P9 q11

JGD

Model Answer, Information Theory and Coding, Question 1. 2000

A. The information measure assigns $\log_2(p)$ bits to the observation of an event whose probability is p. The probability of the combination of N independent events whose probabilities are $p_1 .... p_N$ is $\prod_{i=1}^{N} p_i$

Thus the information content of such an combination is:

$$\log_2\left(\prod_{i=1}^{N} p_i\right) = \log_2(p_1) + \log_2(p_2) + \cdots + \log_2(p_N)$$

which is the sum of the information of each of the independent events. [4 marks]

B.

Shannon's *Source Coding Theorem* tells us that the entropy of the distribution is the lower bound on average code length, in bits per symbol. This alphabet has entropy

$$H = -\sum_{i=1}^{6} p_i \log_2 p_i = (1/2)(1) + (1/4)(2) + (1/8)(3) + (1/16)(4) + (1/32)(5) + (1/32)(5) =$$

31/16 ~~1.75~~ bits per average symbol [3 marks]

C.

Givens: $p(B|m) = 3p(B|f)$, $p(m) = p(f) = 0.5$, $p(B) = 0.6$ and so $p(NB) = 0.4$ where m means male, f means female, B means black and NB means non-black. From these givens plus the Sum Rule fact that $p(m)p(B|m) + p(f)p(B|f) = p(B) = 0.6$, it follows that $p(B|f) = 0.3$ and $p(B|m) = 0.9$, and hence that $p(NB|m) = 1 - 0.9 = 0.1$

Now we may apply Bayes Rule to calculate that

$$p(m|NB) = \frac{p(NB|m)p(m)}{p(NB)} = \frac{(0.1)(0.5)}{(0.4)} = 0.125 = 1/8$$

[4 marks]

From the information measure $\log_2(p)$, there are 3 bits worth of information in discovering that a non-black raven is male.

[1 mark]

(a) The colour distribution is { 0.6, 0.4 }
(b) The gender distribution is { 0.5, 0.5 }
(c) The (colour | male) distribution is { 0.9, 0.1 }
(d) The (gender | non-black) distribution is { 0.125, 0.875 }

Uncertainty of a random variable is greater, the closer its distribution is to uniformity. Therefore the rank-order of uncertainty, from greatest to least, is: b, a, d, c.

[3 marks]

D. Modulation of the continuous signal by a complex exponential wave $\exp(i\omega t)$ will shift its entire frequency spectrum upwards by an amount ω.

All of AM broadcasting is based on this principle. It allows many different communications channels to be multi-plexed into a single medium, like the electromagnetic spectrum, by shifting different signals up into separate frequency bands.

The original Fourier spectrum of each of these signals can then be recovered by demodulating then down (this removes each AM carrier). This is equivalent to multiplying the transmitted signal by the conjugate complex exponential, $\exp(-i\omega t)$.

[3 marks]

E. The phase spectrum is the indispensable part. This is demonstrated by crossing the amplitude spectrum of one image with the phase spectrum of another one, and *vice versa.* The new image that you see looks like the one whose phase spectrum you are using, and not at all like the one whose amplitude spectrum you've got.

[2 marks]