Paper 11

Solution

a) i) suppose the first N characters of x form the ~~string~~ word y. Consider the state transitions of M as the ~~[illegible]~~ word y is input, starting in say q_0.

$$| c | h | a | r | a | \ldots | o | f | y |$$
$$q_0 \quad q_1 \quad q_2 \quad \cdots \quad \cdots \quad q_{N-1} \, q_N$$

There are $(N+1)$-state values $\{q_i\}_{i=0}^{N}$, hence at least one state must occur twice; say $q_i = q_j$ where $0 \leq i < j \leq N$. Let u be the initial ~~string~~ word of x up to and including the i^{th} character, v consists characters $(i+1) \ldots j$, and w make up the rest of x.

certainly $x = uvw$, $l(u) < N$, $1 \leq l(v) \leq N$;

suppose word $z_k = u v^k w$ is input to M.

Paper 11

Solution

a) i) ctd) w applied to M in state q_j causes transition to $q^* \in A$, some accepting state, since we know that M accepts x. Hence under x

$$q_0 \xrightarrow[u]{} q_i \xrightarrow[v]{} q_j \xrightarrow[w]{} q^* \in A,$$

where $q_i = q_j$.

Consider behaviour of M in q_i under v^k.

If $k = 0$, $v^k = \varepsilon$ (null string), $q_i \xrightarrow{\varepsilon} q_i = q_j$

Suppose that $q_i \xrightarrow[v^k]{} q_j$; note that

$q_i \xrightarrow[v^k]{} q_j = q_i \xrightarrow[v]{} q_j$, hence $q_i \xrightarrow[v^{k+1}]{} q_j$.

$\therefore q_0 \xrightarrow[u]{} q_i \xrightarrow[v^k]{} q_j \xrightarrow[w]{} q^* \in A \quad \forall k \in \mathbb{N}$, by induction.

Paper 11

Solution

a) ii) Let $E = \{w \in S^* \mid l(w) \geq N, M \text{ accepts } w\}$

Certainly E is non-empty: choose a word $z \in E$ of minimal length $l \geq N$.

Suppose if possible that $l \geq 2N$. By part a) i), $z = uvw$ where $1 \leq l(v) \leq N$ and M accepts $z_0 = uw$. Then certainly $l(z_0) \geq N$, and $l(z_0) < l(z)$ CONTRADICTION.

Hence M accepts some word z, $N \leq l(z) < 2N$.

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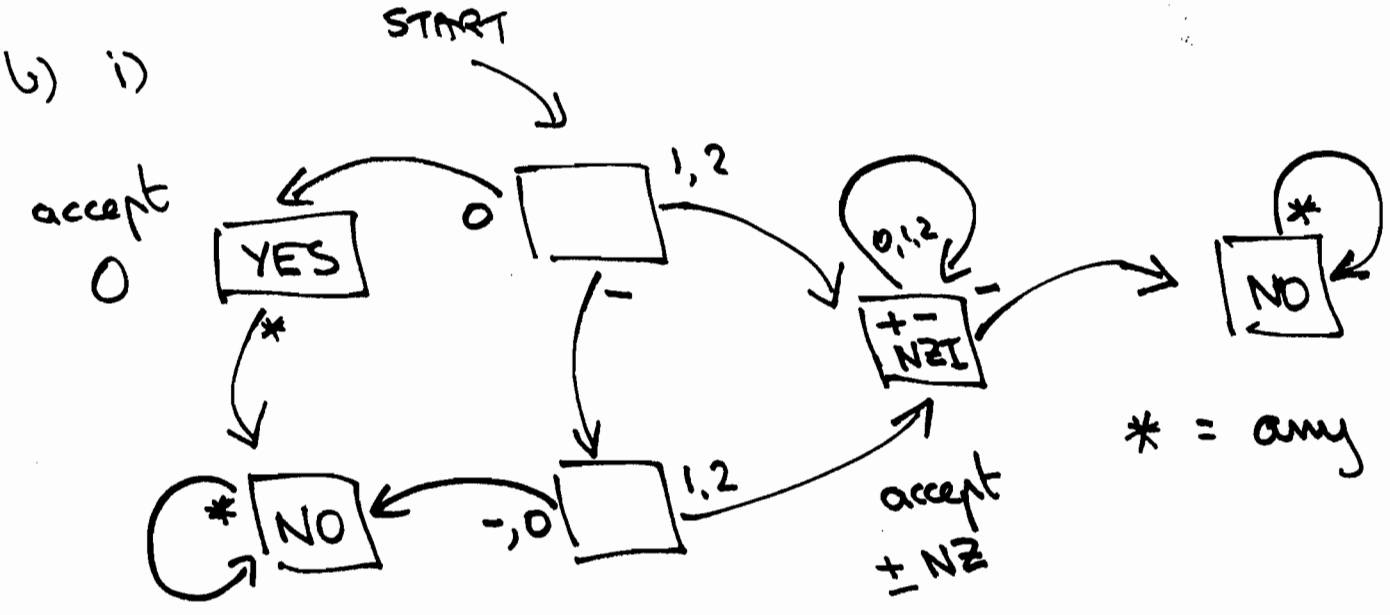
b) i)



Paper II

Solution

6) ii) $"0" + "-"("1"+"2")("0"+"1"+"2")^*$

$+ ("1"+"2")("0"+"1"+"2")^*$