1999 Comm. autom. and Pi Calculus. Question A, solution notes.

Rules (bookwork):

$\text{SUM}_t: \; M + \alpha.P + N \xrightarrow{\alpha} P$ $\qquad$ $\text{REACT}_t: \; \dfrac{P \xrightarrow{\lambda} P' \quad Q \xrightarrow{\bar{\lambda}} Q'}{P|Q \xrightarrow{\tau} P'|Q'}$

$\text{L-PAR}_t: \; \dfrac{P \xrightarrow{\alpha} P'}{P|Q \xrightarrow{\alpha} P'|Q}$ $\qquad$ $\text{R-PAR}_t: \; \dfrac{Q \xrightarrow{\alpha} Q'}{P|Q \xrightarrow{\alpha} P|Q'}$

$\text{RES}_t: \; \dfrac{P \xrightarrow{\alpha} P'}{\text{new}\, a P \xrightarrow{\alpha} \text{new}\, a P'} \; (\alpha \notin \{a, \bar{a}\})$ $\qquad$ $\text{IDENT}_t: \; \dfrac{\{\vec{b}/\vec{a}\}P_A \xrightarrow{\alpha} P'}{A\langle \vec{b} \rangle \xrightarrow{\alpha} P'} \; \left(A(\vec{a}) \stackrel{\text{def}}{=} P_A\right)$

[5 marks]

$P|Q \xrightarrow{\alpha} R$ can be inferred

<u>either</u> (1) REACT_t from $P \xrightarrow{\lambda} P'$ and $Q \xrightarrow{\bar{\lambda}} Q'$, with $\alpha = \tau$ and $R = P'|Q'$
<u>or</u> (2) by L-PAR_t from $P \xrightarrow{\alpha} P'$, with $R = P'|Q$
<u>or</u> (3) by R-PAR_t from $Q \xrightarrow{\alpha} Q'$, with $R = P|Q'$

[5 marks]

In the first case, R_1 is $P'|Q'$. But by taking $\lambda' = \bar{\lambda}$ we have $Q \xrightarrow{\lambda'} Q'$, $P \xrightarrow{\bar{\lambda}'} P'$ and hence by REACT_t we infer $Q|P \xrightarrow{\tau} Q'|P'$, which is structurally congruent to R_1. In the second case R_1 is $P'|Q$. But R-PAR_t can be used to infer from $P \xrightarrow{\alpha} P'$ that $Q|P \xrightarrow{\alpha} Q|P' \equiv R_1$. The third case is similar. [5 marks]

An example is $P = a.P'$, $Q = \bar{a}.Q'$. If $\text{new}\, a\,(P|Q) \xrightarrow{\alpha} R_1$ then we must have that $P|Q \xrightarrow{\alpha} R$, where $R_1 = \text{new}\, a\, R$, $\alpha \notin \{a, \bar{a}\}$. Now impose the condition on P that <u>a is not free in P</u>. Consider the ways in which $P|Q \xrightarrow{\alpha} R$ is inferred

by (1): Then $\alpha = \tau$ and $P \xrightarrow{\lambda} P'$, $Q \xrightarrow{\bar{\lambda}} Q'$, $R = P'|Q'$. Then by the condition on P, λ cannot be a or $\bar{a}$. Hence $\text{new}\, a\, Q \xrightarrow{\bar{\lambda}} \text{new}\, a\, Q'$ by RES_t, and by R-PAR_t we get $P|\text{new}\, a\, Q \xrightarrow{\tau} P|\text{new}\, a\, Q' \equiv R_1$.

by (2) or (3): Then the inference $P|\text{new}\, a\, Q \xrightarrow{\alpha} R_2 \equiv R_1$ is fairly easy to construct (and does not depend on the syntactic condition). Note that α cannot be a or $\bar{a}$.

[5 marks]