p12q10 p8q11

MROD

Solution Notes

Numerical Analysis II 2001

Question A

[2 marks]

(a) By substitution, $\phi(x_j) = y_j$.

(b) $$\phi'(x) = \frac{y_{j+1}-y_j}{d_j} - \frac{(x_{j+1}+x_j-2x)\{(d_j+x_{j+1}-x)\mu_j+(d_j+x-x_j)\mu_{j+1}\}}{6d_j}$$
$$+ \frac{(x-x_j)(x_{j+1}-x)(\mu_{j+1}-\mu_j)}{6d_j}$$

so

$$\phi'(x_j) = \frac{y_{j+1}-y_j}{d_j} - \frac{(2\mu_j+\mu_{j+1})d_j}{6} \quad (1)$$

$$\phi'(x_{j+1}) = \frac{y_{j+1}-y_j}{d_j} + \frac{(2\mu_{j+1}+\mu_j)d_j}{6} \quad (2)$$

[4 marks]

(c) $\phi''(x_j) = \mu_j$. [2 marks]

To force first derivative continuity at x_j, equate formula (2) applied to $[x_{j-1}, x_j]$ with formula (1) applied to $[x_j, x_{j+1}]$:

$$\frac{y_j-y_{j-1}}{d_{j-1}} + \frac{(2\mu_j+\mu_{j-1})d_{j-1}}{6} = \frac{y_{j+1}-y_j}{d_j} - \frac{(2\mu_j+\mu_{j+1})d_j}{6}.$$

Multiply by 6 and rearrange:

$$d_{j-1}\mu_{j-1} + 2(d_{j-1}+d_j)\mu_j + d_j\mu_{j+1} = \frac{6(y_{j+1}-y_j)}{d_j} - \frac{6(y_j-y_{j-1})}{d_{j-1}}.$$

If $\phi(x)$ is linear for $x<x_1$, $x>x_n$ then $\mu_1=\mu_n=0$ so the equations form the system $A\underline{m}=\underline{z}$ where $\underline{m}=\{\mu_2,\mu_3,\ldots\mu_{n-1}\}$,

$$A = \begin{bmatrix} 2(d_1+d_2) & d_2 & & & \\ d_2 & 2(d_2+d_3) & d_3 & \ddots & \\ & d_3 & \ddots & & d_{n-2} \\ & & \ddots & d_{n-2} & 2(d_{n-2}+d_{n-1}) \end{bmatrix}$$

and

$$z_j = \frac{6(y_{j+1}-y_j)}{d_j} - \frac{6(y_j-y_{j-1})}{d_{j-1}}.$$

[10 marks]

A is symmetric positive definite and tridiagonal so is well conditioned for solution. [2 marks]