

- 1  $\lambda x. y$
- 2  $f \text{ true}$
- 3  $\text{isZero}(x)$

$$1) \{y: X_1\} \vdash \lambda x: X_2. y : X_2 \rightarrow X_1$$

$$2) \{f: \text{Bool} \rightarrow X_1\} \vdash f \text{ true} : X_1$$

$$3) \{x: \text{Nat}\} \vdash \text{isZero}(x) : \text{Bool}$$

$$\tau ::= X_k \mid C(\tau_1, \dots, \tau_n)$$

$$\begin{array}{c} \tau_1 \\ \tau_2 \end{array} \begin{array}{c} \tau_3 \\ \tau_4 \end{array}$$

$$1 \text{ MGU}\{X_2 \rightarrow X_1 \rightarrow \text{Bool} \stackrel{?}{=} X_2 \rightarrow X_3\}$$

$$2 \text{ MGU}\{(X_2 \rightarrow X_1) \rightarrow \text{Nat} \stackrel{?}{=} X_2 \rightarrow X_3\}$$

$$3 \text{ MGU}\{X_1 \rightarrow \text{Bool} \stackrel{?}{=} \text{Nat} \rightarrow \text{Bool}, X_2 \stackrel{?}{=} X_1 \rightarrow X_1\}$$

$$4 \text{ MGU}\{X_1 \rightarrow X_2 \stackrel{?}{=} X_3 \rightarrow X_4, X_3 \stackrel{?}{=} X_2 \rightarrow X_1\}$$

$$\{ \textcircled{X_2} \rightarrow (X_1 \rightarrow \text{Bool}) \stackrel{?}{=} \textcircled{X_2} \rightarrow \textcircled{X_3} \}$$

$$\xrightarrow{\text{Dec}} \{ \underline{X_2 \stackrel{?}{=} X_2}, X_1 \rightarrow \text{Bool} \stackrel{?}{=} X_3 \}$$

$$\xrightarrow{\text{Elim pr trivial}} \{ X_1 \rightarrow \text{Bool} \stackrel{?}{=} X_3 \}$$

$$\xrightarrow{\text{Swap}} \{ X_3 \stackrel{?}{=} X_1 \rightarrow \text{Bool} \}$$

$$\xrightarrow{\text{Elim}} \emptyset$$

$$\{ X_3 := X_1 \rightarrow \text{Bool} \}$$

$$S = \{ X_3 := X_1 \rightarrow \text{Bool} \}$$

4 MGU  $\{X_1 \rightarrow X_2 \stackrel{?}{=} X_3 \rightarrow X_4, X_3 \stackrel{?}{=} X_2 \rightarrow X_1\}$

$$\xrightarrow{\text{Elim}} \{X_3 := X_2 \rightarrow X_1\} \quad \{X_1 \rightarrow X_2 \stackrel{?}{=} (X_2 \rightarrow X_1) \rightarrow X_4\}$$

$$\xrightarrow{\text{Dec}} \{X_1 \stackrel{?}{=} X_2 \rightarrow X_1, X_2 \stackrel{?}{=} X_4\}$$

$$\xrightarrow{\text{Occurs}} \text{falla}$$

Ejercicio: Rectificar los siguientes términos

■  $(\lambda f. \lambda x. f(f x))(\lambda f. f) =_{\alpha} (\lambda f. \lambda x. f(f x))(\lambda g. g) \checkmark$

■  $x(\lambda x. \text{succ}(x)) =_{\alpha} x(\lambda y. \text{succ}(y)) \checkmark$

Ejercicio: Anotar los siguientes términos

■  $\lambda f. \lambda x. f(f x)$

■  $x(\lambda y. \text{succ}(y))$

1)  $\Gamma_0 = \emptyset \quad M_0 = \lambda f : X_1. \lambda x : X_2. f(f x)$

2)  $\Gamma_0 = \{x : X_1\} \quad M_0 = x(\lambda y : X_2. \text{succ}(y))$

1)  $\lambda f. \lambda x. f(f x)$

1) Rect,  $\vdash_{cor}$  ✓

2)  $\Gamma_0 = \emptyset$

$M_0 = \lambda f: X_1. \lambda x: X_2. f(f x)$

3)

$$\mathcal{I}(\emptyset \mid \lambda f: X_1. \lambda x: X_2. \overbrace{f(f x)}^M)$$

