## 2 Data structures and algorithms

- (a) Describe the structure of splay trees used to represent a set of keys-value pairs. [5 marks]
- (b) Describe how new key-value pair are added to the tree, how the value associated with a given key can be looked up, and how to delete a pair with a given key. [5 marks]
- (c) State without prove the attractive properties of splay trees. [4 marks]
- (d) Describe the ternary tree structure used to hold a dictionary of key-value pairs where the keys are variable length strings. Illustrate the mechanism by showing the structure after items with keys: MIT, SAD, MAN, APT, MUD, ADD, MAG, MINE, MIKE, MINT, AT, MATE and MINES are added in that order to an initially empty ternary tree. [6 marks]

## ANSWER NOTES:

- (a) Splay tree nodes contain a key-value pair, a left pointer and a right pointer. All nodes in the left subtree have earlier keys and all nodes in the right subtree have later keys. Whenever an operation is perform involving a particular node it is promoted to the root by a sequence of simple transformations, described in the notes.
- (b) (1) Insert the new node as a leaf in the required place then promote it to the root. (2) Find the node with the given key and promote it to the root. (3) Find the node to delete, promote it to the root, then delete it by making one of its children the root and placing the other child at the appropriate leaf position. This node could then be promoted to the root.

## (c) Bookwork

(d) Bookwork. A ternary tree node contains a character, and three pointers left, right and middle. A character of zero marks the end of a string and the corresponding middle pointer points the the corresponding value. The nodes in the path from the root where the middle branch was taken contain then characters of the current prefix.

The left branch holds the structure for all entries that are earlier in dictionary order and and have the same prefix excluding the current character. The right pointer is similarly defined for later prefixes. The structure required is:

