$\sigma, \tau ::= \cdots \mid \sigma \times \tau$

$M, N ::= \cdots \mid \langle M, N \rangle \mid \pi_1(M) \mid \pi_2(M)$

Definir como macro la función $curry_{\sigma,\tau,\delta}$ que sirve para currificar funciones que reciben pares como argumento.

$$curry_{\sigma,\tau,\delta} = \lambda f : \sigma \times \tau \to \delta . \lambda x : \sigma . \lambda y : \tau . f \langle x, y \rangle$$

- $$\frac{\Gamma \vdash M : \sigma \quad \Gamma \vdash N : \tau}{\Gamma \vdash \langle M, N \rangle : \sigma \times \tau}\text{T-PAR} \qquad \frac{\Gamma \vdash M : \sigma \times \tau}{\Gamma \vdash \pi_1(M) : \sigma}\text{T-}\pi_1$$

$$\frac{\Gamma \vdash M : \sigma \times \tau}{\Gamma \vdash \pi_2(M) : \tau}\text{T-}\pi_2$$

$\pi_1(\text{false})$ ✗

$\pi_1(\langle \text{true}, \text{false} \rangle) \longrightarrow$ ___

- $M ::= \cdots \mid \langle M, M \rangle \mid \pi_1(M) \mid \pi_2(M)$

$V, W ::= \text{true} \mid \text{false} \mid \lambda x : \sigma . M \mid \text{zero} \mid \text{succ}(V) \mid \langle V, W \rangle$

$\pi_1(\langle \text{true}, \text{false} \rangle) \longrightarrow \text{true}$

* Determinismo

$$\left.\begin{array}{l} M \to N_1 \\ M \to N_2 \end{array}\right\} \Rightarrow N_1 = N_2$$

* Preservación de tipos

$$\left.\begin{array}{l} \Gamma \vdash M : \sigma \\ M \to N \end{array}\right\} \Rightarrow \Gamma \vdash N : \sigma$$

* Progreso

$$\emptyset \vdash M : \sigma \Rightarrow \begin{cases} \text{o bien } M \text{ es un valor} \\ \text{o bien } \exists N \text{ t.q. } M \to N \end{cases}$$

* Terminación

$\emptyset \vdash M : \sigma \Rightarrow$ no pasa que $\underbrace{M \to N \to \cdots}_{\text{cadena } \infty}$

$\pi_1(\langle \; , \; \rangle)$

$$M ::= \dots \mid \langle M, M\rangle \mid \pi_1(M) \mid \pi_2(M)$$

$$V, W ::= true \mid false \mid \lambda x{:}\sigma.M \mid zero \mid succ(V) \mid \langle V, W\rangle$$

M no es un valor

$$\frac{M \to M'}{\pi_1(M) \to \pi_1(M')}\ \text{E-}\pi_1 \qquad \frac{}{\pi_1(\langle V_1, V_2\rangle) \to V_1}\ \text{E-}\pi_1\text{PAR}$$

$$\pi_1((\lambda x{:}Bool \times Bool.\,x)\langle true, false\rangle)$$

$$\xrightarrow[\text{E-}\pi_1 + \beta]{} \pi_1(\langle true, false\rangle)$$

$$\emptyset \vdash \langle\langle true, false\rangle, zero\rangle : (Bool \times Bool) \times Nat$$

$$\frac{M \to M'}{\pi_2(M) \to \pi_2(M')}\ \text{E-}\pi_2 \qquad \frac{}{\pi_2(\langle V_1, V_2\rangle) \to V_2}\ \text{E-}\pi_2\text{PAR}$$

$$\frac{M \to M'}{\langle M, N\rangle \to \langle M', N\rangle}\ \text{E-PAR1} \qquad \frac{N \to N'}{\langle V, N\rangle \to \langle V, N'\rangle}\ \text{E-PAR2}$$

¿Qué problema introduce agregar la siguiente regla?

