$\forall T1 .\forall T2 .\forall M.\forall N.((Tipo(M, T1 \to T2) \wedge Tipo(N, T1)) \implies Tipo(app(M, N), T2))$

$\forall T_1 \forall T_2 \forall M \forall N ((\neg (Tipo(M, T_1 \to T_2) \wedge Tipo(N, T_1)))$
$\vee Tipo(app(M,N), T_2))$

$\forall T_1 \forall T_2 \forall M \forall N (\neg Tipo(M, T_1 \to T_2) \vee \neg Tipo(N, T_1) \vee Tipo(app(M,N), T_2))$

1 _ $\{\neg Tipo(M, T_1 \to T_2), \neg Tipo(N, T_1), Tipo(app(M,N), T_2)\}$

$\exists M.Tipo(M, \alpha \to (\beta \to \gamma))$

$Tipo(Triple, \alpha \to (\beta \to \gamma))$

2 _ $\{Tipo(Triple, \alpha \to (\beta \to \gamma))\}$

$\exists M.Tipo(M, \alpha \to \beta)$

3 _ $\{Tipo(doble, \alpha \to \beta)\}$

4 _ $\{Tipo(simple, \alpha)\}$ $\exists M.Tipo(M, \alpha)$

$\exists M.Tipo(M, \gamma)$

$\neg \exists M. Tipo(M, \gamma)$

$\forall M. \neg Tipo(M, \gamma)$

5_ $\{\neg Tipo(M_5, \gamma)\}$ 4_ $\{Tipo(simple, \alpha)\}$

3_ $\{Tipo(doble, \alpha \to \beta)\}$ 2_ $\{Tipo(Triple, \alpha \to (\beta \to \gamma))\}$

1_ $\{\neg Tipo(M_1, T_1 \to T_2), \neg Tipo(N, T_1), Tipo(app(M_1, N), T_2)\}$

Con 1 y 4 $\sigma_6 = \{N := simple, T_1 := \alpha\}$

6_ $\{\neg Tipo(M_1, \alpha \to T_2), Tipo(app(M_1, simple), T_2)\}$

Con 3 y 6 $\sigma_7 = \{M_1 := doble, T_2 := \beta\}$

7_ $\{Tipo(app(doble, simple), \beta)\}$

Con 2 y 6 $\sigma_8 = \{T_2 := \beta \to \gamma, M_1 := Triple\}$

8_ $\{Tipo(app(triple, simple), \beta \to \gamma)\}$

Con 1 y 7 $\sigma_9 = \{N := app(doble, simple), T_1 := \beta\}$

9_ $\{\neg Tipo(M_1, \beta \to T_2), Tipo(app(M_1, app(doble, simple)), T_2)\}$

Con 8 y 9 $\sigma_{10} = \{M_1 := app(triple, simple), T_2 := \gamma\}$

10_ $\{Tipo(app(app(triple, simple), app(doble, simple)), \gamma)\}$

Con 5 y 10 $\sigma_{11} = \{M_5 := app(app(triple, simple), app(doble, simple))\}$

$\square$

$\text{foldr } (\lambda x.\text{succ}(x)) :: [\,]\ \text{base} \hookrightarrow [\,];\ \text{rec}(x, r) \hookrightarrow \text{if } p\ x \text{ then } x :: r \text{ else } r$

1) $\text{foldr } (\lambda x.\text{succ}(x)) :: [\,]\ \text{base} \hookrightarrow [\,]; \text{rec}(h, r) \hookrightarrow \text{if } p\ h \text{ then } h :: r \text{ else } r$

2) $\Gamma_0 = \{ p : X_1 \}$

$\Gamma_0 = \text{foldr } (\lambda x : X_2 . \text{succ}(x)) :: [\,]_{X_3}\ \text{base} \hookrightarrow [\,]_{X_4}; \text{rec}(h, r) \hookrightarrow \text{if } p\ h \text{ then } h :: r \text{ else } r$

