

Solving the Traveling Salesman Problem with Genetic Hill Climbing

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Abstract

Mathematical understanding and application of the traveling salesman problem is useful for businesses and logistical applications however, mathematicians and planners have struggled with this problem and its applications for centuries. Currently there are conventional as well innovative approaches to the traveling salesman that use the highest level of technology. Despite this, there is still no optimal approach or best solution. In this paper we propose a method to apply to the traveling salesman by utilizing a combination of a hill climber with aspects of genetic algorithm design to alleviate some of the issues typically found in a traditional hill climber.

Keywords: Algorithms, Hill Climbing, Traveling Salesman, Genetic Algorithms, NP-Hard, Exhaustive Search, Branch and Bound, Heuristic

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1 Introduction

The history of the Traveling Salesman problem is unclear, however since its inception it has been a problem that has continued to puzzle humankind even until this day. The problem builds from finding a Hamiltonian cycle which can be defined as a path in an undirected graph where each vertex is visited exactly once and then returning to the start. Thus, the start vertex is visited twice, once in the beginning

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and lastly once at the end. It further pushes the complication of these tasks by now asking of the many possible paths that exist, which path provides the shortest length of the circuit. This type of problem is critical to not only salesman which the name of the problem comes from, but for many modern-day operations and businesses ranging from delivery trucks to flight paths for planes and even including agricultural machinery operation (Liao [2]). A great example would be a delivery truck that picks up boxes at the start of the day, where the truck then needs to route to every destination in the most optimal manner to save time and gas before returning to the warehouse at the end of the day.

There have been many attempts at solving the Traveling Salesman problem ranging from exact solutions which only work for small problem sizes to heuristic models that give approximations. Once the problem enlarges with many cities these outdated exact solutions are not capable of producing an answer. Here is where suboptimal heuristic algorithms come into play. In this paper, we propose a method to combine two individual heuristic algorithms, Hill Climbing and Genetic algorithm (Harik et al. [1]), to create a Genetic Hill Climbing algorithm that generates an approximate solution to the Traveling Salesman problem within a reasonable time. This type of solution to the traveling salesman is not novel (Yuret [6]), (Wang and Xu [5]). The intuition for this approach is to combine the aspects of each individual algorithm to complement their respective weaknesses. On top of these heuristic approaches to the traveling salesman, there are others that use more new and innovative approaches. As seen by Mele et al. [3], they have been able to obtain a time complexity of $O(n^2 \log n^2)$ using the latest in machine learning. Ren et al. [4] also takes inspiration from biological fields to innovate with a time complexity of $O(n^2)$. These time operations are much quicker than traditional approaches.

2 Problem Definition

The Traveling Salesman problem is modelled as an undirected weighted graph where the vertices represent cities while the graph's edges represent the paths. The distances of the paths are then represented as the weights of the edges. There are no limitations in how the graph is designed other than the requirement that there must be a Hamiltonian circuit. Graphs that are complete, meaning there are paths to

every node from any single node provides a larger complexity given that there are more paths and edge weights to consider. This generates the maximum number of possible paths thus finding the shortest provides a difficult task. Realistically this does not exist as it would be nearly impossible to find a road from one city to another without passing through another city or in this case a vertex or without backtracking and visiting another city or vertex.

Symmetrical Traveling Salesman problems are situations in the graph design where the distances between two cities are the same in each opposite direction. This is well represented in a road path where the route from once city to another and back is simply just the opposite way but same length. This is not to be confused with a graph that is connected but not complete. It is still possible for the Traveling Salesman problem to exist where there are cities without direct paths to other cities that satisfies the condition of the problem. Asymmetrical graph design can include paths that are different weights from one city to another in comparison to the opposite direction. In addition to this, asymmetrical design can also have scenarios where paths only exist in one direction which essentially puts the weighted edge at a length of zero. Real world examples of this may include one-way streets, or flight paths.

