If you just write an answer, you will get minimal credit even if the answer is correct.

Rosen section 6.2

Question A) Rosen 6.2 Exercise 4 (p. 405)

A bowl contains 10 red balls and 10 blue balls. A woman selects balls at random without looking at them.

a) How many balls must she select to be sure of having at least three balls of the same color?

This is an example of the generalized pigeonhole principle. We have 2 colors and we will draw x balls. We know that there will be at least Ceiling(x/2) balls with the same color. We want Ceiling(x/2) = 3. The smallest x that works is 5.

b) How many balls must she select to be sure of having at least three blue balls?

This is not an application of pigeon hole principle. To guarantee that we have at least three blue balls, we must pull 13 balls. In the worst case, we get all 10 red balls and then the 3 blue balls.

Question B) Rosen 6.2 Exercise 20 (p. 406)

Find an increasing subsequence of maximal length and a decreasing subsequence of maximal length in the sequence 22, 5, 7, 2, 23, 10, 15, 21, 3, 17.

Increasing Max. Start from the right and find the largest count where your value is less. And 1 to that count.

22,	5,	7,	2,	23,	10,	15,	21,	3,	17
x	x	x	x	x	x	2	1	2	1

So, to find the count for 10 we find the largest count that has a value greater than 10. There are three choices (marked in green). The one with the largest count is the 15, so the 10 will extend that sequence (10, 15, 17)

Continuing we have

22,	5,	7,	2,	23,	10,	15,	21,	3,	17
2	5	4	4	1	3	2	1	2	1

The longest increasing subsequence is 5, 7, 10, 15, 17

Similarly, we build the longest decreasing sequence.

22,	5,	7,	2,	23,	10,	15,	21,	3,	17
3	2	2	1	3	2	2	2	1	1

The longest decreasing subsequence is of length 3 and is not unique. Here is one 22, 7, 2

Question C) Rosen 6.2 Exercise 36 (p. 406)
A computer network consists of six computers. Each computer is directly connected to at least one of the other computers. Show that there are at least two computers in the

This is a pigeonhole principle problem. The number of connections a computer can have is at least 1 and at most 5. These will be our boxes and there are 5 of them. The pigeons will be computers and there are 6 of them. By the PHP there must be at least 2 computers that share the same number of connections.

network that are directly connected to the same number of other computers.

Rosen section 6.3

Question D) Rosen 6.3 Exercise 10 (p. 413)
There are six different candidates for governor of a state. In how many different orders can the names of the candidates be printed on a ballot?

> Order matters, so this is a permutation problem. In this case n=6 and we will select all of them r=6.
> $P(6,6) = 6!$ (Good for the exam)
>
> $P(6,6) = 720$

Question E) Rosen 6.3 Exercise 18 (p. 413)
A coin is flipped eight times where each flip comes up either heads or tails. How many possible outcomes

a) are there in total?

This is a permutation with repetition: $2^8 = 256$

b) contain exactly three heads?

Here we need to choose 3 of the 8 flips that will be a head.

$$C(8,3) = \frac{8!}{5!3!} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = 8 \cdot 7 = 56$$

c) contain at least three heads?

The easiest way to do this is to subtract out the ones we don't want (0 heads, 1 head, 2 heads)

$$C(8,2) = \frac{8!}{6!2!} = \frac{8 \cdot 7}{2 \cdot 1} = 4 \cdot 7 = 28$$

$$C(8,1) = \frac{8!}{7!1!} = \frac{8}{1} = 8$$

$$C(8,0) = \frac{8!}{8!0!} = \frac{1}{1} = 1$$

So the answer is 256 – 28 – 8 -1 = 219

d) contain the same number of heads and tails?

The only way this can happen is if there are exactly 4 heads and 4 tails.

$$C(8,4) = \frac{8!}{4!4!} = \frac{8 \cdot 7 \cdot 6 \cdot 5}{4 \cdot 3 \cdot 2 \cdot 1} = 2 \cdot 7 \cdot 5 = 70$$

Question F) Rosen 6.3 Exercise 23 (p. 414) **How many ways are there for eight men and five women to stand in a line so that no two women stand next to each other? [*Hint:* First position the men and then consider possible positions for the women.]**

We have our men M1 M2 M3 M4 M5 M6 M7 M8
We have exactly 9 places that we could put a woman.
Task 1: Order the men. 8!
Task 2: Choose the positions of the women

$$C(9,5)=\frac{9!}{4!5!}=\frac{9\cdot 8\cdot 7\cdot 6}{4\cdot 3\cdot 2\cdot 1}=3\cdot 7\cdot 6$$

Task 3: Order the women. 5!

The total is 8! x 3 x 7 x 6 x 5!

Rosen section 6.5

Question G) Rosen 6.1 Exercise 4 (p. 432)
Every day a student randomly chooses a sandwich for lunch from a pile of wrapped sandwiches. If there are six kinds of sandwiches, how many different ways are there for the student to choose sandwiches for the seven days of a week if the order in which the sandwiches are chosen matters?

This is a permutation with repetition. n=6 and r=7
$6^7 = 279{,}936$

Question H) Rosen 6.1 Exercise 9 (p. 432)
A bagel shop has onion bagels, poppy seed bagels, egg bagels, salty bagels, pumpernickel bagels, sesame seed bagels, raisin bagels, and plain bagels. How many ways are there to choose

a) six bagels?

