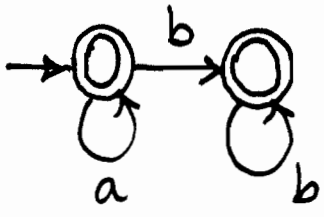
CST IA Regular Languages & Finite Automata
2002, Paper 2, question 9

(a) Is regular, since it is the language of strings accepted by the non-deterministic finite automaton



(b) Is not regular. To see this apply the Pumping lemma. For any regular language L there is a number $l \geq 1$ such that all $w \in L$ with length$(w) \geq l$ can be expressed as $w = u_1 v u_2$ with length$(v) \geq 1$, length$(u_1 v) \leq l$ and $u_1 v^n u_2 \in L$ for all $n \geq 0$.

Language (b) fails to have this property: for any $l \geq 1$, consider $a^l b^l$: it is in the language, its length is $\geq l$, but no matter how we split it up as $a^l b^l = u_1 v u_2$ with length$(v) = r \geq 1$ and length$(u_1 v) = s \leq l$ say, then must have $u_1 = a^{s-r}$, $v = a^r$, $u_2 = a^{l-s} b^l$; hence $u_1 v^2 u_2 = a^{s-r} a^{2r} a^{l-s} b^l = a^{l+r} b^l$ is not in the language [since $r \geq 1$].

(c) Is regular, since it only contains finitely many strings ($w_0, \ldots, w_k$ say — and hence is the language of strings matching the regular expression $w_0 | w_1 | \cdots | w_k$). (2)

(d) Is always regular.

For suppose L is accepted by a deterministic finite automaton M. Let M' be the deterministic finite automaton obtained from M by changing the set of accepting states from $Accept_M$ to $\{q \in States_M \mid q \notin Accept_M\}$. Then w is accepted by M' iff the unique state reached from the start state on inputting w is not accepting for M, iff w is not in L.

So $L(M')$ is the desired language $\{w \mid w \notin L\}$. (4)

(e) Is never regular.

For if it were, then by part (d)

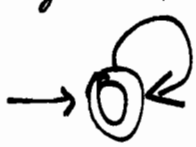
$\{w \in \Sigma^* \mid w \notin \{w' \in \Sigma^* \mid w' \notin L\}\}$

is regular, i.e. $\{w \in \Sigma^* \mid w \in L\}$ is regular — contradiction. (2)

(f) There <u>are</u> infinite subsets of the language in part (b) that are regular: for example

$$\{ b^n \mid n \in \mathbb{N} \} \subseteq \{ a^m b^n \mid m \leq n \}$$

and this $\supseteq$ is regular, being the language of strings accepted by the NFA



Of course not every infinite subset is regular, since we saw in (b) that the language itself is not regular.

(4)