Three ways to show that P and Q are logically equivalent

1) Construct a truth table and show that in every line of P and Q that the truth values are the same (Proof by cases)
2) Construct a truth table and show that $P \leftrightarrow Q$ is a tautology.
3) Apply known logical equivalence to transform both P and Q into X.

DeMorgan's Law $\neg(p \vee q) \equiv \neg p \wedge \neg q$

P or Q is only false when both P and Q are false. Not enough to hand-wave, show with truth table. Values in the yellow columns are the same so they are logically equivalent:

P	Q	$(P \vee Q)$	$\neg(P \vee Q)$	$\neg P$	$\neg Q$	$\neg P \wedge \neg Q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

You should have these logical equivalences memorized. If the name is marked in green, you should know that as well.

Double Negation	$\neg\neg p \equiv p$	
Identity	$p \wedge True \equiv p$	$p \vee False \equiv p$
Domination	$p \wedge False \equiv False$	$p \vee True \equiv True$
Negation	$p \wedge \neg p \equiv False$	$p \vee \neg p \equiv True$
Idempotent	$p \wedge p \equiv p$	$p \vee p \equiv p$
Commutative	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
Associative	$(p \wedge q) \wedge r \equiv q \wedge (p \wedge r)$	$(p \vee q) \vee r \equiv q \vee (p \vee r)$
Distributive	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
De Morgan's	$\neg(p \wedge q) \equiv \neg p \vee \neg q$	$\neg(p \vee q) \equiv \neg p \wedge \neg q$

Conditional	$p \to q \equiv \neg p \vee q$
	$p \to q \equiv \neg q \to \neg p$
Bi-Conditional	$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
	$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

There are more equivalences in Tables 7 and 8 on page 28, but you don't strictly need them. You can always get replace the conditional with a formula that uses and/or/not and work from there. You should review those equivalences, as they might be helpful.

Example Proof 1 : $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$

We could use a truth table, but lets use a direct proof. Can work on one side or the other or both

LHS: $(p \to q) \wedge (p \to r)$

$\equiv (\neg p \vee q) \wedge (\neg p \vee r)$ (by definition of implication)

$\equiv (\neg p \vee q) \wedge (\neg p \vee r)$ (by definition of implication)

RHS: $p \to (q \wedge r)$

$\equiv (\neg p \vee (q \wedge r))$ (by definition of implication)

$\equiv (\neg p \vee q) \wedge (\neg p \vee r)$ (by distributive)

Example Proof 2: $\neg(p \to q) \equiv p \wedge \neg q$

LHS: $\neg(p \to q)$

$\equiv \neg(\neg p \vee q)$ (by definition of implication)

$\equiv (\neg\neg p \wedge \neg q)$ (by De Morgan's)

$\equiv (p \wedge \neg q)$ (by Double negation)

RHS: $p \wedge \neg q$

Satisfiability: Can I find some assignment of truth values to propositional variables that will make a proposition true?

Example

P: Paul has \$100
R: Ron has \$100
S: Sam has \$100
T: Ted has \$100

1) If Paul has \$100 then Ron has \$100 $P \rightarrow R$
2) Sam or Ted has \$100 $S \vee T$
3) If Ron has \$100 then Sam does not have \$100 $R \rightarrow \neg S$
4) Paul has \$100 iff Ted has \$100 $P \leftrightarrow T$

P	R	S	T	$P \rightarrow R$	$S \vee T$	$R \rightarrow \neg S$	$P \leftrightarrow T$	AND
T	T	T	T	T	T	F	T	
T	T	T	F	T	T	F	F	
T	T	F	T	T	T	T	T	T
T	T	F	F	T	F	T	F	
T	F	T	T	F	T	T	T	
T	F	T	F	F	T	T	F	
T	F	F	T	F	T	T	T	
T	F	F	F	F	F	T	F	
F	T	T	T	T	T	F	F	
F	T	T	F	T	T	F	T	
F	T	F	T	T	T	T	F	
F	T	F	F	T	F	T	T	
F	F	T	T	T	T	T	F	
F	F	T	F	T	T	T	T	T
F	F	F	T	T	T	T	F	
F	F	F	F	T	F	T	T	

(Blue markings just delineate chunks of four possibilities of our propositional variables.)

So this problem has two possible solutions
1) Paul, Ron and Ted have \$100 and Sam does not.
2) Paul, Ron and Ted do not have \$100 and Sam does

If we add the proposition: Sam does not have \$100, we end up with a single solution.