Modified Delay Lock Loop Aided by Short Multipath Insensitive Code Loop Discriminator[1]

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BIOGRAPHY

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1 Some of the materials in this paper were from "Modified code tracking loop aided by short multipath insensitive code loop discriminator" which was presented in the ION ITM 2017, Monterey, CA, US, Jan 2017.

ABSTRACT

The S-curve zero-crossing point of Short Multipath Insensitive Code Loop Discriminator (SMICLD) has no bias in the presence of single short multipath with a delay even less than 0.1 chips (GPS L1 C/A). However, SMICLD outputs zero when the code tracking error is negative, which results in large code tracking biases in noise environment. The proposed Modified DLL Aided by SMICLD improves the S-curve by utilizing a modified early minus late discriminator to guarantee a negative output under a negative code phase error, switching back to SMICLD under a positive or zero code phase error, and adding a correlation channel to distinguish the two cases. Test results at both digital intermediate frequency level and radio frequency level in real multipath environment demonstrate that the proposed method has better single short multipath mitigation performance than SMICLD, high resolution correlator (HRC), and multipath estimating delay lock loop (MEDLL) in noise environment.

Index Terms: Short delay multipath, S-curve, Short multipath insensitive code loop discriminator (SMICLD), Code tracking bias, Early minus late discriminator (EMLD).

INTRODUCTION

In a wide range of daily applications of Global Positioning System (GPS), especially the positioning and navigation in urban scenes, meter-level even decimeter-level accuracy in three dimensions has become a normal requirement. In addition to further improving GPS and its augmentation systems, a lot of efforts have been put into the research of GPS positioning error sources. As an important error source in GPS, multipath has always been focused on by many researchers all over the world.

Multipath is the interference to user receivers introduced by the reflection of the direct signal on the ground or other objects. The propagation time of multipath is longer than that of the direct signal, which is directly reflected in the code phase delay of the multipath with respect to the direct signal[1]. Unlike thermal noise and atmospheric interference, multipath cannot be eliminated by differential GPS. This is due to the large differences in multipath rays at different locations[2]. Therefore, multipath mitigation techniques are generally based on receiver and new navigation signal designs.

Mainly three kinds of multipath mitigation techniques are employed in GPS receivers: antenna-based, correlator/discriminator-based, and parameter estimation-based techniques. Choke ring antenna, right-hand circularly polarized antenna, and multi-antenna are typical antenna-based multipath mitigation methods designed to isolate the direct signal from the received signal by reducing the strength of multipath signals[3]. Since the antenna is the front part of a GPS receiver, a good multipath mitigation performance of the antenna can greatly reduce the burdens of the follow-up units in the receiver. The antenna-based multipath mitigation technique, nevertheless, increases the size, weight, and cost.

Multipath distorts the curve of correlation function between the received signal and the local replica of the GPS receiver, which results in the zero-crossing point bias of the S-curve of the discriminator in the code loop. This bias will evolve into the pseudo-range bias. Correlator/discriminator-based techniques mitigate multipath by properly reconstructing or modifying the code loop discriminator or the correlators of the receiver. From this standpoint, some classical algorithms have been developed including Narrow Correlator (NC)[4], Strobe correlator[5], High Resolution Correlator (HRC)[6], and so forth. These techniques show good effects on medium and long multipath, and yet little effects on short multipath.

Most parameter estimation-based techniques perform the maximum likelihood estimation of the parameters of the direct signal and multipath signals. As a typical representative of such methodology, Multipath Estimating Delay Lock Loop (MEDLL) can mitigate not only the code tracking bias but also the carrier tracking bias caused by multipath[7, 8, 9]. Other parameter estimation-based techniques, such as Multipath Mitigating Technology (MMT)[10] and Coupled Amplitude Delay Lock Loops (CADLL)[11], also have great performance in multipath mitigation, especially for medium and long multipath. In terms of short multipath, the parameter estimation becomes an ill-conditioned estimation problem. Therefore, this kind of methodology has poor performance in the case of short multipath.

The short multipath mentioned above has a delay less than 0.1 PRN (pseudo-random noise) code chips (for GPS L1 C/A signal), which means the propagation distance of the short multipath is longer than that of the direct signal by less than 30m. Short multipath usually exists in places with small space size, such as urban canyons where large amount of reflective objects may be close to the user receiver. The short multipath with the delay of 0.1 chips can lead to a C/A code range error of more than 10 meters, which is a large error for urban location applications, such as searching for a public bicycle. It is of great significance but yet very challenging to mitigate short multipath to improve the ranging accuracy in these occasions.

Several short multipath mitigation methods have been proposed so far. A-Posteriori Multipath Estimation (APME) technique proposed by Sleewaegen [2001] is based on the tight relationship between multipath errors and amplitude of received signals and can estimate short-multipath errors by scaling the combination of correlation values with an optimization coefficient[12]. This technique, however, is only suitable for weak multipath of MDR below 0.25, which extremely limits APME's application. The anti-short-multipath methods proposed by Bhuiyan [2010][13] and Gao [2014][14] are similar to APME. Zhang [2005] proposed a short multipath mitigation technique based on virtual multipath[15]. This technique performs well when there is a multipath signal stronger than the direct signal and other multipath signals, which greatly restricts its applications in spite of its low computational complexity. Zhou [2006] improved the short delay multipath mitigation performance of the traditional narrow correlator technique by using Teager-Kaiser operator[16]. Wang [2012] proposed a multipath model with the multipath ray number, and combined it with the adaptive genetic algorithm to mitigate short multipath[17]. A multipath mitigation method based on Fast Orthogonal Search (FOS) proposed by Mo [2018] tries to optimally estimate the correlation function distorted by multipath, which is similar but superior to MEDLL[18]. But large computational loads of this method are high demanding for receiver hardware.

Short multipath insensitive code loop discriminator (SMICLD) is a correlator/discriminator-based short delay multipath mitigation algorithm introduced by Jardak & Samama [2007][19]. The code loop using SMICLD is called SMICL. SMICLD modifies prompt correlation values and the code discriminator formula so that the zero-crossing point of its S-curve has no bias in the presence of short multipath. However, the S-curve of SMICLD is asymmetrical. When the code tracking error is negative, the output of SMICLD is always zero. Therefore, if the code tracking error becomes zero, SMICLD cannot adjust the code phase of the local replica in time, which may cause large code tracking errors in noise environment.

In this paper, we propose a novel and effective code loop, called Modified DLL Aided by SMICLD, to improve the code tracking performance by modifying the S-curve of SMICLD. Modified DLL Aided by SMICLD will use SMICLD as the discriminator when the code tracking error is positive and use modified Early Minus Later Discriminator (EMLD) when the code tracking error is negative, which integrates the strength of SMICLD (the zero-crossing point without any bias) with the strength of modified EMLD (its correct output polarity for negative code tracking error). The traditional EMLD is modified to ensure negative outputs in multipath environment when the code tracking error is negative. To switch the proper discriminator for the code loop, another correlation channel is added to distinguish zero from negative code tracking error.

We then implement the Modified DLL Aided by SMICLD, SMICL, HRC and MEDLL in our software receiver, and test their short multipath mitigation performances by using the digital IF GPS signal simulator to obtain the following results: (1) the code tracking errors with single in-phase or out-of-phase short multipath; (2) the statistical characteristics of code tracking errors with different carrier-to-noise ratio; (3) the multipath code error envelope. Moreover, their positioning errors in a real short multipath scene are tested by using a GPS RF signal simulator and a broadband high-speed data recording system. These results show that Modified DLL Aided by SMICLD can not only mitigate short multipath but also work stably in the presence of noise, and has better single short multipath ray mitigation performance compared with SMICL, HRC and MEDLL.

The paper is organized as follows. The second section briefly introduces the definition of SMICLD and particularizes its multipath mitigation performance in terms of the S-curve symmetry, the multipath ray number, and the multipath delay. The third section proposes Modified DLL Aided by SMICLD and analyzes its feasibility. In the fourth section, the software-defined GPS L1 C/A digital IF signal simulator and receiver developed by our group are utilized to test the performance of Modified DLL Aided by SMICLD as well as SMICL, HRC, and MEDLL in the presence of single short multipath ray and noise; and a real multipath scene at radio frequency signal level is also presented. The fifth section summarizes the study and gives the conclusions.

ASSESSMENT OF SHORT MULTIPATH INSENSITIVE CODE LOOP DISCRIMINATOR

SMICLD was proposed by Jardak & Samama [2007]. By modifying the prompt correlation values and the discriminator expression, SMICLD eliminates the zero-crossing point bias of its S-curve in the presence of single short multipath ray with its relative delay less than min(Δ/2, 1-Δ/2), where Δ denotes the correlator spacing in SMICLD.

