Discrete Math Homework 1
Due Wednesday, January 18 at the beginning of class

General instructions:
- Use standard size paper (8.5 by 11).
- Answer each question in order using a single column.
- Be neat. If we cannot read your solution it is wrong.
- Show your work. If you just write an answer, you will get minimal credit even if the answer is correct.

Question A) Compute the value of the following arithmetic sum using Gauss's method that we discussed in class.

A = 1,000 + 2,000 + 3,000 + 4,000 + …. + 20,000 + 21,000 + 22,000 + 23,000

Question B) Compute the value of the following geometric sum using the method that we discussed in class.

$$G = 1 + 3 + 9 + 27 + \ldots + 3^{40} + 3^{41} + 3^{42}$$

Question C) What are the first 10 values in the following sequence where i starts at 0?

$$a_i = (i-1)(i+1)$$

Question D) Expand the following summation and then compute its value.

$$S = \sum_{i=1}^{5} \frac{1}{i^2}$$

Question E) Give a formula for the arithmetic sequence a_i, that starts at 10 and increases by 5 each time.

Question F) Express the sum of the following geometric sequence using summation notation. Use i as the index variable with a lower limit of one. (*You do not have the evaluate the sum, just write it using summation notation*.)

5, 25, 125, 625, 3125, 15625, 78125, 390625

Question G) Compute the value of the following summation by multiplying out the expression inside the summation and then separating the result. Apply the closed form expressions of the common sequences to get the final answer.

$$\sum_{i=1}^{n} (i+1)^2$$

Question H) Suppose that we have n people sitting in a room numbered from 1 to n. We will award cash to each pair based on the sum of their values. (For example, pair (1, 4) will receive $5.

We will use a double summation to figure out the total amount of cash that will be paid out. The following summation generates the pairs (i, j). [*The limits guarantee that i<j. This means that a person is not allowed to pair with themselves so no (2,2). We also consider that the pair (1,5) is the same as pair(5,1). We will only count one of them.*]

Give a closed form solution for Total.

$$Total = \sum_{j=1}^{n} \sum_{i=1}^{j-1} (i+j)$$

Question I) Decide whether the following are true or false..

a) $2 \bullet 3^2 < 3 \bullet 2^2$
b) $5^3 < 3^5$
c) $3\log(5) = \log(125)$
d) $\log(10) - \log(2) = \log(8)$
e) $\lg(\lg(10^{100})) < 10$ reminder $\lg(x) \equiv \log_2(x)$

Question J) Evaluate the following expressions. No calculator should be needed.

a) $\lfloor 7.5 \rfloor + \lceil 7.5 \rceil$
b) $\lfloor -1.4 \rfloor + \lceil 3 \rceil$
c) $\lfloor -4 \rfloor + \lceil -3 \rceil$
d) $\lfloor n+\varepsilon \rfloor + \lceil n+\varepsilon \rceil$ where n is an integer and $0<\varepsilon<1$
e) $\lfloor n-\varepsilon \rfloor + \lceil n+\varepsilon \rceil$ where n is an integer and $0<\varepsilon<1$

You may choose to solve one (and only one) of the following Extra Credit Problems. If you submit more than one, only the first will be graded.

Extra Credit 1) Suppose we can make our checkerboard larger. In version 1 of the checkerboard problem we increase the number of grains of rice by one in each square. In version 2, we double the number of grains of rice in each square. For both versions, determine how many squares would be required so that the total number of grains of rice will exceed the number of atoms in the universe. (The number of atoms is estimated to be approximately 10^{80}.)

Extra Credit 2) Consider the following summation. Show that the value of the sum converges as n approaches infinity. Hint: Bound the sum by an integral and then show that the integral converges.

$$S_n = \sum_{i=1}^{n} \frac{1}{i^2}$$

Extra Credit 3) Consider the following summation. Evaluate the sum to result in a closed form.

$$T_n = \sum_{i=1}^{n} \log(i)$$