P8 q.4
NAD

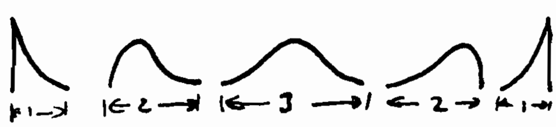
ADV. GRAPHICS - PAPER 8 - SOLUTION NOTES, 2001

A B-spline of order k with open-uniform knot vector has $N+1$ basis functions for $N+1$ points. Assuming that $N+1$ is large enough, the $k-1$ basis functions at each end will be special while the $N+1-2(k-1)$ basis functions in the middle are all identical (and identical to the single order k uniform basis function). Thus, provided $N+1-2(k-1) \geq 0 \Rightarrow N-2k+3 \geq 0 \Rightarrow N \geq 2k-3$, ~~there will be~~ we will need $2(k-1)+1 = 2k-1$ distinct basis functions.

When $N < 2k-3$, there will be a region in the middle where there are special basis functions. e.g. for $k=3$, with the knot vector $[0,0,0,1,1,1]$ there will be a ~~knot vector~~ basis function, defined by $0,0,1,1$ which is not ~~part~~ one of the above set of $2k-1$ basis functions.

$k=2$

$k=3$



OU B-spline $[0,0,0,1,1,1]$

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$$N_{1,1} = 0; \quad N_{2,1} = 0, \quad N_{3,1} = \begin{cases} 1, & 0 \le t < 1 \\ 0, & \text{o/w} \end{cases} \quad N_{4,1} = 0, \quad N_{5,1} = 0$$

$$N_{1,2} = 0 \qquad N_{2,2} = (1-t) \qquad N_{3,2} = t \qquad N_{4,2} = 0$$

$$N_{1,3} = (1-t)^2$$

$$N_{2,3} = t(1-t) + (1-t)t = 2t(1-t)$$

$$N_{3,3} = t^2$$

all valid $0 \le t < 1$ and 0 otherwise

$$p(t) = (1-t)^2 P_0 + 2t(1-t) P_1 + t^2 P_2$$

= bezier quadratic.