The GPS L1 C/A IF signal can be expressed as

$$r(t) = a_0 p(t - \tau_0)\cos(wt + \theta_0) + \sum_{i=1}^{M} a_i p(t - \tau_0 - \tau_i)\cos(wt + \theta_i) \tag{1}$$

where $p(t)$ denotes the C/A code sequence of the transmitted signal, and w is the carrier frequency; a_i and θ_i are the amplitude and carrier phase of path i, respectively. The received signal can be normalized by the amplitude of the direct signal a_0. In this case, a_i (i>0) is called the multipath to direct ratio (MDR) of amplitude. τ_0 represents the propagation time of the direct signal; τ_i is the multipath delay relative to the direct signal; M indicates the number of multipath signals. It is assumed that the navigation message does not change during the observation time considering its period is much longer than that of the C/A code sequence. The local C/A code replica is $p(t\text{-}\tau)$, which is delayed by τ relative to the transmitted signal. The code tracking error ε is the code error between the direct signal and the local replica ($\varepsilon=\tau-\tau_0$). After carrier wipe-off, the signal is correlated with the local replica code to obtain the correlation values. The ideal autocorrelation function is expressed as follows.

$$R(\varepsilon) = \int_{-\infty}^{+\infty} p(t)\, p(t-\varepsilon)\, dt = \begin{cases} 1-|\varepsilon| \ , & |\varepsilon| \le 1 \\ 0 & , \ others \end{cases} \tag{2}$$

Let's denote the in-phase prompt, quadrature prompt, in-phase early, quadrature early, in-phase late, quadrature late correlation values as *IP*, *QP*, *IE*, *QE*, *IL*, *QL*, respectively. SMICLD firstly modifies the prompt correlation values:

$$IP^{'} = IP - \frac{\Delta}{2}\frac{IE + IL}{2-\Delta} \tag{3a}$$

$$QP^{'} = QP - \frac{\Delta}{2}\frac{QE + QL}{2-\Delta} \tag{3b}$$

where Δ denotes the correlator spacing, which is generally no more than 1 chip. Then the adjusted prompt correlation values, *IP'* and *QP'*, are substituted into a novel expression of the discriminator:

$$D_{SMICLD} = (IE^2 + QE^2) - ((IP^{'})^2 + (QP^{'})^2) \tag{4}$$

According to Jardak & Samama [2007], the discriminator function can be written as

$$D_{SMICLD}(\varepsilon)=\begin{cases} 2(2-\Delta)\left[a_0^2\varepsilon+\sum_{i=1}^{M}a_i^2(\varepsilon-\tau_i)+\sum_{i=0}^{M-1}\sum_{j=i+1}^{M}a_ia_j\cos(\theta_i-\theta_j)(2\varepsilon-\tau_i-\tau_j)\right] & 0<\tau_{i,j}<\varepsilon\le\min(\frac{\Delta}{2},1-\frac{\Delta}{2}) \\ 2a_0^2(2-\Delta)\varepsilon+4\sum_{i=1}^{M}a_0a_i\cos(\theta_0-\theta_i)\varepsilon(1+\varepsilon-\tau_i-\frac{\Delta}{2}) & 0<\varepsilon<\tau_i\le\min(\frac{\Delta}{2},1-\frac{\Delta}{2}) \\ 0 & \tau_i-\min(\frac{\Delta}{2},1-\frac{\Delta}{2})<\varepsilon<0<\tau_i\le\min(\frac{\Delta}{2},1-\frac{\Delta}{2}) \end{cases} \quad (5)$$

The characteristics of SMICLD can be illustrated by Table 1. The details can be referred to Weng & Kou [2017][20].

Table 1: SMICLD outputs with different numbers of multipath rays

Code tracking error	Number of multipath rays		
	$M=0$	$M=1$	$M\ge 1$
$\tau_i-\min(\Delta/2,1-\Delta/2)<\varepsilon<0<\tau_i\le\min(\Delta/2,1-\Delta/2)$	$D_{SMICLD}=0$	$D_{SMICLD}=0$	$D_{SMICLD}=0$
$0<\varepsilon<\tau_i\le\min(\Delta/2,1-\Delta/2)$	$D_{SMICLD}>0$	$D_{SMICLD}>0$	D_{SMICLD} is not always greater than zero
$0<\tau_i<\varepsilon\le\min(\Delta/2,1-\Delta/2)$	$D_{SMICLD}>0$	$D_{SMICLD}>0$	D_{SMICLD} is not always greater than zero

In the case of only one short multipath ray, when the code tracking error is positive, the output of SMICLD is positive. When the code tracking error is zero or negative (no less than τ_1-min(Δ/2, 1-Δ/2)), the output of SMICLD is always zero. SMICLD's S-curve in the presence of single multipath ray is displayed in Figure 1. It shows that SMICLD has no zero-crossing point bias, which means that SMICLD can eliminate short multipath error. When the code tracking error is negative, however, SMICLD always outputs zero, which will result in false lock and poor stability in the presence of noise.

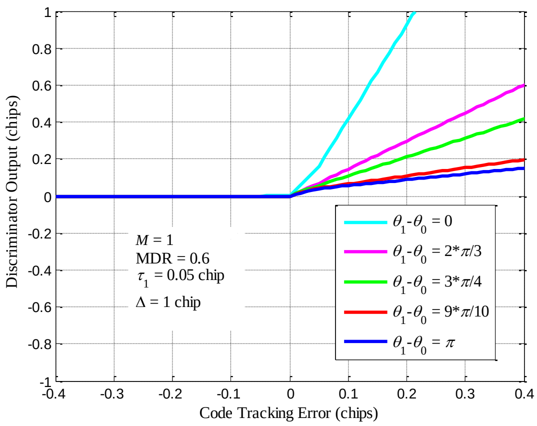


Figure 1 S-curves of SMICLD in the presence of single multipath ray

Table 1 illustrates that in the presence of more than one multipath ray, the output of SMICLD is not always greater than zero when the code tracking error is positive. In this case, the S-curve of SMICLD may have zero-crossing points on the positive axis of the code tracking error, which will cause the code tracking error to be locked into these points. Additionally, SMICLD cannot mitigate the medium and long delay multipath because the equation (5) is based on the premise that the multipath delay is less than or equal to min(Δ/2, 1-Δ/2).

This paper focuses on solving the problem that SMICLD outputs zero when the code tracking error is negative and proposes an effective methodology to improve the S-curve of SMICLD.

MODIFIED DLL AIDED BY SMICLD

Jardak & Samama [2007] suggested to delay the initial prompt replica code phase by 0.5 chips relative to the code phase measured by the acquisition process, which allows SMICLD to work in the positive region (ε>0) in the beginning of the tracking process. Nevertheless, if the satellite signal suffers from thermal noises, the code tracking error is still likely to become negative in the tracking process where SMICLD cannot still work properly. To solve this problem and track the received signal with a robust short multipath mitigation performance in noise environment, a novel code loop called Modified DLL Aided by SMICLD is proposed in this paper.

The discriminator of Modified DLL Aided by SMICLD is composed of SMICLD and modified EMLD (MEMLD). When the code error is positive or zero, the code loop uses SMICLD; when the code error is negative, the code loop uses MEMLD. Moreover, a strategy is proposed to determine whether the code tracking error is negative or not and to switch on an appropriate discriminator.

Modified EMLD (MEMLD)

Although the zero-crossing point of EMLD’s S-curve has a certain bias in the presence of short multipath, EMLD’s S-curve is still symmetrical around it which allows EMLD to work stably in noise environment. From this standpoint, EMLD is used to adjust negative code errors. In some cases, however, the zero-crossing point of traditional EMLD’s S-curve might be negative, which will induce a negative code error. To avoid such false locking, we modify the conventional EMLD to guarantee a negative output of it.

Firstly, a normalized EMLD is defined as follows.

$$D_{norEMLD}(\varepsilon)=\frac{\left(IE_M{}^2+QE_M{}^2\right)-\left(IL_M{}^2+QL_M{}^2\right)}{\left(IE_M{}^2+QE_M{}^2\right)+\left(IP^2+QP^2\right)} \tag{6}$$

where IE_M, QE_M, IL_M, QL_M represent the modified in-phase early, quadrature early, in-phase late, quadrature late correlation values, respectively.

$$
\begin{aligned}
IE_M &= a_0 R(\varepsilon - \Delta_M/2)\cos\left(\theta_0 - \hat{\theta}\right) + a_1 R(\varepsilon - \tau_1 - \Delta_M/2)\cos\left(\theta_1 - \hat{\theta}\right) \\
QE_M &= a_0 R(\varepsilon - \Delta_M/2)\sin\left(\theta_0 - \hat{\theta}\right) + a_1 R(\varepsilon - \tau_1 - \Delta_M/2)\sin\left(\theta_1 - \hat{\theta}\right) \\
IL_M &= a_0 R(\varepsilon + \Delta_M/2)\cos\left(\theta_0 - \hat{\theta}\right) + a_1 R(\varepsilon - \tau_1 + \Delta_M/2)\cos\left(\theta_1 - \hat{\theta}\right) \\
QL_M &= a_0 R(\varepsilon + \Delta_M/2)\sin\left(\theta_0 - \hat{\theta}\right) + a_1 R(\varepsilon - \tau_1 + \Delta_M/2)\sin\left(\theta_1 - \hat{\theta}\right)
\end{aligned}
\tag{7}
$$

