p3q7 AMP ✓

Computation Theory 2003 p3(10), q7

Register machine M specified by

1 - finitely many registers (R0, R1, .., Rn) each capable of storing a natural number ($n = 0, 1, 2, \ldots$)

1 - a program, consisting of finite list

L0 : I0
L1 : I1
⋮
Ln : In

of labelled instructions I_i of one of three forms:

1 $R^+ \to L$: add 1 to contents of R & jump to instruction labelled L

1 $R^- \to L, L'$: if contents of $R > 0$, subtract 1 from it & jump to L, otherwise jump to L'

1 HALT : Stop

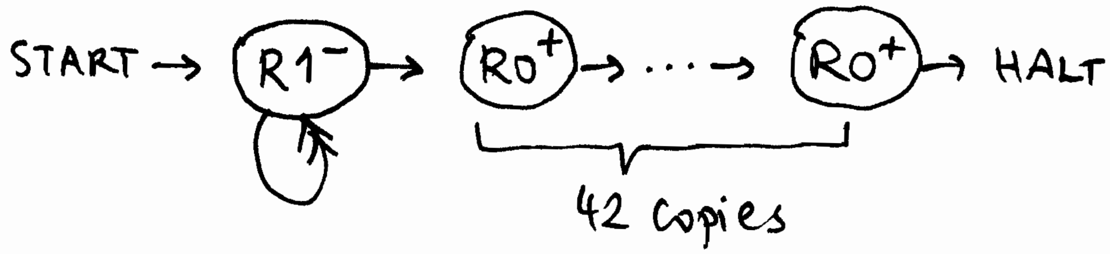
1 M computes by carrying out the instructions in its program, starting at the first one and continuing until a HALT is reached (if ever).

(6)

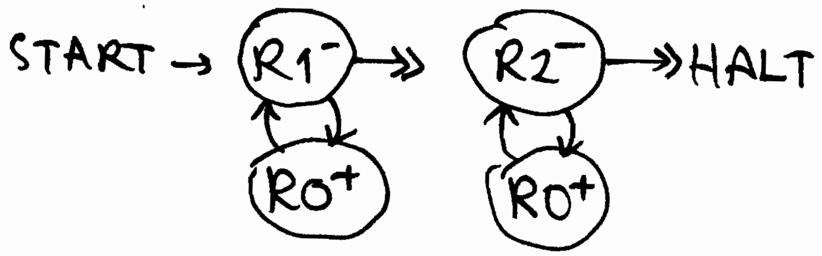
$f(x_1, \ldots, x_n)$ is RM computable if there is a register machine M such that starting M with Ri containing x_i and all other registers containing 0, M halts iff $f(x_1, \ldots, x_n)$ is defined and in that case R0 contains the value of $f(x_1, \ldots, x_n)$.

(3)

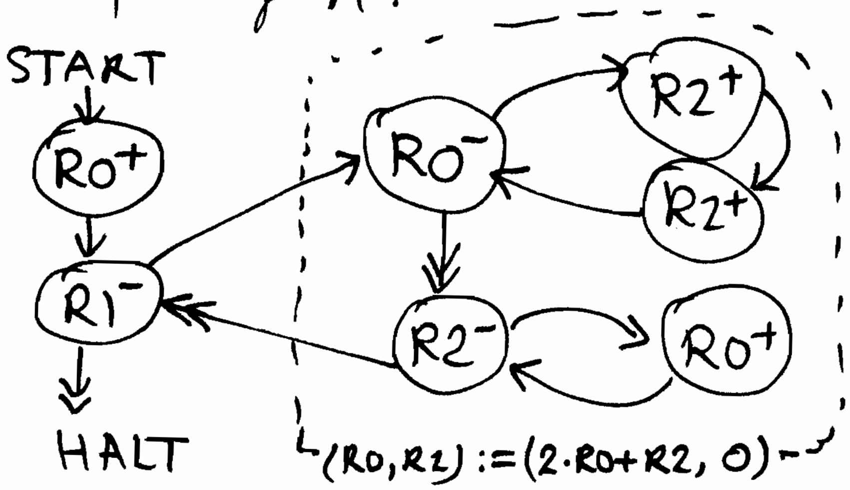
(2) RM computing g:



(2) RM computing f:



(4) RM computing h:



(3) Use: <u>undecidability of Halting Problem</u> – there is no RM computing the function

$$H(x,y) = \begin{cases} 1 & \text{if } y = [a_0, \dots, a_n] \text{ say \& the } x^{th} \text{ RM (in some encoding) started with } R0 = a_0, \dots, Rn = a_n \text{ \& all other } Rs = 0, \text{ eventually halts} \\ 0 & \text{otherwise} \end{cases}$$

Commentary

- The first 9 marks are for bookwork from lecture 2.
- A register machine for f was given in lectures, but not for functions g & h.
- The last part can be answered by appealing to undecidability of the Halting Problem, covered in lecture 5.