2002 Topics in Concurrency (1)

[Uses the ch. on model checking from "Topics in Concurrency"]

(4) (a) A model checker for the modal μ-calculus

As a reduction relation:

$p \models \nu X\{\vec{r}\}A \longrightarrow$ true if $p \in \{\vec{r}\}$.

$p \models \nu X\{\vec{r}\}A \longrightarrow p \models A[\nu X\{\vec{r}, p\}A/X]$ if $p \notin \{\vec{r}\}$

$p \models A \vee B \longrightarrow (p \models A)$ or $(p \models B)$

$\wedge$ — and —

$p \models \neg A \longrightarrow$ not $(p \models A)$

$p \models \langle a \rangle A \longrightarrow (q_1 \models A$ or $\cdots$ or $q_n \models A)$

where $\{q_1 \cdots q_n\} = \{q \mid p \xrightarrow{a} q\}$.

$p \models [a] A \longrightarrow (q_1 \models A$ and $\cdots$ and $\quad)$

Above $\nu X\{\vec{r}\}A$ stands for the maximum fixed point $\nu X.(\{\vec{r}\} \vee A)$

(2) (b) $\mu X. A = \neg \nu X. \neg A[\neg X/X]$.

(7) (c) From the model checker

$p \models \nu X\{\vec{r}\} A \iff (p \in \{\vec{r}\}$ or $p \models A[\nu X\{\vec{r}, p\} A/X])$

Hence

$p \models \mu X\{\vec{r}\} A \iff p \models \mu X(\neg\{\vec{r}\} \wedge A)$

$\iff p \models \neg \nu X(\{\vec{r}\} \vee \neg A[\neg X/X])$

$\iff p \not\models \nu X\{\vec{r}\} \neg A[\neg X/X]$

$\iff (p \notin \{\vec{r}\}$ and $p \not\models \neg A[\neg \nu X\{\vec{r}, p\} \neg A[\neg X/X]/X])$

$\iff p \notin \{\vec{r}\}$ and $p \models A[\mu X\{\vec{r}, p\} A/X]$

From which the two equivalences follow.

(2) (d) Suggests:

$p \models \mu X\{\vec{r}\} A \longrightarrow$ false if $p \in \{\vec{r}\}$

$p \models \mu X\{\vec{r}\} A \longrightarrow p \models A[\mu X\{\vec{r}, p\} A/X]$ if $p \notin \{\vec{r}\}$

(5)

$$\begin{tikzcd} P \arrow[r, bend left, "a"] & Q \arrow[l, bend left, "a"] \arrow[d, "a"] \\ & \mathit{nil} \end{tikzcd}$$

$P \models \mu X. [a]F \vee \langle a \rangle X$

$\longrightarrow P \models [a]F \vee \langle a \rangle \mu X\{P\}([a]F \vee \langle a \rangle X)$

$\longrightarrow P \models [a]F$ or $P \models \langle a \rangle \mu X\{P\} \ldots$

$\longrightarrow P \models \langle a \rangle \mu X\{P\} \ldots$ as $P \not\models [a]F$

$\longrightarrow Q \models \mu X\{P\}([a]F \vee \langle a \rangle X)$

$\longrightarrow Q \models [a]F \vee \langle a \rangle \mu X\{P, Q\}([a]F \vee \langle a \rangle X)$

$\longrightarrow Q \models \langle a \rangle \mu X\{P, Q\} \ldots$ as $Q \not\models [a]F$

$\longrightarrow P \models \mu X\{P, Q\} \ldots$ or $\mathit{nil} \models \mu X\{P, Q\} \ldots$

Now $\mathit{nil} \models \mu X\{P, Q\}([a]F \vee \langle a \rangle X)$

$\longrightarrow \mathit{nil} \models [a]F \vee \langle a \rangle \mu X\{P, Q, \mathit{nil}\} \ldots$

$\longrightarrow$ is true as $\mathit{nil} \models [a]F$

Hence whole reduces to true. $\boxtimes$