For our implementation purposes and the design our of solutions to the Traveling Salesman problem we have designed a variety of graph environments that start at a minimum with five vertices however, these graphs eventually scale to a maximum of 500 vertices. Our created graph dataset uses an adjacency matrix as the data structure to store the values of the graph where the edges are randomized with an upper bound weight of 100. Instead of a complete graph our graph environment uses a randomized pattern to delete nodes from a complete graph to create a symmetrical graph with some edges missing. This can be showcased in fig. 1. We have created a function that uses the squared number of vertices multiplied by a constant of .22 to assign a value as the number of deletions to occur within a specified graph. The constant of .22 allows us to generate sufficiently small graphs that still have a solution to the problem, whilst maintaining a similar order of complexity in larger graphs.

Even though a complete graph causes difficulty in finding the most optimal path due to the sheer number of possibilities, implementing a graph environment where there are missing edges allows our system to be a greater challenge. By introducing missing edges, it creates an environment where the genetic hill climber has a higher chance of reaching local minimums. In other words, due to the missing edges, the population of hill climbers could reach dead ends. It creates a situation where all moves are to previously visited nodes, thus excluding them from the possible solution. In addition to this, having dead ends in the graph environment also allows a closer simulation to the real world where there may not be a route from one city to another without the use of

back tracking or routing through another city or in this case another vertex. We believe that instead of using an imported data set, our custom generated dataset will provide an optimal learning environment for our Genetic Hill Climbers to display their abilities to traverse the graph and create an optimal solution. This was done to test the proposed algorithm more rigorously against other heuristic solutions.

The Traveling Salesman problem is an NP-Hard problem. Exact solutions to the problem are feasible from a variety of proven conventional algorithms. Of course, a brute force exhaustive search will yield an optimal path for the Traveling Salesman, given its time complexity this method will only work on small graphs. Trying all the permutations would lie in the polynomial factor of $O(n!)$. Better exact solutions such as Held-Karp, branch and bound, cutting-plane, and dynamic programming often yield a slightly better result with a solution to the problem in time $O(n^n)$. In worse case scenarios depending on the graph environment, these solutions are no better than an exhaustive search. We have included an exhaustive search and an implementation of the branch and bound algorithm to compete with our proposed approach at an optimal solution to the Traveling Salesman problem that can be seen in our results and analysis section. Because the Genetic Hill Climber is a heuristic solution, it is only possible to provide an approximate solution. These suboptimal algorithms deliver approximated solutions in a reasonable time greatly reduced from $O(n!)$ or even eclipsing the time of established exact algorithm's $O(n^n)$ time. We believe that this type of approach is appropriate once the graph sizes enlarge to a point where the earlier mentioned exact solutions would be rendered useless.

Example 10 node Graph Environment

```
[0, 0, 35.2859, 0, 59.246, 84.1872, 55.9504, 0, 0, 0]
[0, 0, 0, 89.6323, 73.7106, 68.9749, 0, 8.0663, 0, 87.6504]
[35.2859, 0, 0, 24.5824, 0, 88.1017, 0, 0, 44.7347, 0]
[0, 89.6323, 24.5824, 0, 59.3391, 15.5779, 6.9986, 0, 72.9665, 69.883]
[59.246, 73.7106, 0, 59.3391, 0, 0, 93.9489, 0, 0, 0]
[84.1872, 68.9749, 88.1017, 15.5779, 0, 0, 33.6037, 0, 0, 0]
[55.9504, 0, 0, 6.9986, 93.9489, 33.6037, 0, 0, 91.0531, 47.9972]
[0, 8.0663, 0, 0, 0, 0, 0, 0, 74.5876]
[0, 0, 44.7347, 72.9665, 0, 0, 91.0531, 0, 0, 68.544]
[0, 87.6504, 0, 69.883, 0, 0, 47.9972, 74.5876, 68.544, 0]
```

Figure 1. Example graph of ten vertices utilizing random node deletion in an adjacency matrix

