Discrete Math Homework 2 Solution
Revised 1/27/2017 – Mistake in G part d, e

Question A) Consider functions f and g. For small positive values of x, which function is larger? For large positive values of x, which function is larger? Find the value of x where the two functions cross each other. You may solve part 4, by trying integer values with a calculator.

1) $$f(x) = x^2$$
$$g(x) = 100$$

g(x) is larger for small positive values

f(x) is larger for large positive value

We solve

$$x^2 = 100$$

to find the crossover at x=10.

2) $$f(x) = \log x$$
$$g(x) = 100$$

g(x) is larger for small positive values

f(x) is larger for large positive value

We solve

$$\log x = 100$$
$$10^{\log x} = 10^{100}$$
$$x = 10^{100}$$

to find the crossover

3) $$f(x) = 2^x$$ $$g(x) = 100$$

g(x) is larger for small positive values

f(x) is larger for large positive value

We solve

$$2^x = 100$$
$$\lg 2^x = \lg 100$$
$$x = \lg 100 \approx 6.64$$

to find the crossover .

4) $$f(x) = x^x$$ $$g(x) = 100$$

g(x) is larger for small positive values

f(x) is larger for large positive value

We solve

$$x^x = 100$$
$$1^1 = 1 < 100$$
$$2^2 = 4 < 100$$
$$3^3 = 27 < 100$$
$$4^4 = 256 > 100$$

to find the crossover at approximately 3 and a half.

Question B) The Baskin-Robbins chain famously claimed that they had 31 flavors of ice cream (one for each day in the month).

1) How many different pairs of ice-cream flavors can we create? We can only use a flavor once in a pair, e.g. (Vanilla, Vanilla) is not a legal pair. The order of flavors in the pair does not matter, e.g. (Chocolate, Vanilla) is the same as (Vanilla, Chocolate) *Hint: Think about creating a grid that lists each flavor on the side and on the top.*

	Choc	Van	CC	Straw	…
Choc	x	x	x	x	
Van	count	x	x	x	
CC	Count	count	x	x	
Straw	Count	Count	Count	x	
…					

If we total up the count we get

$$1+2+3+\cdots+29+30=(1+30)\frac{30}{2}=465$$

2) Suppose that the number of flavors can change… Lets call it n. Give an exact formula for the number of pairs of flavors.

$$Pairs(n)=$$

Generalizing from the picture above we get the sum

$$1+2+3+\cdots+(n-2)+(n-1)=(1+n-1)\frac{n-1}{2}$$

$$=(n)\frac{n-1}{2}=\frac{n^2}{2}-\frac{n}{2}$$

3) Express the formula from part 2 using Big-Theta notation.

We can drop the lower order term (n/2) and ignore the constant. We write

$$\frac{n^2}{2}-\frac{n}{2}=\Theta\left(n^2\right)$$

Question C) Suppose that we have a 3 symbol passcode protecting our computer. If there are n possible symbols, the number of passcodes is n^3 and we can express the time to try all the passcodes as $Time(n)=kn^3$.

1) Using technology from 2015, we could try all passcodes with n=10 in 1 hour. Find the value of k.

We put the known values into the equation and solve for k.

$$1=k(10)^3$$

or

$$k=\frac{1}{10^3}$$

2) At the start of 2017 we received new technology that is twice as fast. Find the new value of k.

The new value of k (call it k') will be one half the old value.

$$k' = \frac{1}{2}k = \frac{1}{2}\cdot\frac{1}{10^3}$$

3) Using the new technology, for how big an n can we try all passcodes in an hour?

The new formula is

$$Time'(n) = k'n^3 = \frac{1}{2}\cdot\frac{1}{10^3}n^3$$

We will solve this equation for n given that the time is 1 hour.

$$1 = \frac{1}{2}\cdot\frac{1}{10^3}n^3$$

$$n^3 = 2\cdot 10^3$$

$$n = \sqrt[3]{2\cdot 10^3} = 10\cdot\sqrt[3]{2} \approx 10\cdot 1.26 \approx 126$$

If we have growth that is cubic, doubling the speed of the computation allows us to solve a problem that is approximately 26% larger.

Question D) Suppose that the time to complete a problem depends on its size.

$$Time(n) = \Theta(2^n)$$

We know that a problem of size 10 requires 3 minutes to solve. Find an approximate formula for Time(n) and then predict how long it will take to solve a problem of

1) size n=11
2) size n=20
3) size n=30

First, write the time as proportional to the growth

$$Time(n) \approx k2^n$$

Plug in the known values and solve for k

$$3 \approx k2^{10}$$

$$k \approx \frac{3}{2^{10}}$$

We now have a formula for the time

$$Time(n) \approx \frac{3}{2^{10}} 2^n = 3 \cdot 2^{n-10}$$

Now we can plug in n=11, 20, 30

$$Time(11) \approx 3 \cdot 2^{11-10} = 3 \cdot 2^1 = 6$$

$$Time(20) \approx 3 \cdot 2^{20-10} = 3 \cdot 2^{10} = 3072$$

$$Time(30) \approx 3 \cdot 2^{30-10} = 3 \cdot 2^{20} = 3145728$$

Each time we add 10 to the size, the time goes up by a factor of about 1000.

The following questions are over material from Rosen section 1.1. If you want to try other similar problems when you study, note that the odd problems have answers in the back of the book.

