

2002  
Topics in Concurrency (1)

[Uses the ch. a model checking  
from "Topics in Concurrency"]

(4) (a) A model checker for the model  $\mu$ -calculus

as a reduction relation:

$$p \models \forall X \{ \vec{r} \} A \longrightarrow \text{true if } p \in \{ \vec{r} \}$$

$$p \models \forall X \{ \vec{r} \} A \longrightarrow p \models A[\forall X \{ \vec{r} \}, p \} A / X] \text{ if } p \notin \{ \vec{r} \}$$

$$p \models A \vee B \longrightarrow (p \models A) \vee (p \models B)$$

$\wedge$  - and -

$$p \models \neg A \longrightarrow \text{not } (p \models A)$$

$$p \models \langle a \rangle A \longrightarrow (q_1 \models A \vee \dots \vee q_n \models A)$$

where  $\{q_1 \dots q_n\} = \{q \mid p \xrightarrow{a} q\}$

$$p \models [c] A \longrightarrow (q_1 \models A \text{ and } \dots \text{ and } q_n \models A)$$

where  $\forall X \{ \vec{r} \} A$  stands for the maximum  
fixed point  $\forall X. (\{ \vec{r} \} \vee A)$

$$(2) (b) \quad \mu X. A = \neg \nu X. \neg A[\neg X/X].$$

(7) (c) From the model checker

$$p \models \nu X \{ \vec{r} \} A \Leftrightarrow (p \in \{ \vec{r} \} \vee p \models A[\nu X \{ \vec{r}, p \} A / X])$$

hence

$$p \models \mu X \{ \vec{r} \} A \Leftrightarrow p \models \mu X (\neg \{ \vec{r} \} \wedge A)$$

$$\Leftrightarrow p \models \neg \nu X (\{ \vec{r} \} \vee \neg A[\neg X/X])$$

$$\Leftrightarrow p \not\models \nu X \{ \vec{r} \} \neg A[\neg X/X]$$

$$\Leftrightarrow (p \notin \{ \vec{r} \} \text{ and } p \not\models \neg A[\nu X \{ \vec{r}, p \} \neg A[\neg X/X] / X])$$

$$\Leftrightarrow p \notin \{ \vec{r} \} \text{ and } p \models A[\mu X \{ \vec{r}, p \} A / X]$$

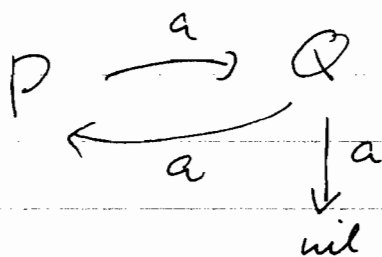
From which the two equivalences follow.

(2) (d) suggests:

$$p \models \mu X \{ \vec{r} \} A \longrightarrow \text{false if } p \in \{ \vec{r} \}$$

$$p \models \mu X \{ \vec{r} \} A \longrightarrow p \models A[\mu X \{ \vec{r}, p \} A / X] \text{ if } p \notin \{ \vec{r} \}$$

(5)



$$P \models \mu X. [a]F \vee \langle a \rangle X$$

$$\rightarrow P \models [a]F \vee \langle a \rangle \mu X \{P\} ([a]F \vee \langle a \rangle X)$$

$$\rightarrow P \models [a]F \text{ or } P \models \langle a \rangle \mu X \{P\} \dots$$

$$\rightarrow P \models \langle a \rangle \mu X \{P\} \dots \text{ as } P \not\models [a]F$$

$$\rightarrow Q \models \mu X \{P\} ([a]F \vee \langle a \rangle X)$$

$$\rightarrow Q \models [a]F \vee \langle a \rangle \mu X \{P, Q\} ([a]F \vee \langle a \rangle X)$$

$$\rightarrow Q \models \langle a \rangle \mu X \{P, Q\} \dots \text{ as } Q \not\models [a]F$$

$$\rightarrow P \models \mu X \{P, Q\} \dots \text{ or } nil \models \mu X \{P, Q\} \dots$$

$$\text{Now } nil \models \mu X \{P, Q\} ([a]F \vee \langle a \rangle X)$$

$$\rightarrow nil \models [a]F \vee \langle a \rangle \mu X \{P, Q, nil\} \dots$$

$$\rightarrow \text{is true as } nil \not\models [a]F$$

Hence whole reduces to true.