The Traveling Salesman problem is an NP-Hard problem. Exact solutions to the problem are feasible from a variety of proven conventional algorithms. Of course, a brute force exhaustive search will yield an optimal path for the Traveling Salesman, given its time complexity this method will only work on small graphs. Trying all the permutations would lie in the polynomial factor of $O(n!)$. Better exact solutions such as Held-Karp, branch and bound, cutting-plane, and dynamic programming often yield a slightly better result with a solution to the problem in time $O(n^n)$. In worse case

scenarios depending on the graph environment, these solutions are no better than an exhaustive search. We have included an exhaustive search and an implementation of the branch and bound algorithm to compete with our proposed approach at an optimal solution to the Traveling Salesman problem that can be seen in our results and analysis section. Because the Genetic Hill Climber is a heuristic solution, it is only possible to provide an approximate solution. These suboptimal algorithms deliver approximated solutions in a reasonable time greatly reduced from $O(n!)$ or even eclipsing the time of established exact algorithm's $O(n^n)$ time. We believe that this type of approach is appropriate once the graph sizes enlarge to a point where the earlier mentioned exact solutions would be rendered useless.

3 Solution

The solution that is outlined here includes two major concepts, Hill Climbing and Genetic algorithms. Both are heuristic search techniques used to commonly solve combinatorial optimization problems. Individually they are both frequently used for solving the Traveling Salesperson problem. However, the most effective and tested approaches currently used are neither of these. Those solutions include Cutting-Plane, Simulated Annealing, Branch and Cut to name a few. As such, the two algorithms have lost the focus of researchers attempting to optimize the Traveling Salesman problem. For this reason, these two algorithms will be the focus of this solution.

A Hill Climbing algorithm is one that uses a state space model to traverse a problem incrementally through taking steps that minimize cost of the overall solution. In the simplest form, this is a greedy technique. Where the algorithm simply evaluates all steps from its current state, and chooses that which satisfies some cost function the best. There are common improvements on this principle, one of which is a Stochastic Hill Climber. This is a hill climber that may not choose the next state that satisfies the cost function the best. Depending on the implementation this can vary. Some common ways to implement this model are to choose a random state that improves the current state or has higher probabilities for choosing better moves than worse ones. It is worth noting that if you are familiar with Simulated Annealing, this differs in that the likelihood of accepting a bad move according to the cost function does not change over time.

A Genetic Algorithm is any algorithm that uses a common set of principles derived from the theory of evolution. Given a population of potential solutions, assess each according to some function. Then create a new population of potential solutions derived in some form from the solutions in the previous population that satisfy the given function the best. Complex implementations of this algorithm will include two additional functions, crossover, and mutations. Crossover is as it is in conventional genetics, in this case combining

$$\sum_{k=1}^G \sum_{j=1}^P \sum_{i=1}^N \sum_{q=1}^N 1 = O(G \times P \times N^2) \approx O(N^3)$$

Figure 2. Theoretical time complexity of the Genetic Hill Climber with inputs being number of Generations, Populations, and Nodes

random parts of two solutions in the population to create a new solution for the next population. Mutations would be introducing new random parts of the solution to a new one regardless of what is reflected in the 'parent' solutions. Over each generation in a Genetic Algorithm, the best individuals of each generation will approach the best solution.

The concept for this implementation is that when the problem consists of a complex, large, non-complete graph the starting position of a hill climber can widely influence the final solution. Here, the starting position of the hill climber is being optimized by a Genetic algorithm. Each generation introduces new instances of hill climbers to the population whose starting position is based relative to the best individual hill climbers of previous populations.

