Numerical Analysis II   Question A   2003   p 8q 11 MROD

Solution Notes

(a) Descartes' rule of signs states that

$$0 \le c - n_+ = 2k$$

for some non-negative integer k.   [2 marks]

(b) If $p_3(x) = x^3 + 13x^2 + 54x + 72$ then

$$c = 0 \Rightarrow n_+ = 0.$$

And $p_3(-x) = -x^3 + 13x^2 - 54x + 72$ so

$$c = 3 \Rightarrow n_+ = 1 \text{ or } 3.$$

$\therefore$ There are 1 or 3 negative real roots of $p_3(x)$, and no positive real roots.   [2 marks]

(c) If $q_5(x) = x^5 + 5x^4 + 32x^3 + 160x^2 + 256x + 1280$ then

$$c = 0 \Rightarrow n_+ = 0.$$

And $q_5(-x) = -x^5 + 5x^4 - 32x^3 + 160x^2 - 256x + 1280$ so

$$c = 5 \Rightarrow n_+ = 1 \text{ or } 3 \text{ or } 5.$$

Search for a negative root

$$q_5(0) = 1280 > 0$$

$$q_5(-1) = -1 + 5 - 32 + 160 - 256 + 1280 > 0$$

$$q_5(-10) = -100000 + 50000 - 32000 + 16000 - 2560 + 1280 < 0$$

$$\Rightarrow \text{ a root in } (-10, -1).$$

Try $x = -5$ (because leading terms cancel):

$$q_5(-5) = \cancel{-5^5} + \cancel{5^5} - 32.5^3 + 160.5^2 - 256.5 + 1280 = 0$$

$\Rightarrow -5$ is a root, so $(x+5)$ is a factor.

Factorise

$$q_5(x) = (x+5)(x^4 + 32x^2 + 256)$$
$$= (x+5)(x^2+16)^2$$

$\Rightarrow x = -5$ or $\pm 4i$ (twice).

[7 marks]

PTO.

(d) $f(x) = 3x^4 - 28x^3 + 24x^2 + 144x + 432$

$f'(x) = 12x^3 - 84x^2 + 48x + 144$

For $f(x)$, $c = 2 \Rightarrow n_+ = 0$ or 2.

For $f'(x)$, $c = 2 \Rightarrow n_+ = 0$ or 2.

There is a possibility of division by zero, so find the roots of $f'(x)$.

$f'(-x) = -12x^3 - 84x^2 - 48x + 144$

So

$$c = 1 \Rightarrow n_+ = 1$$

i.e. $f'(x)$ has one negative root.

Observe that

$$f'(-1) = -12 - 84 - 48 + 144 = 0$$

and factorise

$$\begin{aligned} f'(x) &= (x+1)(12x^2 - 96x + 144) \\ &= 12(x+1)(x^2 - 8x + 12) \\ &= 12(x+1)(x-2)(x-6). \end{aligned}$$

We are interested in $x > 3$. Since $f'(6) = 0$ there is a potential problem if the required root of $f(x)$ is close to 6. Check $f(6)$:

$$\begin{aligned} f(6) &= 3.6^4 - 28.6^3 + 24.6^2 + 144.6 + 432 \\ &= 0. \end{aligned}$$

The Newton-Raphson formula leads to a $0/0$ calculation at the solution $x = 6$, so convergence is impossible.

[9 marks]