Set Theory

A collection of elements → is a set.

- $\in$ (Belongs to)

$A = \{1, 2, \ldots\ldots\}$ $\qquad$ $B = \{1, 2, 3\}$

$1 \in A, B$

$4 \notin B$

Relations

A: names of students of class III CSE II

$= \{a_1, a_2, a_3, \ldots\ldots, a_{110}\}$

B: courses of III CSE II

$= \{CSE\ 121, CSE\ 122, CSE\ 123\}$

$A\ R\ B$ $\qquad$ or $\qquad$ $A \not R\ B$

$a_1\ R\ CSE\ 121$

$C = \{CSE\ 211, CSE\ 222\}$

$a_1 \not R\ CSE\ 211$

If sets are related then each element of each set is related to one another.

- Cartesian Product of 2 sets :

$A \times B$: is the set of all ordered pairs of the form (a,b) where $a \in A$ and $b \in B$.

Example :-

$A = \{a, b\}$ $\quad$ $B = \{c, d\}$

$A \times B = \{(a,c), (a,d), (b,c), (b,d)\}$

* For 3 sets - (a, b, c)

- Binary relation -

If R is a binary relation from A to B & if the ordered pair (a,b) is in R, then we would say that the element 'a' is related to element 'b'.

A : set of elements $= \{a, b, c, d\}$ $\quad$ B : set of courses $= \{CSE 121, CSE 122, CSE 123\}$

$R = \{(a, CSE 121), (b, CSE 122), \ldots\}$ (opted courses)

$T = \{(a, CSE 123), (b, CSE 121), \ldots\}$ (difficult courses)

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Tabular form and Graphical form –

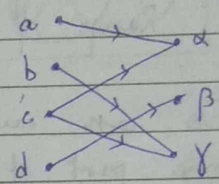
$A = \{a, b, c, d\}$ $\quad$ $B = \{\alpha, \beta, \gamma\}$

$R : A \rightarrow B$

$R = \{ (a,\alpha), (b,\gamma), (c,\alpha), (c,\gamma), (d,\beta) \}$

R	α	β	γ
a	✓		
b			✓
c	✓		✓
d		✓	

Tabular form



Graphical form

• Domain and Range –

The domain of a relation R is the set of all first elements of the ordered pair which belongs to R and the range of R is the set of second elements.

Let $A = \{1, 2, 3\}$ $\quad$ $B = \{x, y, z\}$

$R = \{(1,y), (1,z), (3,y)\}$

Domain of $R = \{1, 3\}$

Range of $R = \{y, z\}$

Q/ $A = \{eggs, milk, corn\}$
$B = \{cows, goats, hens\}$
$R : A \to B$; $(a,b) \in R$ if a is produced by b

- $R = \{(eggs, hens), (milk, cows), (milk, goats)\}$
  Domain of $R = \{eggs, milk\}$
  Range of $R = \{hens, cows, goats\}$

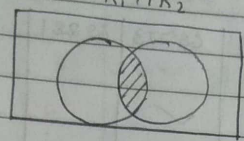
We can say 2 countries are adjacent if they have some part of their boundaries in common. Then "is adjacent to" is a relation R on the countries on the Earth.

(Italy, Switzerland) $\in R$
(Canada, Mexico) $\notin R$

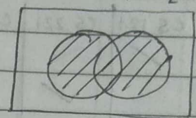
- Suppose R_1 & R_2 : Binary relations from set A to set B. then,

  $R_1 \cap R_2$: Intersection
  $R_1 \cup R_2$: Union
  $R_1 \oplus R_2$: Symmetric Difference.
  $R_1 - R_2$: Difference

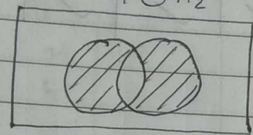
$R_1 \cap R_2$



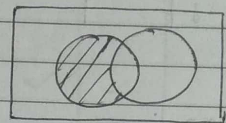
$R_1 \cup R_2$



$R_1 \oplus R_2$



$R_1 - R_2$



For example:

$A = \{a, b, c, d\}$: set of students

$B = \{CS121, CS221, CS251, CS264, CS273, CS281\}$: set of courses

R_1 : Binary rel^n from A to B ~~des~~ describing the courses the students are taking

R_2 : Binary rel^n from A to B describing the courses the students are interested in.

R_1 :

	CS121	CS221	CS251	CS264	CS273	CS281
a	✓					
b		✓	✓			
c		✓			✓	✓
d				✓		✓

R_2

	CS 121	CS 221	CS 257	CS 264	CS 273	CS 281
a	✓			✓		
b		✓			✓	
c						
d				✓	✓	✓

~~$R_1 \cap R_2 = 4$~~

~~$R_1 \cup R_2 =$~~

~~$R_1 \oplus R$~~

Q/ Let $A = \{a, b, c, d\}$: set of students

$B = \{$ BT&T, Comp. Comm., GEE, IBM, Orange $\}$: set of companies that come to the university to interview for job.

$R_1 : A \to B$ = interviews the companies had with the students.

$R_2 : A \to B$ = job offers companies made to the students.

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R_1	BT&T	Comp. Comm.	GEE	IBM	Orange
a	✓	✓		✓	
b	✓	✓	✓	✓	✓
c				✓	✓
d	✓		✓	✓	

R_2					
a					
b			✓	✓	
c				✓	✓
d			✓	✓	

$R_1 \cap R_2$: ~~(b, GEE), (b, IBM), (c, IBM), (c, Orange)~~, R_2

$R_1 \cup R_2$: R_1

$R_1 \oplus R_2$: $R_1 - R_2$

$R_1 - R_2$: $R_1 - R_2$

Q/ ~~Let A be~~ :

Q/ Let $A = \{1, 2, 3, 4\}$ $\quad R = \{(a, b) : a$ is divisible by $b\}$

$R = \{(2,1), (3,1), (4,1), (4,2)\}$

Q/ $A = \{a, b, c\}$ & $B = \{a, b, c, d\}$

$R = \{(a,a), (b,b), (c,c)\}$

$S = \{(a,a), (a,b), (a,c), (a,d)\}$

– $R \cup S = \{(a,a), (a,b), (a,c), (a,d), (b,b), (c,c)\}$

$R \cap S = \{(a,a)\}$

$R - S = \{(b,b), (c,c)\}$

Q/ A : set of CS students

B : set of CS languages

R : all ordered pairs (a, b) where student 'a' is interested to learn language 'b' in a course

S : (a, b) such that 'a' has learnt language 'b'

Describe the ordered pairs in the following relations.

$R \cup S$, $R \cap S$, $R \oplus S$, $R - S$, $S - R$

– ~~RUS~~

– $R \cup S$: all ordered pairs (a, b) where student 'a' is interested to learn language 'b' in a course

$R \cap S$: all ordered pairs (a, b) such that 'a' has learnt language 'b'.

$R \oplus S$: all ordered pairs (a, b) such that 'a' ~~has learnt language~~ is interested to learn language 'b' but has not ~~yea~~ yet learnt.

$R - S$: same as $R \oplus S$

$S - R$: ϕ

- **Ternary Relations:**

  Let A, B, C be the sets.

  $(A \times B) \times C$: is a set of all ordered triples of the form $((a,b),c)$ s.t $(a,b) \in A \times B$ & $c \in C$

- **Quaternary Relations:**

  A, B, C, D be the sets

  $((A \times B) \times C) \times D$

- In general - $A_1, A_2, \ldots, A_n$

  $((((A_1 \times A_2) \times A_3) \times A_4) \ldots \times A_n)$

- **Inverse of a relation:-**

  $R : A \to B$ then,

  $R^{-1} : B \to A$

  $R = \{(1,y), (1,z), (3,y)\}$

  $R^{-1} = \{(y,1), (z,1), (y,3)\}$

Composition of Relations

~~Let~~

Let R & S be two relations. $R \to a-b$; $S \to b-c$

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \qquad M_S = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array}\begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

Then, composition of R & S,

$$M_R \cdot M_S = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

- A, B, C, D

  $R: A \to B$; $S: B \to C$; $T: C \to D$ then,

  $$(R.S).T = R.(S.T)$$

Q/ Let, ~~(R.S).T~~

$R = \{(a,b), (c,d), (b,b)\}$

$S = \{(d,b), (b, [illegible]), (c,a), (a,c)\}$

Compute: (1) R.S (4) R.(S.R) (7) R.R.R

(2) S.R (5) R.R

(3) (R.S).R (6) S.S

(1)

$$R = \begin{array}{c c} & \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \qquad S = \begin{array}{c c} & \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

$$R.S = \begin{array}{c c} & \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$R.S = \{(a,c), (b,c), (c,b)\}$

(2)

$S.R = \{(d,b), (b,d), (c,b), (a,d)\}$

(3) $(R.S).R = \{(a,d), (b,d), (c,b)\}$

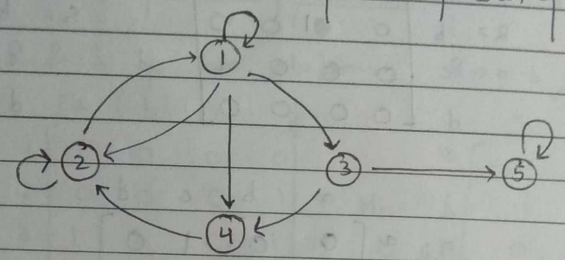
(4) $R.(S.R) = \{(a,d), (c,b), (b,d)\}$

(5) $R.R = \{(a,b)\}$

(6) $SS = \{(d,c), (b,a), (c,c), (a,a)\}$

(7) $R.R.R = (R.R).R = \{(a,b)\}$

Q/ Find the relation determined by the following diagram -



$A = \{1, 2, 3, 4, 5\}$

- $R : A \to A$

$R = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,1), (3,4), (3,5), (4,2), (5,5)\}$

Q/ Let R & S be 2 relations of set of +ve integers I,

$R = \{(a, 3a) / a \in I\}$ $(1,3), (2,6), (3,9) \ldots$

$S = \{(a, a+1) / a \in I\}$ $(1,2), (2,3), (3,4) \ldots$

Compute: (i) R.S (ii) R.R (iii) R.R.R (iv) R.S.R

- (i) $R.S = \{a, \cancel{2(a+1)}\ 3a+1) / a \in I\}$

(ii) $R.R = \{a, 9a / a \in I\}$

(iii) $R.R.R = \{a, 27a / a \in I\}$

(iv) $R.S.R = \{a, 3(3a+1) / a \in I\}$

Types of Relations :-

1) Reflexive – A relation R on set A is reflexive if aRa; for every $a \in A$ i.e if $(a,a) \in R$ for every $a \in A$.

And,

not reflexive – if there exists an $a \in A$ s.t. $(a,a) \notin R$

2) Irreflexive – A relation R on set A is irreflexive if $(a,a) \notin R$ for every $a \in A$.

And,

not irreflexive – if there exists atleast one $a \in A$ s.t $(a,a) \in R$.

Eg:- $A = \{1, 2, 3, 4\}$

$R_1 = \{(1,1), (1,2), (2,3), (1,3), (4,4)\}$ not reflexive, not irreflexive

$R_2 = \{(1,1), (1,2), (2,1), (2,2), (3,3), (4,4)\}$ reflexive, not irreflexive

$R_3 = \{(1,3), (2,1)\}$ not reflexive, irreflexive

$R_4 = \phi$ empty relation not reflexive, irreflexive

$R_5 = A \times A$, the universal relation. reflexive, not irreflexive

Q/ $A = \{a, b, c\}$
$R = \{(a,a)\}$ not reflexive, not irreflexive

3) Symmetric ~~& Anti-Symmetric~~ — A relation R on a set A is symmetric if whenever aRb, then bRa i.e if whenever $(a,b) \in R$ then, $(b,a) \in R$

And,

not symmetric - if there exists $(a,b) \in R$ s.t $(b,a) \notin R$.

4) Anti-Symmetric — A relation R on a set A is anti-symmetric if whenever aRb & bRa then, $a=b$ i.e, whenever $(a,b), (b,a) \in R$, then, $a=b$

And,

not anti-symmetric - if there exists $(a,b) \in R$ such that (a,b) & (b,a) belongs to R, but $a \neq b$.

5) Transitive — A relation R on a set A is transitive if whenever aRb & bRc, then, aRc i.e whenever $(a,b), (b,c) \in R$, then $(a,c) \in R$

And,

not transitive - if there exists $a, b, c \in A$ s.t $(a,b), (b,c) \in R$ but $(a,c) \notin R$

Q/ $R_1 = \phi$ $A = \{a, b, c\}$

$R_2 = A \times A$

$R_3 = \{(a,a), (b,b), (c,c)\}$

$R_4 = \{(a,b), (b,a), (a,a), (b,b)\}$

$R_5 = \{(a,a), (b,b), (c,c), (a,b), (b,c)\}$

$R_6 = \{(a,b), (b,c), (a,c)\}$

- R_1 = Not reflexive / Irreflexive / Symmetric / Anti-Symmetric / Transitive

R_2 = Reflexive / Not irreflexive / Symmetric / Not anti-symmetric / Transitive

R_3 = Reflexive / ~~Irref~~ Not-irreflexive / Symmetric / Anti-Symmetric / Transitive

R_4 = Not reflexive / Not-irreflexive / Symmetric / Note anti-symmetric / Transitive

R_5 = ~~Sym~~ Reflexive / Not-irreflexive / Not-symmetric / ~~Not anti-symmetric~~ Anti-Symmetric / Not transitive

R_6 = Not Reflexive / Irreflexive / Not-symmetric / Anti-Symmetric / Transitive

Equivalence Relation

A relation R ~~from~~ on set A is said to be equivalent if R is
(i) Reflexive
(ii) Symmetric and
(iii) Transitive

Eg:- $A = \{1, 2, 3\}$

$R_1 = \phi$ Not equivalent

$R_2 = \{(1,1), (2,2), (3,3)\}$ Equivalent

$R_3 = \{(1,1), (2,2), (3,3), (2,1)\}$ Not equivalent

$R_4 = \{(1,1), (1,3), (2,1), (3,1)\}$ Not equivalent

$R_5 = \{(1,1), (2,2), (3,3), (1,2), (1,3), (2,1), (3,1)\}$ ~~Equivalent~~ Not equivalent (2,3) x

$R_6 = \{(1,1), (1,2), (2,1), (2,3), (3,1), (3,2), (3,3)\}$ Not equivalent

$R_7 = A \times A$ Equivalent

Equivalence Class

Equivalence Class of x is denoted by $[x]$ and,

$[x] = \{y \,/\, y \in A \ \& \ (x,y) \in R\}$

Q. $A = \{1, 2, 3, 4, 5\}$

$R = \{(1,1), (2,2), (3,3), (4,4), (5,5), (1,2), (2,1), (4,5), (5,4)\}$

$$\left.\begin{array}{l}[1] = \{1, 2\} \\ [2] = \{2, 1\}\end{array}\right\} P_1$$

$[3] = \{3\}$ — P_2

$$\left.\begin{array}{l}[4] = \{4, 5\} \\ [5] = \{5, 4\}\end{array}\right\} P_3$$

For verification,

$\boxed{P_1 \cup P_2 \cup P_3 = A}$ & $\boxed{P_1 \cap P_2 \cap P_3 = \phi}$

Partial Ordering Relation

A relation R on set A is said to be partial order relation if it is

(i) Reflexive

(ii) Anti-symmetric

(iii) Transitive.