MAL

$$\pi_1(\langle M, N\rangle) \longrightarrow M$$

$$\pi_1((\lambda x{:}Bool \times Nat.\,x)\langle true, 0\rangle)$$

$$\frac{M \to M'}{\pi_1(M) \to \pi_1(M')}\ \text{E-}\pi_1 \qquad \frac{}{\pi_1(\langle V_1, V_2\rangle) \to V_1}\ \text{E-}\pi_1\text{PAR}$$

$$\frac{M \to M'}{\pi_2(M) \to \pi_2(M')}\ \text{E-}\pi_2 \qquad \frac{}{\pi_2(\langle V_1, V_2\rangle) \to V_2}\ \text{E-}\pi_2\text{PAR}$$

$$\frac{M \to M'}{\langle M, N\rangle \to \langle M', N\rangle}\ \text{E-PAR1} \qquad \frac{N \to N'}{\langle V, N\rangle \to \langle V, N'\rangle}\ \text{E-PAR2}$$

$$\pi_1(\langle(\lambda x{:}Bool.\,x)\ true, false\rangle) \xrightarrow{\text{MAL}} (\lambda x{:}Bool.\,x)\ true$$

$$\pi_1(\langle(\lambda x{:}Bool.\,x)\ true, false\rangle) \longrightarrow \pi_1(\langle true, false\rangle)$$

$$
\cfrac{\cfrac{\cfrac{\cfrac{}{x : Nat \vdash x : Nat}\ \text{T-VAR} \qquad \cfrac{}{x : Nat \vdash true : Bool}\ \text{T-TRUE}}{x : Nat \vdash \langle x, true \rangle : Nat \times Bool}\ \text{T-PAR}}{\emptyset \vdash \lambda x : Nat . \langle x, true\rangle : Nat \to Nat \times Bool}\ \text{T-ABS} \qquad \cfrac{}{\emptyset \vdash 0 : Nat}\ \text{T-ZERO}}{\cfrac{\emptyset \vdash (\lambda x : Nat . \langle x, true \rangle)\, 0 : Nat \times Bool}{\emptyset \vdash \pi_1((\lambda x : Nat.\langle x, \mathrm{True}\rangle)\ 0) : Nat}\ \text{T-}\pi_1}\ \text{T-APP}
$$

$$
\cfrac{\Gamma \vdash \ \ \sigma \qquad \Gamma \vdash \ \ \tau}{\Gamma \vdash \ \ \sigma \wedge \tau}\ \text{T-PAR}
\qquad
\cfrac{\Gamma \vdash \ \ : \sigma \wedge \tau}{\Gamma \vdash \ \ \sigma}\ \text{T-}\pi_1
\qquad
\cfrac{\Gamma \vdash \ \ \sigma \wedge \tau}{\Gamma \vdash \ \ : \tau}\ \text{T-}\pi_2
$$

$$
\pi_1((\lambda x : Nat.\langle x, \mathrm{True}\rangle)\ 0) \xrightarrow[\beta]{E\text{-}\pi_1,} \pi_1(\langle 0, true \rangle) \xrightarrow[E\text{-}\pi_1 PAR]{} 0
$$

$$
\pi_1(\pi_1\ (\lambda x : Nat, \langle\langle x, x\rangle, true\rangle)\ 0) \xrightarrow[E\text{-}\pi_1,\ E\text{-}\pi_1,\ \beta]{} \pi_1(\pi_1(\langle\langle 0, 0\rangle, true\rangle))
$$

$$
\cfrac{M \to M'}{\pi_1(M) \to \pi_1(M')}\ E\text{-}\pi_1
\qquad
\cfrac{}{\pi_1(\langle V_1, V_2\rangle) \to V_1}\ E\text{-}\pi_1 PAR
$$

$$
\cfrac{M \to M'}{\pi_2(M) \to \pi_2(M')}\ E\text{-}\pi_2
\qquad
\cfrac{}{\pi_2(\langle V_1, V_2\rangle) \to V_2}\ E\text{-}\pi_2 PAR
$$

$$
\cfrac{M \to M'}{\langle M, N\rangle \to \langle M', N\rangle}\ E\text{-PAR1}
\qquad
\cfrac{N \to N'}{\langle V, N\rangle \to \langle V, N'\rangle}\ E\text{-PAR2}
$$

$$
\begin{array}{lcl}
\sigma & ::= & \dots \mid \sigma + \sigma \quad \text{"="} \quad \sigma, \tau ::= \dots \mid \sigma + \tau \\
M & ::= & \dots \mid \mathrm{left}_\sigma(\boxed{M}) \mid \mathrm{right}_\sigma(\boxed{M}) \mid \\
 & & \mathrm{case}\ \boxed{M}\ \mathrm{of}\ \mathrm{left}(x) \rightsquigarrow \boxed{M} \,\|\, \mathrm{right}(y) \rightsquigarrow \boxed{M}
\end{array}
$$

$$\frac{\Gamma \vdash M : \tau}{\Gamma \vdash \mathrm{left}_\sigma(M) : \tau + \sigma} \; \text{T-LEFT}$$

