The candidate is being asked to prove the Pumping Lemma (or rather, its contrapositive).

Suppose L has the given property. We assume L is regular and derive a
1 contradiction:
1 If L is regular, then $L = L(M)$, the language accepted by some DFA M.
2 Consider $\ell \triangleq$ number of states in M.
If $w \in L$ with $\mathrm{length}(w) \geq \ell$, say $w = a_1 a_2 \cdots a_n$ $(n \geq \ell)$, since w is accepted by M, we have a transition sequence in M of the form

1 $$s_M = \underbrace{q_0 \xrightarrow{a_1} q_1 \xrightarrow{a_2} q_2 \cdots \xrightarrow{a_\ell} q_\ell}_{\ell+1 \text{ states}} \cdots \xrightarrow{a_n} q_n \in Accept_M$$

Since $\ell+1$ is one more than the number of states
2 in M, $q_0, \ldots, q_\ell$ can't all be distinct states: so $q_i = q_j$ for some $0 \leq i < j \leq \ell$ and the above transition sequence looks like

$$s_M = q_0 \xrightarrow{u_1}{}^{*} q_i = q_j \xrightarrow{u_2}{}^{*} q_n \in Accept_M$$

$$q_i \xrightarrow{v}{}^{*} q_j$$

3 where $u_1 \triangleq a_1 \cdots a_i$, $v = a_{i+1} \cdots a_j$, $u_2 = a_{j+1} \cdots a_n$
1 Thus $\mathrm{length}(u_1) = i \leq \ell$, $\mathrm{length}(v) = j-i \geq 1$ and
12 1 $u_1 v^n u_2 \in L(M) = L$, for all n. Contradicting the property of L.

2 (a) Not regular. Show that L_1 has the property mentioned in the first part of the question.
Given any $\ell \geq 1$, consider

1 $$w = a^\ell b a^\ell b \in L_1$$

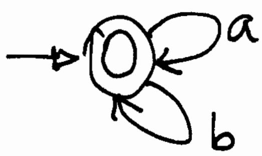
$\text{length}(w) = 2(\ell+1) \geq \ell$ ✓

For any splitting $w = u_1 v u_2$ with $\text{length}(u_1 v) \leq \ell$, $\text{length}(v) \geq 1$, can only have

$$\left.\begin{aligned} u_1 &= a^p \\ v &= a^q \\ u_2 &= a^{\ell-p-q} b a^\ell b \end{aligned}\right\} \text{some } p, q \text{ with } p+q \leq \ell \ \& \ q \geq 1$$

So $u_1 v^0 u_2 = a^{\ell-q} b a^\ell b \notin L_1$ because if $a^{\ell-q} b a^\ell b = uu$ for some u, then u ends in b ('cos uu does), so can only have
2 $u = a^{\ell-q} b$, $u = a^\ell b$; but $q \geq 1$, so $a^{\ell-q} b \neq a^\ell b$.
So L does not have the Pumping Lemma Property.
(5) So is not regular.

1 (b) Is regular. For taking $w = \varepsilon$, we have
that $L_2 \supseteq \{\varepsilon v \varepsilon \mid v \in \Sigma^*\} = \Sigma^*$ and
1 hence $L_2 = \Sigma^*$ — which is regular: e.g.

1 

is a FDA accepting Σ^*.

(3)