3)

$$\Big(\overbrace{\{p:X_1\}}^{\Gamma} \;\Big|\; \text{foldr } (\lambda x:X_2.\, \text{succ}(x))::[\,]_{X_3} \text{ base} \Rightarrow [\,]_{X_4};\ \text{rec}(h,r) \Rightarrow \text{if } p\ h \text{ then } h::r \text{ else } r\Big)$$

$$= \big([X_4] \;\big|\; E1 \cup E2 \cup \{[X_3] \doteq [X_h],\ [X_4] \doteq X_r,\ X_r \doteq X_r\}\big)$$

$$\big(\Gamma \mid (\lambda x:X_2.\,\text{succ}(x))::[\,]_{X_3}\big) = \big([X_3] \mid \underbrace{\{[X_3] \doteq [X_2 \to N],\ X_2 \doteq N\}}_{E1}\big)$$

$$\big(\Gamma \mid [\,]_{X_4}\big) = \big([X_4] \mid \emptyset\big)$$

$$\big(\overbrace{\{p:X_1,\ h:X_h,\ r:X_r\}}^{\Gamma'} \mid \text{if } p\ h \text{ then } h::r \text{ else } r\big) = \big(X_r \mid \overbrace{\{X_5 \doteq Bool,\ X_r \doteq X_r,\ X_1 \doteq X_h \to X_5,\ X_r \doteq [X_h]\}}^{E2}\big)$$

$$\big(\Gamma \mid \lambda x:X_2.\,\text{succ}(x)\big) = \big(X_2 \to N \mid \{X_2 \doteq N\}\big)$$

$$\big(\Gamma \mid [\,]_{X_3}\big) = \big([X_3] \mid \emptyset\big)$$

$$\big(\{p:X_1, x:X_2\} \mid \text{succ}(x)\big) = \big(Nat \mid \{X_2 \doteq Nat\}\big)$$

$$\big(\{p:X_1, x:X_2\} \mid x\big) = \big(X_2 \mid \emptyset\big)$$

$$\big(\Gamma' \mid p\ h\big) = \big(X_5 \mid \{X_1 \doteq X_h \to X_5\}\big)$$

$$\big(\Gamma' \mid p\big) = \big(X_1 \mid \emptyset\big) \qquad \big(\Gamma' \mid h\big) = \big(X_h \mid \emptyset\big)$$

$$\big(\Gamma' \mid h::r\big) = \big(X_r \mid \{X_r \doteq [X_h]\}\big)$$

$$\big(\Gamma' \mid h\big) = \big(X_h \mid \emptyset\big) \qquad \big(\Gamma' \mid r\big) = \big(X_r \mid \emptyset\big)$$

$$\big(\Gamma' \mid r\big) = \big(X_r \mid \emptyset\big)$$

$$S = \{X_4 := N \to N,\ X_r := [N \to N],\ X_1 := (N \to N) \to B,\ X_h := N \to N,\ X_3 := N \to N,\ X_5 := B,\ X_2 := N\}$$

$$\boxed{\{p:(N \to N) \to B\} \vdash \text{foldr } (\lambda x:N.\,\text{succ}(x))::[\,]_{N \to N} \text{ base} \Rightarrow [\,]_{N \to N};\ \text{rec}(h,r) \Rightarrow \text{if } p\ h \text{ then } h::r \text{ else } r : [N \to N]}$$

$$S = mgu\left(\{ [X_3] \doteq [X_2 \to N],\ X_2 \doteq N,\ X_1 \doteq X_w \to X_5,\ X_r \doteq [X_w],\ X_5 \doteq Bool,\ \cancel{X_r \doteq X_r},\ [X_3] \doteq [X_w],\ [X_4] \doteq X_r,\ \cancel{X_r \doteq X_r} \}\right)$$

$$\xrightarrow[\{X_5 := B,\ X_2 := N\}]{} \{ [X_3] \doteq [N \to N],\ X_1 \doteq X_w \to B,\ X_r \doteq [X_w],\ [X_3] \doteq [X_w],\ [X_4] \doteq X_r \}$$

$$\xrightarrow[\{X_3 := N \to N\}]{} \{ X_1 \doteq X_w \to B,\ X_r \doteq [X_w],\ \underbrace{[N \to N] \doteq [X_w]},\ [X_4] \doteq X_r \}$$