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Ex:- $A = \{1, 2, 3\}$

$R_1 = \{\ \}$ Not partial order

$R_2 = \{(1,1), (2,2), (3,3)\}$ Partial Order

$R_3 = \{(1,1), (2,2), (3,3), (1,2), (2,1)\}$ Not partial order

$R_4 = \{(1,1), (2,2), (3,3), (1,3), (2,3)\}$ ~~Not~~ partial order

$R_5 = \{(1,1), (1,2), (2,3), (1,3)\}$ Not partial order

$R_6 = \{(1,1), (1,3), (2,2), (2,3), (3,3)\}$ Partial order

$R_7 = A \times A$ Not partial order

Q/ Which of the following is not a partial order relation?

$R_1 = \{(a,b) / a, b \in Z,\ a < b\}$ No

$R_2 = \{(a,b) / a, b \in Z,\ a <= b\}$ Yes

$R_3 = \{(A,B) / A, B \in P(X),\ A \subseteq B\}$ Yes

~~$R_4 = \{(A,B) /$~~ .

$R_4 = \{(a,b) / a, b \in Z,\ a/b \in Z\}$ Yes

- R_1

Q/ Which of the following relations is symmetric or anti-symmetric?

$R_1 = \{(a,a), (c,c)\}$ Anti-symmetric, Symmetric

$R_2 = \{(a,b), (b,a), (a,c)\}$

$R_3 = \{(a,b), (b,c), (a,c)\}$ Anti-symmetric

$R_4 = \{(a,b), (b,a), (c,c)\}$ Symmetric

Q/ $A = \{1,2,3,4\}$

$R = \{(1,1), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4)\}$

a) Equivalence

b) Irreflexive & anti-symmetric

c) Symmetric but not reflexive

d) Transitive

Q/ Consider the following five relations on $A = \{1,2,3\}$

$R = \{(1,1), (1,2), (1,3), (3,3)\}$

$S = \{(1,1), (1,2), (2,1), (2,2), (3,3)\}$

$T = \{(1,1), (1,2), (2,2), (2,3)\}$

ϕ = empty relation

$A \times A$ = universal relation

Determine whether or not each of the above relation is

a) Reflexive - S, $A \times A$

b) Symmetric - S, ϕ, $A \times A$

c) Transitive - R, S, ϕ, $A \times A$

d) Anti-symmetric - R, T, ϕ

Q/ $A = \{1, 2, 3, 4\}$

$R = \{(1,1), (2,1), (2,2), (2,3), (2,4), (3,1)$

a) Equivalence
b) Irreflexive & anti-symmetric
c) Symmetric ~~& reflexi~~ but
d) Transitive

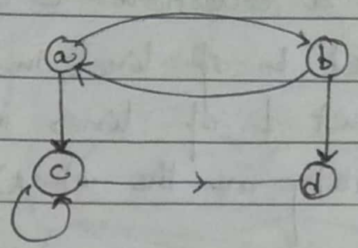
Q/ Let R be the following equivalence relation on $A = \{1, 2, 3, 4, 5, 6\}$

$R = \{(1,1), (1,5), (2,2), (2,3), (2,6), (3,2), (3,3), (3,6), (4,4), (5,1), (5,5), (6,2), (6,3), (6,6)\}$

Find the partitions ~~cartesian~~ of A induced by R.

- $[1] = \{1, 5\}$ – P_1
$[2] = \{2, 3, 6\}$ – P_2
$[3] = \{2, 3, 6\}$
$[4] = \{4\}$ – P_3
$[5] = \{1, 5\}$
$[6] = \{2, 3, 6\}$

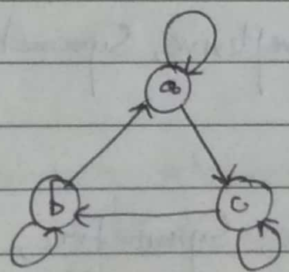
Q/ From the following diagrams write relations as the sets of ordered pairs and check equivalence or partial ordering ~~orderness~~.



— R = {(a,b), (a,c), (b,a), (b,c), (c,c), (c,d)}

Reflexive — X

⇒ Not equivalence & not partial ordering



— R = {(a,a), (b,b), (c,c), (a,c), (c,b), (b,a)}

Transitive — X

⇒ Neither equivalence nor partial ordering

Q/ Consider the following relations:

1. Relation $\leq$ on the set Z of integers
2. Set inclusion $\subseteq$ on a collection C of sets
3. Relation $\perp$ on the set L of lines in the plane
4. Relation // on the set L of lines in the plane.
5. Relation / of divisibility on the set N of +ve integers.

- 1) Reflexive, Not irreflexive, Not symmetric, Anti-symmetric, Transitive.

2) Reflexive, Not irreflexive, Not symmetric, Anti-symmetric, Transitive.

3) ~~Reflex~~ Not reflexive, Irreflexive, Symmetric, Not anti-symmetric, Not transitive.

4) Reflexive, Not irreflexive, Symmetric, Not anti-symmetric, Transitive.

5) Reflexive, Not irreflexive, Not symmetric, Anti-symmetric, Transitive.

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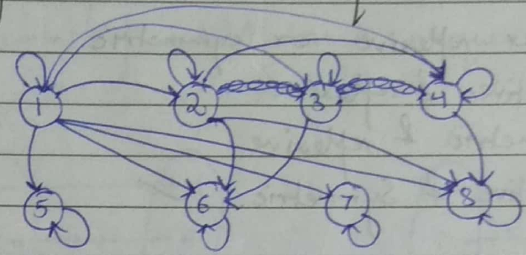
Q/ Draw a diagram for relation R.

$A = \{1, 2, 3, 4, 5, 6, 7, 8\}$

xRy : whenever y is divisible by x.

and say if the relation is equivalent or partial ordered.



Reflexive ✓
Symmetric ✗
Anti-symmetric ✓
Transitive ✓

⇒ Partial Ordered Relation.

Q/ Let R be a binary relation on A, such that $(a,b) \in R$ if book a costs more & contains fewer pages than book b.

(1) Reflexive ✗
(2) Symmetric ✗
(3) Anti-symmetric ✓
(4) Transitive ✓

~~Q/ Let S be a binary relation on set A, such that~~

Q/ The binary relation $S = \phi$ (empty set) on $A = \{1, 2, 3\}$.

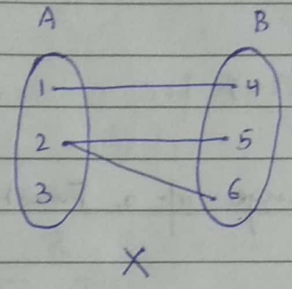
(1) Neither reflexive nor symmetric
(2) Transitive & reflexive
(3) Symmetric & reflexive
(4) Transitive & Symmetric.

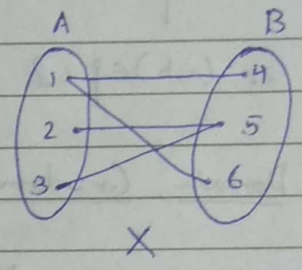
Q/ A relation R is defined on Z as xRy iff $(x+y)$ is even.

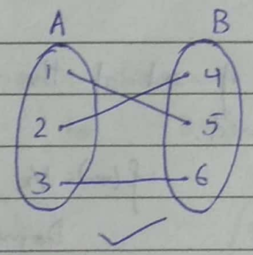
(1) R is not an equivalence
(2) R is an equivalence relatⁿ having 1 equivalence class
(3) ———————— " ——— 2 equivalence classes
(4) ———————— " ——— 3 equivalence classes

FUNCTIONS

A function is a rule / manner / correspondance / mapping defined from set A to set B such that each and every element of set A has a unique image in set B.







- $f : A \rightarrow B$

  $y = f(x)$

  $x \xrightarrow{f} f(x) \rightarrow y$

  $x \in A \; ; \; y \in B$

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- **Properties of a Function :-**

1. f is a collection of ordered pairs.
2. f is a subset of $A \times B$ i.e. $f \subset (A \times B)$

f has same no. of elements as $(A \times B)$ when there is exactly 1 element in B.

3. $(a,b) \in f$ & $(a,c) \in f \Rightarrow b = c$

- **Domain, Co-domain & Range of a Function :-**

$$f : A \rightarrow B$$

Domain: Collectⁿ of set of the values for which a function is defined.

$f(x) = 1/x \quad ; \quad x \neq 0$

Domain: $x \in R - \{0\}$

$f(x) = \dfrac{1}{\sqrt{x-2}} \quad ; \quad x > 2$

Domain: $x = (2, \infty)$

Mathematically,

$\boxed{\text{Domain of } f : \{a \mid a \in A : (a, f(a)) \in f\}}$

Co-domain: Set B

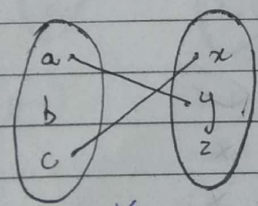
Range: Set of the values used by the f^n as per the domain.

$$\boxed{\text{Range of } f: \{b \mid b \in B : (a, \underbrace{f(a)}_{b}) \in f\}}$$

Q/ $f(x) = \sin x$ $\quad f: R \to R$.

- Domain: $x \in R$
  Co-domain: R
  Range: $[-1, 1]$

Q/ State whether or not each diagram defines a f^n from $A = \{a, b, c\}$ to $B = \{x, y, z\}$



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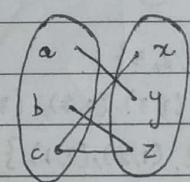

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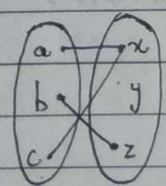


✓

Q/ Let $X = \{1, 2, 3, 4\}$. Determine whether or not each relation below is a f^n from X to X.

a) $f = \{(2,3), (1,4), (2,1), (3,2), (4,4)\}$ ✗
b) $g = \{(3,1), (4,2), (1,1)\}$ ✗
c) $h = \{(2,1), (3,4), (1,4), (2,1), (4,4)\}$ ✓

Q/ Let A be the set of students in a school. Determine which of the following assignments define a function on A.

a) To each student assign his age ✓
b) ——————" —————— teacher ✓ if each student has 1 teacher

✗ if any student has >1 teacher

Q/ Let $A = \{1, 2, 3\}$
$f = \{(1,3), (2,3), (3,1)\}$ ✓
$g = \{(1,2), (3,1)\}$ ✗
$h = \{(1,3), (2,1), (1,2), (3,1)\}$ ✗

Properties of Functions:-

1) One to One : A f^n $f: A \to B$ is said to be one to one if different elements in the domain A have distinct images

~~So~~ Injection

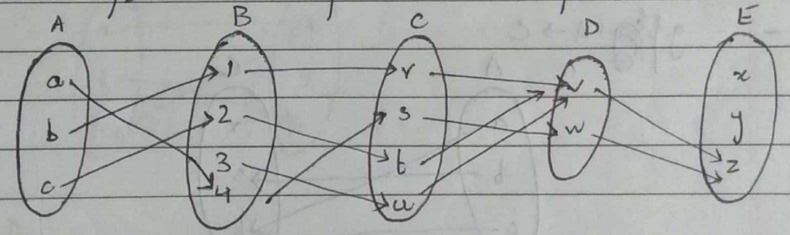
2) Onto : A f^n $f: A \to B$ is said to be onto if each element of B is the image of element of A.

Surjection

3) Invertible : A f^n $f: A \to B$ is invertible if its inverse relation f^{-1} is a function from B to A. i.e, if f is both one-one & onto.

Bijection

Q/ $f_1 : A \to B$ $f_2 : B \to C$ $f_3 : C \to D$ $f_4 : D \to E$

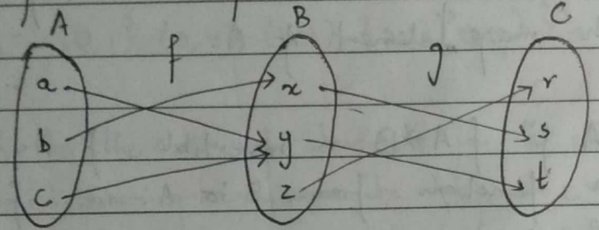


(1-1) ~~(1-1)~~ ~~onto~~ (1-1) onto onto none

invertible

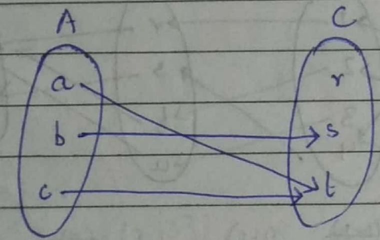
\# A $f: A \to B$ is a relatⁿ from A to B i.e, a subset of $(A \times B)$ such that $a \in A$ belongs to a unique ordered pair (a, b) in f.

Q/ Let $f: A \to B$ & $g: B \to C$



Find the composition fⁿ ($\cancel{f \cdot g}$)

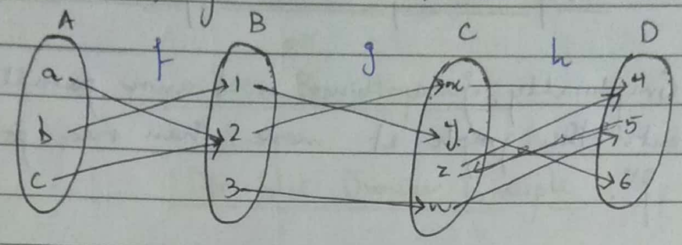
– $g \cdot f : A \to C$



Q/ Let the fⁿs f & g be defined as
$f(x) = 2x + 1$ $\quad$ $g(x) = x^2 - 2$
Find ~~com~~ $g \cdot f$

– $g \cdot f = g[f(x)] = g(2x+1)$
$= (2x+1)^2 - 2$
$= 4x^2 + 4x - 1$

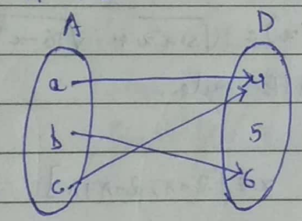
Q/ $f: A \to B$, $g: B \to C$, ~~h~~ $h: C \to D$



Determine if each f^n is onto & find the composition f^n ~~f.g.~~ $h.g.f.$

— h is onto.

