Foundations of Functional Programming 1999

p6q12

AD

Paper 6

1. Y is given by: $\lambda f.(\lambda x.f(xx))(\lambda x.f(xx))$.

   K is given by $\lambda xy.x$.

   T is given by $\lambda xy.yx$.

   I is given by $\lambda x.x$. [1 mark each]

2. $Y \to \lambda f.f(\lambda x.f(xx)\lambda x.f(xx))$. The latter terms is in head normal form. Thus, we conclude that Y is defined. [2]

   Proceeding by left most reductions on YK, we obtain:

$$\begin{aligned} YK &\to (\lambda x.K(xx))(\lambda x.K(xx)) \\ &\to (\lambda xy.x)((\lambda x.K(xx))(\lambda x.K(xx))) \\ &\to \lambda y.(\lambda x.K(xx))(\lambda x.K(xx)) \end{aligned}$$

   at which point, it is clear that no head normal form will be reached. Thus, YK is not defined. [2]

   For YT, again proceeding by left-most reductions:

$$YT \to T(YT) \equiv (\lambda xy.yx)(YT) \to \lambda y.y(YT).$$

   The last term above is in head normal form, and therefore YT is defined. [2]

   Applying left-most reductions to YI gives us:

$$YI \to I(YI) \to YI$$

   and therefore YI does not have a head normal form. [2]

3. Let N be the term $\lambda gx.x$. Then,

$$\begin{aligned} YN &= (\lambda f.f((\lambda x.f(xx))(\lambda x.f(xx))))N \\ &\to N((\lambda x.N(xx))(\lambda x.N(xx))) \\ &\to \lambda x.x = I. \end{aligned}$$

   [3]

   As YK does not have a head normal form, by Wadsworth's theorem, it is not solvable. The same applies to YI. [2]

For YT, we have seen above that $YT \to^\star \lambda y.y(YT)$. Thus, if we let N be the term $\lambda gx.x$, then

$$YTN \to (\lambda y.y(YT))N \to N(YT) \to \lambda x.x = I.$$

[3]