$$= (X_1 \rightarrow X_2 \rightarrow X_4 \mid \overbrace{E}^{\text{Abs}})$$

$$\mathcal{I}(\{f: X_1\} \mid \lambda x: X_2. \overbrace{f(f x)}^M) \checkmark$$

$$= (X_2 \rightarrow X_4 \mid \overbrace{E})$$

$$\mathcal{I}(\underbrace{\{f: X_1, x: X_2\}}_{\Gamma} \mid \underbrace{f}_{M_1}(\underbrace{f x}_{M_2})) = (X_4 \mid \overbrace{\{X_1 \stackrel{?}{=} X_3 \rightarrow X_4, X_1 \stackrel{?}{=} X_2 \rightarrow X_3\}}^E)$$

$$\mathcal{I}(\overbrace{\Gamma}^{f: X_1} \mid f) = (X_1 \mid \emptyset) \checkmark$$

$$\mathcal{I}(\Gamma \mid f x) = (X_3 \mid \{X_1 \stackrel{?}{=} X_2 \rightarrow X_3\})$$

$$\mathcal{I}(\Gamma \mid f) = (X_1 \mid \emptyset) \checkmark$$

$$\mathcal{I}(\Gamma \mid x) = (X_2 \mid \emptyset) \checkmark$$

App

4) MGLU

$$\mathcal{I}(\emptyset \mid \lambda f: X_1. \lambda x: X_2. f(f x)) = (X_1 \rightarrow X_2 \rightarrow X_4 \mid \overbrace{E})$$

$S = MGLU(E)$

$$\begin{aligned} & \{X_1 \stackrel{?}{=} X_3 \rightarrow X_4, \underbrace{X_1 \stackrel{?}{=} X_2 \rightarrow X_3}_{\text{circled}}\} \\ & \xrightarrow{E_{lin}} \{X_1 := X_3 \rightarrow X_4, X_3 \rightarrow X_4 \stackrel{?}{=} X_2 \rightarrow X_3\} \\ & \xrightarrow{Dec} \{X_3 \stackrel{?}{=} X_2, X_4 \stackrel{?}{=} X_3\} \xrightarrow{E_{lin}} \{X_4 \stackrel{?}{=} X_2\} \end{aligned}$$

$$\xrightarrow{EI_i} \{X_4 := X_2\} \emptyset$$

$$f \circ g(x) = f(g(x))$$

$$S = \{X_4 := X_2\} \circ \{X_3 := X_2\} \circ \{X_1 := \cancel{X_3} \rightarrow X_4\}$$

$$= \{X_4 := X_2\} \circ \{X_3 := X_2, X_1 := X_2 \rightarrow X_4\}$$

$$S = \{X_4 := X_2, X_3 := X_2, X_1 := X_2 \rightarrow X_2\}$$

$$S(\Gamma_0) \vdash S(M_0) : S(\tau)$$

posos 1+2

poso 3

$$\emptyset \vdash \lambda_1: X_2 \rightarrow X_2. \lambda_1: X_2. \overset{X_1}{f}(f(x)) : (X_2 \rightarrow X_2) \rightarrow X_2 \rightarrow X_2$$

$\lambda x. x y x$

1) Rectification ✓

2)  $\Gamma_0 = \{y: X_1\}$

$M_0 = \lambda x: X_2. (x y) x$

3)  $\mathcal{I}(y: X_1 \mid \lambda x: X_2. \underline{x y x}) = (X_2 \rightarrow X_4 \mid E)$

Abs

$\mathcal{I}(y: X_1, x: X_2 \mid (x y) x) = (X_4 \mid \{X_3 \stackrel{?}{=} X_2 \rightarrow X_4, X_2 \stackrel{?}{=} X_1 \rightarrow X_3\})$

App

$\mathcal{I}(\Gamma \mid x y) = (X_3 \mid \{X_2 \stackrel{?}{=} X_4 \rightarrow X_3\})$  ✓  $\mathcal{I}(\Gamma \mid x) = (X_2 \mid \emptyset)$  ✓

$\mathcal{I}(\Gamma \mid x) = (X_2 \mid \emptyset)$  ✓  $\mathcal{I}(\Gamma \mid y) = (X_1 \mid \emptyset)$