$h.g.f : A \to D$



~~Q/ Dete (2) D~~

Q/ Determine if each f^n is one to one:

(a) To each person on the earth, assign the no. which corresponds to his age. ✗

(b) To each country in the world, assign the latitude & longitude of its capital ✓

(c) To each book written only by one author, assign the author. ✗

(d) To each country in the world which have a PM, assign the PM. ✓

Graphical Representation of a Function

Graphically, if a line is drawn parallel to y-axis & it cuts the graph at more than one point, then its not a f^n.

Q/ Find the domain of a f^n $f(x) = \sqrt{\sin x}$

$x \in [0, \pi]$

$x \in [2n\pi, 2n\pi + \pi]$, $n \in Z$

Q/ Find the domain of $f(x) = \sqrt{\sin x} + \sqrt{16 - x^2}$

Follow ILATE rule.

- $\sqrt{\sin x}$: $\cancel{x \in [0, \pi]}$ $x \in [2n\pi, 2n\pi + \pi]$

$\sqrt{16 - x^2}$: $x \in [-4, 4]$

$x \in [-4, -\pi] \cup [0, \pi]$

Q/ Range = ? $f(x) = 2\sin x + 3$

$y \in [1, 5]$

Pigeon Hole Principle
or
Shoe Box Argument
or
Dirichlet Drawer Principle

If there are many pigeons & a few pigeon holes, then, there must be some pigeon holes occupied by two or more pigeons.

Generally,

for any f^n f: D to R, there exists 'i' elements $d_1, d_2, \ldots, d_i$ in D ~~, then,~~ where $i = \lceil |D|/|R| \rceil$ such that

~~$i = \lceil |D|/|R| \rceil$ such that~~

$f(d_1) = f(d_2) = \ldots = f(d_i)$

SET THEORY

- **Principle of Extension:**
  2 sets A and B are equal iff they have the same members.

- Set can be described by the property which characterises its elements. Based on this statement, we have

Principle of Abstraction

Given any set U and any property P, there is a set A, s.t., the elements of A are exactly those members of U, which have the property P.

Venn Diagram : (Graphical representation)

Set Operations :

1) UNION
2) INTERSECTION
3) COMPLEMENT
4) RELATIVE COMPLEMENT - $A/B = \{x / x \in A, x \notin B\}$
5) FUNDAMENTAL PRODUCT -

Consider 'n' distinct sets $A_1, A_2, \ldots, A_n$. A fundamental product of the set is a set of the form.

$$A_1^* \cap A_2^* \cap A_3^* \ldots \cap A_n^*$$

where, A_i^* is either A_i or A_i^c

(i) There are 2^n such fundamental products
(ii) Any 2 such fundamental products are disjoint.
(iii) The universal set U is the union of all the fundamental products

6) SYMMETRIC DIFFERENCE -

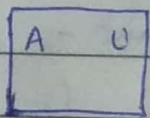
~~A~~ Symmetric difference of 2 sets A & B $(A \oplus B)$ consists of those elements which belong to A or B but not both.

$$A \oplus B = (A \cup B) / (A \cap B)$$

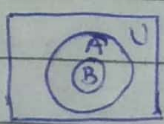
Q/ For each of the following conditions, what relation must be held b/w sets. Draw the venn diagrams.

(1) $A^c \cap U = \phi$ ~~→ A is~~
(2) $(A \cap B)^c = B^c$
(3) $(A \cap B) = (A \cap C)$
$(A^c \cap B) = (A^c \cap C)$

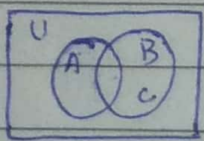
- (1) A is a universal set



(2) B is a subset of A



(3) B=C



Algebra of Sets :-

1) Idempotent Law - $A \cap A = A$ $\quad$ $A \cup A = A$

2) Associative Law - $(A \cap B) \cap C = A \cap (B \cap C)$

3) Commutative Law - $A \cap B = B \cap A$ $\quad$ $A \cup B = B \cup A$

4) Distributive Law - ~~$A \cup C$~~ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

5) Identity Law - $A \cap U = A$ $\quad$ $A \cap \phi = \phi$

$A \cup \phi = A$ $\quad$ $A \cup U = U$

6) Involution Law - $(A^c)^c = A$

7) Complement Law - $A \cap A^c = \phi$ $\quad$ $A \cup A^c = U$

$\phi^c = U$ $\quad$ $U^c = \phi$

8) De Morgan's Law - $(A \cap B)^c = A^c \cup B^c$

$(A \cup B)^c = A^c \cap B^c$

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Duality

~~The identities above are arranged in~~

The identities above are arranged in pairs.

Suppose,

E is a set of algebraic equations, then, the dual of E i.e, E^* is obtained by replacing

$\cup \quad \cap \quad U \quad \phi$

by $\cap \quad \cup \quad \phi \quad U$, respectively

Ex:- $(U \cap A) \cup (B \cap A) = A$

$\Rightarrow (\phi \cup A) \cap (B \cup A) = A$

The set algebra called the Principle of Duality, ~~set~~ states- if any equation E is an identity, then, its dual E^* is also an identity.

Finite Sets:- (Numerable sets)

A set is said to be finite if it contains exactly 'n' distinct elements, where, n is non-negative integer. Here, n = cardinality of the set.

Infinite Sets:- ~~(De-numerable sets)~~

A set which is not finite is infinite.

→ Countable
→ Countable infinite / denumerable
→ Uncountable infinite

The Inclusion Exclusion Principle

Let A & B be any finite sets, then

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

i.e, to find the number $n(A \cup B)$ of elements in the union $A \cup B$, we add $n(A)$ & $n(B)$ and then, we substract $n(A \cap B)$,

i.e, we include $n(A)$ & $n(B)$ and exclude $n(A \cap B)$

Similarly,

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

→ Classes of set
→ Power Set
→ Partitions of set

Principle of Mathematical Induction I

Let P be a proposition defined on the +ve integers N i.e., $P(n)$ is either true / false for each 'n' in N.

Suppose,
P has the following 2 properties -
(i) $P(1)$ is true
(ii) $P(n+1)$ is true, whenever $P(n)$ is true
~~(iii) If $P(1)$ is~~
Then,
P is true for every +ve integer

Principle of Mathematical Induction II

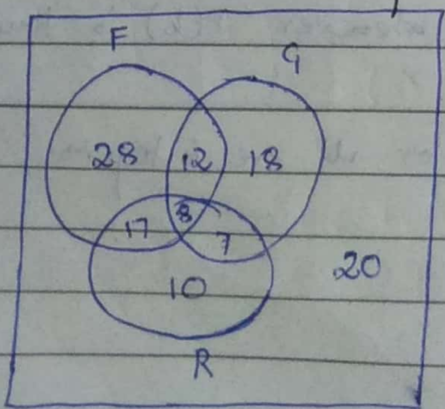
Let P be a proposition defined on the +ve integers N such that,
(i) $P(1)$ is true
(ii) $P(n)$ is true whenever $P(k)$ is true for all $1 \leq k \leq n$
Then,
P is true for all +ve integers

Q/ Find the number of mathematics students at a college taking atleast one of the languages - French, German & Russian, given the following data:-

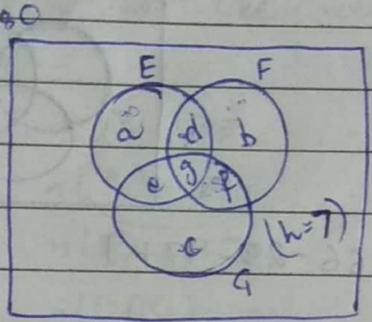
65 study French
45 study German
42 study Russian
20 study French & German
25 study French & Russian
15 study German & Russian
~~25~~ 8 study all three languages

$n(F \cup G \cup R) = 65 + 45 + 42 - 20 - 25 - 15 + 8$
$= 152 - \cancel{55}\ 60 + 8$
$= 100$

Q/ Consider the following data for 120 mathematics students at a college learning the languages French, German & Russian as in the above order. Fill in the correct no. of students in each of the 8 regions of the venn diagram.



Q/ In a class of 80 students, ~~50~~ $\overset{50}{\cancel{45}}$ students know English, 55 know French & 46 know German.
37 know English & French
28 know French & German   25 know German & English
7 know none of the languages.
Find out - (i) how many students know all the languages.
(ii) how many students know exactly 2 languages.
(iii) how many students know exactly 1 language.



$n(E\cup F\cup G) = n(E) + n(F) + n(G) - n(E\cap F) - n(F\cap G) - n(G\cap E) + n(F\cap G\cap E)$

$\Rightarrow 73 = 50 + 55 + 46 - 37 - 28 - n(G\cap E) + n(E\cap F\cap G)$

$\Rightarrow n(E\cap F\cap G) = 73 - \cancel{105}\ 151 + 66 + n(G\cap E)$

$\Rightarrow n(E\cap F\cap G) = 12 + n(G\cap E)$

$n(G\cap E) = n(E\cap F\cap G)$

ATP, $a+d+e+g=50$ $d+g=37$
$d+b+g+f=55$ $g+f=28$
$c+e+f+g=46$ $a+b+c+d+e+f+g=73$

Q/ Among integers ~~one to~~ 1-1000,

(a) how many of them are not divisible by 3 nor by 5 nor by 7.

(b) how many are not divisible by 5 or 7 but divisible by 3.

– $n(3) =$ ~~100-33=67~~ ~~1000-333=667~~ 333

$n(5) =$ ~~100-2~~ ~~1000-200=800~~ 200

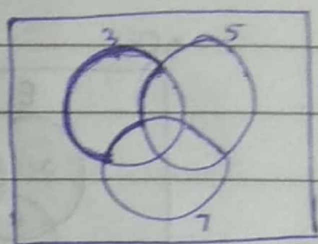
$n(7) =$ ~~1000-142=858~~ 142

$n(3\cap 5) = 66$

$n(5\cap 7) = 28$

$n(3\cap 7) = 47$

$n(3\cap 5\cap 7) = 9$



$\therefore n(3\cup 5\cup 7) = 333+200+142-66-28-47+9$

$= 684 - 141$

$= 543$

(a) $\therefore$ Not divisible by any $= 1000 - 543 = 457$

~~= 456~~

(b) ~~n(3) - n(5~~ ~~No. of numbers in~~

No. of integers = ~~n(3) - n(5) - n(7) + n(3∩~~

$= n(3) - n(5\cap 3) - n(3\cap 7) + n(3\cap 5\cap 7)$

$= 333 - 66 - 47 + 9$

$= 342 - 113$

$= 229$

Date
Page

Q/ A survey among 1000 people, 595 are democrats, 595 wear glasses, 550 like icecream, 395 of them are democrats who wear glasses, 350 of them are democrats who like icecream, 400 of them wear glasses and like icecream & 250 of them are all the three.

(a) How many of them are not democrats, ~~no~~ do not wear glasses & do not like icecream.

(b) How many of them are democrats who do not wear glasses & do not like icecream.

— $n(D) = 595$

$n(G) = 595$

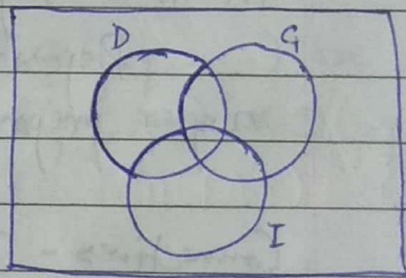
$n(I) = 550$

$n(D \cap G) = 395$

$n(D \cap I) = 350$

$n(G \cap I) = 400$

$n(D \cap G \cap I) = 250$



$\therefore n(D \cup G \cup I) = 595 + 595 + 550 - 395 - 350 - 400 + 250$

$= 1990 - 1145$

$= 845$

(a) No. of people $= 1000 - 845 = 155$

(b) No. of people $= n(D) - n(D \cap G) - n(D \cap I) + n(D \cap G \cap I)$

$= 595 - 395 - 350 + 250$

$= 200 - 100 = 100$

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PREPOSITIONAL CALCULUS

↓

statement/declarative

A ~~#~~ prepositional st. is a simple statement which has either value True or False.

Composite / Compound Prepositions –

Many prepositions are composite i.e composed of sub-prepositions, alongwith various connectors. Such composite prep. are called compound prep.

Primitive Prepositions –

Prepositions which cannot be broken down into simpler prepositions, i.e, they are not compound prep.

Connectives –

Basic logical operators that act as connectives to the primitive prep.

- Conjunction $\wedge$
- Disjunction $\vee$
- Negation $\neg$

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- Tautologies and Contradictions -

Some prepositions $P(p, q, \ldots)$ contain only T in the last col. of their truth table, i.e, they are true for any truth value of their variables. Such prepositions are called as tautologies. Ex:- $P \vee \neg P$

A preposition $P(p, q, \ldots)$ is called a contradiction if it contains only ~~false~~ F in the last col. of ~~the~~ its truth table, i.e., if it is false for any truth value of its variables. Ex:- $P \wedge \neg P$

- Logical Equivalence -

Two prepositions $P(p, q, \ldots)$ & $Q(p, q, \ldots)$ are said to be logically equivalent, denoted by $P(p, q, \ldots) \equiv Q(p, q, \ldots)$ if they have identical truth tables.

Ex:- $\neg(P \wedge Q) \equiv \neg P \vee \neg Q$

Ex:- It is not the case that $\underbrace{\text{roses are red}}_{P}$ and $\underbrace{\text{violets are blue}}_{Q}$.

