Coherent Control of Quantum Fluctuations Using Cavity Electromagnetically Induced Transparency

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(Received 15 May 2013; published 11 September 2013)

We study the all-optical control of the quantum fluctuations of a light beam via a combination of single-atom cavity quantum electrodynamics (CQED) and electromagnetically induced transparency (EIT). Specifically, the EIT control field is used to tune the CQED transition frequencies in and out of resonance with the probe light. In this way, photon blockade and antiblockade effects are employed to produce sub-Poissonian and super-Poissonian light fields, respectively. The achievable quantum control paves the way towards the realization of a prototype of a novel quantum transistor which amplifies or attenuates the relative intensity noise of a light beam. Its feasibility is demonstrated by calculations using realistic parameters from recent experiments.

DOI: 10.1103/PhysRevLett.111.113602 PACS numbers: 42.50.Pq, 42.50.Gy, 42.50.Lc

Quantum engineering aims to develop new concepts and tools for the controlled application of quantum phenomena, in particular, at the most fundamental level of single particles. In optical physics, it is cavity quantum electrodynamics (CQED) which provides the ideal setting to engineer the interaction between individual quanta of light and matter. Indeed, the large optical nonlinearity provided by a single emitter strongly coupled to a high-finesse cavity allows for novel tasks ranging from quantum-coherent networking [1–3] to quantum simulation [4,5]. Exchanging quantum states between quantum network nodes, and generating a continuous photon stream which reflects the steady-state particle distribution of a simulated quantum system, respectively, requires the development of increasingly more powerful control capabilities for genuine quantum systems.

Here, we theoretically demonstrate that by means of a single three-level atom in a high-finesse cavity it is possible to optically control the quantum fluctuations of a probe beam via a combination of electromagnetically induced transparency (EIT) [6–8] and photon blockade in CQED [9–12]. Exploiting the anharmonic energy-level structure of the atom-cavity system [13], an incoming laser beam is controllably converted into an outgoing light field with photon number fluctuations above or below the shot-noise level, depending on the control-laser parameters.

Our system can be considered a step towards a quantum transistor which amplifies or attenuates the relative intensity noise of a light field, in contrast to a classical transistor which controls the average intensity. Such a concept of a quantum transistor extends the well-known definition of a classical transistor into the quantum domain where the unconditional amplification of quantum states is forbidden by the no-cloning theorem [14].

We consider a single atom with three energy levels in Λ configuration [see Figs. 1(a) and 1(b)] strongly coupled to a single electromagnetic mode of frequency ω_{Cav} of a high-finesse optical resonator. The cavity is driven by a coherent field (probe) of strength ε and frequency ω_P. The atomic transitions $|1\rangle \leftrightarrow |3\rangle$ (frequency ω_{31}) and $|2\rangle \leftrightarrow |3\rangle$ (frequency ω_{32}) are coupled by the quantum cavity mode with vacuum Rabi frequency $2g$ and by a classical field (control), with frequency ω_C and Rabi frequency $2\Omega_C$, respectively.

Introducing the electric dipole and rotating-wave approximations, the time-independent Hamiltonian which describes the atom-field coupling in a rotating frame is given by

$$H = \Delta_1\sigma_{33} + (\Delta_1 - \Delta_2)\sigma_{22} - \Delta\sigma_{11} + \Delta a^\dagger a + (\varepsilon a + g a\sigma_{31} + \Omega_C\sigma_{32} + \mathrm{H.c.}),$$

where a, $a^\dagger$ are photon annihilation and creation operators, $\sigma_{ij} = |i\rangle\langle j|$, $i, j = 1, 2, 3$ are the atomic raising and lowering operators ($i \neq j$), and atomic energy-level population operators ($i = j$). The detunings are given by $\Delta = \omega_{\mathrm{Cav}} - \omega_P$, $\Delta_1 = \omega_{31} - \omega_{\mathrm{Cav}}$, and $\Delta_2 = \omega_{32} - \omega_C$. The dynamics of the system is obtained numerically by solving the master equation for the atom-cavity density operator:

$$\dot\rho = -i[H, \rho] + \kappa(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a) + \sum_{m=1,2}\Gamma_{3m}(2\sigma_{m3}\rho\sigma_{3m} - \sigma_{33}\rho - \rho\sigma_{33}),$$

where κ is the cavity-field decay rate and Γ_{31}, Γ_{32} the polarization decay rates of the excited level $|3\rangle$ to the levels $|1\rangle$ and $|2\rangle$, respectively. We solve for the steady state of ρ by truncating the Fock basis of the cavity field according to the probe strength following the method presented in Ref. [15]. We study the system in the strong-coupling regime $g \gg (\kappa, \Gamma_{ij})$ while driving it with a probe strength

