

Solution to Q2

$$\begin{aligned}
\text{Opt}_\alpha &= \forall \beta (\beta \rightarrow (\alpha \rightarrow \beta) \rightarrow \beta) \\
\text{None}_\alpha &= \Lambda \beta (\lambda n : \beta (\lambda s : \alpha \rightarrow \beta (n))) \\
\text{Some}_\alpha &= \lambda x : \alpha (\Lambda \beta (\lambda n : \beta (\lambda s : \alpha \rightarrow \beta (s x))))
\end{aligned}$$

$$\frac{\frac{\frac{\overline{\{\alpha, \beta\}, n : \beta, s : \alpha \rightarrow \beta \vdash n : \beta} \text{ var}}{\{\alpha, \beta\}, n : \beta \vdash \lambda s : \alpha \rightarrow \beta (n) : (\alpha \rightarrow \beta) \rightarrow \beta} \text{ fn}}{\{\alpha, \beta\}, \emptyset \vdash \lambda n : \beta (\lambda s : \alpha \rightarrow \beta (n)) : \beta \rightarrow (\alpha \rightarrow \beta) \rightarrow \beta} \text{ fn}}{\{\alpha\}, \emptyset \vdash \text{None}_\alpha : \text{Opt}_\alpha} \text{ gen}$$

$$\text{Lift}_{\gamma\delta} = \lambda f : \gamma \rightarrow \delta (\lambda g : \text{Opt}_\gamma (\Lambda \beta (\lambda n : \beta (\lambda s : \delta \rightarrow \beta (g \beta n (\lambda x : \gamma (s (f x))))))))$$

$$\begin{aligned}
& (\text{Lift}_{\gamma\delta} f) (\text{Some}_\gamma x) \\
& \rightarrow^2 \Lambda \beta (\lambda n : \beta (\lambda s : \delta \rightarrow \beta ((\text{Some}_\gamma x) \beta n (\lambda x : \gamma (s (f x)))))) \\
& \rightarrow^4 \Lambda \beta (\lambda n : \beta (\lambda s : \delta \rightarrow \beta ((\lambda x : \gamma (s (f x))) x))) \\
& \rightarrow \Lambda \beta (\lambda n : \beta (\lambda s : \delta \rightarrow \beta (s (f x)))) \\
& = \text{Some}_\delta (f x)
\end{aligned}$$

$$\text{Compose}_{\gamma\delta\epsilon} = \lambda f : \gamma \rightarrow \text{Opt}_\delta (\lambda g : \delta \rightarrow \text{Opt}_\epsilon (\lambda x : \gamma ((f x) \text{Opt}_\epsilon \text{None}_\epsilon g)))$$