Both the hill climbing individuals and the Genetic algorithm instantiating them have a stochastic element. The hill climbers have a fixed chance of taking a random move from the available moves from their current position on the graph. This allows them to potentially make it over a local maxima and wider variance of paths from the same or similar starting positions of the previous fittest individual hill climbers. Within the genetic portion of the algorithm, there is the additional layer of stochasticism by which the creation of a generation has a fixed chance of implementing a new hill climber from a completely random point on the graph, this is the mutation portion of the Genetic algorithm. This allows the algorithm to constantly test completely new areas of the graph that it had not considered before in the creation of climber.

There are several variables involved in addition to the input for the solution. There is the number of generations that will be used (G) and the number of individuals in each population (P). The input size can be measured as the number of nodes within the graph that represents the state space of the problem. Given this, the time complexity of the implementation of the algorithm described can be expressed as the following. Note that the approximation is the value obtained in the testing shown in the next section of the paper.

4 Performance Evaluation

Firstly, before analysis of our developed Genetic Hill Climber, the results of the conventional approaches to the Traveling Salesman problem should be looked at first. Using our graph environments that were created and discussed earlier in the problem definition section, we were unable to obtain

optimal solutions to the graphs once the number of nodes passed 15. Graphs of size 20 vertices and higher, no optimal solutions were able to be found. This could be attributed to the circumstances of our testing machine. In attempts to find the optimal solution either using an exhaustive search or the Branch and Bound algorithm, our machine ran for several hours until an abort was conducted due to a limitation of memory. For the graph of 15 nodes, utilizing an exhaustive search, the solution was found after a surplus of many hours though the exact runtime was not recorded. For the Branch and Bound algorithm running on a graph of 15 nodes, the run time was a 339.85 seconds. Additionally, it must be pointed out that the solutions that were obtained via branch and bound were not as optimal and exact in comparison to the exhaustive search. Our accuracy metric for evaluating these solutions uses a fitness definition that can be defined as the total amount of length found in the circuit divided by the total amount of nodes. This could be due to our implementation of the Branch of Bound algorithm where certain branches are excluded from the optimal solution due to the graphs not being complete. These dead ends in the graph could result in the algorithm failing to find the global maximum. This can be seen where only the optimal fitness is equal for the graph of five nodes. Once the graph increases to a size of ten and 15 nodes, the fitness differs between the branch and bound algorithm compared exhaustive search.

These results show that these traditional solutions to the Traveling Salesman problem are not reliable once a graph scales to a certain size. It is extremely easy for real world applications to require more than 15 nodes. Once again it must be emphasized that in this situation the limiting factor would be our machine when conducting these tests. This data and conclusion can be referenced in table 1 where there are blanks in columns one through five from a lack of data that could not be obtained for graphs of nodes greater than 15. Attempting to fit the Branch and Bound algorithm on a time complexity chart shows poorly without our test data of three points. As can be seen in figure 3, due to scaling, even if a fourth point were to be obtained with an X-axis of 20 nodes, the line of best fit for a graph of n^n would project to be a near vertical line.

Next, upon examination, our Stochastic Hillclimber, which eventually will be developed as a Genetic Hill Climber, provided us with appalling results. The Stochastic Hill Climber was able to quickly traverse the graphs and provide solutions where the other conventional methods were not. At one end of the spectrum with a graph of five vertices, it was able to obtain the same optimal fitness as both the Branch and Bound algorithm as well as the exhaustive search with a fitness of 74.08426. Although this is not much of an achievement given the small size of the graph. The run times were comparable as well within ten thousandths of a second. These times were kept until a graph of 50 nodes was introduced where the hill

Branch and Bound				
Nodes	Time(seconds)	Fitness	Optimal Fitness	% Error
5	0.0004961	74.08426	74.08426	0
10	0.0168638	57.99643	45.86036	0.26463
15	339.8524076	50.03704	18.30926	1.73288
20	-	-	-	-
25	-	-	-	-
50	-	-	-	-
75	-	-	-	-
100	-	-	-	-
125	-	-	-	-
150	-	-	-	-
175	-	-	-	-
200	-	-	-	-
300	-	-	-	-
400	-	-	-	-
500	-	-	-	-