It is noted that the correlation spacing Δ_M used in this normalized EMLD differs from Δ used in SMICLD. The in-phase prompt and quadrature prompt correlation values used in both SMICLD and this normalized EMLD are displayed in equation (8)

$$
\begin{aligned}
IP &= a_0 R(\varepsilon)\cos\left(\theta_0 - \hat{\theta}\right) + a_1 R(\varepsilon - \tau_1)\cos\left(\theta_1 - \hat{\theta}\right) \\
QP &= a_0 R(\varepsilon)\sin\left(\theta_0 - \hat{\theta}\right) + a_1 R(\varepsilon - \tau_1)\sin\left(\theta_1 - \hat{\theta}\right)
\end{aligned}
\tag{8}
$$

When τ_1- min ($\Delta_M/2$, 1-$\Delta_M/2$) $\le \varepsilon < 0$, autocorrelation functions in above equations can be written as follows

$$
\begin{aligned}
&R(\varepsilon) = \varepsilon + 1 \\
&R(\varepsilon - \tau_1) = \varepsilon - \tau_1 + 1 \\
&R(\varepsilon - \Delta_M/2) = \varepsilon - \Delta_M/2 + 1 \\
&R(\varepsilon + \Delta_M/2) = 1 - \left(\varepsilon + \Delta_M/2\right) \\
&R(\varepsilon - \tau_1 - \Delta_M/2) = \varepsilon - \tau_1 - \Delta_M/2 + 1 \\
&R(\varepsilon - \tau_1 + \Delta_M/2) = 1 - \left(\varepsilon - \tau_1 + \Delta_M/2\right)
\end{aligned}
\tag{9}
$$

In order to simplify the calculation, k and b are defined as follows.

$$
\begin{aligned}
k &= a_0^2 + a_1^2 + 2a_0 a_1 \cos\left(\theta_0 - \theta_1\right) \\
b &= a_1^2 \tau_1 + a_0 a_1 \tau_1 \cos\left(\theta_0 - \theta_1\right)
\end{aligned}
\tag{10}
$$

Combining equation (6) ~ (10), the normalized EMLD can be expressed as follows.

$$
D_{norEMLD}(\varepsilon) = \frac{4\left(1 - \Delta_M/2\right)\left(k\varepsilon - b\right)}{2k\varepsilon^2 + 2\left[\left(2 - \Delta_M/2\right)k - 2b\right]\varepsilon + \left[\left(1 - \Delta_M/2\right)^2 + 1\right]k - 2\left(2 - \Delta_M/2\right)b + 2a_1^2\tau_1^2}
\tag{8}
$$

The global maximum of $D_{norEMLD}(\varepsilon)$ is denoted as $D_{norEMLD}(\varepsilon_e)_{max}$ and given in Equation (12).

$$
D_{norEMLD}\left(\varepsilon_e\right)_{\max} = \frac{2 - \Delta_M}{\sqrt{2\left[1 + (1 - \Delta_M/2)^2\right]} + 2 - \Delta_M/2}
\tag{9}
$$

where:

$$
\varepsilon_e = \frac{2b + \sqrt{2\left\{\left[1 + \left(1 - \Delta_M/2\right)^2\right]k^2 + 2a_1^2\tau_1^2 k - 2b^2\right\}}}{2k}
\tag{10}
$$

$$
\cos\left(\theta_0 - \theta_1\right) = 1 \text{ or } -1
\tag{11}
$$

Hence, MEMLD can be acquired by subtracting $D_{norEMLD}(\varepsilon_e)_{\max}$ from $D_{norEMLD}(\varepsilon)$, which is shown in Equation (15).

$$D_{MEMLD}(\varepsilon) = \frac{\left(IE_M{}^2 + QE_M{}^2\right) - \left(IL_M{}^2 + QL_M{}^2\right)}{\left(IE_M{}^2 + QE_M{}^2\right) + \left(IP^2 + QP^2\right)} - \frac{2-\Delta_M}{\sqrt{2\left[1+(1-\Delta_M/2)^2\right]} + 2 - \Delta_M/2} \tag{12}$$

Additionally, to remove amplitude sensitivity, SMICLD is normalized by its early and adjusted prompt correlation values, which is given by:

$$D_{norSMICLD} = \frac{(IE^2 + QE^2) - ((IP')^2 + (QP')^2)}{(IE^2 + QE^2) + ((IP')^2 + (QP')^2)} \tag{16}$$

Discussion

According to Equation (15) and (16), properties of the S-curve of Modified DLL Aided by SMICLD are discussed in this section.

(1) S-curves with different Δ_M

Figure illustrates the S-curves of Modified DLL Aided by SMICLD in the presence of single multipath ray. The MDR and multipath delay are 0.6 and 0.05 chips respectively. The carrier phase difference between the multipath and the direct signal is $\pi/3$. The correlator spacing for SMICLD Δ is 1 chip. As shown in Figure , the S-curve for positive and zero code tracking errors belongs to SMICLD while the left half part belongs to MEMLD. Compared with the S-curve of SMICLD shown in Figure 1, the asymmetry has been improved, which allows Modified DLL Aided by SMICLD to operate properly when the code tracking error is negative. However, these S-curves are still discontinuous and have a gap at their zero crossing point, which is induced by the modification of the normalized EMLD according to Equation (15).

As shown in Fig. 2, the gap at the zero-crossing point is related to the correlation spacing, which can be explained by Equation (12):

$$D_{norEMLD}(\varepsilon_e)_{\max} = \frac{2-\Delta_{\mathrm{M}}}{\sqrt{2\left[1+(1-\Delta_{\mathrm{M}}/2)^2\right]} + 2 - \Delta_{\mathrm{M}}/2} = \frac{2}{\sqrt{2\left[1+\frac{1}{(1-\Delta_{\mathrm{M}}/2)^2}\right]} + \frac{1}{1-\Delta_{\mathrm{M}}/2} + 1} \tag{17}$$

Therefore, the gap can be diminished by increasing the correlation spacing Δ_M of MEMLD as shown in Fig.2.

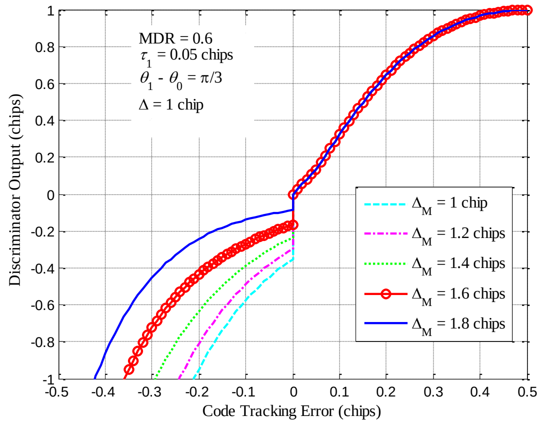
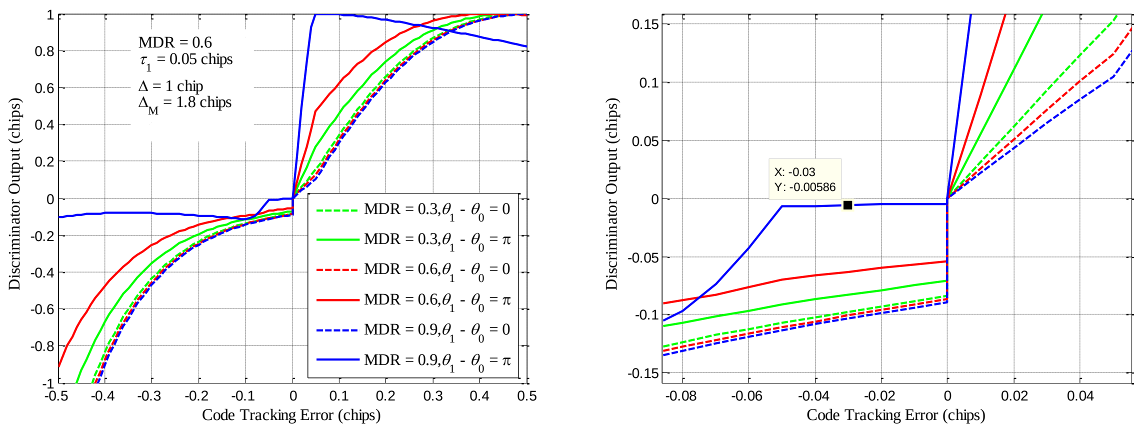


Figure 2 S-curves of Modified DLL Aided by SMICLD in the presence of single short multipath ray with different ΔM

(2) S-curves with different MDR and carrier phases

Figure displays the S-curves of Modified DLL Aided by SMICLD in the presence of single multipath ray. The multipath delay is 0.05 chips. The MDR of multipath is chosen from 0.3, 0.6 or 0.9. The carrier phase difference between the multipath and the direct signal is 0 or π. The correlator spacing for SMICLD Δ is 1 chip while the correlator spacing for MEMLD ΔM is 1.8 chips. Figure 3 (b) zooms in the region around the zero point of the S-curves shown in Figure 3 (a).



