Probability 2004 Paper 2 Question 5 (FHK)

Much of this question involves simple use of Bayes's Theorem and elementary arithmetic. Part (c) requires use of the Trinomial Distribution.

It is convenient to represent the probabilities given in the preamble as follows:

$$P(A) = \frac{2}{10} \qquad P(B) = \frac{3}{10} \qquad P(C) = \frac{5}{10}$$

$$P(dud \mid A) = \frac{3}{1000} \qquad P(dud \mid B) = \frac{8}{1000} \qquad P(dud \mid C) = \frac{4}{1000}$$

(a) The Probability of a randomly selected component being faulty is simply the denominator of the first fraction given for P(dud | A) in (a) above:

$$\frac{3}{1000} \times \frac{2}{10} + \frac{8}{1000} \times \frac{3}{10} + \frac{4}{1000} \times \frac{5}{10}$$

$$= \frac{6 + 24 + 20}{10000} = \frac{50}{10000} = \frac{1}{200}$$

[5 marks]

(b) Given the above value, Bayes's Theorem shows:

$$P(A \mid dud) = \frac{\dfrac{3}{1000} \times \dfrac{2}{10}}{\dfrac{3}{1000} \times \dfrac{2}{10} + \dfrac{8}{1000} \times \dfrac{3}{10} + \dfrac{4}{1000} \times \dfrac{5}{10}} = \frac{6}{6 + 24 + 20}$$

$$= \frac{3}{3 + 12 + 10} = \frac{3}{25}$$

$P(B \mid \text{dud})$ and $P(C \mid \text{dud})$ likewise.

In summary: $P(A \mid \text{dud}) = \frac{3}{25}$ $\quad P(A \mid \text{dud}) = \frac{12}{25}$ $\quad P(A \mid \text{dud}) = \frac{10}{25}$

[9 marks]

(c) The Trinomial Distribution applies here and gives the result:

$$\frac{6!}{2!\ 2!\ 2!} \left(\frac{3}{25} \right)^2 \left(\frac{12}{25} \right)^2 \left(\frac{10}{25} \right)^2$$

There is no need to simplify this. [6 marks]