This is a combination with repetition. n=8 r=6

I got the values of n and r correct, and then used 5 instead of 6 for r in the formula. The correct application is as shown now.

b) a dozen bagels?

This is a combination with repetition. n=8 r=12

$$C(7+12,12)=\frac{19!}{7!12!}=\frac{19\cdot 18\cdot 17\cdot 16\cdot 15\cdot 14\cdot 13}{7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}=19\cdot 3\cdot 17\cdot 2\cdot 2\cdot 13$$

c) two dozen bagels?

This is a combination with repetition. n=8 r=24

$$C(7+24,24)=\frac{31!}{7!24!}=\frac{31\cdot 30\cdot 29\cdot 28\cdot 27\cdot 26\cdot 25}{7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}=31\cdot 29\cdot 9\cdot 13\cdot 25$$

d) a dozen bagels with at least one of each kind?

This is also a combination with repetition, but we know what 8 of the bagels are. The remaining 4 are a combination with repetition.

This is a combination with repetition. n=8 r=4

$$C(7+4,4)=\frac{11!}{7!4!}=\frac{11\cdot 10\cdot 9\cdot 8}{4\cdot 3\cdot 2\cdot 1}=11\cdot 10\cdot 3$$

e) a dozen bagels with at least three egg bagels and no more than two salty bagels?

Since at least three will be egg bagels, we only have 9 left to select. We compute each salty bagel possibility separately and them add the results. The number we have to select from will include egg and not salty leaving 7.

No salty bagel. N=7 r=9

$$C(6+9,9)=\frac{15!}{6!9!}=\frac{15\times14\times13\times12\times11\times10}{6\times5\times4\times3\times2\times1}=7\times13\times11\times5$$

One salty bagel. N=7 r=8

$$C(6+8,8)=\frac{14!}{6!8!}=\frac{14\times13\times12\times11\times10\times9}{6\times5\times4\times3\times2\times1}=7\times13\times11\times3$$

Two salty bagel. N=7 r=7

$$C(6+7,7)=\frac{13!}{6!7!}=\frac{13\times12\times11\times10\times9\times8}{6\times5\times4\times3\times2\times1}=13\times11\times3\times4$$

Total is 7x13x11x5 + 7x13x11x3 + 13x11x3x4

Question I) Rosen 6.1 Exercise 12 (p. 432)

How many different combinations of pennies, nickels, dimes, quarters, and half dollars can a piggy bank contain if it has 20 coins in it?

This is a combination with repetition. We are selecting from n=5 items. We are selecting r=20 items

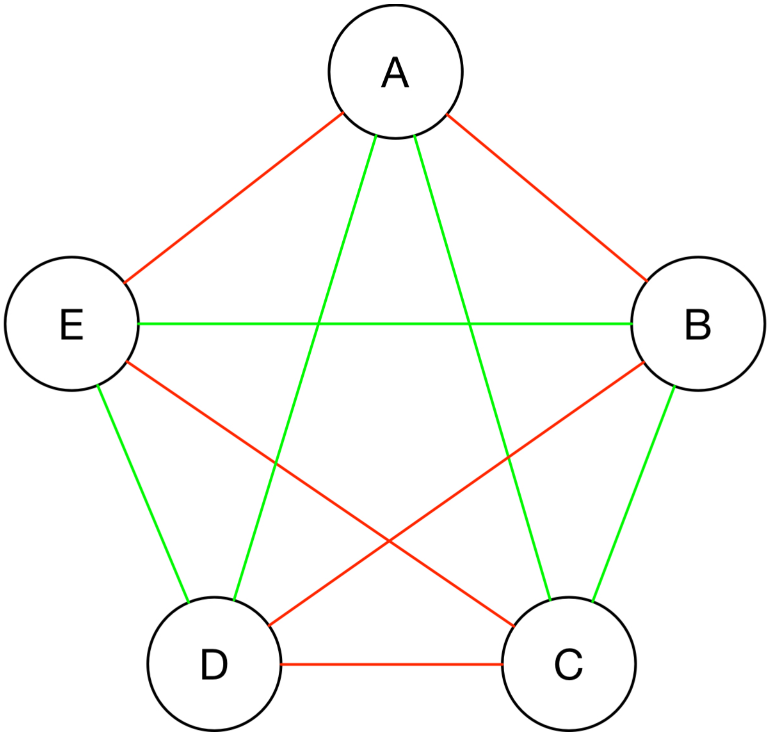
$$C(5-1+20,20)=\frac{24!}{20!4!}=\frac{24\cdot23\cdot22\cdot21}{4\cdot3\cdot2\cdot1}=6\cdot23\cdot11\cdot7=10{,}626$$

You may choose to solve one (and only one) of the following Extra Credit Problems. If you submit more than one, only the first will be graded.

Extra Credit 1) Rosen 6.2 Exercise 26 (p. 406)

Show that in a group of five people (where any two people are either friends or enemies), there are not necessarily three mutual friends or three mutual enemies.

The following graph with green for friends and red for enemies does not have 3 mutual friends or three mutual enemies



Extra Credit 2) Rosen 6.5 Exercise 14 (p. 432)
How many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 = 17$,

where x_1, x_2, x_3, and x_4 are nonnegative integers?

We need to choose 17 items from 4 categories (x1, x2, x3, x4) with repetition.

$$C(4-1+17,17) = \frac{20!}{17!3!} = \frac{20 \cdot 19 \cdot 18}{3 \cdot 2 \cdot 1} = 20 \cdot 19 \cdot 3 = 1{,}140$$