$\neg(P \wedge Q)$

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Algebra of Prepositions :-

1) Idempotent Law : ~~PxP~~ $P \vee P \equiv P$
$P \wedge P \equiv P$

2) Associative Law: $(P \vee Q) \vee R \approx P \vee (Q \vee P)$
$(P \wedge Q) \wedge R \approx P \wedge (Q \wedge R)$

3) Commutative Law: $P \vee Q \approx Q \vee P$
$P \wedge Q \approx Q \wedge P$

4) Distributive Law: $P \vee (Q \wedge R) \approx (P \vee Q) \wedge (P \vee R)$
$P \wedge (Q \vee R) \approx (P \wedge Q) \vee (P \wedge R)$

5) Identity Law: $P \vee F \approx P$ $\quad P \wedge F \approx F$
$P \vee T \approx T$ $\quad P \wedge T \approx P$

6) Complement Law: $P \vee \neg P : T$
$P \wedge \neg P : F$

7) Involution Law: $\neg\neg P \approx P$

8) De Morgan's Law: $\neg(P \vee Q) \approx \neg P \wedge \neg Q$
$\neg(P \wedge Q) \approx \neg P \vee \neg Q$

Conditional & BiConditional Statements:-

~~Con~~ If P then Q / P only if Q

$P \rightarrow Q$

$(\neg P \vee Q)$

P if and only if Q

$P \leftrightarrow Q$

$P \rightleftarrows Q$

$(P \rightarrow Q) \wedge (Q \rightarrow P)$

When P is T & Q is F, then, $P \rightarrow Q$ is F otherwise T

When P & Q both have ~~the same truth~~ the same T values, then, $P \rightleftarrows Q$ is T, otherwise F.

P	Q	$P \rightarrow Q$	$P \rightleftarrows Q$
T	T	T	T
T	F	F	F
F	T	T	F
F	F	T	T

Let P be "It is cold" & Q be "It is raining". Give a simple verbal st. which describes each of the following sts.

(i) $\neg P$ – It is not cold

(ii) $P \wedge Q$ – It is cold and it is raining

(iii) $P \vee Q$ – ~~It is either co~~ Either it is cold or it is raining

(iv) $Q \vee \neg P$ – Either it is raining or it is not cold

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Q/ Let P: Erik reads News Week
Q: Erik reads The New Yorker
R: Erik reads Times
Write each of the following in symbolic form.

(a) Erik reads Newsweek or the New Yorker but not Times.
(b) Erik reads Newsweek and The New Yorkers or he does not read Newsweek and Times.
(c) It is not true that Erik reads Newsweek but not Times.
(d) It is not true that Erik reads Times or The New Yorker but not Newsweek.

- (a) $(P \vee Q) \wedge \neg R$
(b) $(P \wedge Q) \vee \neg(P \wedge R)$
(c) $\neg(P \wedge \neg R)$
(d) $\neg((R \vee Q) \wedge \neg P)$

Q1/ Verify the preposition
$P \vee \neg(P \wedge Q)$ is a tautology.

Q2/ Show that the prepositions
$\neg(P \wedge Q)$ and $\neg P \vee \neg Q$ are logically equivalent.

Q3/ Use Algebra Laws to show that
$\neg(P \vee Q) \vee (\neg P \wedge Q) \equiv \neg P$

- **Arguments:**

An argument is an assertion that a given set of preposition $P_1, P_2, \ldots P_n$, called premises, yields another preposition Q, called the conclusion. Such an argument is denoted by

$$P_1, P_2, \ldots\ldots, P_n \vdash Q$$

An argument $P_1, P_2, \ldots P_n \vdash Q$ is said to be valid if Q is True whenever all the premises $P_1, P_2, \ldots, P_n$ are True. An argument which is not valid is called a fallacy.

Q/ Find out whether following arguments are valid or not.

(i) $P, P \rightarrow Q \vdash Q$ [Law of Detachment]

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

$\therefore$ Valid

(ii) $P \to Q, Q \vdash P$

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

$\therefore$ Not valid

Theorem:-

The argument $P_1, P_2, \ldots, P_n \vdash Q$ is valid if & only if the preposition $((P_1 \wedge P_2 \wedge P_3 \ldots \wedge P_n) \to Q)$ is a tautology.

- Law of Syllogism:-

If P implies Q & Q implies R, then, P implies R.

$P \to Q, Q \to R \vdash P \to R$

P	Q	R	① $P \to Q$	② $Q \to R$	③ ①$\wedge$②	④ $P \to R$	③ $\to$ ④
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

- **Logical Implication :-**

A preposition $P(p,q,\ldots\ldots)$ is said to ~~be~~ logically imply a preposition $Q(p,q,\ldots\ldots)$, written as

$$P(p,q,\ldots\ldots) \Rightarrow Q(p,q,\ldots\ldots)$$

if $Q(p,q,\ldots\ldots)$ is True whenever $P(p,q,\ldots\ldots)$ is True.

Ex :-

$$p \Rightarrow p \vee q$$

p	q	$p \vee q$
T	T	T ✓
T	F	T ✓
F	T	T
F	F	F

Theorem :-

For any preposition $P(p,q,\ldots\ldots)$ & $Q(p,q,\ldots\ldots)$, the following sets are equivalent :-

① $P(p,q,\ldots\ldots)$ logically implies $Q(p,q,\ldots\ldots)$

② The argument $P(p,q,\ldots\ldots) \vdash Q(p,q,\ldots\ldots)$ is valid

③ The preposition $P(p,q,\ldots\ldots) \rightarrow Q(p,q,\ldots\ldots)$ is a tautology.

$p \to q$

$\neg q \to \neg p$

Rules of Inference:

1. Law of Detachment : (Modus Ponen)

$$\begin{array}{l} P \to Q \\ P \\ \hline Q \end{array}$$

or $P \to Q, P \vdash Q$

or $[(P \to Q) \land P] \to Q$

2. Law of Contrapositive : (Modus Tallen)

$$\begin{array}{l} P \to Q \\ \neg Q \\ \hline \neg P \end{array}$$

P	Q	P→Q	¬Q	¬P
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

3. Disjunctive Syllogism :

$$\begin{array}{l} P \lor Q \\ \neg P \\ \hline Q \end{array}$$

4) Hypothetical Syllogism :

$$\begin{array}{l} P \to Q \\ Q \to R \\ \hline P \to R \end{array}$$

- $P \to Q$ simple
  $Q \to P$ converse
  $\neg P \to \neg Q$ inverse
  $\neg Q \to \neg P$ contrapositive

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Q/ Show that the following argument is a fallacy.

$p \to q, \neg p \vdash \neg q$

-

p	q	$p \to q$	$\neg p$	$\neg q$
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F ×
F	F	T	T	T ✓

$\therefore$ Fallacy

Q/ Determine the validity of the following argument.

① $p \to q, \neg q \vdash \neg p$

② $p \to \neg q, r \to q \vdash \neg p$

- ①

p	q	$p \to q$	$\neg q$	$\neg p$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T ✓

$\therefore$ Valid

②

p	q	r	$\neg q$	$p \to \neg q$	$r \to q$	$\neg p$
T	T	T	F	F	T	F
T	T	F	F	F	T	F
T	F	T	T	T	F	F
T	F	F	T	T	T	F ×
F	T	T	F	T	T	T ✓
F	T	F	F	T	T	T ✓
F	F	T	T	T	F	T
F	F	F	T	T	T	T ✓

$\therefore$ Fallacy

Answers:-

1) $P \vee \neg(P \wedge Q)$

P	Q	$P \wedge Q$	$\neg(P \wedge Q)$	$P \vee \neg(P \wedge Q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

$\Rightarrow$ Tautology

2)

P	Q	$P \wedge Q$	$\neg(P \wedge Q)$	$\neg P$	$\neg Q$	$\neg P \vee \neg Q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

$\Rightarrow$ Equivalent

3) $\neg(P \vee Q) \vee (\neg P \wedge Q) \equiv \neg P$

$\Rightarrow (\neg P \wedge \neg Q) \vee (\neg P \wedge Q) \equiv \neg P$

$\Rightarrow \neg P \wedge (\neg Q \vee Q) \equiv \neg P$

$\Rightarrow \neg P \wedge T \equiv \neg P$

$\Rightarrow \neg P \equiv \neg P$, which is true

Hence, proved

Q/ $[(P\to Q)\wedge(Q\to R)\wedge(\neg R)]\to\neg P$

$P\to Q$
$Q\to R$
$\neg R$
—
$\neg P$

Law of transitivity

$P\to R$
$\neg R$
—
$\neg P$

From modus tollen, it is valid.

Q/ $R\to S$
~~P~~

$R\to S$
$P\to Q$
$R\vee P$
—
$S\vee Q$

⇒ left side of these are true
⇒ right " " " " "

→ as this is true,

implies conclusion is true

OR

$\neg R\vee S$
$\neg \cancel{P}\vee Q$
$\cancel{R}\vee \cancel{P}$
—
$S\vee Q$

Q/ $\neg P\to\neg R$
$\neg S$
$P\to W$
$R\vee S$
—
W

Law of Contrapositive

$R\to P$
$\neg S$
$P\to W$
$R\vee S$
—
W

Valid

OR

$\neg R\vee P$
$\neg S$
$\neg P\vee W$
$R\vee S$
—
W

Q/

$P \vee Q$	$\cancel{P} \vee \cancel{Q}$
$P \to R$	$\cancel{\neg P} \vee R$
$Q \to R$	$\cancel{\neg Q} \vee R$
	$R \vee R$
R	R

Q/

$P \to R$	$\neg P \vee \cancel{R}$
$Q \to S$	$\neg Q \vee \cancel{S}$
$\neg R \vee \neg S$	$\cancel{\neg R} \vee \cancel{\neg S}$
$\neg P \vee \neg Q$	$\neg P \vee \neg Q$

Q/

$P \to (R \to S)$	$\cancel{\neg P} \vee \cancel{\neg R} \vee S$
$\neg R \to \neg P$	$\cancel{R} \vee \cancel{\neg P}$
P	$\cancel{P}$
S	S

Q/

$P \to R$	$\neg P \vee \cancel{R}$
$Q \to S$	$\neg Q \vee \cancel{S}$
$\neg R \vee \neg S$	$\cancel{\neg R} \vee \cancel{\neg S}$
$\neg P \vee \neg Q$	$\neg P \vee \neg Q$

Q/

$\neg P \vee Q$	$\cancel{\neg P} \vee Q$
$R \to S$	$\cancel{\neg R} \vee \cancel{S}$
$P \vee R$	$\cancel{P} \vee \cancel{R}$
$\neg S$	$\cancel{\neg S}$
Q	Q

Date ______
Page ______

Q/ $\neg P$
Q
$Q \to (P \to R)$

$\neg R$

$\neg P$
Q
$\neg Q \vee \neg P \vee R$

✗ Fallacy

P	Q	R	¬P	P→R	Q→(P→R)	¬R
T	T	T	F	T	T	F
T	T	F	F	F	F	T
T	F	T	F	T	T	F
T	F	F	F	F	T	T
F	T	T	T	T	T	F
F	T	F	T	T	T	T
F	F	T	T	T	T	F
F	F	F	T	T	T	T

Q/ $(Q \wedge R) \to P$
$\neg Q \vee P$

R

$\neg(Q \wedge R) \vee P \Rightarrow \neg Q \vee \neg R \vee P$
$\neg Q \vee P$ $\quad$ $\neg Q \vee P$

Q/ P
$P \to R$
$Q \vee \neg R$

Q

$\cancel{P}$
$\cancel{\neg P} \vee \cancel{R}$
$Q \vee \cancel{\neg R}$

Q

Q/ $(Q \wedge R) \to P$
$\neg Q \vee P$

R

$\neg Q \vee \neg R \vee P$
$\neg Q \vee P$

P	Q	R	Q∧R	(Q∧R)→P	Q→P
T	T	T	T	T	T
T	T	F	F	T	T
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	F	F
F	T	F	F	T	F
F	F	T	F	T	T
F	F	F	F	T	T

✗ Fallacy

Q/ S_1: If today is Gandhi's ~~#~~ birthday the today is ~~8~~ 2nd october.

S_2: Today is 2nd october

C: Today is Gandhi's birthday

$$\begin{array}{l} S_1 \rightarrow S_2 \\ \underline{S_2} \\ S_1 \end{array}$$

Fallacy

Q/ S_1: If Canada is a country, then London is a city

S_2: London is not a city

C: Canada is not a country

$$\begin{array}{l} S_1 \rightarrow S_2 \\ \underline{\neg S_2} \\ \neg S_1 \end{array}$$

Valid

Q/ S_1: If candidate is known to be corrupt then he will not be elected.

S_2: If a candidate is kind, then he will be elected.

(a) If a person is known to be corrupt, he is kind.

(b) If a person is not known to be corrupt, he is not kind.

(c) If a person is kind, he is not known to be corrupt.

(d) If a person is not kind, he is not known to be corrupt.

$C \rightarrow \neg E$ $\Rightarrow$ $\neg C \vee \cancel{\neg E}$

$K \rightarrow E$ $\quad$ $\neg K \vee \cancel{E}$

$\neg C \vee \neg K$

$\Rightarrow C \rightarrow \neg K$ or $K \rightarrow \neg C$

Q/ P: Good mobile phones are not cheap

Q: Cheap mobile phones are not good

L: $P \rightarrow Q$

M: $Q \rightarrow P$

N: $P \cong Q$

$G \rightarrow \neg C$ $\Rightarrow$ $\neg G \vee \neg C$ (P)

$C \rightarrow \neg G$ $\quad$ $\neg C \vee \neg G$ (Q)

a) L

b) M

c) N

d) L, M, N

G	C	¬G	¬C	¬G ∨ ¬C	¬C ∨ ¬G
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	T	T

Q/ S_1: If it rains then the cricket match will not be played.
S_2: The match was played.
C: There was no rain

- $R \to \neg C$
C
$\overline{\neg R}$ Valid
(Modus Tollen)

Q/ S_1: If it rains then the cricket match will not be played
S_2: It did not rain
C: Match was played.

- $R \to \neg C$
$\neg R$
$\overline{C}$

$\Rightarrow \neg R \vee \neg C$
$\neg R$
$\overline{C}$ X Fallacy

Q/ It is not raining & it is pleasant & it is not pleasant only if it is rainy & it is cold.

P: It is raining

Q: It is cold

R: It is pleasant

a) $(\neg P \wedge R) \wedge (\neg R \rightarrow (P \wedge Q))$

b) $(\neg P \wedge R) \wedge ((P \wedge Q) \rightarrow \neg R)$

c) $(\neg P \wedge R) \vee ((P \wedge Q) \rightarrow \neg R)$

d) $(\neg P \wedge R) \vee (R \rightarrow (P \wedge Q))$

Normal Forms

The problem of finding whether a given statement is a tautology or a contradiction is called a decision problem. For decision problem, the constructⁿ of truth table may not be practical always. Thus, there should be an alt. procedure known as "reduction to normal forms".

