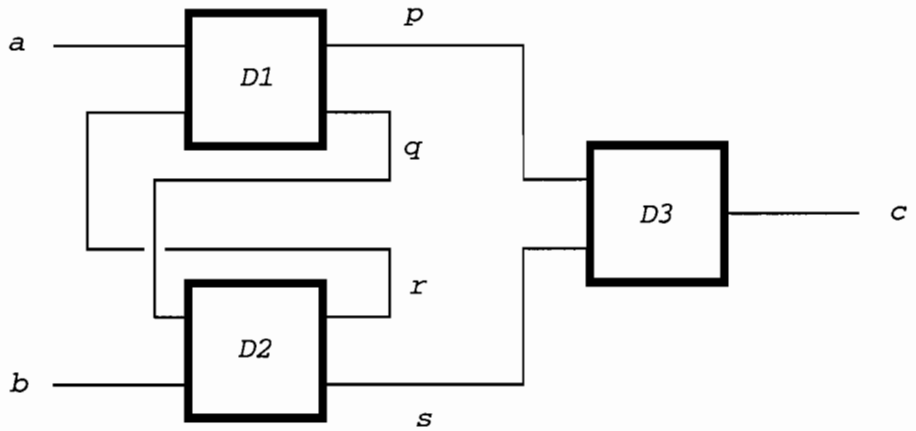
p9q12
MJCG

Specification and Verification II 2001

SV2.2: Solution Notes

Suppose definitions of $D1(a,r,p,q)$, $D2(q,b,r,s)$ and $D3(p,s,c)$ are given. Using $\exists$ and $\wedge$ write down a definition of a predicate D such that $D(a,b,c)$ defines the relation between a, b and c when $D1$, $D2$ and $D3$ are connected as in the following diagram. [4 marks]

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D(a,b,c) = ∃p q r s. D1(a,r,p,q) ∧ D2(q,b,r,s) ∧ D3(p,s,c)
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Suppose now that $D1$, $D2$ and $D3$ are defined in terms of functions f_1, f_2, f_3, f_4 and f_5 by

$$\begin{aligned} D1(a,r,p,q) &= (p = f_1\ a\ r) \wedge (q = f_2\ a) \\ D2(q,b,r,s) &= (r = f_3\ q\ b) \wedge (s = f_4\ q\ b) \\ D3(p,s,c) &= (c = f_5\ p\ s) \end{aligned}$$

Write down an equation expressing c in terms of a and b. [4 marks]

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c = f5(f1 a (f3 (f2 a) b))(f4 ≈ (f2 a) b)]
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Show the logical steps needed to derive the equation from the definition of D. [12 marks]

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D(a,b,c) = ∃p q r s. D1(a,r,p,q) ∧ D2(q,b,r,s) ∧ D3(p,s,c)
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D(a,b,c) = ∃p q r s. ((p = f1 a r) ∧ (q = f2 a)) ∧
                     ((r = f3 q b) ∧ (s = f4 q b)) ∧
                     (c = f5 p s)
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$$D(a,b,c) = \exists p\ q\ r\ s.\ ((p = f_1\ a\ r) \wedge (q = f_2\ a)) \wedge ((r = f_3\ q\ b) \wedge (s = f_4\ q\ b)) \wedge (c = f_5\ (f_1\ a\ r)\ (f_4\ q\ b))$$

$$D(a,b,c) = \exists p\ q\ r\ s.\ ((p = f_1\ a\ r) \wedge (q = f_2\ a)) \wedge ((r = f_3\ q\ b) \wedge (s = f_4\ q\ b)) \wedge (c = f_5\ (f_1\ a\ r)\ (f_4\ q\ b))$$

$$D(a,b,c) = \exists p\ q\ r\ s.\ ((p = f_1\ a\ r) \wedge (q = f_2\ a)) \wedge ((r = f_3\ q\ b) \wedge (s = f_4\ q\ b)) \wedge (c = f_5\ (f_1\ a\ (f_3\ q\ b))\ (f_4\ q\ b))$$

$$D(a,b,c) = \exists p\ q\ r\ s.\ ((p = f_1\ a\ r) \wedge (q = f_2\ a)) \wedge ((r = f_3\ q\ b) \wedge (s = f_4\ q\ b)) \wedge (c = f_5\ (f_1\ a\ (f_3\ (f_2\ a)\ b))\ (f_4\ (f_2\ a)\ b))$$

$$D(a,b,c) = \exists p\ q\ r\ s.\ ((p = f_1\ a\ r) \wedge (q = f_2\ a)) \wedge ((r = f_3\ (f_2\ a)\ b) \wedge (s = f_4\ (f_2\ a)\ b)) \wedge (c = f_5\ (f_1\ a\ (f_3\ (f_2\ a)\ b))\ (f_4\ (f_2\ a)\ b))$$

$$D(a,b,c) = ((\exists p.\ p = f_1\ a\ r) \wedge (\exists q.\ q = f_2\ a)) \wedge ((\exists r.\ r = f_3\ (f_2\ a)\ b) \wedge (\exists\ s.\ s = f_4\ (f_2\ a)\ b)) \wedge (c = f_5\ (f_1\ a\ (f_3\ (f_2\ a)\ b))\ (f_4\ (f_2\ a)\ b))$$

$$D(a,b,c) = (T \wedge T) \wedge (T \wedge T) \wedge (c = f_5\ (f_1\ a\ (f_3\ (f_2\ a)\ b))\ (f_4\ (f_2\ a)\ b))$$

$$D(a,b,c) = (c = f_5\ (f_1\ a\ (f_3\ (f_2\ a)\ b))\ (f_4\ (f_2\ a)\ b))$$