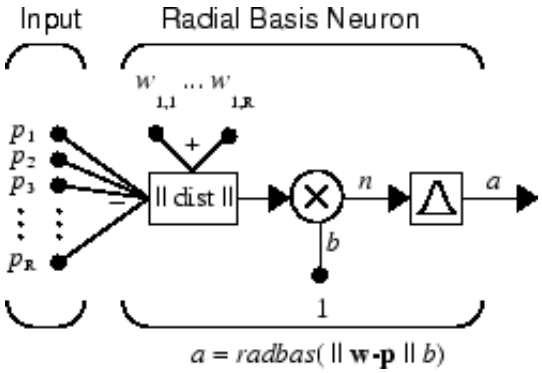
# Radial Basis Neural Networks

## Neuron Model

Here is a radial basis network with $R$ inputs.

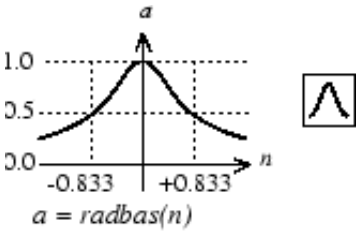



Notice that the expression for the net input of a `radbas` neuron is different from that of other neurons. Here the net input to the `radbas` transfer function is the vector distance between its weight vector **w** and the input vector **p**, multiplied by the bias $b$. (The `||dist||` box in this figure accepts the input vector **p** and the single row input weight matrix, and produces the dot product of the two.)

The transfer function for a radial basis neuron is

$$radbas(n) = e^{-n^2}$$

Here is a plot of the `radbas` transfer function.



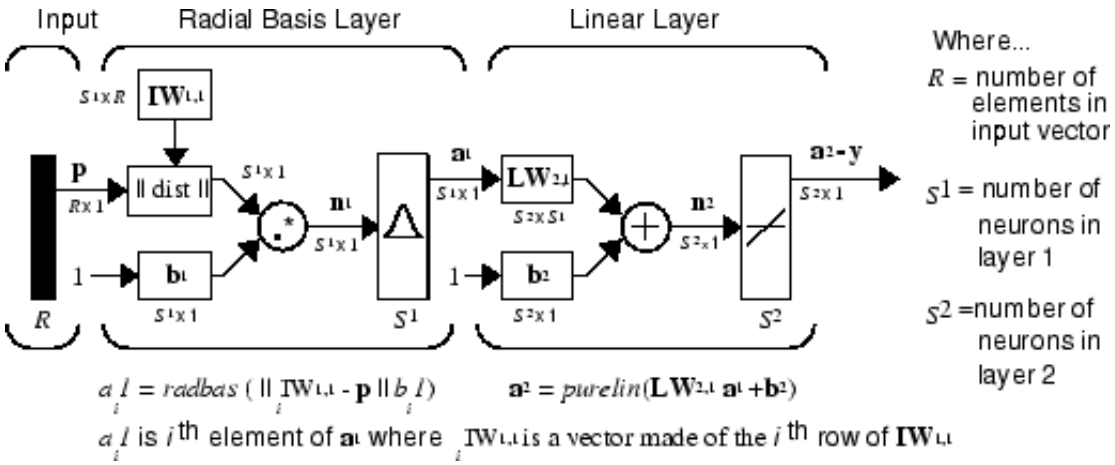

Radial Basis Function

The radial basis function has a maximum of 1 when its input is 0. As the distance between **w** and **p** decreases, the output increases. Thus, a radial basis neuron acts as a detector that produces 1 whenever the input **p** is identical to its weight vector **w**.

The bias $b$ allows the sensitivity of the `radbas` neuron to be adjusted. For example, if a neuron had a bias of 0.1 it would output 0.5 for any input vector **p** at vector distance of 8.326 (0.8326/$b$) from its weight vector **w**.

## Network Architecture

Radial basis networks consist of two layers: a hidden radial basis layer of $S^1$ neurons, and an output linear layer of $S^2$ neurons.