$\vdash \mathrm{left}(\mathrm{zero}) : \mathrm{Nat} + \bigcirc$?

$\vdash \mathrm{left}_{\underline{\underline{\mathrm{Bool}}}}(\mathrm{zero}) : \mathrm{Nat} + \underline{\underline{\mathrm{Bool}}}$

$$\frac{\Gamma \vdash M : \tau}{\Gamma \vdash \mathrm{right}_\sigma(M) : \sigma + \tau} \; \text{T-RIGHT}$$

$\vdash \mathrm{left}_{\mathrm{Nat} \to \mathrm{Bool}}(\mathrm{zero}) : \mathrm{Nat} + (\mathrm{Nat} \to \mathrm{Bool})$

$\vdash \mathrm{zero} : \mathrm{Nat}$

$$\frac{\Gamma, x:\sigma \vdash M : \tau}{\Gamma \vdash \lambda x:\sigma. M : \sigma \to \tau} \; \text{T-ABS}$$

$$\frac{\Gamma \vdash M : \sigma + \tau \qquad \Gamma, x:\sigma \vdash N : \rho \qquad \Gamma, y:\tau \vdash O : \rho}{\Gamma \vdash \mathrm{case}\ M\ \mathrm{of}\ \mathrm{left}(x) \rightsquigarrow N \ [\!]\ \mathrm{right}(y) \rightsquigarrow O : \rho} \; \text{T-CASE}$$

- $V ::= \dots \mid \mathrm{left}_\sigma(V) \mid \mathrm{right}_\sigma(V)$

true

$\mathrm{left}_{\mathrm{Bool}}(\mathrm{zero})$

$\mathrm{left}_{\mathrm{Bool}}(\mathrm{succ}(\mathrm{zero}))$

$\mathrm{left}_{\mathrm{Bool}}(\text{if true then } \underline{0} \text{ else } \underline{0})$

$\downarrow$

$\vdash \mathrm{left}_{\mathrm{Bool}}(\underline{0}) : \mathrm{Nat} + \mathrm{Bool}$

$\cancel{\downarrow} \qquad \not\vdash$

$\vdash \underline{0} : \mathrm{Nat}$

$$\frac{M \to M'}{\mathrm{left}_\sigma(M) \to \mathrm{left}_\sigma(M')} \; \text{E-LEFT}$$

$$\frac{M \to M'}{\mathrm{right}_\sigma(M) \to \mathrm{right}_\sigma(M')} \; \text{E-RIGHT}$$

$$\frac{M \to M'}{\begin{array}{c}\mathrm{case}\ M\ \mathrm{of}\ \mathrm{left}(x) \rightsquigarrow N \ [\!]\ \mathrm{right}(y) \rightsquigarrow O \\ \longrightarrow \\ \mathrm{case}\ M'\ \mathrm{of}\ \mathrm{left}(x) \rightsquigarrow N \ [\!]\ \mathrm{right}(y) \rightsquigarrow O\end{array}} \; \text{E-CASE}$$

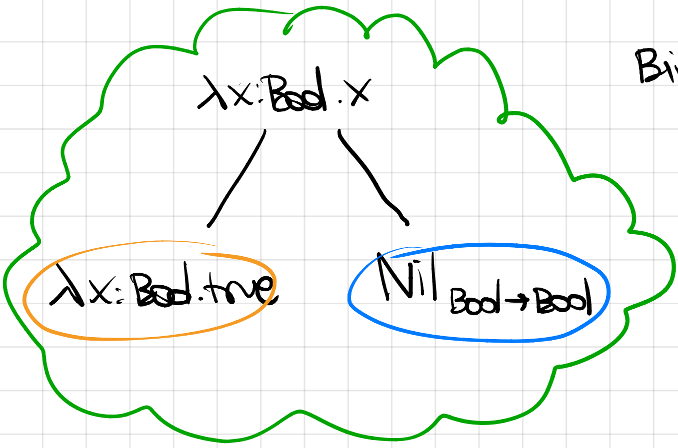
$$\frac{}{\text{case } \text{left}_\sigma(V) \text{ of } \text{left}(x) \rightsquigarrow N \;[]\; \text{right}(y) \rightsquigarrow O \longrightarrow N\{x := V\}}\ \text{E-CASELEFT}$$