Table 1. Runtimes and fitness of the Branch and Bound algorithm in comparison to the optimal fitness found in an exhaustive search

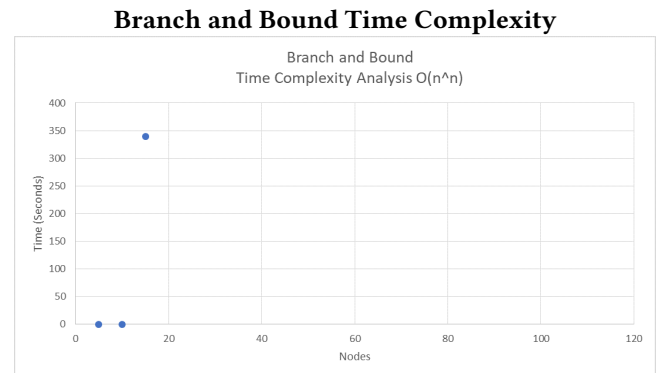
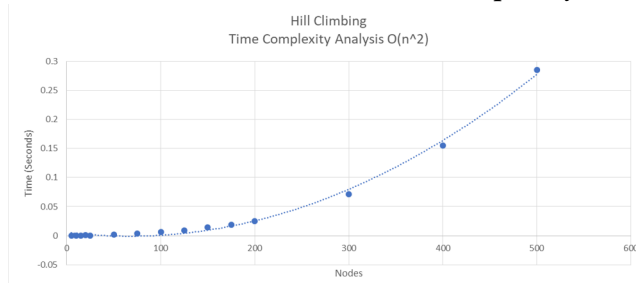


Figure 3. Time complexity of the Branch and Bound algorithm using limited runtime data

climber took a thousandth of a second to calculate a heuristic solution. These quick times continued all the way to the other end of the spectrum with a 0.28519 second run time for a graph of 500 nodes. These data values can be seen in table 2. As mathematically defined in the solutions section, this a glaring obvious difference between a time complexity of $O(n^n)$ and a time complexity of $O(n^2)$. The Stochastic Hill Climber time complexity chart can be seen in figure 4. Despite these quick times, due to our limitations of the other conventional approach, we are unsure of how our fitness that was obtained from the Stochastic Hill Climber compares to an optimal fitness. In other words, even though a solution was generated, we are unsure of how accurate and optimal this solution is. This just goes to show the complexity of the NP-Hard problem that is the Traveling Salesman.

Stochastic Hill Climber

Nodes	Time(seconds)	Fitness	Optimal Fitness	% Error
5	0.0004804	74.08426	74.08426	0
10	0.0004966	55.173	45.86036	0.20299
15	0.0004954	28.21844	18.30926	0.54121
20	0.0009851	21.96413	-	-
25	0.0004961	23.15343	-	-
50	0.0014879	18.45314	-	-
75	0.0034711	13.36534	-	-
100	0.0064475	13.24880	-	-
125	0.0089280	14.08632	-	-
150	0.0143837	11.71591	-	-
175	0.0183513	9.423754	-	-
200	0.0247995	11.53126	-	-
300	0.0709278	12.72170	-	-
400	0.1547515	7.936304	-	-
500	0.2851998	9.040675	-	-

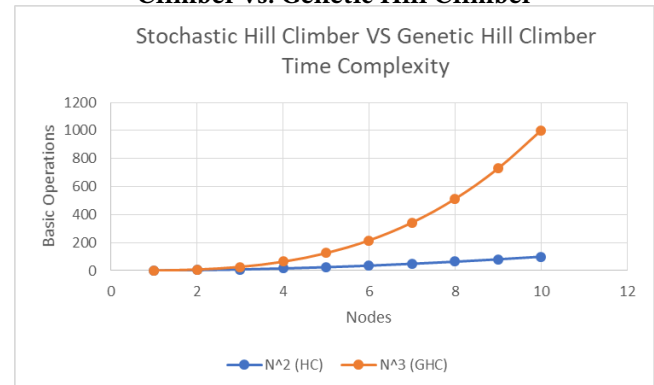
Table 2. Run times of the Stochastic Hill Climber and obtained Fitness**Stochastic Hill Climber Time Complexity****Figure 4.** Time complexity of the Stochastic Hill Climber using our run time data