(a) **(b)**

Figure 3 S-curves of Modified DLL Aided by SMICLD in the presence of single short multipath ray with different MDR and carrier phases

Figure 3 illustrates that the gap at the zero-crossing point of the S-curve distorted by in-phase multipath is greater than that of the S-curve distorted by out-of-phase multipath, which can be explained by calculating the derivative of Equation (15) with respect to $\cos(\theta_0 - \theta_1)$ when the code tracking error is zero:

$$\frac{\partial D_{MEMLD}(0)}{\partial\cos(\theta_0-\theta_1)} = -4(1-\Delta_M/2)\frac{\left[1+(1-\Delta_M/2)^2\right]a_0 a_1\tau_1\left(a_0^2-a_1^2\right)+2a_0 a_1^3\tau_1^3}{\left(2k\varepsilon^2+2\left[(2-\Delta_M/2)k-2b\right]\varepsilon+\left[(1-\Delta_M/2)^2+1\right]k-2(2-\Delta_M/2)b+2{a_1}^2{\tau_1}^2\right)^2} < 0 \quad (18)$$

Therefore, $D_{MEMLD}(0)$ reaches its minimum values when $\cos(\theta_0 - \theta_1)$ equals to 1 ($\theta_0 - \theta_1 = 0$), which results in the largest gap.

Figure 3 also shows that when out-of-phase multipath with large MDR exists, the output of MEMLD is close to zero but still a negative value of about -0.005 chips. In real scenario, however, the power of a multipath channel can hardly reach such high level due to the channel attenuation. Even if such multipath ray exists, it will counterbalance the most power of the direct signal and result in a weak combined signal that is difficult for the GPS receiver to acquire. Even if the receiver is sensitive enough to acquire such weak signal, MEMLD still outputs negative values and adjusts the negative code error in right direction. As shown in Figure 3 also shows that the output of MEMLD is negative when ε is τ_1- min ($\Delta_M/2$, 1-$\Delta_M/2$).

Strategy for Switching Discriminators

The algorithm of Modified DLL Aided by SMICLD is illustrated in Figure 4. When the code tracking error is positive, the code loop uses SMICLD; when the code tracking error is negative, the code loop uses MEMLD. The key point is how to distinguish a zero code tracking error from a negative code error. A zero-crossing point searching strategy with controllable errors will be proposed in the following.

Let CP0, CE0 and *d* represent the local replica code phase, the code tracking error, and the zero-crossing position resolution, respectively. When SMICLD outputs zero, the code phase of the local replica is reduced to CP0-*d*, which means that the code tracking error ε = CE0 is shifted to CE0+*d*. The correlation and SMICLD will be calculated again. If the new SMICLD output still remains zero, CE0 will be considered as negative. Then the code loop is switched into the MEMLD mode.

Figure simply demonstrates the code tracking process of Modified DLL Aided by SMICLD. The zero-crossing position resolution *d* is 0.01 chips. It is assumed that the initial code tracking error is negative and less than –*d* chips, which is represented by point A. In this case, SMICLD outputs zero both in the prompt correlation. In addition, SMICLD still outputs zero after the local code phase error increases 0.01, which is considered as the sign of negative code tracking error. Thus, MEMLD is employed to adjust the code phase of the local replica. The current state of the code loop is switched to point B. Then, MEMLD outputs a negative discrimination value to adjust the code tracking error in the positive direction, which switches the state of the code loop to point C. Now, SMICLD outputs a positive value that pulls the code tracking error to zero then.

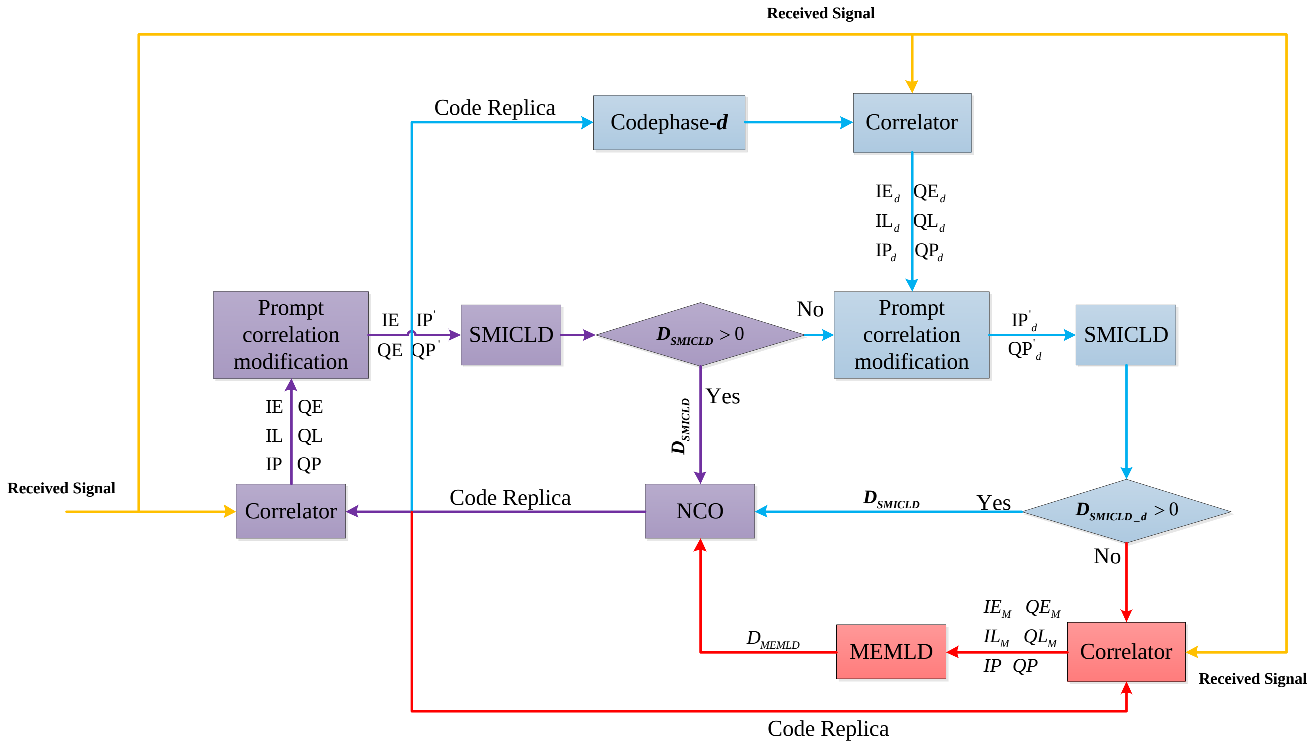


Figure 4 Algorithm Diagram of Modified DLL Aided by SMICLD

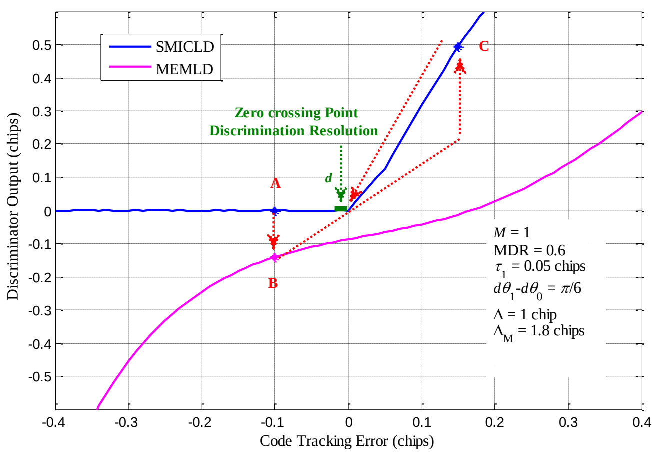


Figure 5 Code Phase Tracking Process of Modified DLL Aided by SMICLD

In Figure **6** 6, the tracking process of SMICL, Modified DLL aided by SMICLD and NC has be simulated in the presence of single short multipath ray with a multipath delay of 0.05 chips, the MDR of 0.6 and a carrier phase difference between the direct and multipath signals of $\pi/4$. The initial code tracking error for the three code loops are 0.03 chips. In addition, the code tracking error jumps to -0.01chips at the time of 5 second. The correlator spacing Δ is set to 1 chip for SMICL and Modified DLL Aided by SMICLD, 0.1 chips for HRC. The correlator spacing Δ_M of MEMLD in Modified DLL Aided by SMICLD is set to 1.8 chips.