$$\frac{}{\text{case } \underbrace{\text{right}_\sigma(V)}_{\sigma+\boxed{\tau}} \text{ of } \text{left}(x) \rightsquigarrow N \;[]\; \text{right}(y) \rightsquigarrow \underline{O} \longrightarrow \boxed{O\{y := V\}}}\ \text{E-CASERIGHT}$$

$$\frac{\Gamma \vdash M : \sigma+\tau \qquad \Gamma, x:\sigma \vdash N : \rho \qquad \Gamma, y:\tau \vdash O : \rho}{\Gamma \vdash \text{case } M \text{ of } \text{left}(x) \rightsquigarrow N \;[]\; \text{right}(y) \rightsquigarrow O : \rho}\ \text{T-CASE}$$

$\sigma ::= \cdots \mid AB_\sigma$

$M, N, O ::= \cdots \mid \text{Nil}_\sigma \mid \text{Bin}(M, N, O) \mid \text{raíz}(M) \mid \text{der}(M) \mid \text{izq}(M) \mid \text{esNil}(M)$



$$\text{Bin}(\underbrace{\text{Bin}(\text{Nil}_{B\to B}, \lambda x:\text{Bool}.\text{true}, \text{Nil}_{B\to B})}, \lambda x:\text{Bool}.x, \underbrace{\text{Nil}_{\text{Bool}\to\text{Bool}}})$$

$$\frac{}{\Gamma \vdash \text{Nil}_\sigma : AB_\sigma}\ \text{T-NIL}$$

$$\frac{\Gamma \vdash M : AB_\sigma \qquad \Gamma \vdash N : \sigma \qquad \Gamma \vdash O : AB_\sigma}{\Gamma \vdash \text{Bin}(M, N, O) : AB_\sigma}\ \text{T-BIN}$$

$\text{Nil}_{\text{Bool}} \neq \text{Nil}_{\text{Nat}}$

$$\frac{\Gamma \vdash M : AB_\sigma}{\Gamma \vdash \text{raíz}(M) : \sigma}\ \text{T-RAÍZ}$$

$$\frac{\Gamma \vdash M : AB_\sigma}{\Gamma \vdash \text{izq}(M) : AB_\sigma}\ \text{T-IZQ}$$

$$\frac{\Gamma \vdash M : AB_\sigma}{\Gamma \vdash \text{der}(M) : AB_\sigma}\ \text{T-DER}$$

$$\frac{\Gamma \vdash M : AB_\sigma}{\Gamma \vdash \text{esNil}(M) : \text{Bool}}\ \text{T-ESNIL}$$

$\sigma ::= \cdots \mid AB_\sigma$

$M, N, O ::= \cdots \mid \mathsf{Nil}_\sigma \mid \mathsf{Bin}(M, N, O) \mid \mathsf{raíz}(M) \mid \mathsf{der}(M) \mid \mathsf{izq}(M) \mid \mathsf{esNil}(M)$

$V ::= \cdots \mid \mathsf{Nil}_\sigma \mid \mathsf{Bin}(V, V, V)$

$$\frac{M \to M'}{\mathsf{Bin}(M,N,O) \to \mathsf{Bin}(M',N,O)}\ \text{E-BIN1}$$

$$\frac{N \to N'}{\mathsf{Bin}(V,N,O) \to \mathsf{Bin}(V,N',O)}\ \text{E-BIN2}$$

$$\frac{O \to O'}{\mathsf{Bin}(V_1,V_2,O) \to \mathsf{Bin}(V_1,V_2,O')}\ \text{E-BIN3}$$

$$\frac{M \to M'}{\mathsf{raíz}(M) \to \mathsf{raíz}(M')}\ \text{E-RAÍZ}$$

$$\frac{}{\mathsf{raíz}(\mathsf{Bin}(V_1,V_2,V_3)) \to V_2}\ \text{E-RAÍZBIN}$$

COMENTARIO

$\mathsf{raíz}(\mathsf{Nil}_\sigma) \not\to$

$\mathsf{izq}(\mathsf{Nil}_\sigma) \not\to$

$\mathsf{der}(\mathsf{Nil}_\sigma) \not\to$

perdemos progreso.

