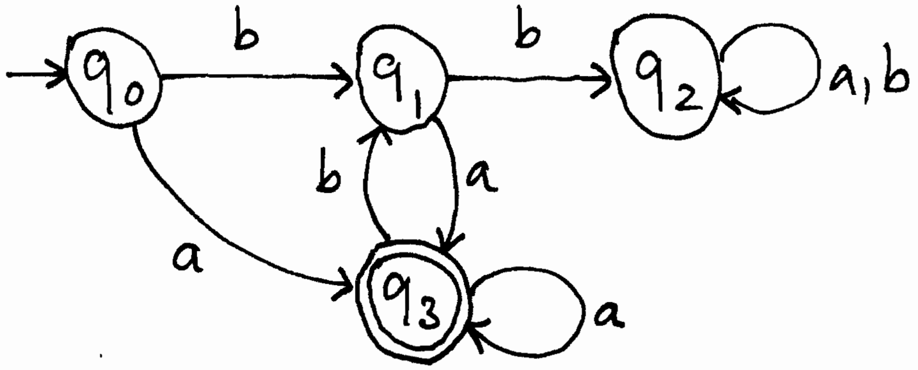
Reg. Lang. & Finite Automata 2003 p2, q 9

1 (a)

Define M to be the DFA given by:

2 

<u>Claim</u> $L(M) = L \triangleq \{w \in \{a,b\}^* \mid w$ ends in a & doesn't contain bb $\}$

3 Justification: note that

$$q \xrightarrow{bb} q' \Rightarrow q' = q_2$$

& hence

$$q \xrightarrow{w} q' \text{ with } w \text{ containing } bb \Rightarrow q' = q_2.$$

Also $q_0 \xrightarrow{w} q_3 \Rightarrow w$ ends in a

Hence $L(M) \subseteq L$. Conversely, note that

$$q_0 \xrightarrow{w} q_2 \Rightarrow w \text{ contains } bb$$

$$q_2 \neq q \xrightarrow{a} q' \Rightarrow q' = q_3$$

(5) and thus $L \subseteq L(M)$.

(b) Regular expressions over Σ & the languages they determine:

1 • each $a \in \Sigma$ is a RE, and $L(a) = \{a\}$

1 • ε is a RE, and $L(\varepsilon) = \{\varepsilon\}$

1 • $\emptyset$ is a RE, and $L(\emptyset) = \emptyset$

• if r, s are REs, then so are $r|s$, rs & r^*, and

1 $L(r|s) = L(r) \cup L(s)$

1 $L(rs) = \{uv \mid u \in L(r) \ \& \ v \in L(s)\}$

1 $L(r^*) = \bigcup_{n \geq 0} L(r^n)$ where $\begin{cases} r^0 \triangleq \varepsilon \\ r^{n+1} = rr^n \end{cases}$.

(6)

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2 If r does not contain any occurrences of ϕ, then $L(r) \neq \phi$.

not required

[because, reasoning by induction on the size of r,

- statement is true for a, ε (ϕ is excluded)
- if statement is true for r, s, then
  - if $r|s$ does not contain ϕ, then r & s do not so by hyp. $L(r) \neq \phi \neq L(s)$, so $L(r|s) = L(r) \cup L(s) \neq \phi$
  - if rs does not contain ϕ, then r & s do not so by hyp. $L(r) \neq \phi \neq L(s)$, so $L(rs) \neq \phi$
  - $L(r^*) \neq \phi$ whatever r.

(2) so statement holds for $r|s$, rs & r^*.]

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2 Use <u>Kleene's Theorem</u>: the collection of languages over Σ of the form $L(r)$ for some regular expression r, is exactly the same as the collection of languages accepted by some deterministic finite automaton (i.e. of the form $L(M)$ for some DFA M).

Given r, by Kleene's Theorem we can find a DFA M with $L(M) = L(r)$.

Construct a new DFA M' from M by interchanging the rôle of accepting & non-accepting states in M. Thus for any $w \in \Sigma^*$, letting $\delta(w)$ be the unique state in M (or M') reached from the start via w, we have

$$\begin{aligned} w \in L(M) &\iff \delta(w) \text{ is accepted by } M \\ &\iff \delta(w) \text{ is not accepted by } M' \\ &\iff w \notin L(M') \end{aligned}$$

(note how we rely on the <u>determinism</u> of M for this argument to work).

Thus $L(M') = \Sigma^* - L(M)$.

By Kleene's Theorem we can find a RE r^* with $L(r^*) = L(M')$. Then

$$L(r^*) = L(M') = \Sigma^* - L(M) = \Sigma^* - L(r)$$

7 as required.