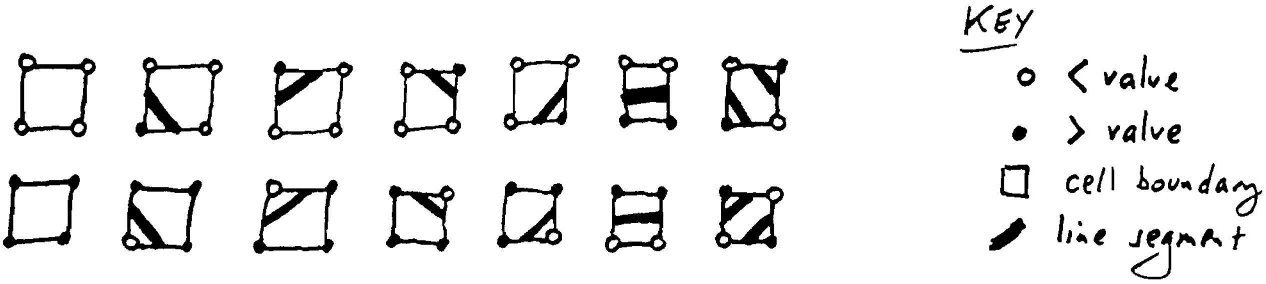
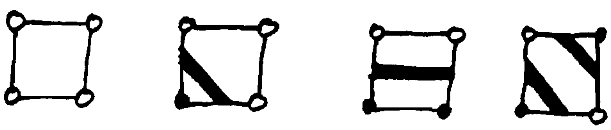
ADVANCED GRAPHICS – PAPER 7 – SOLUTION NOTES 2001 P7Q1 NAD

1. Take a regular grid of sample values and a height value, h. Mark each sample value as to whether it is less than (<) or greater than (>) h. Values equal to h are arbitrarily denoted as >. Fill each cell with the appropriate line segment, where a cell is the square region bounded by four sample values. There are 16 possible cells:



These reduce, by reflection and inversion (geometric reflection and inversion of < and >), to just four cases:



If you want to be clever you can join up the line segments into polylines.

Another piece of cleverness is to find one non-empty cell (i.e. at least one of its four values is < and one other is >) and the <u>march</u> around the grid from one non-empty cell to another until you return to the start or fall off the edge of the grid. You need to be careful, if you do this, because there may be more than one, disconnected, contour for height value, h, and the naïve algorithm will only draw the first it comes to.

4. (a) ray: $x(t) = x_E + t x_D$; $y(t) = y_E + t y_D$; $z(t) = z_E + t z_D$

cone: $y^2 + z^2 = x^2$ $\quad x_{min} \le x \le x_{max}$

(technically I should have said $y^2+z^2=kx^2$, but that doesn't affect parts (a) and (b)) to any great extent)

solve the quadratic equation in t formed by substituting the ray equation in the cone equation:

$$(y_E + t y_D)^2 + (z_E + t z_D)^2 = (x_E + t x_D)^2$$

this will give two values of t: $t_1 \le t_2$

if t_1 and t_2 are imaginary $\rightarrow$ no intersection
if $t_1 \le t_2 < 0$ then no intersection
if $t_1 < 0 \le t_2$ then t_2 is the intersection with the infinite double cone
if $0 \le t_1 \le t_2$ then t_1 and t_2 are both intersections with the infinite double cone

substitute the intersections with the infinite double cone into
$x_i = x(t_i) = x_E + t_i y_D$

if $x_{min} \le x_i \le x_{max}$ then t_i represents an intersection with the finite-length open-ended cone.
the lowest value of t_i for which this condition is true is the first intersection and the first intersection point can be found by substituting t_i into the ray equation.

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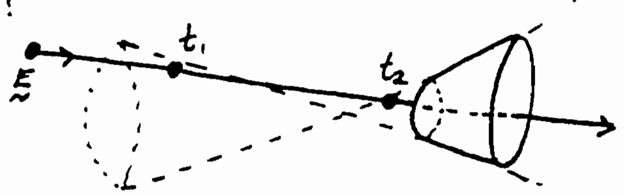
(b) To handle a closed cone we cannot just do what I described in lectures for a closed cylinder because of cases like this:



where the ray intersects the end caps but the intersections with the cone both lie on the same side of the finite cone (cont). The easiest way to handle this is to find:

$$t_3 = \frac{x_{min} - x_E}{x_D} \quad \text{and} \quad t_4 = \frac{x_{max} - x_E}{x_D}$$

from these, substitute into the ray equations to get (y_3, z_3) and (y_4, z_4)

if $(y_3^2 + z_3^2) \leq x_{min}^2$ and $t_3 \geq 0$ } then t_3 represents an intersection with the circular end-cap at $x = x_{min}$ of radius x_{min}

if $(y_4^2 + z_4^2) \leq x_{max}^2$ and $t_4 \geq 0$ } then t_4 represents an intersection with the circular endcap at $x = x_{max}$

Now, take all of the t_is which represent valid intersection points: the smallest of these is the one you want (it will be non-negative and smaller in value than the others).

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4(c) Assume that $\underset{\sim}{E}$ is outside the cone.

If t_3 is the intersection, the normal is $[-1, 0, 0]$

If t_4 " " " " " $[1, 0, 0]$

If t_1 or t_2 is the intersection then find x, y, z from the ray equations. The normal points at $45°$ to a perpendicular drawn from $(x, 0, 0)$ to (x, y, z) and is:

$$\left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\left(\frac{y}{\sqrt{y^2+z^2}}\right), \frac{1}{\sqrt{2}}\left(\frac{z}{\sqrt{y^2+z^2}}\right)\right]^{*}$$

Now, if $\underset{\sim}{E}$ is inside the cone (check by seeing if all of these conditions are true: $y_E^2 + z_E^2 < x_E^2$, $x_{min} < x_E < x_{max}$) then the normal is the negative of those given above.

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* N.B. if the cone is $y^2 + z^2 = kx^2$ then we get $\left[-\sin(\tan^{-1}k), \frac{(\cos(\tan^{-1}k))\,y}{\sqrt{y^2+z^2}}, \frac{(\cos(\tan^{-1}k))\,z}{\sqrt{y^2+z^2}}\right]$