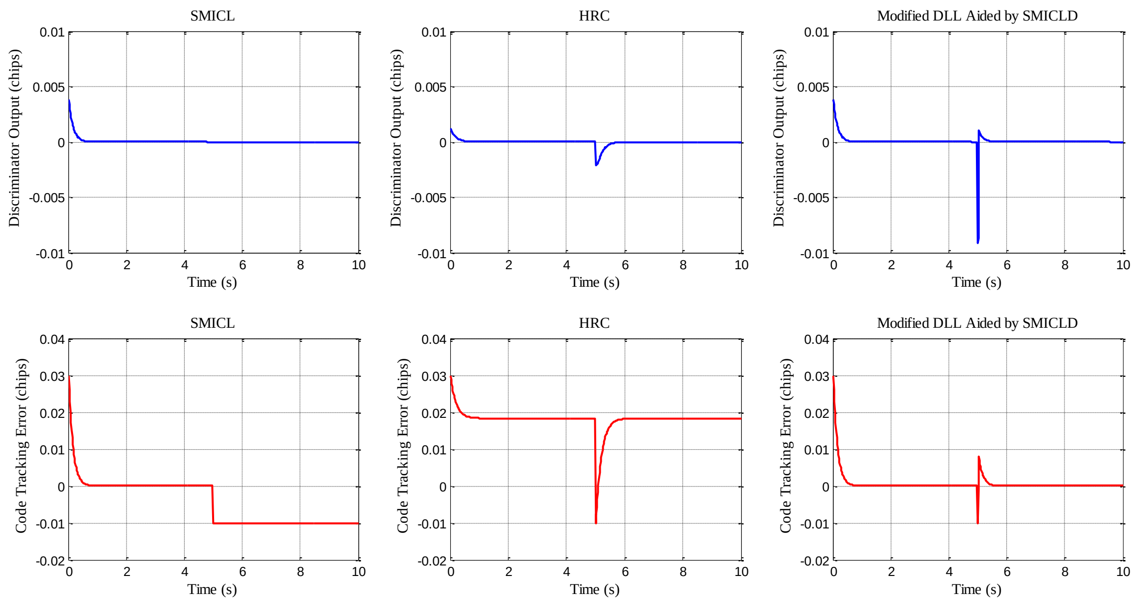


Figure 6 Discriminator Outputs and Code Tracking Errors of SMICL, HRC and Modified DLL Aided by SMICLD

As shown in Figure **6** 6, all the three discriminators can work stably in the first five seconds. The code tracking errors of SMICL and Modified DLL Aided by SMICLD are zero, whereas HRC is affected by short multipath and has a certain tracking bias. After the abrupt code phase changes, SMICLD cannot find this negative-going code phase error and thus locks to the point with a tracking bias of -0.01chip. HRC recognizes this code phase change and adjusts the code phase of the local replica promptly to track the received signal with the same bias as before. Modified DLL aided by SMICLD can not only catches the phase jump, but also pulls the code tracking error to zero without any bias.

EXPERIMENTAL RESULTS

SMICL, Modified DLL Aided by SMICLD, HRC and MEDLL have been implemented in software-defined GPS L1 C/A digital intermediate frequency (IF) signal receivers respectively, the short multipath mitigation performance of which has been tested with by means of the software-defined GPS L1 C/A digital IF signal simulator. Both these receivers and the simulator are developed by our group with C++ in Windows. Moreover, the GPS Radio Frequency (RF) signal simulator and broadband high-speed data recording system are used to further test the short multipath mitigation performance of Modified DLL Aided by SMICLD in a real short multipath scene

Tests Results at Digital IF level

Firstly, to demonstrate the superiority of Modified DLL Aided by SMICLD, its code tracking errors are compared with those of SMICL, HRC and MEDLL in short multipath and noise environment. Moreover, the multipath envelope of Modified DLL Aided by SMILCD is provided to further assess its multipath mitigation performance.

Code tracking errors in short multipath environment

The following four scenarios are simulated using our digital IF GPS signal simulator to test the performance against short delay multipath of the code loops mentioned above:

Scenario A. Weak noise environment with single in-phase short multipath ray;

Scenario B: Strong noise environment with single in-phase short multipath ray;

Scenario C: Weak noise environment with single out-phase short multipath ray;

Scenario D: Strong noise environment with single out-phase short multipath ray.

The sampling frequency and the carrier frequency are 100MHz and 1.405MHz, respectively. MDR and multipath delay of the single short multipath signal in these four scenarios are 0.6 and 0.05 chips respectively. The carrier-to-noise ratio (CN0) in scenario A and C is 65 dB•Hz, where the short multipath mitigation performance of these code loops is mainly tested. The CN0 in scenario B and D is 45 dB•Hz, where the anti-noise and anti-short multipath abilities of these code loops are both tested. The correlator spacing Δ of the software receiver is set to 1 chip for SMICL and Modified DLL Aided by SMICLD, 0.2 chips for MEDLL and 0.1 chips for HRC. The correlator spacing Δ_M of MEMLD in Modified DLL Aided by SMICLD is set to 1.5 chips.

Code tracking errors of SMICL, Modified DLL Aided by SMICLD, HRC and MEDLL in the four scenarios are displayed in Fig. For a quantitative comparison, means and standard deviations of code tracking errors of above code loops in the four scenarios are summarized in Table 2.

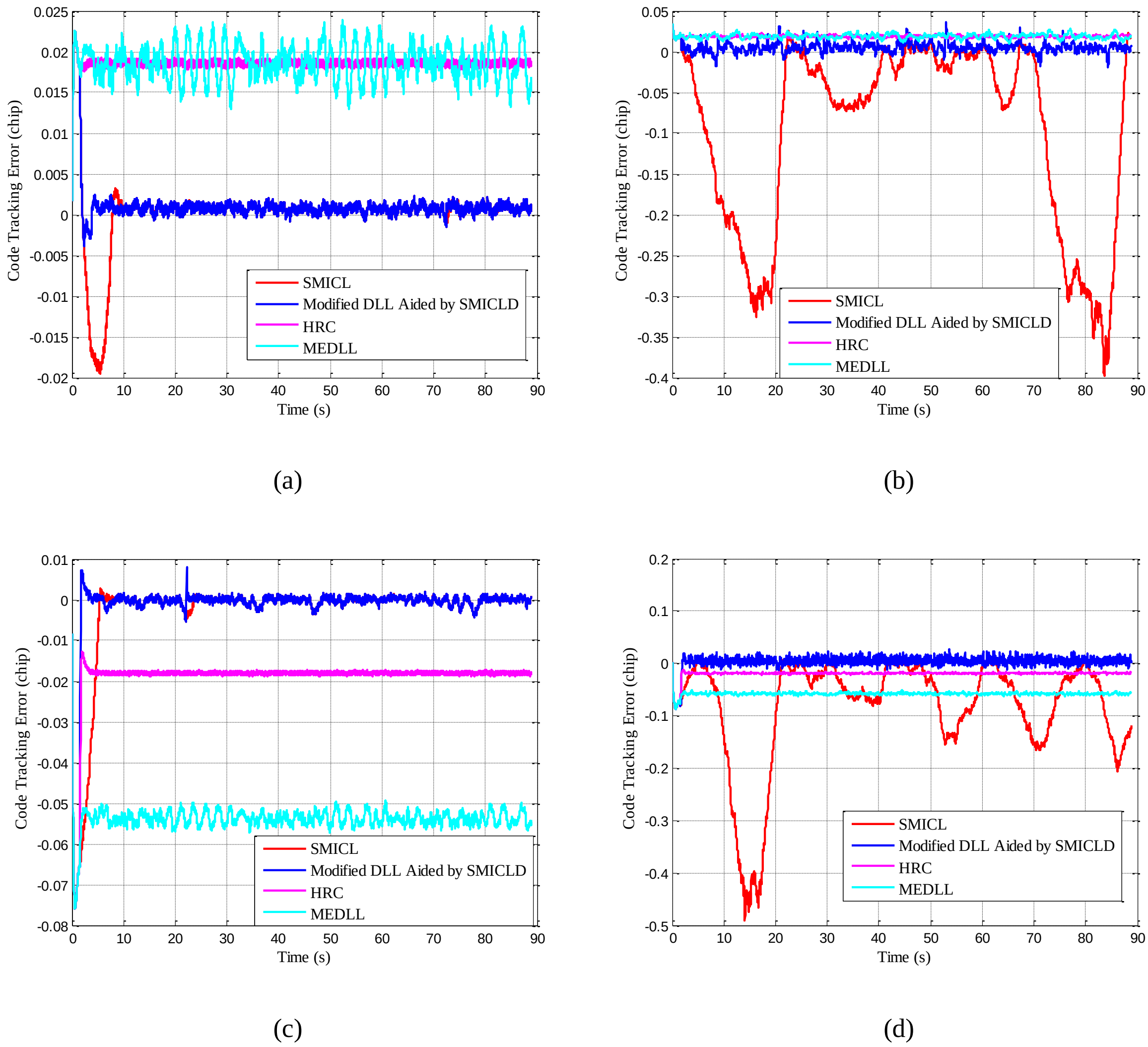


Figure 7 Code Tracking Errors of SMICL, Modified DLL Aided by SMICLD, HRC and MEDLL in (a) scenario A, (b) scenario B, (c) scenario C and (d) scenario D

Fig (a) and (c) also illustrate that the code tracking error of SMICL fluctuates in the beginning. Because the practical correlation function is not the same as the ideal correlation function expressed by equation (2), SMICLD actually outputs about -0.001chips rather than zero when the code tracking error is negative. Thus, SMICL can still converge its code tracking error to zero but very slowly. Therefore, the code tracking error of SMICL is doomed to have large fluctuations in noise environment because noise will turn the code tracking error into negative, which has been proved in Figure 7(b) and (d). The means of the code tracking error of SMICL in strong noise are more than 100 times larger than those in weak noise. By contrast, Modified DLL aided by SMICLD can instantly adjust the local replica code phase to be aligned with the received signal.

