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Paper 5
Foundations of Functional Programming

1. The Church-Rosser theorem states that any two terms that are equal reduce to the same normal form. Thus, if we take two terms that are both in normal form, say $\lambda x.x$ and y which are clearly not congruent, then they cannot be equal. [2 marks]

2. Let M and N be any two terms. The following is a sequence of reductions and expansions transforming M into N.

$$M \leftarrow (\lambda xy.x)MN \rightarrow (\lambda xy.y)MN \rightarrow N$$

[3 marks]

3. To show that Θ is a fixed point combinator, we need to show that, for any term M, $\Theta M = M(\Theta M)$. This is established through the following sequence of equalities:

$$\Theta M \equiv AAM \equiv \lambda xy.y(xxy)AM \rightarrow M(AAM) \equiv M(\Theta M)$$

[3 marks]

4. To construct the term **rev**, we proceed by constructing a series of auxiliary operations on lists.

   Let **isnull** denote the term:

$$\lambda l.l(\lambda xy.\mathbf{false})\mathbf{true}.$$

[2 marks]

   Let **append** denote the term:

$$\lambda l_1 l_2.(\lambda fx.l_1 f(l_2 fx)).$$

[2 marks]

   Let **head** denote the term:

$$\lambda l.l(\lambda xy.x)x.$$

[2 marks]

Let **tail** denote the term:

$$\lambda l.\mathbf{snd}l(\lambda uv.\mathbf{pair}u\mathbf{append}(\lambda fx.f(\mathbf{fst}v)x)(\mathbf{snd}v))(\lambda fx.x).$$

[3 marks]

Then, **rev** can be defined as the term:

$$\Theta(\lambda rl.\mathbf{if}(\mathbf{isnull}l)l(\mathbf{append}(\mathbf{rev}(\mathbf{tail}l))(\lambda fx.f(\mathbf{head}l)x))).$$

[3 marks]