$$\frac{M \to M'}{\mathsf{izq}(M) \to \mathsf{izq}(M')}\ \text{E-IZQ}$$

$$\frac{}{\mathsf{izq}(\mathsf{Bin}(V_1,V_2,V_3)) \to V_1}\ \text{E-IZQBIN}$$

$$\frac{M \to M'}{\mathsf{esNil}(M) \to \mathsf{esNil}(M')}\ \text{E-ESNIL}$$

$$\frac{M \to M'}{\mathsf{der}(M) \to \mathsf{der}(M')}\ \text{E-DER}$$

$$\frac{}{\mathsf{der}(\mathsf{Bin}(V_1,V_2,V_3)) \to V_3}\ \text{E-DERBIN}$$

$$\frac{}{\mathsf{esNil}(\mathsf{Bin}(V_1,V_2,V_3)) \to \mathsf{false}}\ \text{E-ESNILBIN}$$

$$\frac{}{\mathsf{esNil}(\mathsf{Nil}_\sigma) \to \mathsf{true}}\ \text{E-ESNILNIL}$$

$\sigma ::= \cdots \mid AB_\sigma$

$M, N, O ::= \cdots \mid Nil_\sigma \mid \mathsf{Bin}(\boxed{M}, \boxed{N}, \boxed{O}) \mid$
$\mathsf{case}\ \boxed{M}\ \mathsf{of}\ Nil \rightsquigarrow \boxed{N}; \mathsf{Bin}(i, r, d) \rightsquigarrow \boxed{O}$

$$\frac{}{\Gamma \vdash Nil_\sigma : AB_\sigma}\ \text{T-NIL} \qquad \frac{\Gamma \vdash M : AB_\sigma \quad \Gamma \vdash N : \sigma \quad \Gamma \vdash O : AB_\sigma}{\Gamma \vdash \mathsf{Bin}(M, N, O) : AB_\sigma}\ \text{T-BIN}$$

$$\frac{\Gamma \vdash M : AB_\tau \qquad \Gamma \vdash N : \sigma \qquad \Gamma, i : AB_\tau, r : \tau, d : AB_\tau \vdash O : \sigma}{\Gamma \vdash \mathsf{case}\ M\ \mathsf{of}\ Nil \rightsquigarrow N; \mathsf{Bin}(i, r, d) \rightsquigarrow O : \sigma}\ \text{T-CASEAB}$$

- $V ::= \cdots \mid Nil_\sigma \mid \mathsf{Bin}(V, V, V)$ ✓
-

$$\frac{M \to M'}{\mathsf{Bin}(M, N, O) \to \mathsf{Bin}(M', N, O)}\ \text{E-BIN1} \qquad \frac{N \to N'}{\mathsf{Bin}(V, N, O) \to \mathsf{Bin}(V, N', O)}\ \text{E-BIN2}$$

$$\frac{O \to O'}{\mathsf{Bin}(V_1, V_2, O) \to \mathsf{Bin}(V_1, V_2, O')}\ \text{E-BIN3}$$

$$\frac{M \to M'}{\begin{array}{l}\mathsf{case}\ M\ \mathsf{of}\ Nil \rightsquigarrow N; \mathsf{Bin}(i, r, d) \rightsquigarrow O \\ \qquad \to \mathsf{case}\ M'\ \mathsf{of}\ Nil \rightsquigarrow N; \mathsf{Bin}(i, r, d) \rightsquigarrow O\end{array}}\ \text{E-CASEAB}$$

$$\frac{}{\mathsf{case}\ Nil_\sigma\ \mathsf{of}\ Nil \rightsquigarrow N; \mathsf{Bin}(i, r, d) \rightsquigarrow O \to N}\ \text{E-CASENIL}$$