Table 2: Means and standard deviations of code tracking errors of HRC, MEDLL, SMICL and Modified DLL Aided by SMICLD

Multipath Mitigation Methods	Code tracking errors							
	Mean (chip)				Standard Deviation (chip)			
	Scenario A	Scenario B	Scenario C	Scenario D	Scenario A	Scenario B	Scenario C	Scenario D
HRC	0.0186	0.0187	-0.0180	-0.0197	5.05e-4	0.0013	3.14e-4	0.0075
MEDLL	0.0187	0.0190	-0.0537	-0.0579	0.0022	0.0029	0.0015	0.0034
SMICL	7.38e-4	-0.0969	-2.49e-4	-0.0853	5.12e-4	0.1151	0.0011	0.1059
Modified DLL Aided by SMICLD	7.44e-4	0.0053	-1.83e-4	0.0028	5.05e-4	0.0064	0.0010	0.0123

Table 2 indicates that the proposed Modified DLL Aided by SMICLD has the smallest means of code tracking errors among these four code loops in noise scenario B and D. Compared with SMICL, the means of code tracking errors of Modified DLL Aided by SMICLD in scenario B and D have decreased by more than 94% and 96% respectively. At the same time, the standard deviations are also reduced by 2 and 1 order of magnitude respectively. Moreover, the code tracking errors of HRC and MEDLL both have larger means than Modified DLL Aided by SMICLD, even though they have smaller standard deviations. In scenario B, the mean of the code error of Modified DLL Aided by SMICLD is smaller than those of HRC and MEDLL by more than 71% and 72% respectively. In scenario D, the mean of the code error of Modified DLL Aided by SMICLD is smaller than those of HRC and MEDLL by more than 85% and 95% respectively. Therefore, the proposed Modified DLL Aided by SMICLD has the best anti-short multipath performance in noise environment among the four code loops. The calculation and comparison about code tracking errors of code loops mentioned above are all referred to absolute values.

Noise performance

To further assess the short multipath mitigation performance of Modified DLL Aided by SMICLD and SMICL in noise environment, Figure 8 provides means and standard deviations of code tracking errors in different noise backgrounds. The CN0 in Figure 8 varies from 43dB•Hz to 59dB•Hz. The multipath ray in this test has the MDR of 0.6, multipath delay of

0.05chips. The difference of carrier phase between the short multipath and direct signal is 0.75π. The settings of receivers are the same as before.

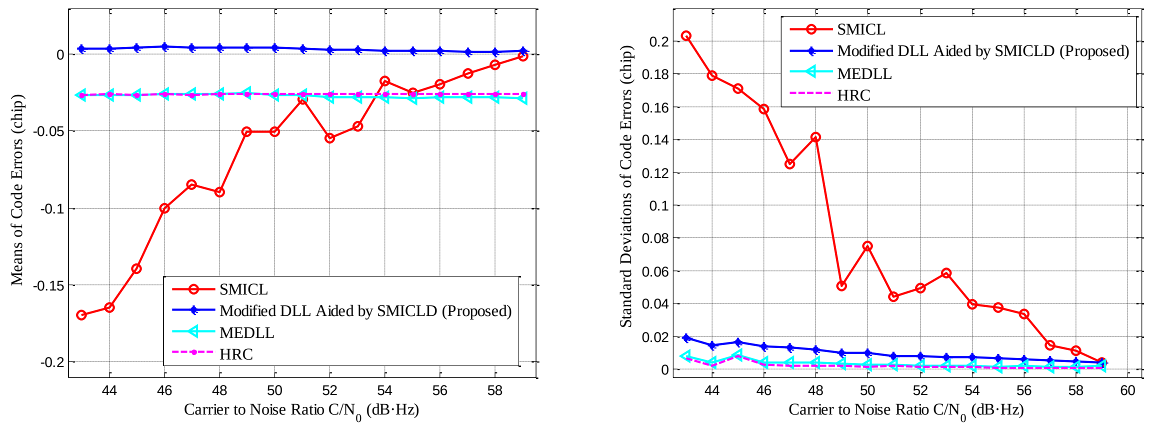


Figure 8 (a) Means and (b) Standard Deviations of Code Tracking Errors of SMICL, Modified DLL Aided by SMICLD, MEDLL and HRC in Different Noise Backgrounds

Figure 8 (a) indicates that SMICL has the largest code tracking error mean among the four code loops when CN0 is lower than 54 dB•Hz. As it is explained before, the poor anti-noise ability of SMICLD counterbalances its anti-short multipath ability.

Compared with the other three code loops, the proposed Modified DLL Aided by SMICLD can maintain the code tracking error mean of less than 0.01 chips in noise environment with any CN0. Figure 8 (b) indicates that the standard deviation of the code tracking error of Modified DLL Aided by SMICLD is also much smaller than that of SMICL. Although the standard deviation of the code tracking error of the proposed code loop is still higher than that of HRC and MEDLL, the difference between them is no more than 0.02 chips. Thus, its anti-noise ability is close to HRC and MEDLL.

In terms of multipath mitigation performance, we are more concerned about the bias of the code tracking error mean. Therefore, Modified DLL Aided by SMICLD has the best short multipath mitigation capability among the above four code loops.

Multipath code error envelope

As can be seen from Figure 2 and Figure 3, the S-curve of the Modified DLL Aided by SMICLD has a gap at its zero-crossing point. In noise environment, when the negative code tracking error is close to zero, the adjustment value will be too large relative to the current negative code tracking error, which makes the code tracking error excessively adjusted towards the positive direction and eventually results in a positive code tracking error.

The multipath code error envelope depends on the gap at the zero-crossing point of Modified DLL Aided by SMICLD's S-curve, which is caused by the output of MEMLD when the code tracking error is zero. The upper boundary of the multipath code error envelope of Modified DLL Aided by SMICLD appears where the gap is the largest. In contrast, the lower boundary of multipath code error envelope occurs when the gap is minimal.

(1) $\tau_1 \leq \min(\Delta_M/2, 1-\Delta_M/2)$

According to Equation (18), $D_{MEMLD}(0)$ reaches its minimum value when $\cos(\theta_0 - \theta_1)$ equals to 1 or ($\theta_0 - \theta_1 = 0$), which results in the largest gap and the upper boundary of the multipath code error envelope of Modified DLL Aided by SMICLD. $D_{MEMLD}(0)$ reaches its maximum value when $\cos(\theta_0 - \theta_1)$ equals to -1 ($\theta_0 - \theta_1 = \pi$), which results in the smallest gap and the lower boundary.

(2) $\min(\Delta_M/2, 1-\Delta_M/2) \leq \tau_1 \leq \min(\Delta/2, 1-\Delta/2)$

In this case, the derivative of Equation (15) with respect to $\cos(\theta_0 - \theta_1)$

$$\frac{\partial D_{MEMLD}(0)}{\partial \cos(\theta_0 - \theta_1)} = -2a_0 a_1 (1-\Delta_M/2+\tau_1) \frac{a_0^2(1-\Delta_M/2)\left[(1-\Delta_M/2)^2+1\right] - a_1^2(1-\tau_1)\tau_1(2-\Delta_M/2)}{\left[a_0^2(1-\Delta_M/2)^2 + a_0^2 + a_1^2(1-\tau_1)^2 + 2a_0 a_1(1-\tau_1)\cos(\theta_0-\theta_1)\right]^2} \tag{19}$$

The MDR of the multipath ray tested in this section is 0.6. And the correlation spacing Δ_M in MEMLD is 1.8 chips. In this case, Equation (19) is greater than zero, which means the largest gap and the upper boundary of the multipath code error envelope of Modified DLL Aided by SMICLD appear when $\cos(\theta_0 - \theta_1)$ equals to -1 or ($\theta_0 - \theta_1 = \pi$). On the contrary, the smallest gap and the lower boundary are reached when $\cos(\theta_0 - \theta_1)$ equals to 1 or ($\theta_0 - \theta_1 = 0$).