Immediately looking at the data for the developed Genetic Hill Climber, run times are drastically higher compared to the Stochastic Hill Climber. For the graph of five nodes, the Genetic Hill Climber took approximately 250 times longer compared to the Stochastic Hill Climber. Once the graphs enlarged to a size of 500, the Stochastic Hill Climber had a faster run time by a factor of nearly 100. We deliberately decided to exclude a runtime graph to compare these two algorithms due to the vast differences in performance. Using our data, this type of chart would show the Genetic Hill Climber as exponential while the Stochastic Hill Climber would appear linear despite it being $O(n^2)$. However, there is an included graph which can show the estimates of basic operations per given number of nodes with these two algorithms side by side (fig. 5). This estimate of using theoretical number of computations more closely resembles how these two graphs should look when compared with one another.

However, there is still a misleading linear type of line and exponential line can clearly be seen. Table 3 shows these run times side by side where the comparison can be more easily seen with the Stochastic Hill Climber being much quicker compared to the slower Genetic Hill Climbing algorithm.

**Genetic Hill Climber
Vs.
Stochastic Hill Climber
(seconds)**

Nodes	Genetic Hill Climber	Stochastic Hill Climber
5	0.10316	0.00048
10	0.11755	0.00049
15	0.12796	0.00049
20	0.14284	0.00098
25	0.19344	0.00049
50	0.33231	0.00148
75	0.57833	0.00347
100	0.89924	0.00644
125	1.32828	0.00892
150	1.77617	0.01438
175	2.52414	0.01835
200	3.27260	0.02479
300	8.49250	0.07092
400	16.37443	0.15475
500	30.20190	0.28519

Table 3. Run times between the Genetic Hill Climber and the Stochastic Hill Climber**Computed Time Complexity of Stochastic Hill Climber vs. Genetic Hill Climber****Figure 5.** Computed time complexity of our Stochastic Hill Climber compared to our Genetic Hill Climber using theoretical mathematical data

As for fidelity to the solutions, the Genetic Hill Climber was able to best the Stochastic Hill Climber in every scenario except for the case of a graph with five nodes, as can be seen

in table 4 where they managed to obtain the same optimal fitness. This is a strong indication that our Genetic Hill Climber would be able to further find more optimal solutions with an even larger graph than the sizes than presented. It must be noted that our variation of the Genetic Hill Climber started with a population of ten individuals propagating within ten generations. After this initial phase of testing, we were able to generate the test data that is shown above and within the variance chart as well as below in the standalone Genetic Hill Climbing data. Since we were able to best the Stochastic Hill Climber early on, we did not further progress to see the optimal number of populations and generations as our initial proposal was to simply best a basic Hill Climber in fitness. It can safely be said that the Genetic Hill Climber exchanges time complexity for accuracy. Obtaining better results with a lower fitness than the Stochastic Hill Climber, however the Genetic Hill Climber slowed down a large amount. The standalone data for the Genetic Hill Climber follows below.