4)  $\mathcal{I}(y: X_1 \mid \lambda x: X_2. x y x) = (X_2 \rightarrow X_4 \mid E)$

$\{X_3 \stackrel{?}{=} X_2 \rightarrow X_4, X_2 \stackrel{?}{=} X_1 \rightarrow X_3\}$

Elim

$\{X_3 \stackrel{?}{=} X_2 \rightarrow X_4\} \{X_2 \stackrel{?}{=} X_1 \rightarrow X_2 \rightarrow X_4\}$

Occurs check

falla

$$\mathcal{I}(\Gamma \mid \text{map}_{\sigma, \tau}) = ((\sigma \rightarrow \tau) \rightarrow [\sigma] \rightarrow [\tau] \mid \emptyset)$$

$$\mathcal{I}(\Gamma \mid \text{foldr}_{\sigma, \tau}) = ((\sigma \rightarrow \tau \rightarrow \tau) \rightarrow \tau \rightarrow [\sigma] \rightarrow \tau \mid \emptyset)$$

1) foldr map ✓

$$2) \Gamma_0 = \emptyset$$

$$\gamma_0 = \text{foldr}_{X_1, X_2} \text{map}_{X_3, X_4}$$

3)

$$\mathcal{I}(\emptyset \mid \text{foldr}_{X_1, X_2} \text{map}_{X_3, X_4}) =$$

$$= (X_5 \mid E)$$

$$\begin{aligned} \mathcal{I}(\emptyset \mid \text{foldr}_{X_1, X_2}) &= \mathcal{I}(\emptyset \mid \text{map}_{X_3, X_4}) = \\ &= ((X_3 \rightarrow X_4) \rightarrow [X_3] \rightarrow [X_4] \mid \emptyset) \\ &= ((X_1 \rightarrow X_2 \rightarrow X_2) \rightarrow X_2 \rightarrow [X_1] \rightarrow X_2 \mid \emptyset) \end{aligned}$$

$$E = \{ (X_1 \rightarrow X_2 \rightarrow X_2) \rightarrow X_2 \rightarrow [X_1] \rightarrow X_2 = (X_3 \rightarrow X_4) \rightarrow [X_3] \rightarrow [X_4] \rightarrow X_5 \}$$

$$S = \text{MGU} \{ E \}$$

$$\text{neg} \} (X_1 \rightarrow X_2 \rightarrow X_2) \rightarrow (X_2 \rightarrow [X_1] \rightarrow X_2) \doteq ((X_3 \rightarrow X_4) \rightarrow [X_3] \rightarrow [X_4]) \rightarrow \{$$

$$\xrightarrow{\text{dec}} \} (X_1 \rightarrow X_2 \rightarrow X_2) \doteq (X_3 \rightarrow X_4) \rightarrow [X_3] \rightarrow [X_4],$$

$$X_2 \rightarrow [X_1] \rightarrow X_2 \doteq X_5 \}$$

$$\xrightarrow{\text{swap}} \} X_1 \rightarrow X_2 \rightarrow X_2 \doteq (X_3 \rightarrow X_4) \rightarrow [X_3] \rightarrow [X_4],$$

$$X_5 \doteq X_2 \rightarrow [X_1] \rightarrow X_2 \}$$

$$\xrightarrow{\text{elim}} \} X_1 \rightarrow (X_2 \rightarrow X_2) \doteq (X_3 \rightarrow X_4) \rightarrow ([X_3] \rightarrow [X_4])$$

$$\} X_5 \doteq X_2 \rightarrow [X_1] \rightarrow X_2 \}$$

$$\xrightarrow{\text{dec}} \} X_1 \doteq X_3 \rightarrow X_4, X_2 \rightarrow X_2 \doteq [X_3] \rightarrow [X_4] \}$$

$$\xrightarrow{\text{elim}} \} X_1 \doteq X_3 \rightarrow X_4 \} X_2 \rightarrow X_2 \cdot [X_3] \rightarrow [X_4] \}$$

