P79 15
G-W
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Topics in Concurrency Qu 2.

(1)
$$\frac{\Gamma \vdash u : \mathbb{P} \qquad \Gamma, x : \mathbb{P} \vdash t : \mathbb{Q}}{\Gamma \vdash [u > x \Rightarrow t] : \mathbb{Q}}$$

(2)
$$\Pi_a\Big(\sum_i t_i\Big) \xrightarrow{p} t' \quad \text{iff} \quad \sum_i t_i \xrightarrow{ap} t'$$
$$\text{iff} \quad \exists j.\ t_j \xrightarrow{ap} t'$$
$$\text{iff} \quad \exists j.\ \Pi_a(t_j) \xrightarrow{p} t'$$
$$\text{iff} \quad \sum_i \Pi_a(t_i) \xrightarrow{p} t'$$

Thus $\Pi_a\big(\sum_i t_i\big) \sim \sum_i \Pi_a(t_i)$.

$$\Big(\sum_i t_i\Big)u \xrightarrow{p} t' \quad \text{iff} \quad \Big(\sum_i t_i\Big) \xrightarrow{u \mapsto p} t'$$
$$\text{iff} \quad \exists j.\ t_j \xrightarrow{u \mapsto p} t'$$
$$\cancel{\text{iff} \quad \exists j\ \sum_i t_i \xrightarrow{u \mapsto p} t'}$$
$$\text{iff} \quad \cancel{\text{iff}}\ \exists j.\ t_j u \xrightarrow{p} t'$$
$$\text{iff} \quad \sum_i (t_i u) \xrightarrow{p} t'$$

Thus $\big(\sum_i t_i\big)u \sim \sum_i (t_i u)$.

$$[\textstyle\sum_i t_i > .y \Rightarrow u) \xrightarrow{q} u' \quad \text{iff} \quad \underline{\textstyle\sum_i t_i \xrightarrow{\cdot} t' \ \& \ u[t'/y] \xrightarrow{q} u'}$$

$$\text{iff} \quad \exists j.\ t_j \xrightarrow{\cdot} t' \ \& \ u[t'/y] \xrightarrow{q} u'$$

$$\text{iff} \quad \exists j.\ [t_j > .y \Rightarrow u] \xrightarrow{q} u'$$

$$\text{iff} \quad \textstyle\sum_i [t_i > .y \Rightarrow u] \xrightarrow{q} u'$$

Thus $[\sum_i t_i > .y \Rightarrow u) \sim \sum_i [t_i > .y \Rightarrow u]$.

Let x have type $a.\mathbb{O} + b.\mathbb{O}$.

$$.(a.nil + b.nil) \not\sim .a.nil + .b.nil$$

as lhs $\xrightarrow{\cdot}$ $a.nil + b.nil$

while rhs $\xrightarrow{\cdot}$ $a.nil$ or rhs $\xrightarrow{\cdot}$ $b.nil$.

Neither $a.nil$ nor $b.nil$ is bisimilar to $a.nil + b.nil$. Hence $.x$ is not linear in x.

(3)
$$\dfrac{\dfrac{u \xrightarrow{a.} u'}{\Pi_a(u) \xrightarrow{\cdot} u'} \qquad t[u'/x] \xrightarrow{q} v}{[\Pi_a(u) > .x \Rightarrow t] \xrightarrow{q} v}$$

$$['' u > a.x \Rightarrow t]$$

I.e. we can derive

$$\frac{u \xrightarrow{a} u' \qquad t[u'/x] \xrightarrow{q} v}{[u > a.x \Rightarrow t] \xrightarrow{q} v.}$$

(4) For CCS, take

$$\mathbb{P} = \tau.\mathbb{P} + \sum_{a \in A} a.\mathbb{P} + \sum_{\bar{a} \in A} \bar{a}.\mathbb{P}$$

where a ranges over the non-τ actions of CCS.

$$Par : \mathbb{P} \to (\mathbb{P} \to \mathbb{P})$$

is defined by

$$Par \equiv rec\, Pl.\, \lambda X, \lambda Y.$$

$$\sum_{\alpha} [X > \alpha.x \Rightarrow \alpha.\cancel{x}\, Pl(x,Y)] +$$

$$\sum_{\beta} [Y > \beta.y \Rightarrow \beta.\, Pl(X,y)] +$$

$$\sum_{a} [X > a.x \Rightarrow [Y > \bar{a}.y \Rightarrow \tau.\, Pl(x,y)]]$$

where α, β range over all CCS actions while a ranges over non-τ actions.

(5) (a) 'Late' value-passing:

$$\mathbb{P} = \tau.\mathbb{P} + \sum_a a.\sum_{v\in V} v\,\mathbb{P} + \sum_{\bar{a}} \bar{a} \sum_{v \in V} v.\mathbb{P}$$

or 'early' value passing

$$\mathbb{P} = \tau.\mathbb{P} + \sum_{a,v} av.\mathbb{P} + \sum_{\bar{a},v} \bar{a}v.\mathbb{P}$$

[Either will do].

(b) $$\mathbb{P} = \tau.\mathbb{P} + \sum_a a.(\mathbb{P} \to \mathbb{P}) + \sum_{\bar{a}} \bar{a}.(\mathbb{P} \& \mathbb{P})$$

where $\mathbb{P} \& \mathbb{P}$ abbreviates $1\,\mathbb{P} + 2\,\mathbb{Q}$.

(It has pairing $(t,u) = 1\,t + 2\,u$ and projections $\text{fst}\ t = \pi_1(t)$
$\text{snd}\ t = \pi_2(t)$.)