p7q1
MJCG

Solution notes for SV1.1 2000

The Assignment Axiom assumes that only the variable being assigned to is modified. If the evaluation of the right hand side of an assignment has a side effect then it might change other variables. For example, an instance of the Assignment Axiom is

```
 |-  {Y=1} X := (Y := 2; 2) {Y=1}
```

but this is semantically wrong, as Y is changed to 2 as a side effect of evaluation (Y := 2; 2).

Here are two approaches to handling expressions with side-effects.

Approach 1: Rewrite expressions (C;E) away.

```
  V := (C;E)  -->  C; V := E

  IF (C;E) THEN C1 ELSE C2 --> C; IF E THEN C1 ELSE C2

  WHILE (C1;E) DO C2  --> C1; WHILE E DO (C2; C1)
```

Approach 2: Add rules to cope with (C;E)

```
     |- {P}C{Q[E/V]}
  --------------------
  |- {P} V := (C;E) {Q}


  |- {P}C{Q}, |- {Q /\ E} C1 {R}, |- {Q /\ ~E} C2 {R}
  ---------------------------------------------------
           |- {P} IF (C;E) THEN C1 ELSE C2


   |- {P}C1{Q},  |- {Q /\ E} C2; C1 {Q}
  --------------------------------------
  |- {P} WHILE (C1;E) DO C2;C1 {Q /\ ~E}
```

Using approach 1, the example expands to:

```
{Y <= X}
 R := X; Q := 0;
 R := R-Y; Q := Q+1;
 WHILE Y <= R DO SKIP; R := R-Y; Q := Q+1
{X = R+(Y*Q) /\ R<Y}
```

This is easily verified by showing

```
1. {Y <= X}
    R := X; Q := 0;
    R := R-Y; Q := Q+1;
   {X = R+(Y*Q)}
```

and

```
2. {X = R+(Y*Q) /\ Y <= R}
    WHILE Y <= R DO SKIP; R := R-Y; Q := Q+1
   {X = R+(Y*Q)}
```

The VC for 1 is:

```
   Y <= X ==> (X = (R-Y) + (Y*(Q+1)))[X,0/R,Q]
```

i.e. `Y <= X ==> (X = (X-Y) + (Y*(0+1)))`

i.e. `Y <= X ==> (X = (X-Y) + Y)`

i.e. `Y <= X ==> (X = X)`

The VCs for 2 are bookwork (example occurs in notes).