$$\xrightarrow[\{X_w := N \to N\}]{} \{ X_1 \doteq (N \to N) \to B,\ X_r \doteq [N \to N],\ [X_4] \doteq X_r \}$$

$$\xrightarrow[\{X_1 := (N \to N) \to B,\ X_r := [N \to N]\}]{} \{ [X_4] \doteq [N \to N] \} \xrightarrow[\{X_4 := N \to N\}]{} \square$$

$$S = \{ X_4 := N \to N,\ X_r := [N \to N],\ X_1 := (N \to N) \to B,\ X_w := N \to N,\ X_3 := N \to N,\ X_5 := B,\ X_2 := N \}$$

$$\{ X_1 := \overset{N}{\cancel{X_2}} \to B,\ X_2 := N \}$$

b) Demostrar el siguiente teorema usando deducción natural, sin utilizar principios clásicos:

$$(\exists X.P(X)) \Rightarrow (\forall Y.(P(Y) \Rightarrow Q(Y))) \Rightarrow \exists Z.Q(Z)$$

$$\dfrac{}{\Gamma, P(x) \vdash \forall y\,(P(y) \Rightarrow Q(y))}\ \text{Ax}$$

$$\dfrac{\Gamma, P(x) \vdash \forall y\,(P(y) \Rightarrow Q(y))}{\Gamma, P(x) \vdash P(x) \Rightarrow Q(x)}\ \forall e \qquad \dfrac{}{\Gamma, P(x) \vdash P(x)}\ \text{Ax}$$

$$\dfrac{\Gamma, P(x) \vdash P(x) \Rightarrow Q(x) \qquad \Gamma, P(x) \vdash P(x)}{\Gamma, P(x) \vdash Q(x)}\ \Rightarrow e$$

$$\dfrac{\Gamma, P(x) \vdash Q(x) \quad \{Z := x\}}{\Gamma, P(x) \vdash \exists z.Q(z)}\ \exists i$$

$$\dfrac{}{\Gamma \vdash \exists x\, P(x)}\ \text{Ax}$$

$$\dfrac{\Gamma \vdash \exists x\, P(x) \qquad \Gamma, P(x) \vdash \exists z.Q(z)}{\Gamma\{\exists x.P(x), \forall y(P(y) \Rightarrow Q(y)) \vdash \exists z.Q(z)}\ \exists e$$

$$\dfrac{\exists x.P(x), \forall y(P(y) \Rightarrow Q(y)) \vdash \exists z.Q(z)}{\exists x.P(x) \vdash (\forall y.(P(y) \Rightarrow Q(y))) \Rightarrow (\exists z.Q(z))}\ \Rightarrow i$$

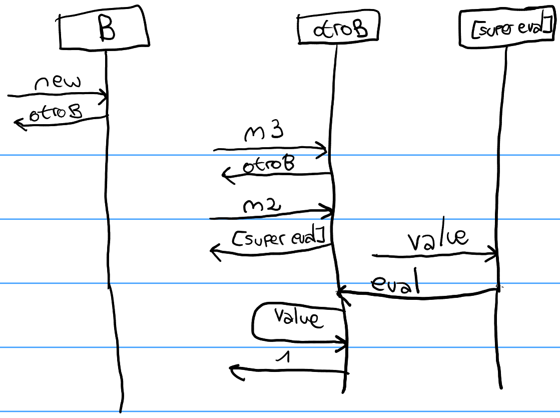
$$\dfrac{\exists x.P(x) \vdash (\forall y.(P(y) \Rightarrow Q(y))) \Rightarrow (\exists z.Q(z))}{\vdash (\exists x.P(x)) \Rightarrow ((\forall y.(P(y) \Rightarrow Q(y))) \Rightarrow \exists z.Q(z))}\ \Rightarrow i$$

B new m1 value

Receptor	mensaje	clase	resultado
B	new	Behavior	unB (instancia de B)
unB	m1	A	[self eval]
[self eval]	value	Constant Block Closure	2
unB	eval	B	2

B new m3 m2 value

Obj	mensaje	clase	Res
B	new	Behavior	otroB
otroB	m3	B	otroB
otroB	m2	B	[super eval]
[super eval]	value	Block Closure	1
otroB	eval	A	1
otroB	value	B	1



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