(3) $\tau_1 \geq \min(\Delta/2, 1-\Delta/2)$

In this case, the multipath code error envelope of Modified DLL Aided by SMICLD only depends on SMICLD. The carrier phase differences between the multipath and the direct signal corresponding to the upper and lower boundaries are the same as situation (2). The details can be referred to Weng & Kou [2017][20].

We measured the multipath code error envelope of Modified DLL Aided by SMICLD with the software-defined GPS L1 C/A IF signal simulator and receiver. The MDR of the multipath ray tested in this section is 0.6. Thermal noise with CN0 of 45 dB•Hz is also added in this test. Table 3 displays the carrier phase differences between the multipath ray and the direct signal with respect to different multipath delays, which are utilized to construct the multipath code error envelope in this test.

Table 3 The carrier phase differences between the multipath ray and the direct signal used in the test of Modified DLL Aided by SMICLD

Multipath delay/chip	0 : 0.01 : 0.1	0.2 : 0.1 : 1.0
The Upper Boundary	θ_0-θ_1=0	θ_0-θ_1=π
The Lower Boundary	θ_0-θ_1=π	θ_0-θ_1=0

Figure 9 exhibits the multipath code error envelopes of SMICL, Modified DLL Aided by SMICLD, HRC and MEDLL. The settings of these receivers are the same as before. Figure 9 shows that the code error envelope of SMICLD fluctuates randomly and seriously in the negative range when multipath delay is shorter than 0.5 chips. In this case, SMICLD always outputs zero and cannot adjust the local replica's code phase timely when the code error becomes negative due to thermal noise. When the multipath delay is larger than 0.5chips, SMICL's multipath code error envelope becomes more stable, because SMICLD now has a symmetrical S-curve around its zero-crossing point but with a certain bias.

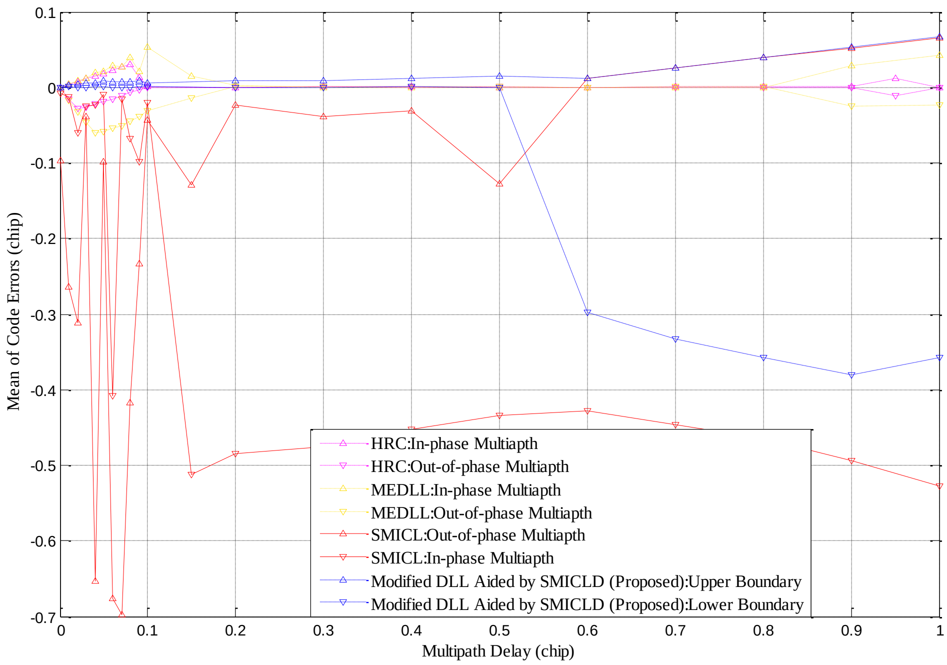


Figure 9 Measured Multipath Code Error Envelopes for SMICL, Modified DLL Aided by SMICLD, HRC and MEDLL

Although HRC can eliminate multipath with delay greater than 0.1 chips, as shown in Figure 9, it cannot mitigate short multipath with delay of less than 0.1 chips. Similarly, MEDLL can mitigate multipath with delay of more than 0.15 chips while useless for short multipath with delay of less than 0.15 chips. To more clearly show the performance of each code loop for short multipath with delay of less than 0.1 chips, the multipath code error envelope within 0.1 chips is zoomed in by Figure 10.

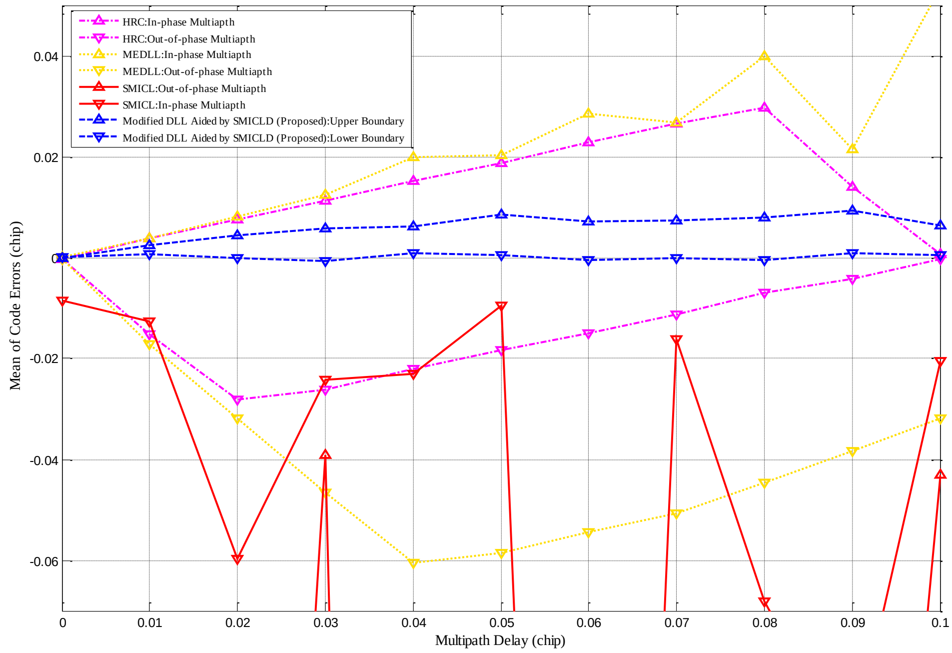


Figure 10 Measured Multipath Code Error Envelopes for SMICL, Modified DLL Aided by SMICLD, HRC and MEDLL with respect to the multipath delay of [0, 0.1chips]

As can be seen from Figure 10, the lower boundary of the multipath code error envelope of Modified DLL Aided by SMICLD is maintained near zero, which is superior to the other three code loops. Its upper boundary of the multipath code error envelope is kept below 0.01 chips, which is also the smallest among the four code loops. For the short multipath of 0.1chips~0.5chips, Modified DLL Aided by SMICLD cannot completely eliminate multipath, but still keep its code tracking error less than 0.015chips. However, it is less effective for medium and long delay multipath with delay over 0.5chips, which is caused by the biased zero-crossing point of SMICLD's S-curve.

Real Multipath Environment Test at RF Signal Level

The GPS Radio Frequency (RF) signal simulator and broadband high-speed data recording system are used to further test the short multipath mitigation performance of Modified DLL Aided by SMICLD in a real short multipath scene displayed in Figure 11.