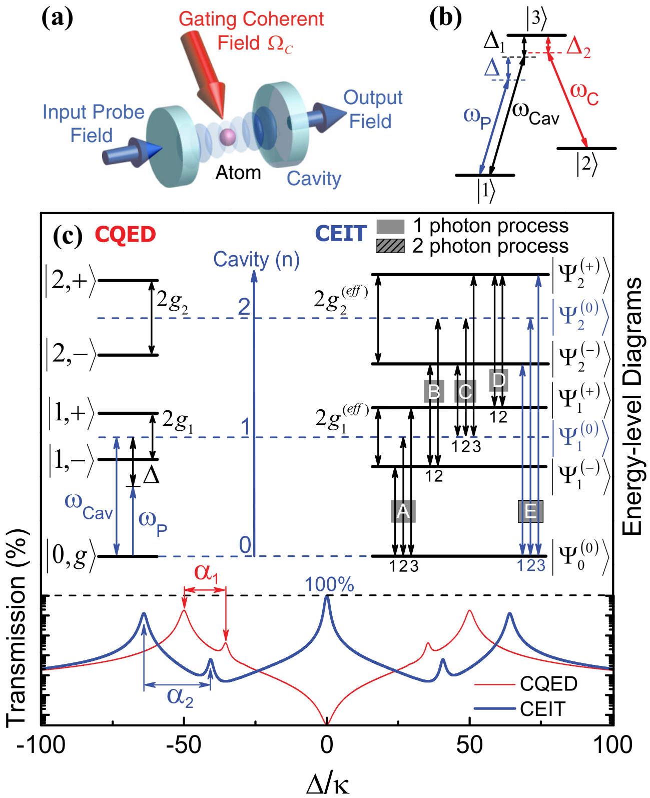

FIG. 1 (color online). (a) Prototype of a quantum transistor for noise control. (b) Three-level atom in Λ configuration showing the cavity (ω_{Cav}) and control (ω_C) fields coupling the transitions $|1\rangle \leftrightarrow |3\rangle$ and $|2\rangle \leftrightarrow |3\rangle$, respectively, and all relevant detunings. (c) Transmission spectra (on a logarithmic scale) vs normalized probe-cavity detuning Δ/κ for the two-level system (CQED, narrow solid red line) using the parameters $g = 50\kappa$, $\varepsilon = 1\kappa$, $\Gamma = \Gamma_{31} + \Gamma_{32} = 0.2\kappa$. For the three-level system (CEIT, broad solid blue line) the parameters are $g = 50\kappa$, $\varepsilon = 1\kappa$, $\Gamma_{ij} = 0.1\kappa$, and $\Omega_C = 40\kappa$. The left and right insets show the atom-cavity eigenstates structure for the two- and three-level configurations, with $g_n = \sqrt{n}g$ and $g_n^{(\mathrm{eff})} = \sqrt{ng^2 + \Omega_C^2}$. The manifolds A, B, C, and D indicate one-photon transitions while the manifold E marks two-photon transitions.

$\varepsilon = 1.0\kappa$, enough to sufficiently populate the energy-levels associated with multiphoton transitions [13].

Before showing how the considered system can be used to coherently modify the photon statistics of the probe beam, it is instructive to emphasize the differences between a cavity EIT (CEIT) system and the standard two-level CQED configuration. Considering perfect cavity and control field resonance [$\Delta_1 = \Delta_2 = 0$ in Fig. 1(b)] we calculate the relative transmission spectrum for both cases. For the CQED system [left inset and red-solid line in Fig. 1(c)], we observe a pronounced vacuum Rabi splitting with side peaks at smaller detunings corresponding to two-photon transitions $|0, g\rangle \leftrightarrow |2, \pm\rangle$ [13] between two different Jaynes-Cummings manifolds. All these transitions occur at different frequencies due to the anharmonic energy-level splitting, $E_n^{(+)} - E_n^{(-)} = 2\sqrt{n}g$, $n = 1, 2, \ldots$ [16]. The presence of EIT now induces an Autler-Townes-like effect which splits the state $|3\rangle$ into a pair of symmetric states given by $|\pm\rangle = (|2\rangle \pm |3\rangle)/\sqrt{2}$ with energies shifted by $\pm\Omega_C$, coupled to the ground state $|1\rangle$.