1) Disjunctive Normal Form (DNF)
2) Conjunctive Normal Form (CNF)

1) Disjunctive Normal Form:

A conjunction of st. variables or ~~and~~ ~~(or)~~ their negation is called a fundamental conjunction. It is ~~alt~~ also known as minterms.

Eg: P, $\neg P$, $\neg P \wedge Q$, $P \wedge \neg P$

A st. form which consists of a disjunction of fundamental conjunctions is called DNF.

$(P \wedge Q \wedge R) \vee (\neg P \wedge Q) \vee (\neg Q \wedge R)$

2) Conjunctive Normal Form:

A disjunction of st. variables or their negation is called a fundamental disjunction. It is also known as maxterms.

Eg: P, $\neg P$, $\cancel{\neg P \wedge Q}$ $\neg P \vee Q$, $P \vee \neg P$

A st. form which consists of a conjunction of fundamental disjunctions is called CNF.

$(P \lor Q \lor R) \land (\neg P \lor Q) \land (\neg Q \lor R)$

Q/ Obtain the DNF of $(P \to Q) \land (\neg P \land Q)$

– $(\neg P \lor Q) \land (\neg P \land Q)$

$= \neg P \land (\neg P \land Q) \lor Q \land (\neg P \land Q)$ [Distributive]

$= (\neg P \land \neg P \land Q) \lor (Q \land \neg P \land Q)$ ~~Idempotent~~

$= (\neg P \land Q) \lor \cancel{(\neg P \land Q)}\ (Q \land \neg P)$ [Idempotent]

$= (\neg P \land Q) \lor (\neg P \land Q)$ [Commutative]

$= \neg P \land Q$

Q/ Obtain the DNF of $\neg(P \to (Q \land R))$

– $\neg(\neg P \lor (Q \land R))$

$= \neg((\neg P \lor Q) \land (\neg P \lor R))$ [Distributive]

$= \neg(\neg P \lor Q) \lor \neg(\neg P \lor R)$ [De-Morgan's]

$= (P \land \neg Q) \lor (P \land \neg R)$ [De-Morgan's]

~~$= P \land (\neg Q$~~

Q/ Obtain CNF of $(P \land Q) \lor (\neg P \land Q \land R)$

– ~~$P \land$~~ $P \lor (\neg P \land Q \land R) \land Q \lor (\neg P \land Q \land R)$

$= (P \lor \neg P) \land (P \lor Q) \land (P \lor R) \land (Q \lor \neg P) \land (Q \lor Q) \land (Q \lor R)$

$= (P \lor Q) \land (P \lor R) \land (Q \lor \neg P) \land (Q \lor R) \land Q$

Q/ Obtain CNF of $(\neg P \to R) \land (P \leftrightarrow Q)$

- $(P \lor R) \land (P \to Q) \land (Q \to P)$

$= (P \lor R) \land (\neg P \lor Q) \land (\neg Q \lor P)$

~~$= (P \lor R) \land$~~

Q/ Obtain the CNF & DNF of the following:-

a) $P \leftrightarrow (\neg P \lor \neg Q)$

b) $(P \lor \neg Q) \to Q$

c) $P \land (P \to Q)$

d) $(P \lor Q) \leftrightarrow (P \land Q)$

e) $P \land (P \to Q)$

f) $(\neg P \lor Q) \leftrightarrow (P \land Q)$

CNF

- (a) $(P \to (\neg P \lor \neg Q)) \land ((\neg P \lor \neg Q) \to P)$

$\Rightarrow (\neg P \lor (\neg P \lor \neg Q)) \land (\neg(\neg P \lor \neg Q) \lor P)$

$\Rightarrow$ ~~$\neg P$~~ $(\neg P \lor \neg Q) \land ((P \land Q) \lor P)$

$\Rightarrow (\neg P \lor \neg Q) \land ((P \lor P) \land (P \lor Q))$

$\Rightarrow (\neg P \lor \neg Q) \land (P \lor Q) \land P$

DNF

$\neg P \land (P \lor (P \land Q)) \lor \neg Q \land (P \lor (P \land Q))$

$\Rightarrow (\neg P \land P) \lor (\neg P \land P \land Q) \lor (\neg Q \land P) \lor (\neg Q \land P \land Q)$

$\Rightarrow F \lor (F \land Q) \lor (\neg Q \land P) \lor (F \land P)$

$\Rightarrow (\neg Q \land P)$

(b) ~~$P \lor \neg Q$~~ $\neg(P \lor \neg Q) \lor Q$

$\Rightarrow (\neg P \land Q) \lor Q$

$\Rightarrow (\neg P \lor Q) \land (Q \lor Q)$

$\Rightarrow (\neg P \lor Q) \land Q$

$\Rightarrow (\neg P \lor Q) \land (F \land \lor Q)$

$\Rightarrow (\neg P \land F) \lor Q \Rightarrow F \lor Q \Rightarrow Q$

~~$(\neg P \land Q) \lor Q$~~

Q

(c) $P \wedge (\neg P \vee Q)$

$(P \wedge \neg P) \vee (P \wedge Q)$

$\Rightarrow F \vee (P \wedge Q) \Rightarrow (P \wedge Q)$

(d) $[(P \vee Q) \rightarrow (P \wedge Q)] \wedge [(P \wedge Q) \rightarrow (P \vee Q)]$

$\Rightarrow (\neg(P \vee Q) \vee (P \wedge Q)) \wedge (\neg(P \wedge Q) \vee (P \vee Q))$

$\Rightarrow ((\neg P \wedge \neg Q) \vee (P \wedge Q)) \wedge ((\neg P \vee \neg Q) \vee (P \vee Q))$

$\Rightarrow \cancel{(\neg P \vee P)}$

$\Rightarrow (\neg P \vee (P \wedge Q)) \wedge (\neg Q \vee (P \wedge Q)) \wedge T$

$\Rightarrow (\neg P \vee P) \wedge (\neg P \vee Q) \wedge (\neg Q \vee P) \wedge (\neg Q \vee Q)$

$\Rightarrow (\neg P \vee Q) \wedge (\neg Q \vee P)$

$[\neg P \wedge (\neg Q \vee P)] \vee [Q \wedge (\neg Q \vee P)]$

$\Rightarrow (\neg P \wedge \neg Q) \vee (\neg P \wedge P) \vee (Q \wedge \neg Q) \vee (Q \wedge P)$

$\Rightarrow (\neg P \wedge \neg Q) \vee (Q \wedge P)$

(e) $P \wedge (\neg P \vee Q)$

$(P \wedge \neg P) \vee (P \wedge Q)$

$\Rightarrow (P \wedge Q)$

(f) $[(\neg P \vee Q) \rightarrow (P \wedge Q)] \wedge [(P \wedge Q) \rightarrow (\neg P \vee Q)]$

$\Rightarrow [\neg(\neg P \vee Q) \vee (P \wedge Q)] \wedge [\neg(P \wedge Q) \vee (\neg P \vee Q)]$

$\Rightarrow [(P \wedge \neg Q) \vee (P \wedge Q)] \wedge [(\neg P \vee \neg Q) \vee (\neg P \vee Q)]$

$\Rightarrow [P \wedge (\neg Q \vee Q)] \wedge (\neg P \vee T)$

$\Rightarrow P \wedge T \wedge T$

$\Rightarrow P$

P

Q/ Show the equivalence of the following:

a) $[d \rightarrow ((\neg a \wedge b) \wedge c)]$ & $\neg[(a \vee (\neg(b \wedge c))) \wedge d]$

b) ~~pvt~~ $p \vee (q \vee r)$ & $(p \wedge q) \wedge (p \vee r)$

– (a) ~~a b c d~~ Writing ~~CNF~~ DNF

$\neg d \vee ((\neg a \wedge b) \wedge c) \Rightarrow \neg d \vee (\neg a \wedge b \wedge c)$

~~$\Rightarrow (\neg d \vee (\neg a \wedge b)) \wedge (\neg d \vee c)$~~

$\Rightarrow$

$\neg[(a \vee (\neg(b \wedge c))) \wedge d]$

$= \neg(a \vee (\neg(b \wedge c))) \vee \neg d$

$= (\neg a \wedge \neg\neg(b \wedge c)) \vee \neg d$

$= (\neg a \wedge b \wedge c) \vee \neg d$

$\therefore$ The DNF(s) are same

$\Rightarrow$ The statements are equivalent.

(b)

p	q	r	$p \wedge q$	$p \vee r$	$q \vee r$	$p \vee (q \vee r)$	$(p \wedge q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	F	T	T	T	F
T	F	F	F	T	F	T	F
F	T	T	F	T	T	T	F
F	T	F	F	F	T	T	F
F	F	T	F	T	T	T	F
F	F	F	F	F	F	F	F

$\therefore$ The statements are not equivalent

Also, by writing DNF's

$p \vee (q \vee r) \Rightarrow p \vee q \vee r$

$(p \wedge q) \wedge (p \vee r)$

$\Rightarrow q \wedge p \wedge (p \vee r)$

$\Rightarrow q \wedge [(p \wedge p) \vee (p \wedge r)]$

$\Rightarrow q \wedge [p \vee (p \wedge r)]$

$\Rightarrow (q \wedge p) \vee (q \wedge p \wedge r)$

$\therefore$ The DNF(s) are not same

$\Rightarrow$ The statements are not equivalent

Q/ Obtain the DNF of the following truth table :

p	q	r	f(p,q,r)
F	F	F	T
F	F	T	F
F	T	F	T
F	T	T	F
T	F	F	T
T	F	T	F
T	T	F	F
T	T	T	T

– ~~$\neg P \wedge \neg$~~ $(\neg p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge \neg r) \vee (p \wedge q \wedge r)$

Q/ Find DNF of $(\neg P \rightarrow R) \wedge (P \leftrightarrow Q)$ using truth table.

P	Q	R	$\neg P$	$\neg P \rightarrow R$	$P \leftrightarrow Q$	$(\neg P \rightarrow R) \wedge (P \leftrightarrow Q)$
F	F	F	T	F	T	F
F	F	T	T	T	T	T
F	T	F	T	F	F	F
F	T	T	T	T	F	F
T	F	F	F	T	F	F
T	F	T	F	T	F	F
T	T	F	F	T	T	T
T	T	T	F	T	T	T

– $(\neg P \wedge \neg Q \wedge P) \vee (P \wedge Q \wedge \neg R) \vee (P \wedge Q \wedge R)$

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Q/ Determine the contrapositive of each st :-

a) If John is a poet then he is poor

b) Only if John studies will he pass the test.

- a) If John is not ~~poor~~ poor then he is not a poet

~~b) If John does not pass the test then he did not study.~~

b) If John does not study then he will not pass the test.

Q/ Consider the following preposition -

Which of the following is a tautology?

a) $(p \vee q) \to p$

b) $p \vee (q \to p)$

c) $p \vee (p \to q)$

d) $p \to (q \to p)$

~~p q p∨q (p∨q)→p q→p p∨(q→p)~~

- (a) $\neg(p \vee q) \vee p \Rightarrow (\neg p \wedge \neg q) \vee p \Rightarrow (p \vee \neg p) \wedge (p \vee \neg q) \Rightarrow T \wedge (p \vee \neg q) \Rightarrow p \vee \neg q$

(b) $p \vee (\neg q \vee p) \Rightarrow p \vee \neg q$

(c) $p \vee (\neg p \vee q) \Rightarrow T \vee q \Rightarrow T$

(d) $\neg p \vee (\neg q \vee p) \Rightarrow T \vee \neg q \Rightarrow T$

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Q/ The preposition $P \wedge (\neg P \vee Q)$ is: $(P \wedge \neg P) \vee (P \wedge Q) \Rightarrow F \vee (P \wedge Q) \Rightarrow (P \wedge Q)$

a) Tautology

b) Contradiction

c) Logically equivalent to $P \wedge Q$

d) None of these

Q/ $(p \wedge p) \wedge (p \to (q \wedge q))$ is logically equivalent to

a) $p \to q$

b) $p \wedge q$

c) $p \vee q$

d) $q \to p$

- $(p \wedge p) \wedge (p \to (q \wedge q))$

$\Rightarrow p \wedge (p \to q)$

$\Rightarrow p \wedge (\neg p \vee q)$

$\Rightarrow (p \wedge \neg p) \vee (p \wedge q)$

$\Rightarrow F \vee (p \wedge q)$

$\Rightarrow p \wedge q$

Q/ Given that $(p \wedge q) \wedge (\neg p \wedge \neg q)$ is false,

The truth value of p and q are:

a) both F

b) both T

c) p is T & q is F

d) p is F & q is T

- $(p \wedge q) \wedge (\neg p \wedge \neg q)$

$\Rightarrow F \wedge F$

$\Rightarrow F$

p	q	$\neg p$	$\neg q$	$p \wedge q$	$\neg p \wedge \neg q$	$(p \wedge q) \wedge (\neg p \wedge \neg q)$
T	T	F	F	T	F	F
T	F	F	T	F	F	F
F	T	T	F	F	F	F
F	F	T	T	F	T	F

Q/ $(p \to q) \wedge (r \to q)$

a) $(p \vee r) \to q$

b) $p \vee (r \to p)$

c) $p \vee (r \to q)$

d) $p \to (q \to r)$

— $(\neg p \vee q) \wedge (\neg r \vee q)$

$\Rightarrow (\neg p \wedge \neg r) \vee q$

$\Rightarrow \neg(p \vee r) \vee q \quad \Rightarrow (p \vee r) \to q$

Q/ Test the validity of the foll. :-

a) S_1: If ~~a tre~~ it rains John will be sick

S_2: It did not rain

C: John will not fall sick.

— $$\begin{array}{c} P \to Q \\ \neg P \\ \hline \neg Q \end{array}$$ $\therefore$ Fallacy

b) S_1: If it rains John will be sick

S_2: John was not sick

C: It did not rain

— $$\begin{array}{c} P \to Q \\ \neg Q \\ \hline \neg P \end{array}$$ $\therefore$ Valid

c) S_1: If I study then will not fail in Maths.
S_2: If I do not play basketball then I will study
S_3: I failed in Maths
C: I must have played basketball.

