2004 p1q2 PR

Discrete mathematics – short question

Given a set A, define the following terms:

- a relation on A
- an equivalence relation on A
- a partial order on A
- a well founded relation on A [4 marks]

If two relations $<_A$ on A and $<_B$ on B are well founded, show that the lexicographic relation and product relation on A×B are both well founded. [6 marks]

Solution

$R \subseteq A \times A$ [1]

Reflexive, symmetric, transitive. [1]

Reflexive, anti-symmetric, transitive. [1]

$\forall S \subseteq A . S \neq \varnothing \Rightarrow \exists m \in S . \forall a \in A . a < m \Rightarrow a \notin S$, or
every descending sequence ultimately constant. [1]

Consider projection of product order into A, [1]
find minimum, [1]
consider projection into B, [1]
find minimum, [1]
show minimal. [1]

Observe $<_L \subseteq <_P$. [1]