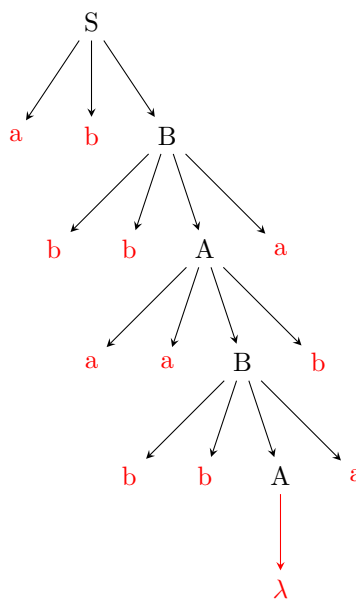


## Problem 4

Give a derivation tree for  $w = \text{abbbaabbaba}$  using the grammar  $G = (\{S\}, \{a, b\}, S, P)$  with productions:  $S \rightarrow abB$ ,  $A \rightarrow aaBb$ ,  $B \rightarrow bbAa$ ,  $A \rightarrow \lambda$



## Problem 9(d)

Find a context free grammar for  $L = \{a^n b^m : 2n \leq m \leq 3n, n \geq 0, m \geq 0\}$

Consider the grammar  $G = (\{S\}, \{a, b\}, S, P)$ , with productions:

$$S \rightarrow aSbb$$

$$S \rightarrow aSbbb$$

$$S \rightarrow \lambda$$

If you only ever apply the first rule, you will have twice as many  $b$ 's as  $a$ 's. If you only ever apply the second rule, you will have three times as many  $b$ 's as  $a$ 's.

If you mix them up, you cannot go below the lower bound of  $2n$ , and cannot go above the upper bound of  $3n$ , so it will be somewhere between  $2n$  and  $3n$ , so the grammar defines the language.

## Problem 12(b)

Find a context free grammar for

$$L = \{a^n b^m c^k : n = m \text{ or } m \neq k, n \geq 0, m \geq 0, k \geq 0\}$$

Consider the grammar  $G = (\{S, E, N, D, A, B^+, C, C^+\}, \{a, b, c\}, S, P)$ , with productions:

$$\begin{array}{ll} S \rightarrow EC \mid AN & A \rightarrow aA \mid \lambda \\ E \rightarrow aEb \mid \lambda & B^+ \rightarrow bB^+ \mid b \\ N \rightarrow B^+D \mid DC^+ & C \rightarrow cC \mid \lambda \\ D \rightarrow bDc \mid \lambda & C^+ \rightarrow cC^+ \mid c \end{array}$$

This one isn't very concise or elegant, but the idea is that you:

- Decide whether you want  $n = m$  ( $E$ ) or  $m \neq k$  ( $N$ )
- If  $E$ qual:
  - Leave room for as many  $c$ 's as you want on the end
  - Nest as many  $aEb$ 's as you want
- If  $N$ ot equal
  - Leave room for as many  $a$ 's as you want at the front
  - Choose which letter is going to have more than the other, generate at least one of those
  - Nest  $Down$  as many  $bDc$ 's as you want

## Problem 18

Show that the following language is context-free:

$$L = \{uvwv^R : u, v, w \in \{a, b\}^+, |u| = |w| = 2\}$$

To show that it's a context-free language, let's construct a context-free grammar for it.

Let that grammar be  $G = \{S, U, W, V, a, b, S, P\}$ , with productions:

$$\begin{array}{l} S \rightarrow UV \\ U \rightarrow aa \mid ab \mid ba \mid bb \\ V \rightarrow aVa \mid bVb \\ V \rightarrow U \end{array}$$

This grammar:

- Starts by letting you generate whatever  $u$  is (using  $U$ , which exhausts all two character strings over  $\{a, b\}$ ).
- Lets you generate  $v$  and  $v^R$  by nesting  $V$ 's
- Only lets you get rid of  $V$  by replacing it with  $w$

## Problem 8

Show that the following grammar is ambiguous:

$$\begin{aligned} S &\rightarrow AB \mid aaaB \\ A &\rightarrow a \mid Aa \\ B &\rightarrow b \end{aligned}$$

Just looking at it, it seems like we'll be able to generate  $aaab$  in two different ways.

The easy way is to go  $S \Rightarrow aaaB \Rightarrow aaab$ .

You can also get there from  $S \Rightarrow AB \Rightarrow AaB \Rightarrow AaaB \Rightarrow aaab \Rightarrow aaab$ .

These derivations are clearly different, you make completely different choices for the replacement of  $S$ , but the generated string is the same.

Because there are two distinct ways to generate this string, the grammar is ambiguous

## Problem 3

Transform the grammar  $S \rightarrow aSaaA \mid A$ ,  $A \rightarrow abA \mid bb$  into Chomsky normal form

CNF requires two non-terminals or a single terminal on the right hand side of every rule.

We can take rules one by one.

$$\begin{aligned} A \rightarrow abA &\implies A \rightarrow TA, T \rightarrow \Sigma\Omega, \Sigma \rightarrow a, \Omega \rightarrow b \\ A \rightarrow bb &\implies A \rightarrow \Omega\Omega \\ S \rightarrow aSaaA &\implies S \rightarrow UV, U \rightarrow \Sigma W, W \rightarrow \Sigma\Sigma, V \rightarrow \Sigma A \\ S \rightarrow A &\implies S \rightarrow A\Lambda, \Lambda \rightarrow \lambda \end{aligned}$$

So our final productions are

$$\Sigma \rightarrow a$$

$$\Omega \rightarrow b$$

$$S \rightarrow UV \mid A\Lambda$$

$$\Lambda \rightarrow \lambda$$

$$U \rightarrow \Sigma W$$

$$W \rightarrow S\Sigma$$

$$V \rightarrow \Sigma A$$

$$A \rightarrow TA \mid \Omega\Omega$$

$$T \rightarrow \Sigma\Omega$$