– ~~P → 7~~

$$\begin{array}{c} S \to \neg M \\ \neg B \to S \\ M \\ \hline B \end{array}$$

∴ Valid

d) S_1: All my friends are musicians
S_2: John is my friend
S_3: None of my neighbours are musicians
C: John is not my neighbour

–

$$\begin{array}{l} F \to M \\ F \\ N \to \neg M \\ \hline \neg N \end{array}$$

∴ Valid

e) S_1: If I like Maths, then I will study
S_2: Either I will study or I will fail.
C: If I fail I do not like Maths.

–

$$\begin{array}{c} M \to S \\ S \oplus F \\ \hline F \to \neg M \end{array}$$

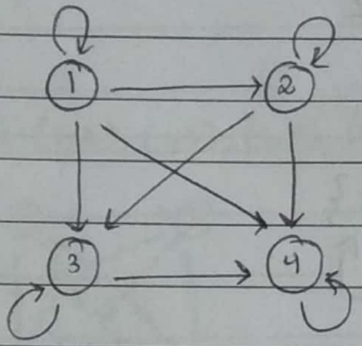
M	S	F	$M \to S$	$S \oplus F$	$F \to \neg M$	$F \to \neg M$
T	T	T	T	F	F	F
T	T	F	T	T	F	T
T	F	T	F	T	F	F
T	F	F	F	F	F	T
F	T	T	T	F	T	T
F	T	F	T	T	T	T
F	F	T	T	T	T	T
F	F	F	T	F	T	T

∴ Valid

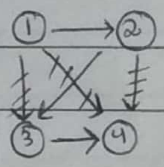
Hasse Diagram / POSET: (Is constructed for only Partial Ordered Relations)

$A = \{1, 2, 3, 4\}$

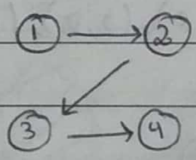
$R = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4), (4,4)\}$



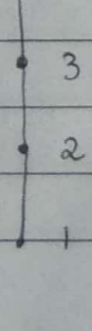
Step 1: Eliminate the reflexive relations



Step 2: Eliminate the transitive relations



Step 3: The Hasse Diagram is drawn as

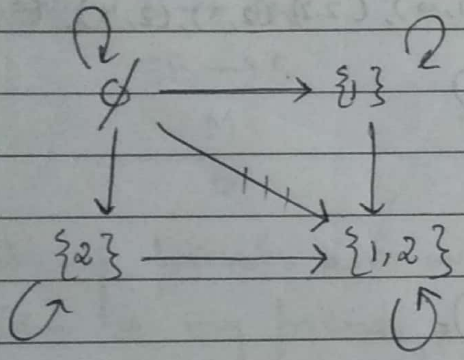


Q/ $A = \{1, 2\}$

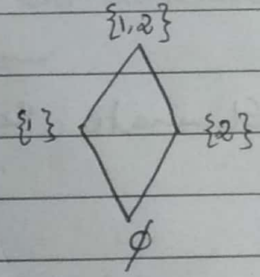
$P(A) = \{ \phi, \{1\}, \{2\}, \{1,2\} \}$

R: Relation of subsets of Power set

$= \{\{\phi,\phi\}, \{\phi, \{1\}\}, \{\phi, \{2\}\}, \{\phi, \{1,2\}\}, \{\{1\},\{1\}\}, \{\{1\},\{1,2\}\},$
$\{\{2\},\{2\}\}, \{\{2\},\{1,2\}\}, \{\{1,2\},\{1,2\}\}\}$

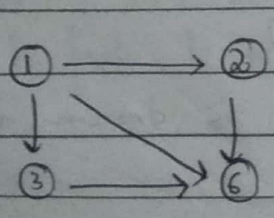


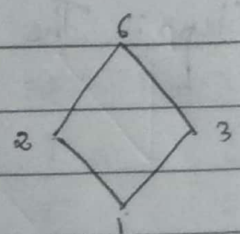
Hasse diagram:



Q/ $A = \{1, 2, 3, 6\}$

$R = \{(1,1), (1,2), (1,3), (1,6), (2,2), (2,6), (3,3), (3,6), (6,6)\}$





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Q1/ [(1, 2, 3, 4, 5), $\leq$]

Q2/ [(1, 2, 3, 4, 6, 12), /]

Q3/ [{ϕ, {1}, {2}, {1,2}}, $\subseteq$]

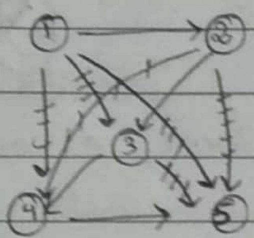
Q4/ [(1, 2, 3, 5, 6, 10, 15, 30), /]

Q5/ [(1, 2, 3, 4, 6, 9), /]

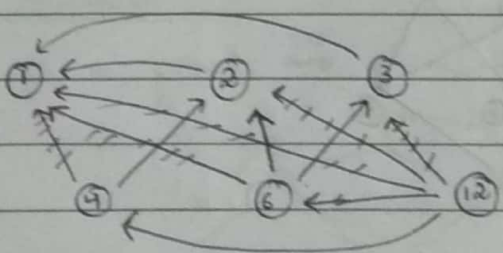
Q6/ [(1, 2, 3, 6, 9, 18), /]

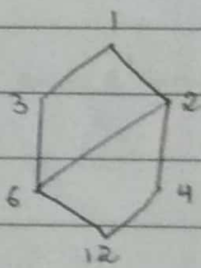
Answers

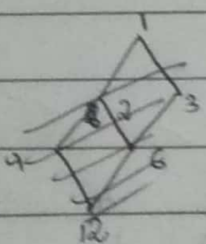
1) ~~R = {(1,1), (1,2), (1,3), (1,4), (1,5),~~

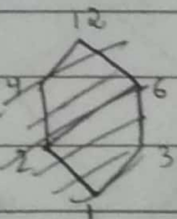


2)

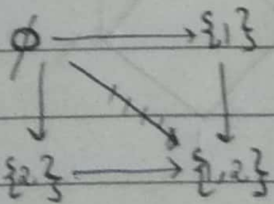


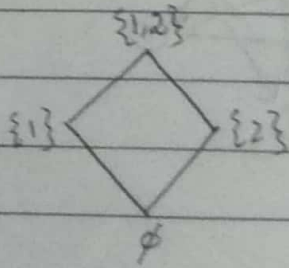




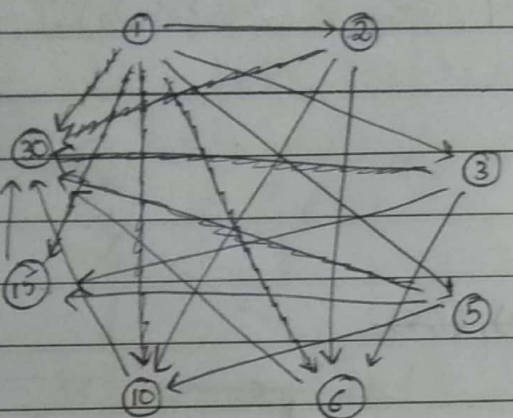


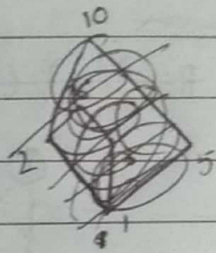
3)

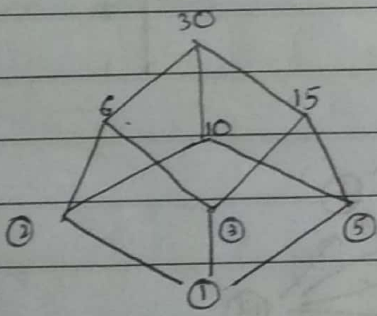




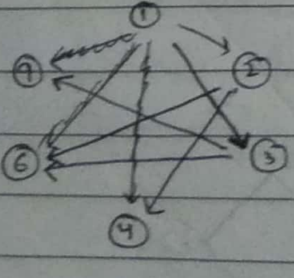
{1, 2, 3, 5, 6, 10, 15, 30}

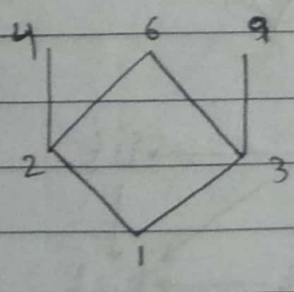




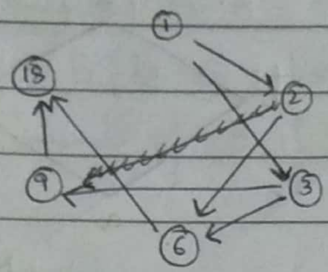


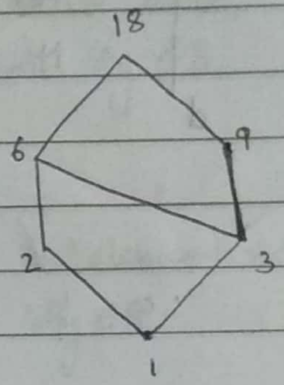
5) (1, 2, 3, 4, 6, 9)





6) $\{1, 2, 3, 6, 9, 18\}$



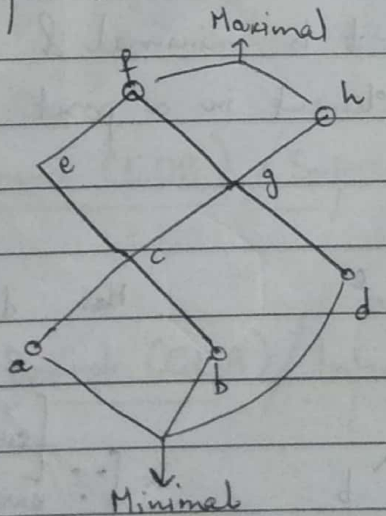


Maximal Element:

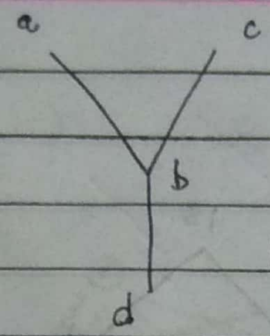
If in a poset, an element is not related to any other element.

Minimal Element:

If in a poset, ~~an~~ no element is related to an element.



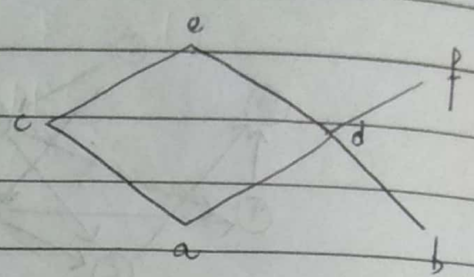
Q/



Max – a, c

Min – d

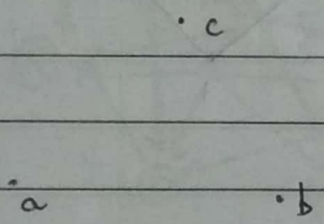
Q/



Max – e, f

Min – a, b

Q/



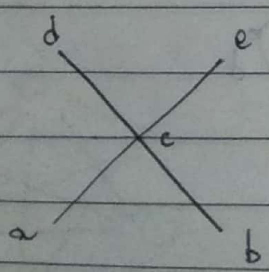
Max – a, b, c

Min – a, b, c

Maximum Element : If it is maximal & ~~ele~~ every element is related to it.

Minimum Element: If it is minimal & it is related to every element in a poset.

Example :-



~~Max~~ d & e are maximal but not maximum.

~~[since d is related to~~

[∵ every element except d e is related to d & vice versa]

Same is the case for minimum element.

Q/

Upper Bound:

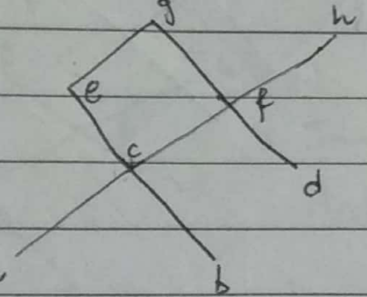
Let B be a subset of a set A. An element $x \in A$ is in the upper bound of B if $(y, x) \in$ poset, $\forall y \in B$.

Lower Bound:

Let B be a subset of a set A. An element $x \in A$ is in the lowerbound of B if $(x, y) \in$ poset, $\forall y \in B$.

Example:-

(c,e) c,e



$B = \{e, c\}$

$UB(B) = \{e, g\}$

$LB(B) = \{a, b, c\}$

$B = \{c, f, d\}$

$UB(B) = \{f, g, h\}$

$LB(B) = \cancel{\{a, b, c, d\}}$ $\emptyset$

- Least Upper Bound (LUB) / Supremum / Join Semi-Lattice / $\vee$ (or)

- Greatest Lower Bound (GLB) / Infimum / Meet Semi-Lattice / $\wedge$ (and)

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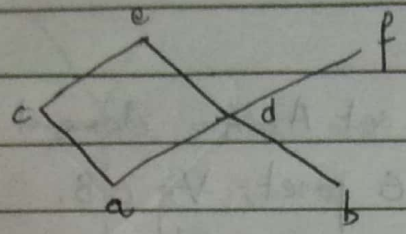
Q/



$B = \{c, d\}$

$LB = \{a\}$ $GLB = \{a\}$

$UB = \{e\}$ $LUB = \{e\}$

$B = \{a, b\}$

$LB = \phi$ $GLB = \phi$

$UB = \{d, e, f\}$ $LUB = \{d\}$

$B = \{e, f\}$

$LB = \{a, b, d\}$ $GLB = \{d\}$

$UB = \phi$ $LUB = \phi$

- Join Semi-Lattice :-

  In a poset, if LUB/ Join/ Supremum exists for every pair of elements then poset is called joint semi-lattice.

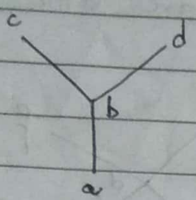
  (Sabse upar ek hi element h aur sabse connected h)

- Meet Semi-Lattice :-

  In a poset, if GLB/ Meet/ Infimum exists for every pair of elements, then poset is called meet semi-lattice.

