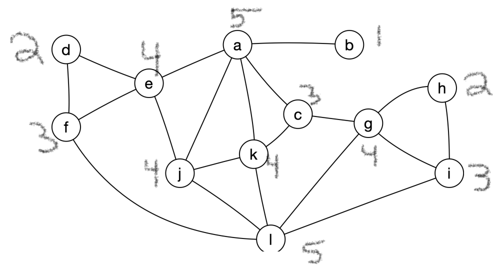
Building Blocks Homework 9
Solution

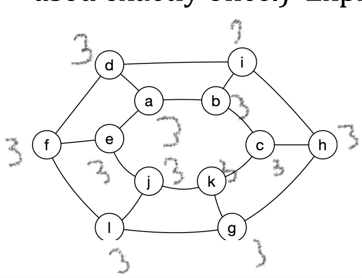
Question A) What are the degrees of the nodes in the following graph?



Question B) Does the previous graph have an Euler circuit? (Definition: A list of nodes where each consecutive pair is connected by an edge, there is an edge from the last to the first node, and every edge is used exactly once) Explain. If so, sketch the circuit. If not, what is the minimum number of edges that must be added so there is an Euler circuit?

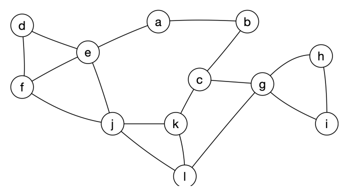
Has odd nodes so no Euler circuit. We have 6 nodes with odd degree. Each edge we add can make two odd nodes even, so we need at least 3 edges.

Question C) Does the following graph have an Euler path? (Definition: A list of nodes where each consecutive pair is connected by an edge and every edge is used exactly once.) Explain. If so, sketch the path.



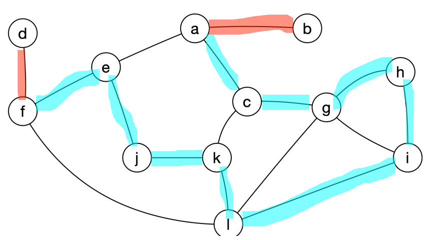
For an Euler path we need zero or two nodes with odd degree. This graph does not, so no Euler path.

Question D) Does the following graph have a Hamiltonian circuit? (Definition: A list of nodes where each consecutive pair is connected by an edge, there is an edge from the last to the first node, edges are used at most once, every node is in the list exactly once.) Explain. If so, sketch the circuit.



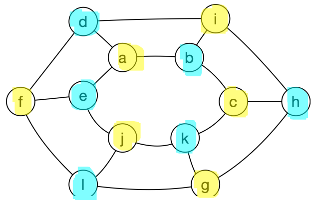
If node G is removed, the graph is disconnected, so no Hamiltonian circuit. (It does have a path though. Path I, H, G, C, B, A, E, D, F, J, L, K is one of them.)

Question E) Does the following graph have a Hamiltonian path? (A list of nodes where each consecutive pair has an edge, edges are used at most once, every node is in the list exactly once.) Explain. If so, sketch the path.



I have marked the disconnecting edges in red. Then fill in the remaining edges of a Hamiltonian path in blue. I started from the ends and did a trial and error process. The first edge I added was FE, and that forced AC. Different choices could have been made.

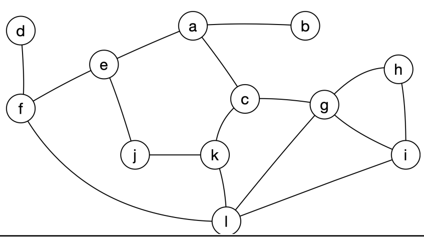
Question F) Is the following graph bipartite? If so, give a partition of the vertices. If not, show that the graph is not 2-colorable.



This graph is Bipartite as demonstrated by the 2-coloring. The partition is {D, H, L, B, K, E} and {I, G, F, A, C, J}

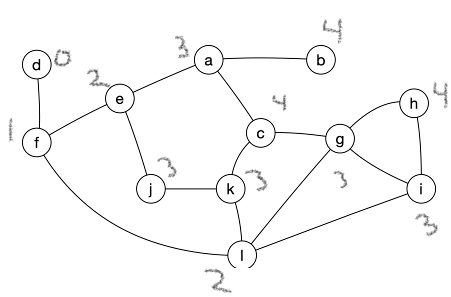
You may choose to solve one (and only one) of the following Extra Credit Problems. If you submit more than one, only the first will be graded.

Extra Credit 1)
Suppose that we have a simple connected graph G (no multi-edges or loops). Between every pair of nodes we can find the length of the shortest path. The diameter of the graph is the largest of these values. What is the diameter of the following graph?

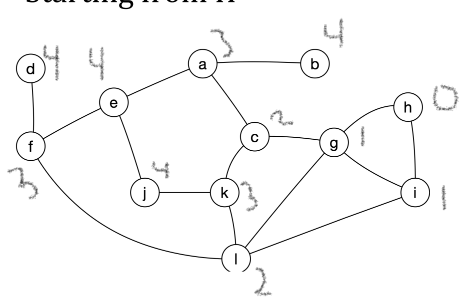


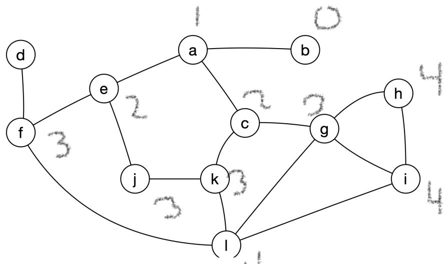
We can start a node labeling process from any given node. Label start as 0. Any neighbor not already labeled gets a 1, and so forth. My expectation is that the longest shortest path will start at an edge node like D, B or H.

Lets start from D.



Starting from H





With a little work, we can show that the interior points can not give us something longer. The answer is four.

Extra Credit 2)

If we have a simple graph we can show that the maximum number of tours (path that visits each node exactly once) is n!. Consider a connected graph where each node has degree 4. What is the maximum number of tours?

We need a list of n nodes. Specifying the path is a sequence of n choices.

Choice 1: Select start	n options	
Choice 2: Select next	4 options	(degree is 4)
Choice 3: Select next	3 options	(Can not go back)
Choice 4: Select next	3 options	(At most)
…		
Choice n-1: Select next	2 options	(At most)
Choice n: Select next	1 options	(At most)

The deeper we go, the more likely that some of the edges lead to nodes we have see before. For the last choice, there can only be one node available if there is an edge to it. Multiply them together and we get $n \times 4 \times 3^{n-4} \times 2$