Some basic cases—Let the problem definition guide you

Odd/Even
Remainder is x when divided by M
Pos/Neg/Zero

Example:
Show for all a, b that are real

$|a+b| \leq |a|+|b|$

Cases:

1. a=0, b anything
2. a anything, b=0
3. a>0, b>0 pos/pos
4. a>0, b<0 pos/neg
5. a<0, b>0 neg/pos
6. a<0, b<0 neg/neg

1) Show $|0+b| \leq |0|+|b|$

$|b| \leq 0+|b|$

$|b| \leq |b|$ **Check**

2) Similar to 1. **Check**

3) a = A, |a| = A, b = B, |b| = B (A, B are positive)

Show $|a+b| \leq |a|+|b|$

$|A+B| \leq A+B$

and since A and B are both positive

$A+B \leq A+B$ **Check**

6) a = -A, |a| = A, b = -B, |b| = B (A, B are positive)

Show $|a+b| \leq |a|+|b|$

$|-A+-B| \leq A+B$

$|-(A+B)| \leq A+B$

$|(A+B)| \leq A+B$

$A+B \leq A+B$ **Check**

5) a = -A, |a| = A, b = B, |b| = B (A, B are positive)

Show $|a+b| \leq |a|+|b|$

$|-A+B| \leq A+B$

3 Subcases:

A) A < B

B) A = B
C) A > B

5B) A = B

$$|-B+B| \le B+B$$

$$|0| \le 2B$$

$$0 \le 2B$$

But B is positive by case
Check

5A) A < B
Write B = A + d where d is positive

$$|-A+(A+d)| \le A+(A+d)$$

$$|d| \le 2A+d$$

$$d \le 2A+d$$

$$0 \le 2A$$

But A is positive by case
Check

5C) A > B
Write A = B + d where d is positive

$$|-(B+d)+B| \le (B+d)+B$$

$$|d| \le 2B+d$$

$$d \le 2B+d$$

$$0 \le 2B$$

But B is positive by case
Check

4) This is similar to case 5 with the roles of a and b swapped. This is legitimate because addition is commutative. **Check**

All cases have been covered
QED.