The `||dist||` box in this figure accepts the input vector **p** and the input weight matrix $\mathbf{IW}^{1,1}$, and produces a vector having $S_1$ elements. The elements are the distances between the input vector and vectors $_i\mathbf{IW}^{1,1}$ formed from the rows of the input weight matrix.

The bias vector $\mathbf{b}^1$ and the output of `||dist||` are combined with the MATLAB® operation .* , which does element-by-element multiplication.

The output of the first layer for a feedforward network `net` can be obtained with the following code:

```
a{1} = radbas(netprod(dist(net.IW{1,1},p),net.b{1}))
```

Fortunately, you won't have to write such lines of code. All the details of designing this network are built into design functions `newrbe` and `newrb`, and you can obtain their outputs with `sim`.

You can understand how this network behaves by following an input vector **p** through the network to the output $\mathbf{a}^2$. If you present an input vector to such a network, each neuron in the radial basis layer will output a value according to how close the input vector is to each neuron's weight vector.

Thus, radial basis neurons with weight vectors quite different from the input vector **p** have outputs near zero. These small outputs have only a negligible effect on the linear output neurons.

In contrast, a radial basis neuron with a weight vector close to the input vector **p** produces a value near 1. If a neuron has an output of 1, its output weights in the second layer pass their values to the linear neurons in the second layer.

In fact, if only one radial basis neuron had an output of 1, and all others had outputs of 0s (or very close to 0), the output of the linear layer would be the active neuron's output weights. This would, however, be an extreme case. Typically several neurons are always firing, to varying degrees.

Now look in detail at how the first layer operates. Each neuron's weighted input is the distance between the input vector and its weight vector, calculated with `dist`. Each neuron's net input is the element-by-element product of its weighted input with its bias, calculated with `netprod`. Each neuron's output is its net input passed through `radbas`. If a neuron's weight vector is equal to the input vector (transposed), its weighted input is 0, its net input is 0, and its output is 1. If a neuron's weight vector is a distance of `spread` from the input vector, its weighted input is `spread`, its net input is sqrt(−log(.5)) (or 0.8326), therefore its output is 0.5.

## Exact Design (newrbe)

You can design radial basis networks with the function `newrbe`. This function can produce a network with zero error on training vectors. It is called in the following way:

```
net = newrbe(P,T,SPREAD)
```

The function `newrbe` takes matrices of input vectors P and target vectors T, and a spread constant SPREAD for the radial basis layer, and returns a network with weights and biases such that the outputs are exactly T when the inputs are P.

This function `newrbe` creates as many `radbas` neurons as there are input vectors in P, and sets the first-layer weights to `P'`. Thus, there is a layer of `radbas` neurons in which each neuron acts as a detector for a different input vector. If there are $Q$ input vectors, then there will be $Q$ neurons.

Each bias in the first layer is set to 0.8326/SPREAD. This gives radial basis functions that cross 0.5 at weighted inputs of +/− SPREAD. This determines the width of an area in the input space to which each neuron responds. If SPREAD is 4, then each `radbas` neuron will respond with 0.5 or more to any input vectors within a vector distance of 4 from their weight vector. SPREAD should be large enough that neurons respond strongly to overlapping regions of the input space.

see the last two paragraphs

The second-layer weights IW $^{2,1}$ (or in code, `IW{2,1}`) and biases b$^2$ (or in code, `b{2}`) are found by simulating the first-layer outputs a$^1$ (`A{1}`), and then solving the following linear expression:

```
[W{2,1} b{2}] * [A{1}; ones(1,Q)] = T
```

Q is the number of hidden neurons

You know the inputs to the second layer (`A{1}`) and the target (T), and the layer is linear. You can use the following code to calculate the weights and biases of the second layer to minimize the sum-squared error.

```
Wb = T/[A{1}; ones(1,Q)]
```

Here `Wb` contains both weights and biases, with the biases in the last column. The sum-squared error is always 0, as explained below.