Question E) Which of the following sentences are propositions. If the sentence is a proposition, give its truth value and its negation.

1) The name of the planet you live on is Mars.
2) x + x = 2x
3) Go to the store.
4) y = 2 + x
5) A fork is used when eating food.

1) Proposition, False, The name of the planet you live on is not Mars
2) Not Proposition, because the variable x is not bound. (But on the edge. Can easily see someone making the claim that this is a true proposition.)
3) Not a proposition – not declarative
4) Not a proposition because the variables x and y are not bound.
5) Proposition. True, A fork is not used when eating food. (On the edge. Can easily see someone making the claim that this is a declarative statement, but the truth value is not determined.)

Question F) Let p and q be the propositions

p: The time is noon.

q: It is warm outside.

Express each of these propositions as an English sentence.

1) $p \vee q$
2) $p \to \neg q$
3) $\neg p \wedge \neg q$
4) $q \wedge (\neg p \vee q)$
5) $p \leftrightarrow q$

> 1) The time is noon or it is warm outside
> 2) If the time is noon then it is not warm outside.
> 3) The time is not noon and it is not warm outside
> 4) It is warm outside and either the time is not noon or it is warm outside
> 5) The time is noon if and only if it is warm outside.

Question G) Rosen 1.1 Exercise 14 a, c, d, e (p. 14)

> P: You get an A on the final exam
> Q: You do every exercise in this book
> R: You get an A in this class
>
> a) Clearly, this statement uses R and NOT Q. There is no implication. It must combine with AND: $R \wedge \neg Q$
>
> c) This statement uses R and P. It has the form P is necessary for R. Which is the same as R implies P: $R \to P$
>
> d) This statement uses all three. There is no implication. It combines with ANDs: [redacted] (Should be R… the nevertheless indicates something that happened.)
>
> e) Has the form P AND Q is sufficient for R: [redacted]

Question H) Rosen 1.1 Exercise 18 (p. 14)

a) if 1+1=3 then unicorns exist	$F \to T$ is T
b) if 1+1=3 then dogs can fly	$F \to F$ is T
c) if 1+1=2 then dogs can fly	$T \to F$ is F
d) if 2+2=4 then 1+2=3	$T \to T$ is T

The premise is marked in color in the original sentence.

a) "It is necessary to wash the boss's car to get promoted."
If you get promoted, then you wash the boss's car.

c) "A sufficient condition for the warranty to be good is that you bought the computer less than a year ago."
If you bought the computer less than a year ago, then the warranty is good.

d) "Wily gets caught whenever he cheats."
If Wily cheats, then he gets caught.

e) "You can access the website only if you pay a subscription fee."
If you can access the website, then you pay a subscription fee.

Question I) Rosen 1.1 Exercise 22 a, c, d, e (p. 14)

You may choose to solve one (and only one) of the following Extra Credit Problems. If you submit more than one, only the first will be graded.

Extra Credit 1) Suppose that we have a coin that has two sentences. On the front of the coin is the sentence: "The statement on the other side is false". On the back of the coin is the sentence: "The statement on the other side is false". Are these two sentences propositions? Explain why or why not.

These sentences can be propositions.

S1 = "S2 is false"
S2 = "S1 is false"

They are declarative, and we can find truth values that will make them consistent. If one is true and the other is false,

Any time we have statements that can reference each other we potentially have a problem.

Example: T: This statement is false.

If T is true, then according to the proposition T is false and we have a contradiction. If T is false, then according to the proposition T is true and we have a contradiction. T can neither be true or false and is not proposition.

Extra Credit 2) Rosen 1.2 Exercise 18 (p. 23)

We have some statements:

1: "If Jasmine attends, she will be unhappy if Samir is there."
2: "Samir will attend only if Kanti will be there."
3: "Kanti will not attend unless Jasmine also does.

We have three propositional variables that we will give somewhat arbitrary names to. We also will assume that a person does not attend if it will make them unhappy.

J is "Jasmine attends"
S is "Samir attends"
K is "Kanti attends"

Now we translate each of the sentences into a compound proposition.

1: Jasmine will be unhappy if Samir is there
$S \to \neg J$
2: $\neg K \to \neg S$ which is the same as $S \to K$
2: $\neg J \to \neg K$ which is the same as $K \to J$

Now we construct a truth table and fill in the truth values

S	J	K	$\neg J$	$S \to \neg J$	$S \to K$	$K \to J$	AND
T	T	T	F	F	T	T	
T	T	F	F	F	F	T	
T	F	T	T	T	T	F	
T	F	F	T	T	F	T	
F	T	T	F	T	T	T	T
F	T	F	F	T	T	T	T
F	F	T	T	T	T	F	
F	F	F	T	T	T	T	T

So there are three possibilities

First: Invite Jasmine and Kanti, but not Samir

Second: Invite just Jasmine

Third: Invite none of them.

Extra Credit 3) Rosen 1.2 Exercise 26 (p. 23)

Statements:

A says "I am the knave"
B says "I am the knave",
C says "I am the knave"

One of them is the knave = always lies.
One of them is the knight = always tells the truth
One of them is the spy = Can lie or tell the truth.

The knight cannot say "I am the knave" because that would be a lie.
The knave cannot say "I am the knave" because that would be the truth.
The spy can say "I am the knave" which would be a lie.

The only possible solution consistent with the three statements would be all three are spies, but we are given that there is one of each kind, so **there are no solutions**.