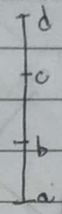
  (Sabse niche ek hi element h aur sabse connected h)

Q/



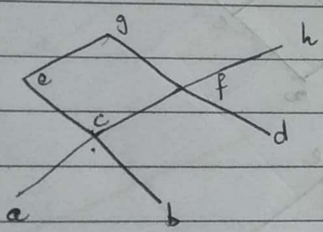
Join ✗
Meet ✓

Q/



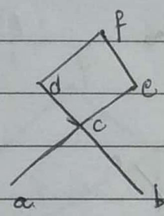
Join ✓
Meet ✓

Q1/



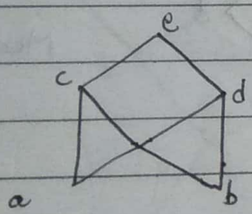
Join ✗
Meet ✗

Q2/



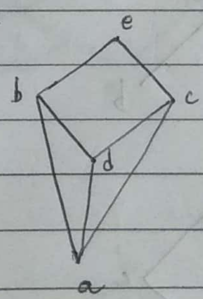
Join ✓
Meet ✗

Q3/



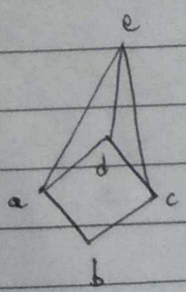
Joint ✓
Meet ✗

Q4/



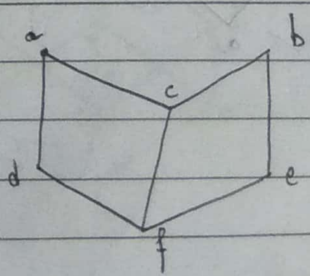
Join ✓
Meet ✓

Q5/



Join ✓
Meet ✓

Q6/

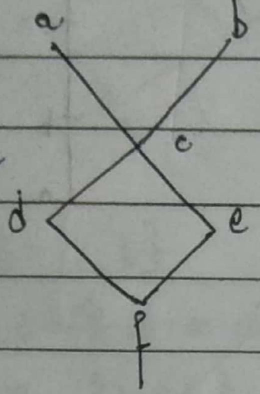


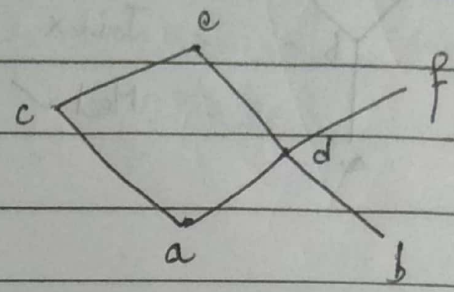
Join ✗
Meet ✓

Q/ Tell whether the following posets are lattices! -

Join ×

Meet ✓



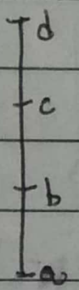


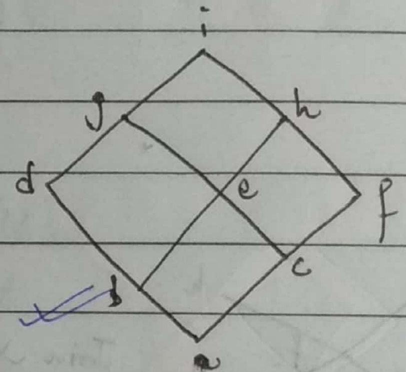
Join ×

Meet ×

Join ✓

Meet ✓



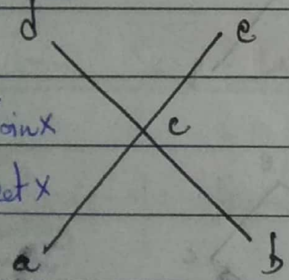


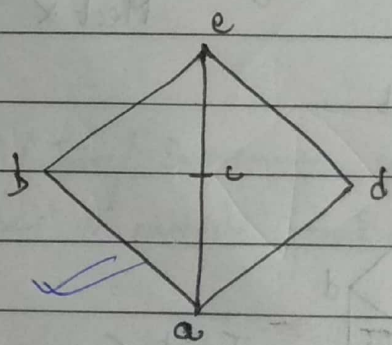
Join ✓

Meet ✓

Join ×

Meet ×

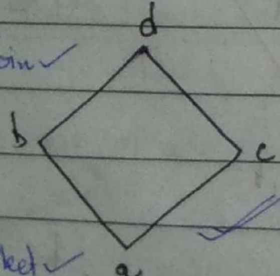




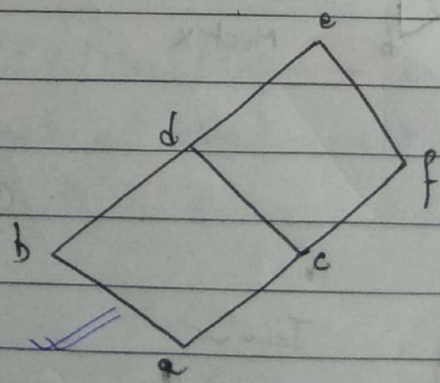
Join ✓

Meet ✓

Join ✓



Meet ✓

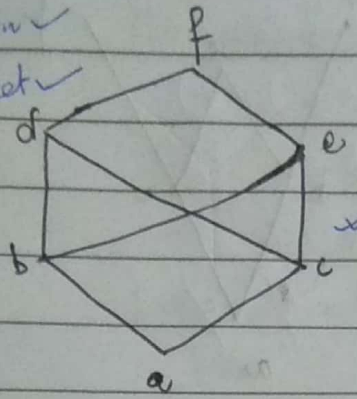


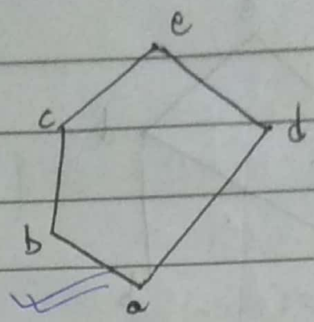
Join ✓

Meet ✓

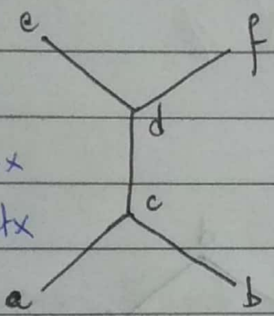
Date
Page

Join ✓
Meet ✓

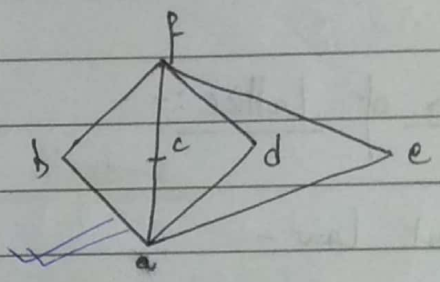




Join ✓
Meet ✓

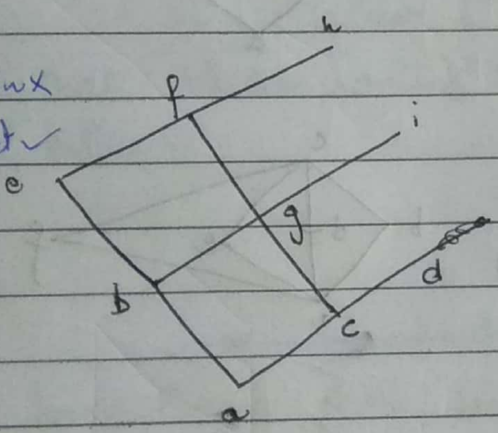


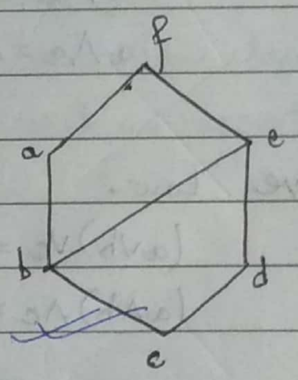
Join ×
Meet ×



Join ✓
Meet ✓

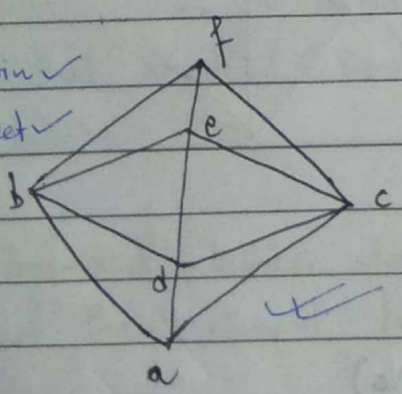
Join ×
Meet ✓

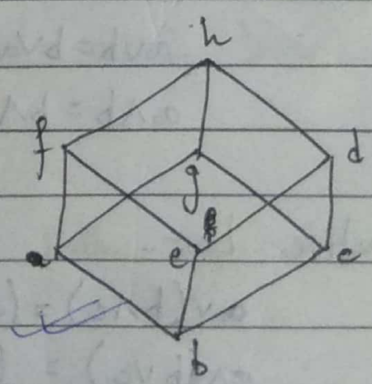




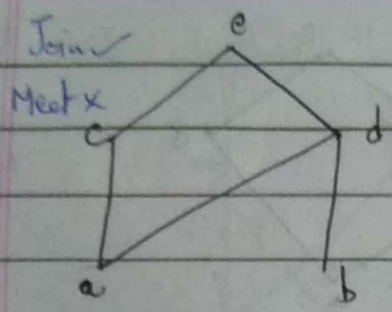
Join ✓
Meet ✓

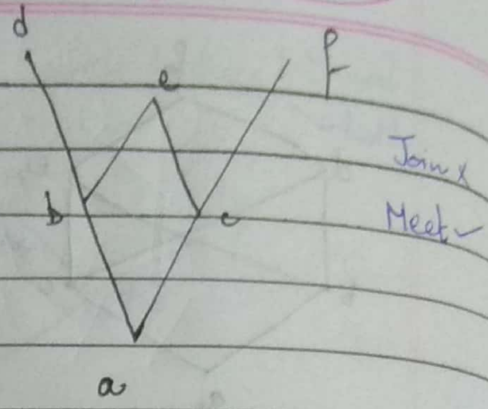
Join ✓
Meet ✓





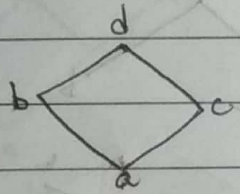
Join ✓
Meet ✓





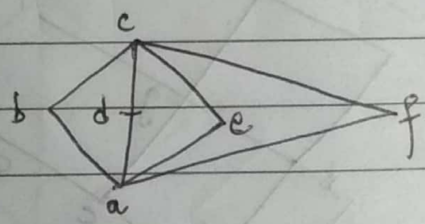
Properties of Lattice :-

1) Idempotent Law -



$$a \vee a = a$$
$$a \wedge a = a$$

2) Associative Law -



$$(a \vee b) \vee c = a \vee (b \vee c)$$
$$(a \wedge b) \wedge c = a \wedge (b \wedge c)$$

3) Commutative Law -

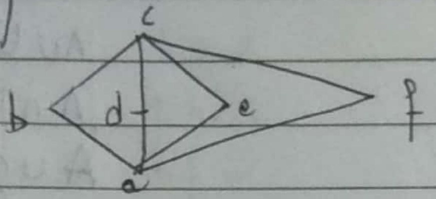
$$a \vee b = b \vee a$$
$$a \wedge b = b \wedge a$$

4) Distributive Law -

$$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$$
$$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$$

* It may hold in some cases & may not in some.

For example:



$b \vee (d \wedge e) \approx (b \vee d) \wedge (b \vee e)$

$b \vee a \qquad c \wedge c$

$b \neq c$

$b \wedge (a \vee e) \approx (b \wedge a) \vee (b \wedge e)$

$b \wedge e = a \vee a$

$a = a$

Boolean Algebra

- Upper Bound of a Lattice (Maximum) [I]:

  In a lattice "L", if there exists an element I such that $\forall a \in L$ (aRL), then I is said as upperbound of the lattice.

~~Lower bound of the lattice~~

- Lower Bound of a Lattice (Minimum) [O]:

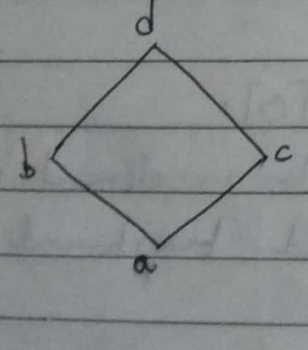
  In a lattice "L", if there exists an element 'O' such that $\forall a \in L (ORa)$, then O is called lowerbound of the lattice.

\# Bounded lattice should have both upper & lower bounds & they should be finite.

# Set Theory	Lattice
$a \vee I = I$ ~~$A \vee I = I$~~	$A \cup U = U$
$a \wedge I = a$ ~~$A \wedge I = a$~~	$A \cap U = A$
$a \vee O = a$	$A \cup \phi = A$
$a \wedge O = O$	$A \cap \phi = \phi$
$a \vee a^c = I$	$A \cup A^c = U$
$a \wedge a^c = O$	$A \cap A^c = \phi$

- Complement of an element in a lattice –

In a bounded lattice $L_{\le}$, for any element $a \in L_{\le}$, if there exists an element $b \in L_{\le}$, such that $a \vee b = I$, $a \wedge b = O$, ~~that~~ then 'b' is called complement of a OR a&b are the complements of each other.