**Genetic Hill Climber
Vs.
Stochastic Hill Climber
Fitness**

Nodes	Genetic Hill Climber	Stochastic Hill Climber
5	74.08426	74.08426
10	45.93808	55.17
15	18.84321	28.21844
20	21.96413	21.96413
25	18.89756	23.15343
50	16.10445	18.45314
75	12.43940	13.36534
100	11.41996	13.24880
125	8.60325	14.08632
150	8.61659	11.71591
175	8.25562	9.42375
200	6.89503	11.53126
300	8.02294	12.72170
400	7.29255	7.93630
500	7.09442	9.04067

Table 4. Fitness of Genetic Hill Climber vs. Stochastic Hill Climber

Genetic Hill Climber Time Complexity

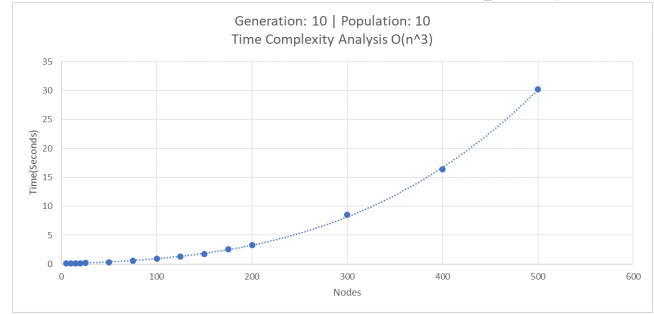


Figure 6. The Genetic Hill Climber time complexity can be seen as $O(n^3)$ taking longer run times compared to the Stochastic Hill Climber

Genetic Hill Climber

Nodes	Time(seconds)	Fitness	Optimal Fitness	% Error
5	0.10316	74.08426	74.08426	0
10	0.11755	45.93808	45.86036	0.20299
15	0.12796	18.84321	18.30926	0.54121
20	0.14284	21.96413	-	-
25	0.19344	18.89756	-	-
50	0.33231	16.10445	-	-
75	0.57833	12.43940	-	-
100	0.89924	11.41996	-	-
125	1.32828	8.60325	-	-
150	1.77617	8.61659	-	-
175	2.52414	8.25562	-	-
200	3.27260	6.89503	-	-
300	8.49250	8.02294	-	-
400	16.37443	7.29255	-	-
500	30.20190	7.09442	-	-

Table 5. Run time data of the Genetic Hill Climber and obtained fitness

**Stochastic Hill Climber VS Genetic Hill Climber
Variance from Optimal Fitness**

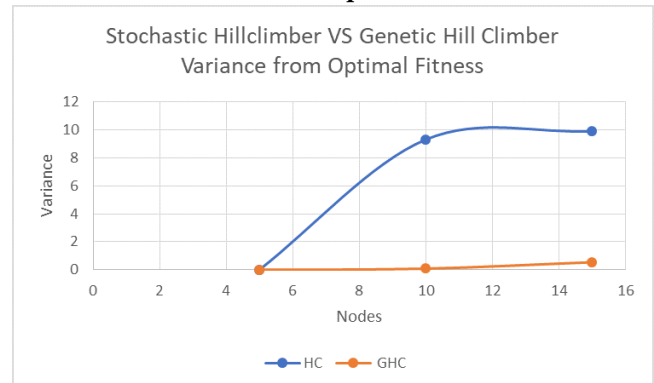


Figure 7. This graph shows the variance of the two algorithms from the optimal fitness. Note: limited graph data

5 Prognosis

All objectives that were planned within the initial start of this project were completed at its conclusion. No difficulties were encountered regarding algorithm design and analysis, however we encountered difficulties with development as the technologies used were unfamiliar.

Our developed Genetic Hill Climbing algorithm was able to obtain suboptimal solutions to the Traveling Salesman problem while conventional approaches were not able to do so. The algorithm yielded solutions, however the optimality of these are unverified because we were unable to obtain exact solutions for our set of large graphs. Despite this, it should be seen on a positive note that any solution could be interpreted as better than no solutions.

Through our runtimes, we were able to conclude that the runtime analysis does not reflect the theoretical time complexity discussed in the solutions section. The graphs and data obtained allowed us to approximate the time complexity and number of basic operations. Perhaps due to the limited number of generations and populations, these variables in the time complexity equation only seemed to result as a single n input instead of an input of n^n . As mentioned earlier, we did not proceed with further testing and optimization to find the optimal number of generations and populations. Further alteration of the algorithm's parameters could affect run times and obtained fitness which we did not test. With that being said, we believe our solution is unable to compete with other forefront heuristic solutions without improvements. This is further expanded upon later.