$$\xrightarrow{\text{dec}} \} X_2 \doteq [X_3], X_2 \doteq [X_4] \}$$

$$\xrightarrow{\text{elim}} \} X_2 \doteq [X_3] \} [X_3] \doteq [X_4] \}$$

$$\xrightarrow{\text{dec}} \} X_3 \doteq X_4 \}$$

$$\xrightarrow{\text{elim}} \} X_3 \doteq X_4 \} \}$$

$$S = \} X_3 \doteq X_4, X_2 \doteq [X_4],$$

$$X_1 \doteq X_4 \rightarrow X_4,$$

$$X_5 \doteq [X_4] \rightarrow [X_4 \rightarrow X_4] \rightarrow [X_4] \}$$

$$S = \{ X_3 := X_4, X_2 := [X_4],$$

$$X_1 := X_4 \rightarrow X_4,$$

$$X_5 := [X_4] \rightarrow [X_4 \rightarrow X_4] \rightarrow [X_4] \}$$

$$\Gamma_0 = \emptyset$$

$$M_0 = \text{foldr}_{X_1, X_2} \text{map}_{X_3, X_4}$$

$$\gamma = X_5$$

$$\phi \vdash \text{foldr}_{X_4 \rightarrow X_4, [X_4]} \text{map}_{X_4, X_4} : [X_4] \rightarrow [X_4 \rightarrow X_4] \rightarrow [X_4]$$

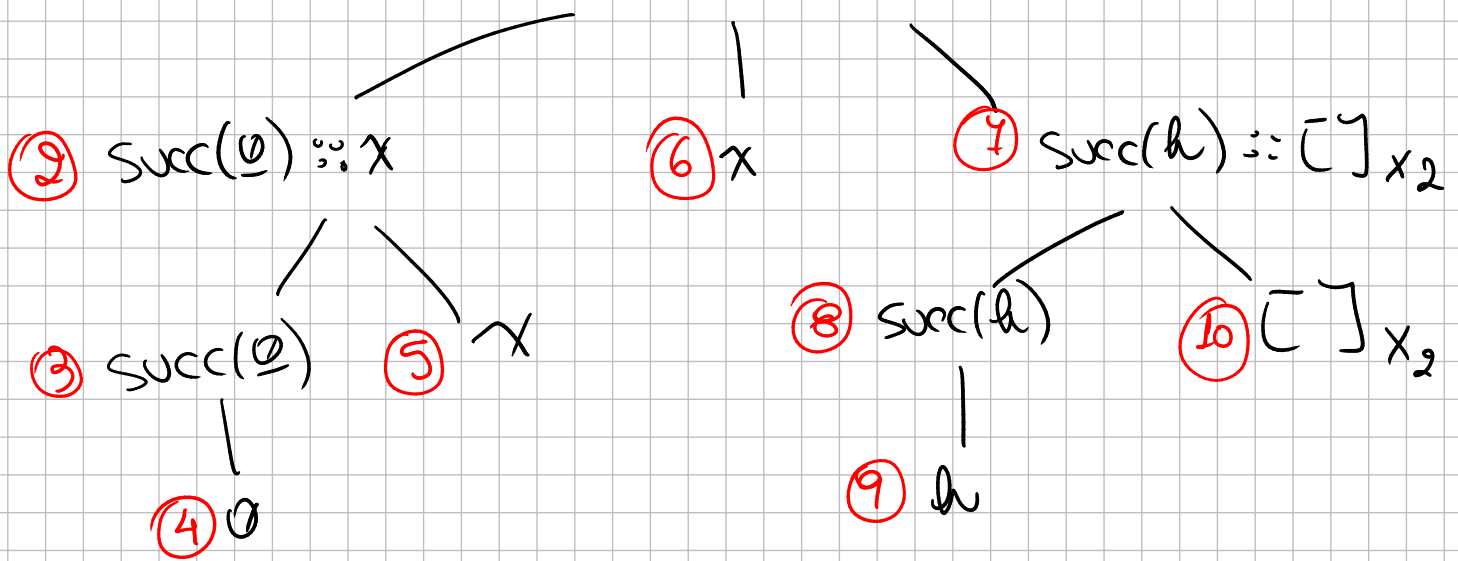


Case  $\text{succ}(\underline{0}) :: x$  of  $[\ ] \rightsquigarrow x ; x :: y \rightsquigarrow \text{succ}(x) :: [\ ]$

1) Case  $\text{succ}(\underline{0}) :: x$  of  $[\ ] \rightsquigarrow x ; h :: t \rightsquigarrow \text{succ}(h) :: [\ ]$