$$\frac{}{\begin{array}{l}\mathsf{case}\ \mathsf{Bin}(V_1, V_2, V_3)\ \mathsf{of}\ Nil \rightsquigarrow N; \mathsf{Bin}(i, r, d) \rightsquigarrow O \\ \qquad \to O\{i := V_1, r := V_2, d := V_3\}\end{array}}\ \text{E-CASEBIN}$$

case if $(\lambda x : Bool.x)$ True then Bin(Nil$_{Nat}$, $\underline{1}$, Nil$_{Nat}$) else Nil$_{Nat}$
of $Nil \rightsquigarrow$ False ; Bin$(i, r, d) \rightsquigarrow$ iszero(r)

$\xrightarrow{\text{E-CASEAB, E-IF, } \beta}$ case if true then Bin(Nil$_{Nat}$, $\underline{1}$, Nil$_{Nat}$) else Nil$_{Nat}$ of
Nil $\rightsquigarrow$ False ;
Bin(i,r,d) $\rightsquigarrow$ iszero(r)

$$\frac{M \to M'}{\text{if } M \text{ then } N \text{ else } O \to \text{if } M' \text{ then } N \text{ else } O}\ \text{E-IF}$$

$$\frac{}{\text{if true then } N \text{ else } O \to N}\ \text{E-IFTRUE}$$

$$\frac{}{\text{if false then } N \text{ else } O \to O}\ \text{E-IFFALSE}$$

$\xrightarrow{\text{E-CASEAB, E-IFTRUE}}$ case Bin(Nil$_{Nat}$, $\underline{1}$, Nil$_{Nat}$) of
Nil $\rightsquigarrow$ False ;
Bin(i,r,d) $\rightsquigarrow$ iszero(r)

$\xrightarrow{\text{E-CASEBIN}}$ iszero(r) $\{i := \text{Nil}_{Nat}, r := \underline{1}, d := \text{Nil}_{Nat}\}$ = iszero($\underline{1}$) = iszero(succ(zero))

$\xrightarrow{\text{E-iszero}_n}$ false

$M, N, O ::= \cdots \mid Nil_\sigma \mid \text{Bin}(M, N, O) \mid$
case M of $Nil \rightsquigarrow N$; Bin$(i, r, d) \rightsquigarrow O$

$esNil_\sigma = \lambda x : AB_\sigma .$ case x of Nil $\rightsquigarrow$ true ; Bin(i,r,d) $\rightsquigarrow$ false

$raiz_\sigma = \lambda x : AB_\sigma .$ case x of Nil $\rightsquigarrow$ $\bot_\sigma$; Bin(i,r,d) $\rightsquigarrow$ r

usa Fix (Punto Fijo, 2da teórica)

$izq_\sigma = \lambda x : AB_\sigma .$ case x of Nil $\rightsquigarrow$ $\bot_{AB_\sigma}$; Bin(i,r,d) $\rightsquigarrow$ i

Definir una nueva extensión que incorpore expresiones de la forma $map(M, N)$, donde N es un árbol y M una función que se aplicará a cada uno de los elementos de N.

$M, N ::= \cdots \mid map(M, N)$

$$\frac{\Gamma \vdash M : \sigma \to \rho \qquad \Gamma \vdash N : AB_\sigma}{\Gamma \vdash map(M,N) : AB_\rho}\ \text{T-MAP}$$

- $V ::= \cdots \mid$ no se agregan valores.

$$\frac{M \to M'}{map(M,N) \longrightarrow map(M',N)} \text{ E-MAP1}$$

$$\frac{N \to N'}{map(V,N) \to map(V,N')} \text{ E-MAP2}$$

E-MAPNIL

$map(\lambda x:Bool.zero, Nil_{Bool}) \longrightarrow Nil_{Nat}$

$$\frac{\vdash V : \tau \to \rho}{map(V, Nil_\tau) \longrightarrow Nil_\rho} \text{ E-MAPNIL}$$

$\vdash \lambda x: Bool.zero : Bool \to Nat$

$$\frac{}{map(V, Bin(V_1, V_2, V_3)) \longrightarrow Bin(map(V, V_1), \; V \; V_2, \; map(V, V_3))} \text{ E-MAPBIN}$$