p6 q12
AD

p13 q13

Paper 6
Complexity Theory 2000

1. Time hierarchy theorem: For every constructible function f, $\textsc{Time}(f(n))$ is properly contained in $\textsc{Time}(f(n)(\log f(n))^2)$.

   Space hierarchy theorem: For every constructible function f, $\textsc{Space}(f(n))$ is properly contained in $\textsc{Space}(f(n)(\log f(n)))$.

2. (a) $\textsc{Dspace}(n^2)$ — closed under linear time reductions, because a linear time function can always be computed in quadratic space. Also contains complete problems, by constuction of the diagonal function.

   (b) L — A linear time function may require more than logarithmic space to compute, so this is unlikely to be closed under such reductions, but proving this would involve separating L from P. It therefore does not contain complete problems.

   (c) P — It is closed under linear time reductions, because linear time is contained in P. It does not contain complete problems with respect to linear time reductions. Suppose there were such a problem A. It would then be computable in time $O(n^k)$ for some k. By composition with the reduction, every problem in P would be computable in time $O(n^k)$, contradicting the time-hierarchy theorem.

   (d) NP — Again closed under linear time reductions, and does not contain complete problems, by arguments analogous to those for P, but this time using the non-deterministic time-hierarchy theorem.