There is a problem with $C$ constraints (input/target pairs) and each neuron has $C$ +1 variables (the $C$ weights from the $C$ `radbas` neurons, and a bias). A linear problem with $C$ constraints and more than $C$ variables has an infinite number of zero error solutions.

Thus, `newrbe` creates a network with zero error on training vectors. The only condition required is to make sure that SPREAD is large enough that the active input regions of the `radbas` neurons overlap enough so that several `radbas` neurons always have fairly large outputs at any given moment. This makes the network function smoother and results in better generalization for new input vectors occurring between input vectors used in the design. (However, SPREAD should not be so large that each neuron is effectively responding in the same large area of the input space.)

The drawback to `newrbe` is that it produces a network with as many hidden neurons as there are input vectors. For this reason, `newrbe` does not return an acceptable solution when many input vectors are needed to properly define a network, as is typically the case.

## More Efficient Design (newrb)

Sequential forward selection algorithm

The function `newrb` iteratively creates a radial basis network one neuron at a time. Neurons are added to the network until the sum-squared error falls beneath an error goal or a maximum number of neurons has been reached. The call for this function is

```
net = newrb(P,T,GOAL,SPREAD)
```

The function `newrb` takes matrices of input and target vectors P and T, and design parameters `GOAL` and `SPREAD`, and returns the desired network.

The design method of `newrb` is similar to that of `newrbe`. The difference is that `newrb` creates neurons one at a time. At each iteration the input vector that results in lowering the network error the most is used to create a `radbas` neuron. The error of the new network is checked, and if low enough `newrb` is finished. Otherwise the next neuron is added. This procedure is repeated until the error goal is met or the maximum number of neurons is reached.

As with `newrbe`, it is important that the spread parameter be large enough that the `radbas` neurons respond to overlapping regions of the input space, but not so large that all the neurons respond in essentially the same manner.

Why not always use a radial basis network instead of a standard feedforward network? Radial basis networks, even when designed efficiently with `newrbe`, tend to have many times more neurons than a comparable feedforward network with `tansig` or `logsig` neurons in the hidden layer.

This is because sigmoid neurons can have outputs over a large region of the input space, while `radbas` neurons only respond to relatively small regions of the input space. The result is that the larger the input space (in terms of number of inputs, and the ranges those inputs vary over) the more `radbas` neurons required.

On the other hand, designing a radial basis network often takes much less time than training a sigmoid/linear network, and can sometimes result in fewer neurons' being used, as can be seen in the next example.

## Examples

The example Radial Basis Approximation shows how a radial basis network is used to fit a function. Here the problem is solved with only five neurons.

Examples Radial Basis Underlapping Neurons and Radial Basis Overlapping Neurons examine how the spread constant affects the design process for radial basis networks.

In Radial Basis Underlapping Neurons, a radial basis network is designed to solve the same problem as in Radial Basis Approximation. However, this time the spread constant used is 0.01. Thus, each radial basis neuron returns 0.5 or lower for any input vector with a distance of 0.01 or more from its weight vector.

Because the training inputs occur at intervals of 0.1, no two radial basis neurons have a strong output for any given input.

Radial Basis Underlapping Neurons showed that having too small a spread constant can result in a solution that does not generalize from the input/target vectors used in the design. Example Radial Basis Overlapping Neurons shows the opposite problem. If the spread constant is large enough, the radial basis neurons will output large values (near 1.0) for all the inputs used to design the network.

If all the radial basis neurons always output 1, any information presented to the network becomes lost. No matter what the input, the second layer outputs 1's. The function `newrb` will attempt to find a network, but cannot because of numerical problems that arise in this situation.

The moral of the story is, choose a spread constant larger than the distance between adjacent input vectors, so as to get good generalization, but smaller than the distance across the whole input space.

For this problem that would mean picking a spread constant greater than 0.1, the interval between inputs, and less than 2, the distance between the leftmost and rightmost inputs.