Topics in Concurrency Qu. 1 2004

(i)
$$\frac{s \xrightarrow{\lambda} s_1}{s \| s' \xrightarrow{\lambda} s_1 \| s'} \qquad \frac{s' \xrightarrow{\lambda} s_1'}{s \| s' \xrightarrow{\lambda} s \| s_1'}$$

$$\frac{s \xrightarrow{a} s_1 \qquad s' \xrightarrow{\bar{a}} s_1'}{s \| s' \xrightarrow{\tau} s_1 | s_1'}$$

Suppose $t \leq u$ and $t' \leq u'$. Let S and S' be the corresponding simulations (here called $\leq$).

Define.

$R = \{ (s \| s', v \| v') \mid (s, v) \in S \ \& \ (s', v') \in S' \}$

We require R a simulation. Assume $(s \| s', v \| v') \in R$.

Suppose $(s \| s') \xrightarrow{\lambda} (s_1 \| s_1')$ Consider cases

Case 1 $\quad s \xrightarrow{\lambda} s_1$ and $s' \equiv s_1'$.

As $(s, v) \in S \quad v \xrightarrow{\lambda} v_1$ and $(s_1, v_1) \in S$.
Then $(v \| v') \xrightarrow{\lambda} (v_1 \| v')$ & $(s_1 \| s_1', v_1 \| v') \in R$.

Case 2. $\quad s \equiv s_1$ & $s' \xrightarrow{\lambda} s_1'$ is very similar.

Case 3. $\quad s \xrightarrow{a} s_1$ & $s' \xrightarrow{\bar{a}} s_1'$.

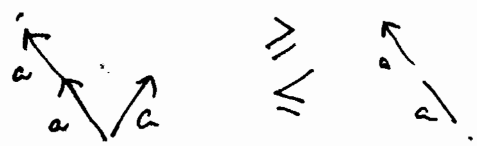
Then as S and S' are simulations

$v \xrightarrow{a} v_1$ and $v' \xrightarrow{\bar{a}} v_1'$ with

$(s_1, v_1) \in S$ and $(s_1', v_1') \in S'$.

Hence $(v, v') \xrightarrow{\tau} (v_1, v_1')$ with

$(s_1 \| s_1', v_1 \| v_1') \in R$.

(ii) 

$a.a.\mathrm{nil} + a.\mathrm{nil} \ \underset{\geq}{\leq}\ a.a.\mathrm{nil}$.

~~(iii)~~ But they are not strongly bisimilar.

(iii) Suppose $t \leq u$ via a simulation S. (def. of $\leq$).

We show for all assertions A,

$\forall t, u.\ (t, u) \in S \ \&\ t \models A \Rightarrow u \models A$

By ~~the~~ structural induction on A.

Suppose $(t, u) \in S$ and $t \models \langle a \rangle B$. Then
$t \xrightarrow{a} t'$ and $t' \models B$. Hence $u \xrightarrow{a} u'$ with

$(t', u') \in S$. By ind. hyp. $u' \models B$ so

$u \models \langle a \rangle B$.

Suppose $(t, u) \in S$ and $t \models \bigwedge_{i \in I} A_i$.

Then $t \models A_i$ all $i \in I$. By ind. hyp. $u \models A_i$ all $i \in I$. So $u \models \bigwedge_{i \in I} A_i$.

Hence if $t \leq u$ and t satisfies an assertion then so does u.

Conversely, suppose that any assertion satisfied by t is satisfied by u.

Define:

$$S = \{ (t, u) \mid \forall A.\ t \models A \Rightarrow u \models A \}$$

We show S is a simulation.

Suppose $t \xrightarrow{a} t'$. We require $u \xrightarrow{a} u'$ with $(t', u') \in S$. Suppose no such u' exists to obtain a contradiction. I.e. suppose for each

~~t' with $t \xrightarrow{a} t'$ that there is $A_{t'}$ st.~~

~~$t' \models A$~~

u' with $u \xrightarrow{a} u'$ there is $A_{u'}$

s.t.

$$t' \models A_{u'} \quad \& \quad u' \not\models A_{u'}.$$

Then $t \models \langle a \rangle \bigwedge_{u' \text{ s.t. } u \xrightarrow{a} u'} A_{u'}$. Consequently

$u \models \langle a \rangle \bigwedge_{u'} A_{u'}$. But then there exists

u'' s.t. $u \xrightarrow{a} u''$ & $u'' \models \bigwedge_{u'} A_{u'}$.

In particular $u'' \models A_{u''}$ — a contradiction $\boxtimes$