2)  $\Gamma_0 = \{x : X_1\}$

$\Gamma_0 =$  case  $\text{succ}(\underline{0}) :: x$  of  $[\ ] \rightsquigarrow x ; h :: t \rightsquigarrow \text{succ}(h) :: [\ ]_{X_2}$



1)  $\mathcal{L}(\{x : X_1\} \mid \text{case } \text{succ}(\underline{0}) :: x \text{ of } [\ ] \rightsquigarrow x ; h :: t \rightsquigarrow \text{succ}(h) :: [\ ]_{X_2})$

$= \left( X_1 \mid \begin{array}{l} \{X_1 \doteq [\ ]_{X_1}, X_1 \doteq [\ ]_{X_2}, X_t \doteq X_1, \\ X_1 \doteq [\ ]_{X_2}, N \doteq N, [\ ]_{X_1} \doteq [\ ]_{X_2}, X_0 \doteq N \end{array} \right)$

2)  $\mathcal{L}(\{x : X_1\} \mid \text{succ}(\underline{0}) :: x) =$

$= \left( X_1 \mid \{X_1 \doteq [\text{Nat}], N \doteq N\} \right)$

$$3) \mathcal{L}(\{x: x_1\} \mid \text{succ}(\underline{0})) = (\text{Nat} \mid \{u \doteq v\})$$

$$4) \mathcal{L}(\{x: x_1\} \mid \underline{0}) = (\text{Nat} \mid \emptyset)$$

$$5) \mathcal{L}(\{x: x_1\} \mid x) = (x_1 \mid \emptyset)$$

$$6) \mathcal{L}(\{x: x_1\} \mid x) = (x_1 \mid \emptyset)$$

$$7) \mathcal{L}(\{x: x_1, u: x_u, v: x_v\} \mid \text{succ}(u) :: [\gamma]_{x_2}) =$$

$$= ([x_2] \mid \{[x_2] \doteq [\text{Nat}], x_u \doteq \text{Nat}\})$$

$$8) \mathcal{L}(\Gamma \mid \text{succ}(u)) = (\text{Nat} \mid \{x_u \doteq \text{Nat}\})$$

$$9) \mathcal{L}(\Gamma \mid u) = (x_u \mid \emptyset)$$

$$10) \mathcal{L}(\Gamma \mid [\gamma]_{x_2}) = ([x_2] \mid \emptyset)$$

$$\lambda (\{x \equiv x_1 \mid \text{case succ}(0) :: x \text{ of } \bar{\square} \mapsto x; \\ h :: t \mapsto \text{succ}(h) :: \bar{\square} x_2\})$$

$$= \left( X_1 \mid \begin{cases} x_1 \equiv \bar{\square} x_a, x_1 \equiv \bar{\square} x_2, x_t \equiv x_1, \\ x_1 \equiv \bar{\square} N, N \equiv N, \bar{\square} x_a \equiv \bar{\square} Nat, x_a \equiv N \end{cases} \right)$$

$$S = \text{MGU} \left\{ \begin{array}{l} x_1 \equiv \bar{\square} x_a, x_1 \equiv \bar{\square} x_2, x_t \equiv x_1, \\ x_1 \equiv \bar{\square} N, N \equiv N, \bar{\square} x_a \equiv \bar{\square} Nat, x_a \equiv N \end{array} \right\}$$

$$= \left\{ x_2 \equiv Nat, x_1 \equiv \bar{\square} Nat, x_a \equiv Nat, x_t \equiv \bar{\square} Nat \right\}$$

$$\{x : \bar{\square} Nat\} \vdash \text{case succ}(0) :: x \text{ of } \bar{\square} \mapsto x; : \bar{\square} Nat \\ h :: t \mapsto \text{succ}(h) :: \bar{\square}_{Nat}$$

$$[\text{succ}(x) \mid x \leftarrow \underline{0} :: \underline{1} :: \underline{2} :: \bar{\square}, \text{true}]$$

$$[\text{if } x \text{ then } \underline{0} \text{ else } \underline{1} \mid x \leftarrow \text{false} :: \text{isZero}(x) :: [\ ], \text{true}]$$

$$[\text{if } y \text{ then } \underline{0} \text{ else } \underline{1} \mid y \leftarrow \text{false} :: \text{isZero}(x) \dots]$$