Building Blocks Homework 3
Revision 1.1 Removed Question J
Due Wednesday, September 15 at the beginning of class

General instructions:

- Submit a file containing your solution. You can submit a word document, image, or PDF.
- Answer each question in order using a single column.
- Be neat. If we cannot read your solution it is wrong.
- Show your work. If you just write an answer, you will get minimal credit even if the answer is correct.

Question A)

a) Give a regular expression over the alphabet {0, 1} (bit strings) that represents strings that have 1100 as a substring.

Examples in the language: 0101011000, 1100101, 101100101,
Examples not in the language: the empty string, 1, 010, 001001000 ,
0011101110011000110

b) Give a regular expression that represents the strings that do not have 1010 as a substring. (This is the complement of the regular expression from part a. Any word that is in the expression from a will not be in the expression for b and vice-versa. This language is a little tricky to specify. Think about units that can be repeated that guarantee we don't have the pattern.)

Question B) Which of the following are propositions. Give the truth value of any proposition if known.

a) "10 > 11"
d) "The dinosaurs were wiped out by a meteor."
e) "What is a mollusk?"
b) "x+y = y+x"
c) "There are farms in Kansas."

Question C) How many rows appear in a truth table for each of the compound propositions.

a) $(r \rightarrow p) \vee (p \rightarrow \neg q) \vee (p \rightarrow \neg s)$
b) $(p \vee \neg t) \wedge (p \vee \neg p) \wedge t$
c) $((q \wedge t) \vee (r \wedge \neg t)) \vee (p \wedge u \wedge \neg w) \vee (\neg u \rightarrow v)$
d) $(\neg s \rightarrow \neg t) \vee (p \wedge r \wedge s) \vee (t \wedge s \wedge s)$

Question D) Construct a truth table for the following formula. Identify if the formula is satisfiable.

$(\neg p \rightarrow q) \wedge \neg q$

Question E) Are the following two formulas logically equivalent?

$(r \rightarrow \neg p) \vee \neg q$ and $\neg(\neg p \wedge r) \vee \neg q$

Question F) Are the following two formulas logically equivalent?

$(r \leftrightarrow p) \wedge q$ and $(q \wedge p) \vee ((r \rightarrow q) \wedge p)$

Question G) Is the following formula a tautology, a contradiction, or neither.

$(p \rightarrow \neg q) \rightarrow (q \vee \neg p)$

Question H) What is the truth value of the following prepositional formulae with a universal quantifier. Give examples/counter examples as needed.

a) $\forall x, y \in N. [2xy \neq 3]$
b) $\forall x, \exists y \in R. [2x = y]$
c) $\forall x \in R. [x < 0 \rightarrow x^3 > 0]$
d) $\exists x, \exists y \in N. [x^3 + y^3 = 10]$

Question I) Consider the following prepositional formulae using the predicates
Pet(x) = "x owns a pet" where x is a person
Drives(x) = "x drives a car" where x is a person

a) $\forall x \in S. [Pet(x) \rightarrow Drives(x)]$ where S is the non-empty domain {Fred, Jack}.
   i) Can this quantified statement be true if Fred does not own a pet?
   ii) Can this quantified statement be true if Jack does not drive a car?
   iii) Can this quantified statement be true if no one drives a car??
b) $\forall x \in S. [Pet(x) \wedge Drives(x)]$ where S is the non-empty domain {Fred, Jack}.
   i) Can this quantified statement be true if Fred does not own a pet?
   ii) Can this quantified statement be true if Jack does not drive a car?
   iii) Can this quantified statement be true if no one drives a car??

You may choose to solve one (and only one) of the following Extra Credit Problems. If you submit more than one, only the first will be graded.

Extra Credit 1)

There is an island on which there are three kinds of people: knights who always tell the truth, knaves who always lie, and spies (called normals by Smullyan) who can either lie or tell the truth. You encounter three people, A, B, and C. You know one of these people is a knight, one is a knave, and one is a spy. Each of the three people knows the type of person each of other two is. For the following set of statements, determine whether there is a unique solution and determine who the knave, knight, and spy are. When there is no unique solution, list all possible solutions or state that there are no solutions.

A says “C is the spy”
B says “A is not the spy”
C says “B is the spy”

Extra Credit 2) Consider the statements S =”Statement T is false.” and T= “Statement S is true”. Show that if the S is false, you have a contradiction. Show that if S is true, you have a contradiction.