SOLUTION NOTES

Advanced Algorithms 2002 Paper 8 Question 8 (ACN)

Syllabus section 2 on Kolmogorov Complexity.

(a) Complexity of a string relative to a (decoding) algorithm A is the length of the shortest input to the algorithm that outputs the string and then halts. Kolmogorov complexity uses a universal TM as the algorithm. For number use the bit-string representation of the number in binary as a string.

(b) If you have a number $N + k$ where k is small you can encode it as the encoding of N plus the encoding of k plus the fixed code to add bit-strings. Thus if k is small the complexity will be close to that of N.

(c) Consider a power of 2, say 2^k, which can be encoded in log(k)+c bits, $=$ log(log(N)), and an incompressible number in the given range which needs log(N) bits.

(d) k is the size of the description of a TM that just copies data from its input tape. In effect the program is `print "101100...0101"`.

(e) Enumerate all strings of complexity up to k. There are just 2^k of them. Find longest!

(f) Consider a number obtained (say) from the Ackerman function. It can be huge even though its complexity is small, and the ratio can exceed the suggested limit.