Maths for Computation Theory 2000

Question 2 Notes A development from 1998 P II q8, with some basic regular algebra to get into the swing, then an example. They may tackle the q. by reasoning directly, in which case play it by ear. Provided they partition as asked it will come out OK.

Solution Definitions

a) $E + F = \{ w \in S^* \mid w \in E \text{ or } w \in F \}$

b) $EF = \{ ww' \in S^* \mid w \in E, w' \in F \}$

c) $E^* = \{\varepsilon\} + \{ w_1 w_2 \ldots w_n \mid w_i \in E, n \in N^+, 1 \le i \le n \}$

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i) $= 1 + \{ w_1 \mid w_1 \in E \}$

$+ \{ w_1 w_2 \ldots w_n \mid n \geq 2, w_i \in E, 1 \le i \le n \}$

$= 1 + E + E . \{ w_2 \ldots w_n \mid n \geq 2, w_i \in E, 2 \le i \le n \}$

Question 2 Solution (ctd)

i) $\therefore E^* = \lambda + EE^*$

(easier with a better definition, as in notes)

ii) $E(FE)^* = \{ w_0(v_1w_1 v_2w_2 .. v_nw_n) \mid n \geq 0, \; w_0 \in E, w_i \in E, v_i \in F, 1 \leq i \leq n \}$

$= \{ (w_0v_1 w_1v_2 .. w_{n-1}v_n) w_n \mid n \geq 0, \; w_n \in E, \; w_{i-1} \in E, v_i \in F, 1 \leq i \leq n \}$

$= (EF)^* E$ 4

Kleene's Theorem An event accepted by some DFA is regular over input alphabet.

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Given $M^* = \begin{pmatrix} (A+BD^*C)^* & A^*B(D+CA^*B)^* \\ D^*C(A+BD^*C)^* & (D+CA^*B)^* \end{pmatrix}$

Question 2 Solution ctd) where

$$
\begin{array}{c|cc|cc}
 & \alpha & \beta & \gamma & \delta \\
\hline
\end{array}
$$

$$
M = \begin{array}{c}
\alpha \\ \beta \\ \gamma \\ \delta
\end{array}
\left(\begin{array}{cc|cc}
b & 0 & a & 0 \\
0 & a & 0 & b \\
\hline
0 & a & 0 & b \\
b & 0 & a & 0
\end{array}\right)
$$

Require to compute $A^*B(D+CA^*B)^*$.

Here $A^* = \begin{pmatrix} b^* & 0 \\ 0 & a^* \end{pmatrix}$, $A^*B = \begin{pmatrix} b^*a & 0 \\ 0 & a^*b \end{pmatrix}$

$$CA^*B = \begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix}\begin{pmatrix} b^*a & 0 \\ 0 & a^*b \end{pmatrix}$$

$$= \begin{pmatrix} 0 & aa^*b \\ bb^*a & 0 \end{pmatrix}$$

$$\therefore D + CA^*B = \begin{pmatrix} 0 & b + aa^*b \\ a + bb^*a & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & a^*b \\ b^*a & 0 \end{pmatrix} \quad \text{by i)}$$

Question 2 Solution ctd) Hence

$$(D+CA^*B)^* = \begin{pmatrix} (a^*bb^*a)^* & a^*b(b^*aa^*b)^* \\ ? & ? \end{pmatrix}$$

and the required matrix

$$= \begin{pmatrix} b^*a & 0 \\ 0 & a^*b \end{pmatrix}\begin{pmatrix} (a^*bb^*a)^* & a^*b(b^*aa^*b)^* \\ ? & ? \end{pmatrix}$$

$$= \begin{pmatrix} b^*a(a^*bb^*a)^* & b^*aa^*b(b^*aa^*b)^* \\ ? & ? \end{pmatrix}$$

Hence event accepted $= (M^*)_{\alpha\gamma} + (M^*)_{\alpha\delta}$

$$= b^*a\{(a^*bb^*a)^* + a^*b(b^*aa^*b)^*\}$$

$$= b^*a(a^*bb^*a)^*\{1+a^*b\} \quad \text{by ii)}$$