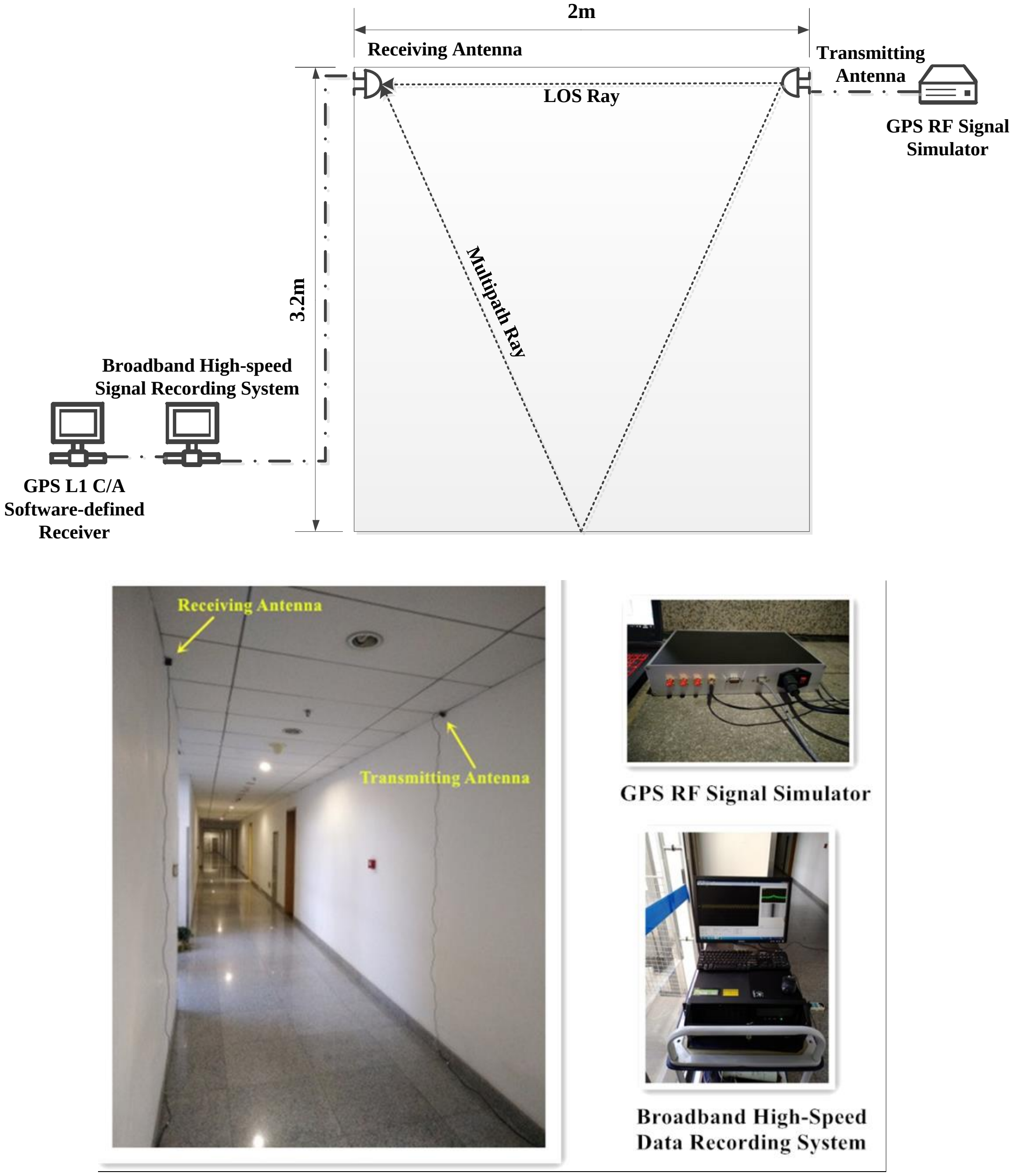


Figure 11 Real Short Delay Multipath Scenario

The GPS RF signal simulator developed by our research group can generate GPS L1 C/A RF signal based on pre-set time, user's location and velocity information. The simulated signal power is adjustable. The broadband high-speed data recording system, a workstation based on the TG-X3600 high-speed data acquisition card and the disk array constituted by solid state disks, can continuously record GPS RF signal with high sampling rate. These two devices have been shown in Figure 11.

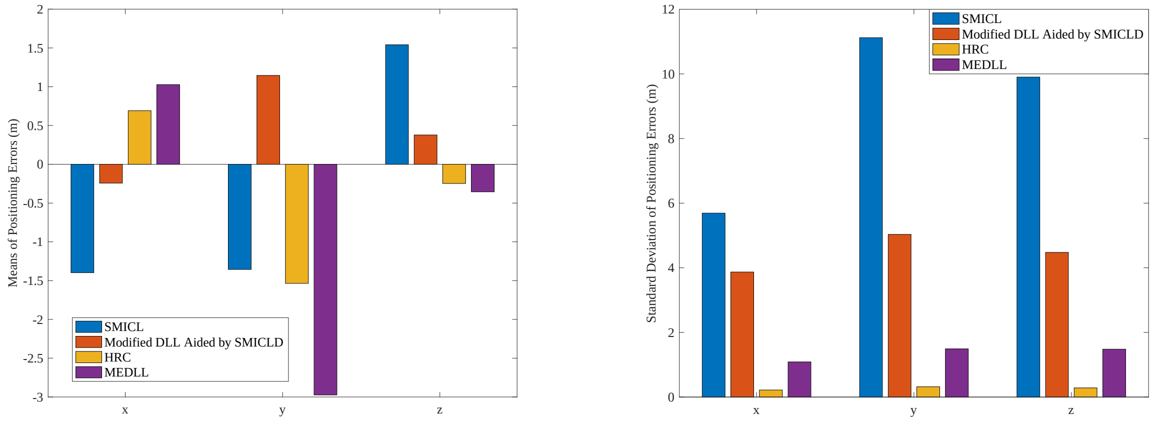
As shown in Figure 11, a real short delay multipath scenario is set up in the corridor of the New Main Building of Beihang University. The length of the corridor is about 280 meters, much longer than the width and the height, so only the following four reflectors are considered: the ground, the ceiling and two walls alongside the corridor. The transmitting antenna connected to the GPS RF signal simulator is attached to the ceiling and the right wall. The receiving antenna connected to a broadband high-speed data continuous system is attached to the ceiling and the left wall. Therefore, the ground becomes the solitary reflection surface without considering multiple reflections whose power has been attenuated seriously. After measurement and calculation, the multipath delay is about 0.016 chips (4.7m). The parameters of signal simulated by the GPS RF signal simulator are displayed in Table 4.

Table 4 Parameters Used in the GPS RF signal simulator

Parameters	Values
Signal Power	9dBm
Radio Frequency fRF	1575.42MHz
Longitude	116º E
Latitude	40º N
Height	100m

The broadband high-speed data recording system stores the collected signal as ".dat" format with sampling frequency of 100MHz. GPS L1 C/A software receivers based on HRC, MEDLL, SMICL and Modified DLL Aided by SMICLD have been used to process the ".dat" files. The settings of these receivers are the same as before.

The positioning results of these receivers are compared with the position coordinates set in the GPS RF signal simulator, from which we can get the positioning errors calculated in xyz-coordinate system. The means and the standard deviation of the positioning errors on the X, Y and Z axes are displayed in Figure 12 (a) and (b) respectively. The epoch interval is 0.02s.



(a) **(b)**

Figure 12 Positioning errors of SMICL, Modified DLL Aided by SMICLD, ESMICL, HRC and MEDLL: (a) means of positioning errors of three axes

As shown in Figure 12, Modified DLL Aided by SMICLD has the smallest positioning errors of each axis among the four receivers. Compared with SMICL, the positioning errors on the X, Y and Z axes of Modified DLL Aided by SMICLD have been diminished by more than 82%, 15% and 87% respectively, which is the contribution from improvement in the S-curve. By contrast, in terms of HRC and MEDLL, code errors induced by short multipath have evolved into positioning errors displayed in Fig.12.

It can be seen that Modified DLL Aided by SMICLD has much smaller standard deviations of positioning errors than SMICL, which means the anti-noise performance has been improved. However, as a result of the discontinuity at the zero-crossing point of its S-curve, the standard deviations of Modified DLL Aided by SMICLD's positioning errors are still larger than the HRC and MEDLL, which implies that Modified DLL Aided by SMICLD is more sensitive to noise than traditional code loops.

The ultimate goal of multipath mitigation techniques is to eliminate the positioning bias caused by multipath. Therefore, Modified DLL Aided by SMICLD is the best choice in above receivers considering both the short multipath mitigation performance and the anti-noise ability.

CONCLUSIONS

This paper has assessed the multipath mitigation performance of SMICLD. Although the zero-crossing point of SMICLD's S-curve has no bias in the presence of single short multipath ray, SMICLD outputs zero when the code tracking error is negative. Such asymmetry of its S-curve can result in large code tracking biases in real noise environment. Additionally, SMICLD cannot deal with two or more multipath rays or with medium and long multipath.

Aiming at improving the S-curve of SMICLD, this paper has proposed a novel code tracking loop called Modified DLL Aided by SMICLD. Instead of using SMICLD, this new code loop utilizes MEMLD to adjust the loop under negative code tracking errors. And a correlation channel that is later than the prompt correlation channel is added to switch the code loop to the proper discriminator. Simulation results show that Modified DLL Aided by SMICLD keeps the short multipath mitigation performance of SMICL while overcoming its disadvantages of instability under severe signal phase fluctuations.

Modified DLL Aided by SMICLD, SMICL, HRC and MEDLL have been implemented in our software receiver and their short multipath mitigation performances have been compared by using the digital IF GPS signal simulator in the following aspects: (1) The code tracking errors with single in-phase or out-of-phase short multipath ray; (2) The mean and standard deviation of code phase errors with respect to different CN0; (3) The multipath code error envelope. Moreover, their positioning errors in a real short multipath scene are tested by using a GPS RF signal simulator and a broadband high-speed data recording system.

Test results show that Modified DLL Aided by SMICLD can not only mitigate the code tracking biases caused by short multipath, but also promptly adjust the local replica code phase to track the received signal. Compared with SMICL, HRC and MEDLL, the proposed Modified DLL Aided by SMICLD has better single short multipath ray mitigation performance in noise environment.

Future work will focus on reducing the gap at the zero-crossing point of the S-curve of Modified DLL Aided by SMICD. The cases with more than one multipath ray will also be investigated.

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