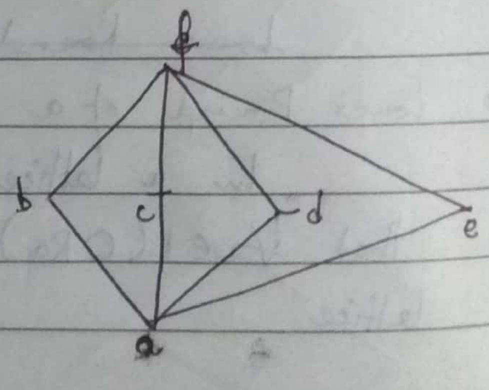


$a^c = d$

$d^c = a$

$b^c = c$

$c^c = a$

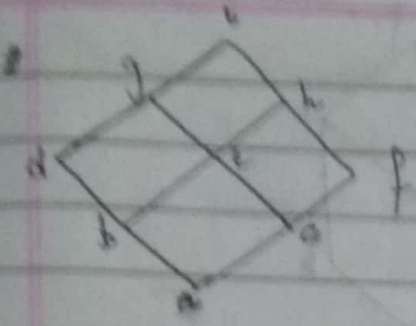


$a^c = f$, $f^c = a$

$b^c = c$, $c^c = b$

[bd, cd, be, de, ce]

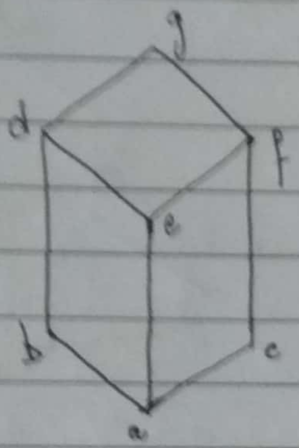
pair of complement elements



a, i ✓
b, c ×
g, h ×
d, f ✓
d, e ×

e, f ×
h, c ×
g, b ×
e, a ×
e, i ×

g, c ×
e, h ×



'e' has no complement

$a^c = g$, $g^c = a$

$b^c = c$, $c^c = b$

$b^c = f$, $f^c = b$

$d^c = c$, $c^c = d$

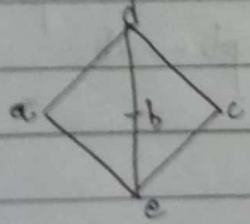
- **Distributive lattice** –

A lattice is said to be distributive if for every $(a, b, c) \in L$,

a) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

b) $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

Ex:-



$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

$a \vee e \qquad d \wedge d$

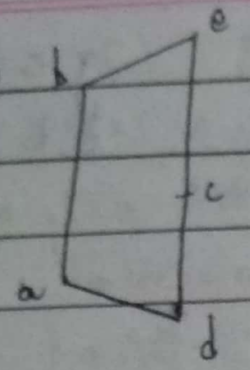
$a \neq d$

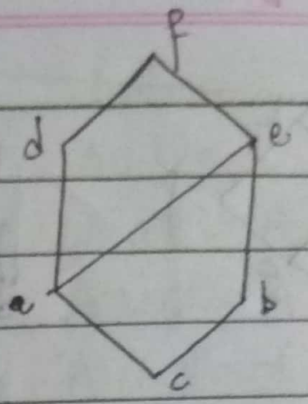
$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

$a \wedge d \qquad e \vee e$

$a \neq e$

6



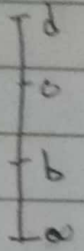


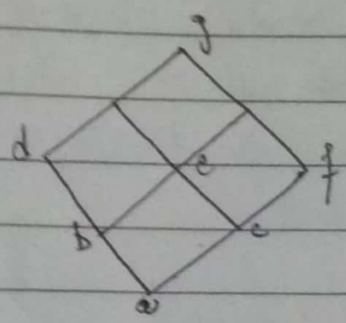
- A lattice is said to be distributive if every element in the lattice L has atmost 1 complement, i.e, either 1 comp. or '0'.

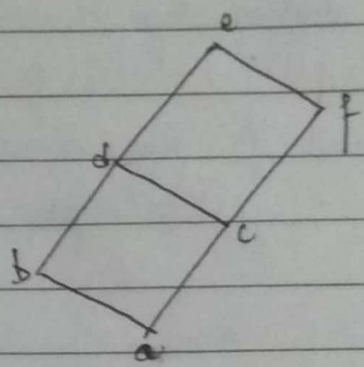
- A lattice L is said to be a complement if every element 'a' $\in$ L must have atleast 1 complement.

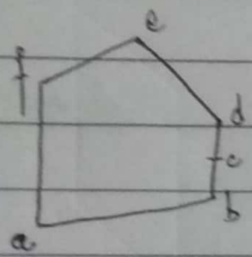
Boolean Algebra :-

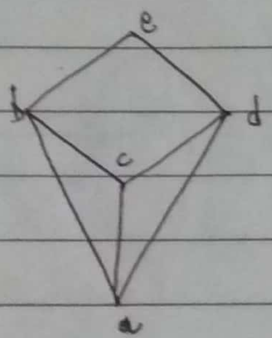
A lattice 'L' is said to be Boolean Algebra if it is complemented and distributive.

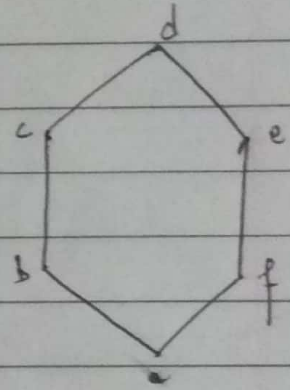












Graph Theory

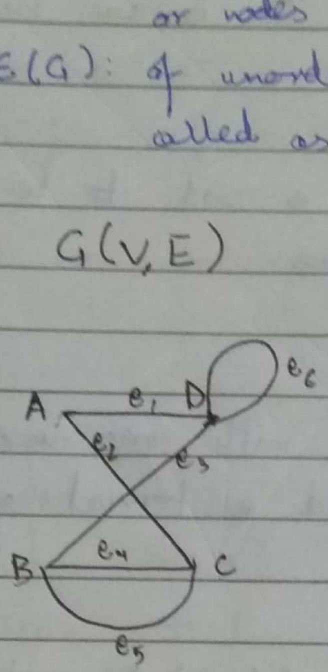
A graph G consists of

(i) A set $V = V(G)$: where elements are called vertices, points, or nodes of G.

(ii) A set $E = E(G)$: of unordered pair of distinct vertices called as edges of G.

$$G(V, E)$$

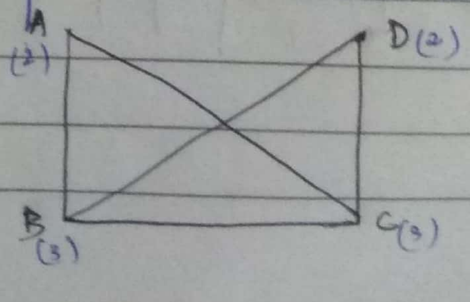
- Multi Graph:



In this graph, edges e_4 & e_5 are called multiple edges as they connect same end points & e_6 is called loop as its end points are the same vertex. Such a diagram is called 'multigraph'.

- Degree of a Vertex:

A degree of a vertex V in a graph G, written as deg(v), is equal to the no. of edges in G which contain V i.e which are incident on V.



- Sum of degrees of vertices of graph G is equal to twice the no. of edges in graph. This is called "handshaking lemma". (Degree of loops is two)

- A vertex with degree '0' is an isolated vertex.

→ Finite Graph:

A multigraph is said to be finite if it has finite no. of vertices & edges.

→ Trivial Graph:-

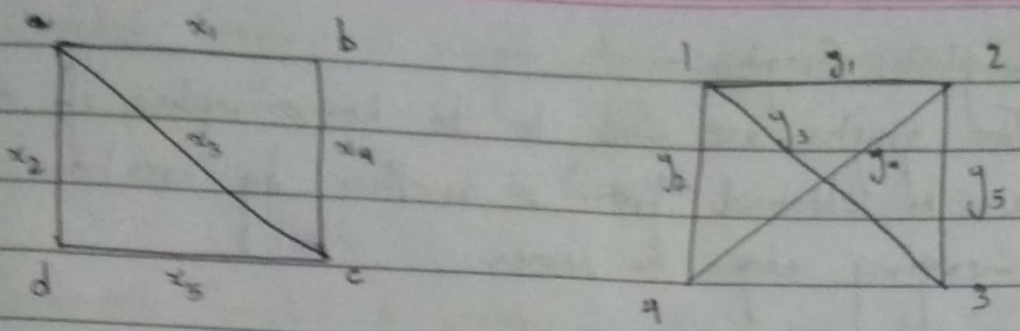
A finite graph with one vertex & no edges, i.e. a single point is called a trivial graph.

→ SubGraph:-

Consider a graph $G(V, E)$, A graph $H(V', E')$ is called a sub graph of G if the vertices & edges of H are contained in the vertices & edges of G i.e, $V' \subseteq V$ & $E' \subseteq E$

→ Isomorphic Graphs:-

Two graphs, G_1 & G_2 are said to be isomorphic if there is one-to-one correspondance b/w their edges & vertices & incidence relationship is preserved i.e, they have identical behaviour for any graph theoritical approaches.



$a=1,\ b=3,\ c=2,\ d=4$

$x_1 = y_3,\ x_2 = y_2,\ x_3 = y_1,\ x_4 = y_5,\ x_5 = y_4$

Q/ How many single non-isomorphic graphs are possible with 3 vertices?

— 3 vertices

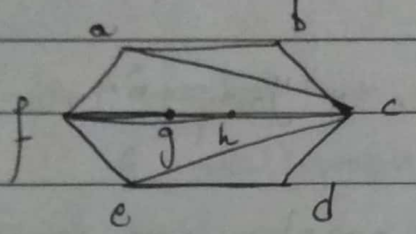
0 - edges → 1
1 edge → 3
2 edges → 3
3 edges → 1

$8 = 2^3$

Q/ How many single non-isomorphic graphs are possible with 4 vertices & 3 edges?

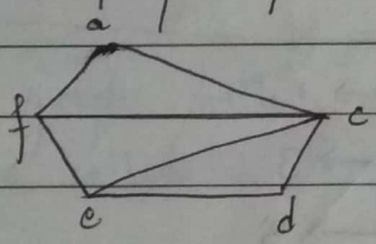
→ Homomorphic Graphs:-

Two graphs are said to be homomophic if one graph can be obtained from another by creation of edges or by merging edges in series.



New edges → fg & fh

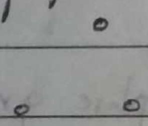
By merging edges,



~~By merging~~

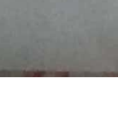
Types of Graphs:-

1) Null Graph: A graph with no edges

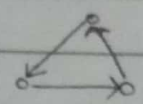


2) Trivial Graph: Only one vertex & no edges

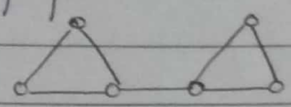
3) Undirected Graph: A graph which has no directions



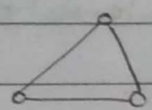
4) Directed Graph: A graph which has directions



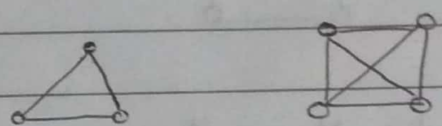
5) Connected Graph: A graph in which one edge connects two parts of a graph.



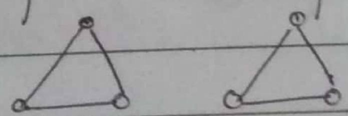
6) Regular Graph: A graph in which all nodes are of the same degree.



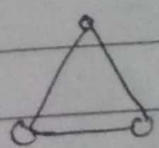
7) Complete Graph: A graph in which there is an edge b/w all the nodes.



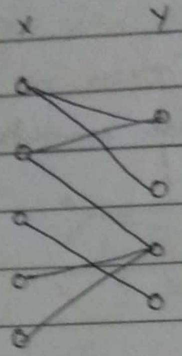
8) Disconnected Graph: A graph in which the connection b/w two sub-parts are missing.



9) Finite Graph: A graph in which there are finite no. of vertices & edges.

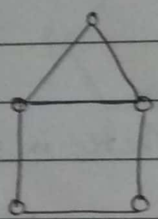


10) Bipartite Graph:

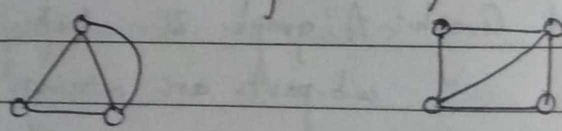


No internal edges.

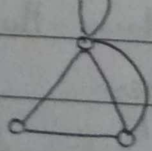
11) Planer Graph: No intersection of edges.



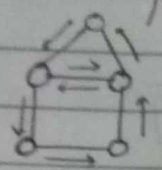
12) Multi Graph: More than one paths b/w 2 vertices.



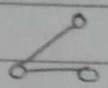
13) Pseudograph: In which there are multiple edges and/or self loops



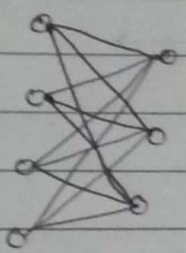
14) Cyclic Graph: A graph which contains a cycle.



15) Acyclic Graph:



16) Complete Bipartite Graph.

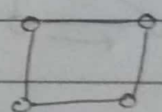


17) N-cube Graph: Q_n

If $n=1$, nodes $= 2^1 = 2$

If $n=2$, nodes $= 2^2 = 4$

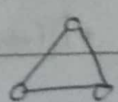
Q_1



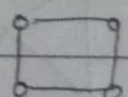
Q_2

18) Cycles: C_n $n \geq 3$

C_3

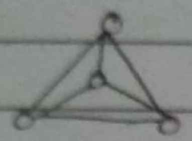


C_4

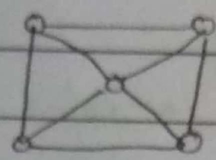


19) Wheels: $W_n = C_n$ + vertex

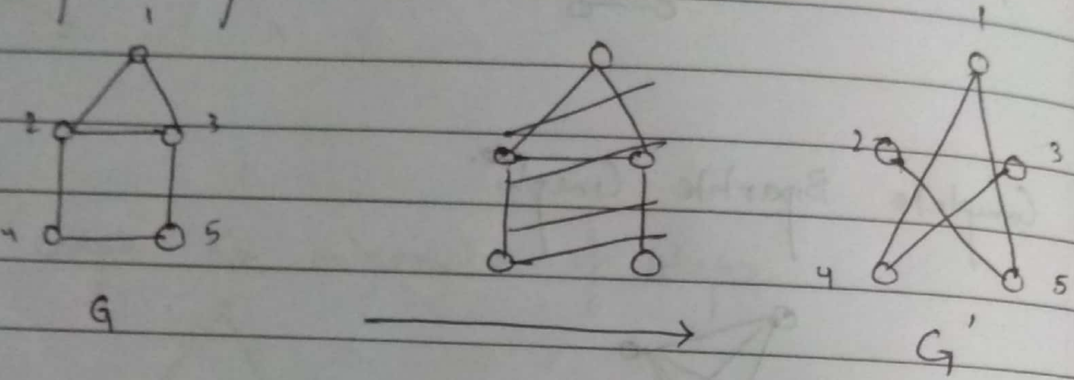
W_3



W_4

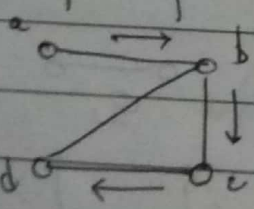


20) Complement of a Graph:



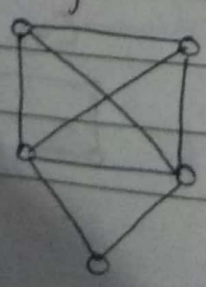
Hamiltonian Path:

It is a path that passes through every vertex exactly once. It may/may not exist in a particular graph.



Hamiltonian Circuit:

In which the first & last nodes should be the same one.



• Dirac Theorem applies here.

→ **Dirac Theorem** :-

① $n \geq 3$

② degree of each vertex should be atleast $n/2$

→ **Ore's Theorem** :-

$\underbrace{deg(u) + deg(v)}_{\text{there should}} \geq n$ [no. of nodes]

~~be edge b/w them~~