Discrete Mathematics 2003

p19 7
PF

Solution A

1. Use contradiction. Suppose that there are k primes, $p_1, p_2, \ldots, p_k$. Let $n = p_1 p_2 \ldots p_k + 1$. Now n is greater than all $p_1, p_2, \ldots, p_k$ and is not exactly divisble by any of them.
2. Expressing the specified numbers less than or equal to n in the form suggested, observe that $m^2 \leq n$, so there at most $\sqrt{n}$ choices for m. There are at most 2^k choices for the ε_i. Hence &c.

   $\sqrt{n}2^k < n$, so $n > 4^k$.
3. $$\left|\bigcup_{s\in S} A_s\right| = -\sum_{\phi\neq I\subseteq S} (-1)^{|I|}\left|\bigcap_{i\in I} A_i\right|$$

   Let $D_k = \{ n \mid 0 < n < 100 \text{ with } k \mid n \}$ for $k = 2, 3, 5$ & 7. So $|D_k| = 100/k$.
   $|U| = 50 + 33 + 20 + 14 - 16 - 10 - 7 - 6 - 4 - 2 + 3 + 2 + 1 + 0 - 0 = 78$.
   So #primes $= 100 - 78 + 4 - 1 = 25$.