Database Theory 2004 – Paper 9 Question 9 (AD)

Consider a database with one binary relation B.

(a) Write programs in stratified Datalog for the following queries:

(i) Give the set of elements x such that there are fewer than three elements y such that $B(x, y)$. [3 marks]

```
A(x) <- B(x,y1),B(x,y2),B(x,y3), y1 =/= y2, y1=/= y3, y3 =/= y2
Q(x) <- - A(x)
```

(ii) Give the set of elements x such that there is no path (in the graph formed by B) from x back to itself. [3 marks]

```
T(x,y) <- B(x,y).
T(x,y) <- B(x,z) <- T(z,y).

L(x) <- T(x,x).

Q(x) <- - L(x).
```

(iii) Give the set of elements x such that $B(x, y)$ for every y. [3 marks]

```
N(x,y) <- - B(x,y).
A(x) <- N(x,y).
Q(x) <- - A(x).
```

(b) Which of the queries defined in part (a) is:

(i) safe? [3 marks]

The first two queries are unsafe. The first one includes all elements that have no neighbours in the graph formed by B, of which there are infinitely many. This is also true of the second one. The third one is safe as only finitely many elements have any neighbours at all.

(ii) domain independent? [4 marks]

All three queries are domain dependent. A new element added to the domain, which participates in no tuple of the relation B would have to be added to the result of

the first two queries, which demonstrates that those two are domain dependent. In the case of the third query, adding such a node to the domain may result in an element that was previously in the result of the query being removed

(*iii*) monotone? [4 marks]

The third query is monotone. If an element is in relation B to all elements in the domain, it will remain so if more tuples are added. The first two queries are not monotone. This can be demonstrated by constructing suitable examples. In the case of the first one, if x has two neighbours and a tuple is added which makes x neighbour to y, which it wasn't before, this would result in x being removed from the result of the query. Similarly, one can show the second query is not monotone with an example where x initially does not appear in any tuple in B, and then the tuple (x, x) is added.

In each case, justify your answer fully.