Rewriting the CEIT Hamiltonian in this new basis $\{|1\rangle, |-\rangle, |+\rangle\}$ and for $\varepsilon = \Delta = 0$, yields the following eigenstates:

$$|\Psi_n^{(0)}\rangle = N_n^{(0)}[|1, n\rangle - \eta(|+, n-1\rangle + |-, n-1\rangle)], \quad (1a)$$

$$|\Psi_n^{(\pm)}\rangle = N_n^{(\pm)}[|1, n\rangle + \lambda_\pm|+, n-1\rangle - \beta_\pm|-, n-1\rangle], \quad (1b)$$

with $\eta = g\sqrt{n/2}/\Omega_C$, $\lambda_\pm = g\sqrt{n/2}/(E_n^{(\pm)} - \Omega_C)$, $\beta_\pm = g\sqrt{n/2}/(E_n^{(\pm)} + \Omega_C)$, and $N_n^{(0,\pm)}$ denoting normalization factors. The eigenstates $|\Psi_n^{(0)}\rangle$, $n = 0, 1, 2, \ldots$ (or any combination of them) with eigenvalues $E_n^{(0)} = 0$ are the intracavity dark states of the CEIT system causing an empty-cavity-like transmission. Additionally, the dressed states $|\Psi_n^{(\pm)}\rangle$, with eigenvalues $E_n^{(\pm)} = \pm\sqrt{ng^2 + \Omega_C^2}$, represent a CEIT atom-cavity system sharing n excitations. Similarly to the CQED system, they compose an anharmonic ladder structure with energy-level splitting $E_n^{(+)} - E_n^{(-)} = 2g_n^{(\mathrm{eff})} = 2\sqrt{ng^2 + \Omega_C^2}$ [right inset in Fig. 1(c)].

These new dressed states can be addressed by adjusting the probe-cavity detuning $\Delta_n^{(\pm)} = \pm\sqrt{ng^2 + \Omega_C^2}/n$. In Fig. 1(c) (broad solid blue line) we show the consequences of this new level structure on the average transmission of the cavity, using the same set of parameters as in the two-level configuration and $\Omega_C = 40\kappa$. We distinguish the shifted vacuum Rabi-splitting (corresponding to $\Delta_1^{(\pm)}$) and a second resonance corresponding to the first multiphoton transition $|\Psi_0^{(0)}\rangle \leftrightarrow |\Psi_2^{(\pm)}\rangle$. In addition, we see a narrow transmission window at zero detuning referred to as the intracavity dark state [6,17,18]. We notice that the frequency difference between the first and second doublets $|1, -\rangle(|\Psi_1^{(-)}\rangle)$ and $|2, -\rangle(|\Psi_2^{(-)}\rangle)$ for the CEIT case is always larger than its CQED counterpart, as $\alpha_1 = g(1 - 1/\sqrt{2}) < \alpha_2 = \sqrt{g^2 + \Omega_C^2} - \sqrt{2g^2 + \Omega_C^2}/2$ for any $\Omega_C \neq 0$.

To characterize the quantum nature of the studied system, we now evaluate equal-time photon-photon correlations, $g^{(2)}(0) = \langle a^\dagger a^\dagger a a\rangle/\langle a^\dagger a\rangle^2$, which were calculated numerically for the steady state $\dot{\rho} = 0$ of the density operator of the atom-cavity system. In Fig. 2(a), we compare the $g^{(2)}(0)$ correlation for the CQED and CEIT cases as a function of the normalized probe-cavity detuning Δ/κ. The minima in $g^{(2)}(0)$ for the CEIT case are associated with the transitions $A(1)$, $A(3)$ (one photon) and $E(1)$, $E(3)$

In EIT case, notice that only the ground states are populated at Δ=as EIT prohibits state excitations ---->