6 Conclusion and Future Work

In closing, it is clear that a lot of improvements would be necessary to be made for this to compete with other heuristic algorithms used for solving the Traveling Salesman problem. The data has shown that this solution does allow for the generation of good sub-optimal solutions in an efficient manner compared to conventional approaches. At best any answer is better than no answer. It is also possible this methodology might be more effective for other combinatorial optimization problems.

The representation used for the Traveling Salesman Problem in this implementation was an adjacency matrix as described in a previous section. Implementing the representation as an adjacency list would have significantly cut down on the overhead of the entire implementation. Quicker run times could have potentially been obtained. It may also improve the time complexity to create graphs allowing for more testing data of larger sizes. Creating Graphs of large sizes took a significant amount of time.

In the future, modifying the algorithm such that the size of the population and the number of generations is determined by the complexity of the problem might allow for further time reduction. Similarly, the algorithm could be modified

to continue creating new generations until the cost function yields a particular level of optimality. This would allow the algorithm to end sooner if a near goal state that meets the needs of the problem has already been found.

If instead of allowing the hill climbers in each generation of our algorithm run to completion, there was an evaluation function to end and replace hill climbers that have a low probability of improving the next generation, a large amount of run time could be avoided. Similarly, if the population existed with hill climbers that are all at various stages of completion, there would be only one continuous population through each generation. Though this might not make the complexity of the algorithm any different, it would improve the ease of implementing some of the earlier suggested improvements to dynamic set the population size and number of hill climbers created. In short, these improvements would allow for the same amount of compute time to be spent on solutions with a higher probability of improving upon the best solution.

Similarly, to Simulated Annealing, it would be possible and more optimal to change the level of stochasticity as the algorithm progressively gets closer to the optimal solution. During the beginning, allowing more movement around the graph for starting positions would result in finding the way around large global minimums early. Furthermore, it would also be possible to implement this design using a population of Simulated Annealing Climbers, though it is unsure if this would be any improvement.

References

- [1] G.R. Harik, F.G. Lobo, and D.E. Goldberg. 1999. The compact genetic algorithm. *IEEE Transactions on Evolutionary Computation* 3, 4 (Nov. 1999), 287–297. <https://doi.org/10.1109/4235.797971>
- [2] Huichuan Liao. 2021. Research on agricultural machinery operation path optimization based on traveling salesman problem (TSP). *Journal of Physics: Conference Series* 1976, 1 (July 2021), 012051. <https://doi.org/10.1088/1742-6596/1976/1/012051>
- [3] Umberto Junior Mele, Luca Maria Gambardella, and Roberto Montemanni. 2021. A New Constructive Heuristic Driven by Machine Learning for the Traveling Salesman Problem. *Algorithms* 14, 9 (Sept. 2021), 267. <https://doi.org/10.3390/a14090267>
- [4] Xiaomin Ren, Xiaoming Wang, Zhaocai Wang, and Tunhua Wu. 2020. Parallel DNA Algorithms of Generalized Traveling Salesman Problem-Based Bioinspired Computing Model. *International Journal of Computational Intelligence Systems* 14, 1 (2020), 228. <https://doi.org/10.2991/ijcis.d.201127.001>
- [5] Xiang Wang and Guoyi Xu. 2011. Hybrid Differential Evolution Algorithm for Traveling Salesman Problem. *Procedia Engineering* 15 (2011), 2716–2720. <https://doi.org/10.1016/j.proeng.2011.08.511>
- [6] Deniz Yuret. 2014. Dynamic Hill climbing: Overcoming the limitations of optimization techniques. https://www.academia.edu/9313355/Dynamic_Hill_Climbing_Overcoming_the_limitations_of_optimization_techniques