# Solution notes for Advanced Graphics 2002

(a) ray, defined by eyepoint $\underset{\sim}{E}$ and direction vector $\underset{\sim}{D}$

disc, defined by centre point $\underset{\sim}{C}$, normal vector $\underset{\sim}{N}$ and radius, $r$

first find the intersection point between the ray and the plane in which the disc lies.

plane is defined as: $\underset{\sim}{P} : (\underset{\sim}{P} - \underset{\sim}{C}) \cdot \underset{\sim}{N} = 0$

ray is defined as $\underset{\sim}{P}(t) = \underset{\sim}{E} + t\underset{\sim}{D}$, $t \geq 0$

$\therefore (\underset{\sim}{E} + t\underset{\sim}{D} - \underset{\sim}{C}) \cdot \underset{\sim}{N} = 0$

$\Rightarrow t\underset{\sim}{D} \cdot \underset{\sim}{N} = (\underset{\sim}{C} - \underset{\sim}{E}) \cdot \underset{\sim}{N}$

So: if $\underset{\sim}{D} \cdot \underset{\sim}{N} = 0 \Rightarrow$ no intersection point

else $t = \dfrac{(\underset{\sim}{C} - \underset{\sim}{E}) \cdot \underset{\sim}{N}}{\underset{\sim}{D} \cdot \underset{\sim}{N}}$

if $t < 0 \Rightarrow$ no intersection point

else $\underset{\sim}{P} = \underset{\sim}{E} + t\underset{\sim}{D}$

Now, if $|\underset{\sim}{P} - \underset{\sim}{C}| \leq r$ then $\underset{\sim}{P}$ is the point of intersection between the ray and disc

otherwise there is no intersection point

(b) Defining a closed cylinder as an open cylinder and two discs requires three ray-object intersection calculations while having a closed cylinder primitive requires only a single ray-object intersection calculation. The latter also only considers the two ends when strictly necessary and is therefore more efficient in all respects.

In terms of accuracy, the closed cylinder primitive guarantees that the correct intersection will be found, if one exists, whereas the "three primitive" version allows for the possibility for a ray to "slip between" the disc and cylinder owing to floating point inaccuracies

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(c) A winged edged data structure should be used when the geometry may need to be altered: e.g. in a 3D geometry editor or a subdivision scheme.
It is not sensible when all we want to do is draw the objects (i.e. when there is no possibility of changing the mesh).
The minimum information required is the position of each vertex and which vertices define each polygon (an ordered list with a consistent ordering: clockwise or counter clockwise). From this we can extract the normal vector for each polygon and hence each vertex. We then need only the colour at each vertex.

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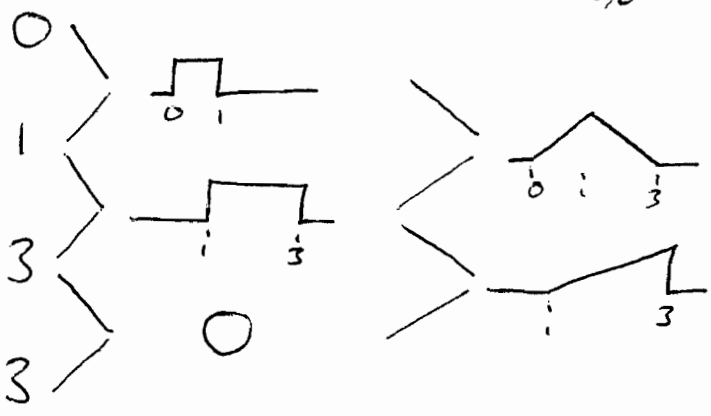
(d) $N_{3,3}(t)$ depends on the 3rd, 4th, 5th & 6th knots, which are 0 1 3 3

| $i$ | $t_i$ | $N_{i,1}(t)$ | $N_{i,2}(t)$ |
|---|---|---|---|
| 3 | 0 | | |
| 4 | 1 | | |
| 5 | 3 | | |
| 6 | 3 | | |

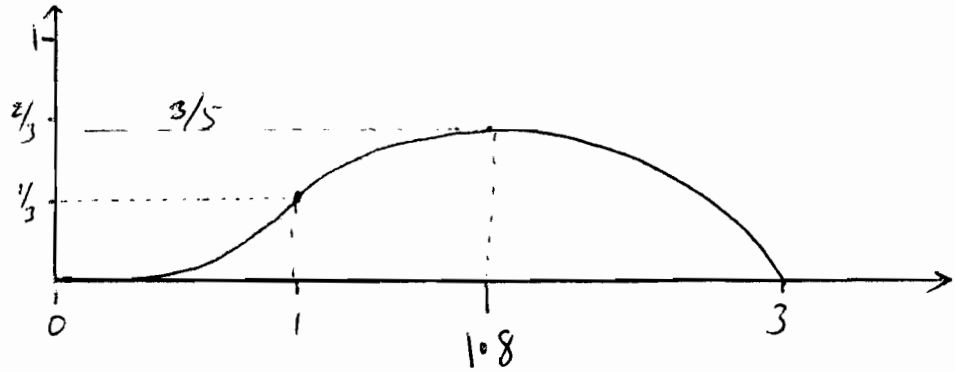



$$N_{3,2}(t) = \begin{cases} t, & 0 \le t < 1 \\ \frac{1}{2}(3-t), & 1 \le t < 3 \end{cases}$$

$$N_{4,2}(t) = \tfrac{1}{2}(t-1), \quad 1 \le t \le 3$$

$$N_{3,3}(t) = \frac{t}{3} N_{3,2}(t) + \frac{3-t}{2} N_{4,2}(t)$$

$$= \begin{cases} \frac{1}{3}t^2, & 0 \le t < 1 \\ \frac{1}{6}t(3-t) + \frac{1}{4}(t-1)(3-t), & 1 \le t < 3 \\ 0, & \text{otherwise} \end{cases}$$



# Marking scheme and notes for Advanced Graphics

(a) tests the "Other geometric primitives" material
(b) " " "The polygon" and "Revision" "
(c) " " "The polygon" material
(d) " " "Splines" material

## Marking scheme — preliminary

(a) definition of ray to disc 1
find intersection with plane 1
equation to do this 1
check for parallel plane/ray ($\underline{D} \cdot \underline{N} = 0$) 1
check for $t < 0$ 1
check for $|\underline{P} - \underline{C}| \leq r$ 1

(b) correct reasoning for efficiency 2
" " " accuracy 2

(c) correct reasons for when needed 1
" " " not needed 1
" minimum information requirements 2

(d) correct method 3
correct equation for $N_{3,3}(t)$ 2
correct graph 1