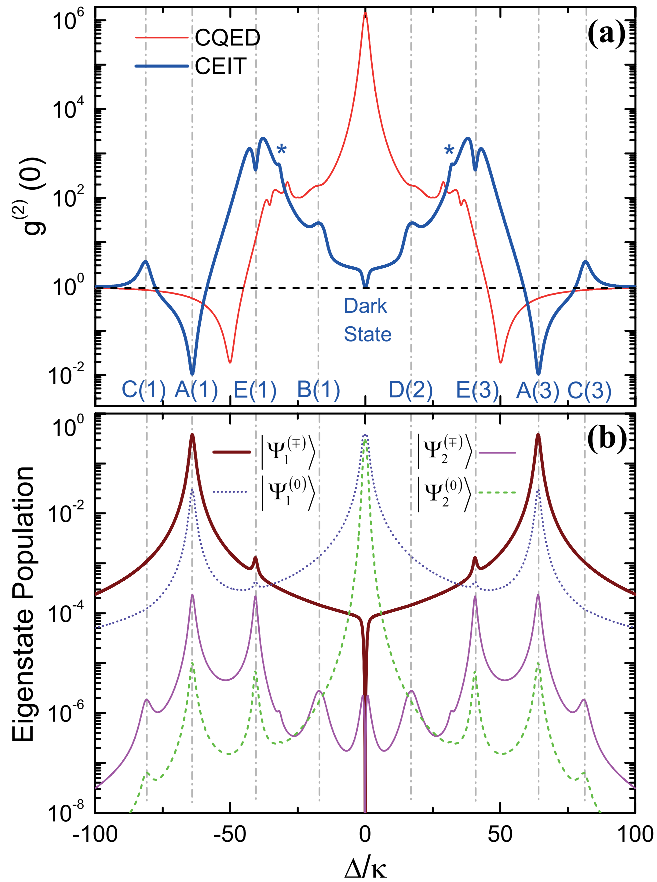

FIG. 2 (color online). (a) $g^{(2)}(0)$ correlation vs normalized probe-cavity detuning Δ/κ for the CQED case with $g = 50\kappa$, $\varepsilon = 1\kappa$, and $\Gamma = 0.2\kappa$ (red line) and for the CEIT configuration for the same g and ε, $\Gamma_{31} = \Gamma_{32} = 0.1\kappa$ and $\Omega_C = 40\kappa$. (b) Population of the CEIT eigenstates $|\Psi_{1,2}^{(-)}\rangle$ for $\Delta < 0$, $|\Psi_{1,2}^{(+)}\rangle$ for $\Delta > 0$, and respective dark states $|\Psi_{1,2}^{(0)}\rangle$. The vertical dashed-dotted lines connect the maxima and minima in the $g^{(2)}(0)$ correlation with the populations of the relevant eigenstates.

(two-photon) [see right-inset in Fig. 1(c)] akin to our observations in the transmission spectrum. The correlations, however, also show signatures of other transitions. For example, the maxima are associated with the one-photon transitions $C(1)$, $C(3)$, $B(1)$, $D(2)$ and the multi-photon transition $|\Psi_0^{(0)}\rangle \leftrightarrow |\Psi_3^{(\pm)}\rangle$ [marked with asterisks in Fig. 2(a)]. Another key aspect of the CEIT system is the presence of a coherent field [$g^{(2)}(0) = 1$] at $\Delta = 0$ [horizontal dashed line in Figs. 2(a), 3(a), and 4] and photon-bunching around $\Delta = \pm 80\kappa$. This is in stark contrast to the strong photon-bunching behavior [$g^{(2)}(0) \gg 1$] [19] and an almost coherent field for the respective detunings in the CQED case. The behavior of $g^{(2)}(0)$ is strongly correlated to the maxima in the frequency spectrum of the eigenstates populations [plotted in Fig. 2(b); see vertical dashed-dotted lines]. A prominent feature is the improvement of the maximum achievable photon antibunching (photon

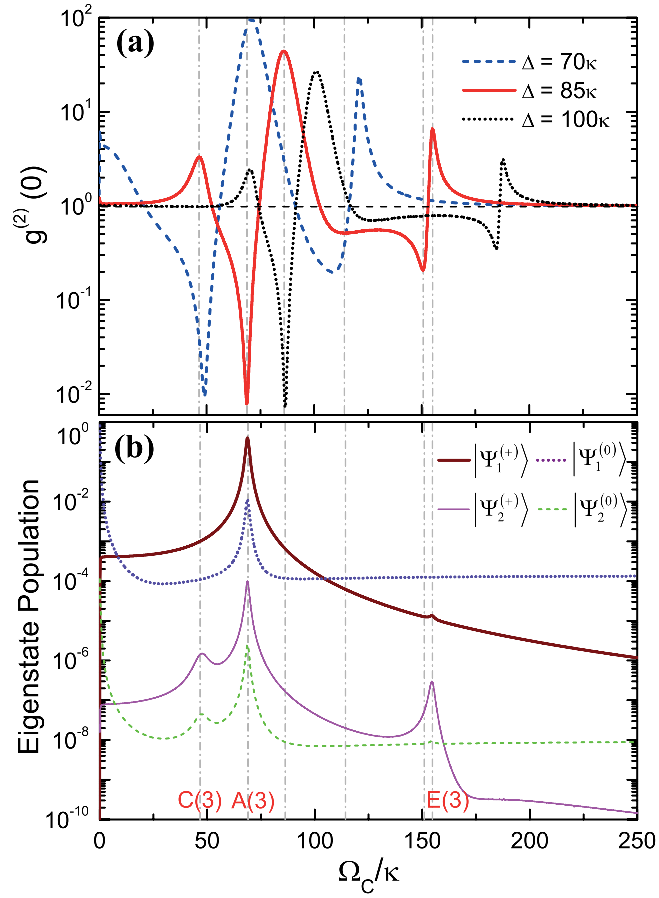

FIG. 3 (color online). (a) $g^{(2)}(0)$ correlation vs normalized Rabi frequency Ω_C/κ of the control field for the CEIT system for different probe-cavity detunings Δ using the same set of parameters as in Fig. 2. (b) Population of the CEIT eigenstates $|\Psi_{1,2}^{(+)}\rangle$ and respective dark-states $|\Psi_{1,2}^{(0)}\rangle$ for $\Delta = 85\kappa$. The vertical dashed-dotted lines connect the features in the $g^{(2)}(0)$ correlation with the populations of the relevant eigenstates.

blockade) due to the larger frequency difference between the first and second doublets in the CEIT system (thus, decreasing the probability of exciting the second manifold), as compared to the CQED case.

So far we have shown that it is possible to tune the photon statistics of the outgoing field by changing the frequency of the input field. In order to design a quantum control device, this possibility has to be provided by an external control field. This is demonstrated in Fig. 3(a) where the $g^{(2)}(0)$ correlation is plotted against the normalized Rabi frequency of the control field, Ω_C/κ, for different probe-cavity detunings. From the CEIT system level-structure [right-inset in Fig. 1(c)] it can be seen that varying the control-field Rabi frequency changes the frequency of transition resonances. When keeping Δ constant, we, thus, probe the system over regions in which sub-Poissonian [$g^{(2)}(0) < 1$] and super-Poissonian [$g^{(2)}(0) > 1$] light are created. This is, to our knowledge, the first time optical control of photon statistics is predicted, a unique result provided by the strongly coupled single-atom CEIT system. Similar to the analysis of Fig. 2(a), the first (left-most) super-Poissonian peak seen in Fig. 3(a) for each detuning Δ is associated with a local maximum in the population of the second dark state $|\Psi_2^{(0)}\rangle$ and the multiphoton state $|\Psi_2^{(+)}\rangle$, corresponding to transition $C(3)$, as depicted in Fig. 3(b) for $\Delta = 85\kappa$ (see vertical dashed-dotted lines).

When further increasing Ω_C the first and most pronounced minimum of $g^{(2)}(0)$ is obtained when the transition $A(3)$, $|\Psi_0^{(0)}\rangle \leftrightarrow |\Psi_1^{(+)}\rangle$, is resonantly driven by the probe field, resulting in sub-Poissonian light as a consequence of single-photon blockade. Once this resonance condition is violated for some larger Ω_C, a super-Poissonian field is generated [19]. We emphasize that in contrast to the first super-Poissonian peak discussed above no maxima are observed in any of the eigenstate populations. A similar scenario occurs when the control Rabi frequency is increased to a value $\Omega_C = 150\kappa$ (for $\Delta = 85\kappa$). Here a pronounced sub-Poissonian minimum is observed with $g^{(2)}(0) = 0.2$, without any maxima in the eigenstate populations. The situation changes when Ω_C is slightly increased so that the two-photon transition $E(3)$, $|\Psi_0^{(0)}\rangle \leftrightarrow |\Psi_2^{(+)}\rangle$, is resonantly driven [13], as shown by the local maximum in the population of state $|\Psi_2^{(+)}\rangle$ at $\Omega_C = 155\kappa$ (for $\Delta = 85\kappa$). But, opposite to what happens on the single-photon resonance $A(3)$, $|\Psi_0^{(0)}\rangle \leftrightarrow |\Psi_1^{(+)}\rangle$, we now observe super-Poissonian statistics. The change from sub-Poissonian to super-Poissonian statistics occurs over a very small range of the control field, between $\Omega_C = 150\kappa$ and $\Omega_C = 155\kappa$ (for $\Delta = 85\kappa$). This demonstrates the unique tuning possibility offered by the control field. Note that if only two-photon transitions would be allowed, the light would exhibit sub-Poissonian statistics. The observed super-Poissonian statistics on the two-photon resonance results from the presence of single-photon transitions both in the excitation and the deexcitation of the system. Another remarkable characteristic of our system is the presence of a broad region with $g^{(2)}(0) < 1$, independently of Ω_C. The width of this plateau region can be extended by increasing the probe-cavity detuning Δ, although at the expense of a less pronounced sub-Poissonian statistics.

In all cases discussed so far, truly strong light-matter coupling was assumed for the achievement of sub-Poissonian fields. As shown above, the introduction of CEIT relaxes this constraint. In fact, our calculations show that it is possible to obtain similar optical control as presented in Fig. 3(a) when using a realized set of parameters from recent CQED experiments, $\{g, \kappa, \Gamma_{ij}\}/2\pi = \{12, 1.5, 1.5\}$ MHz [20] (see Fig. 4). This shows the feasibility of the proposed quantum device using currently available technology.

In summary, we have studied a novel quantum system based upon the faithful combination of EIT and CQED.

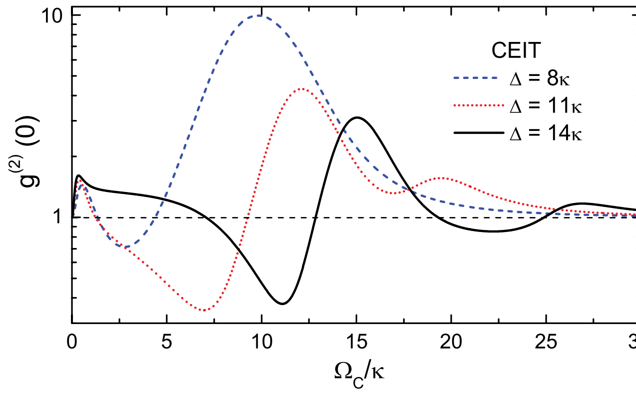

FIG. 4 (color online). $g^{(2)}(0)$ correlation vs normalized Rabi frequency Ω_C/κ of the control field for the CEIT system using a realized set of parameters $g = 8\kappa$, $\varepsilon = 1\kappa$, and $\Gamma_{31} = \Gamma_{32} = 1\kappa$ for different probe-cavity detunings Δ.

We have shown that the photon-photon correlation features more structure than the transmission and, thus, provides a deeper insight into the system. Moreover, the ability to optically control the quantum fluctuations of a light beam makes our system a crucial step towards the realization of a quantum transistor which amplifies or attenuates the relative quantum noise.

Several applications could be envisioned. For example, our CEIT system with its ability to continuously produce correlated photons could be integrated into a network of cascaded atom-cavity systems, with the goal to control the predicted phase transition of light into an ordered Mott-insulator-like state of photons [21–23]. Further avenues include extending our analysis to configurations in which both the probe and the control field are quantized, thus, providing optical control using single photons as "gate" fields. This might allow one to realize photonic quantum gates in which the interaction between photons is controlled with single atoms.

We acknowledge fruitful discussions with S. Ritter and S. Rebic. This work was supported by the Deutsche Forschungsgemeinschaft [Research Unit 635] and the European Union [IST program AQUTE]. J. A. S. and C. J. V.-B. acknowledge support from CAPES (Project Grant No. 2496/11-9), CNPq, FAPESP (Project Grants No. 2010/17609-0, No. 2012/00176-9, and No. 2013/01182-5), and the Brazilian National Institute for Science and Technology of Quantum Information (INCT-IQ). E. F. acknowledges support from the Alexander von Humboldt Foundation.

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