

Information Theory and Coding 2004, Paper 7 Question 8 (JGD and MGK)

(a)

A	B	C	D	E	F	G	H
$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{128}$	$\frac{1}{128}$

(i) For this symbol distribution, the most efficient sequence of questions to ask (until a ‘yes’ is obtained) would be just: (1) Is it A? (2) Is it B? (3) Is it C? (Etc.) [2 marks]

(ii) Each such 1-bit question is maximally informative because the remaining uncertainty is reduced by half (1 bit). [2 marks]

(iii) The probability of terminating successfully after exactly N questions is 2^{-N} . Therefore the weighted average of interrogation durations is:

$$\frac{1}{2} + (2)\left(\frac{1}{4}\right) + (3)\left(\frac{1}{8}\right) + (4)\left(\frac{1}{16}\right) + (5)\left(\frac{1}{32}\right) + (6)\left(\frac{1}{64}\right) + (7)\left(\frac{1}{128}\right) + (8)\left(\frac{1}{128}\right) = 1\frac{127}{128}$$

In other words, on average just slightly less than two questions need to be asked in order to learn which of the 8 symbols it is. [2 marks]

(iv) The entropy of the above symbol set is coincidentally exactly the same calculation:

$$H = - \sum_{i=1}^8 p_i \log_2 p_i = 1\frac{127}{128}$$

[2 marks]

(v) A natural code book to use would be the following:

A	B	C	D	E	F	G	H
1	01	001	0001	00001	000001	0000001	0000000

It is uniquely decodable because each code corresponds to a unique letter rather than any possible combination of letters; and it has the prefix property because the code for no letter could be confused as the prefix for another letter. [2 marks]

(vi) The bit strings in the above prefix code for each letter can be interpreted as the history of answers to the ‘yes/no’ questions. [2 marks]

(b) Functions which have exactly the same form as their Fourier transforms are called “self-Fourier.” Examples of such pairs include: the Gaussian; the Gabor wavelet; the sampling Comb function; and the hyperbolic secant. [3 marks]

(c) (i) The sensation limit of a sense is the lowest amplitude of a stimulus that can be perceived. [1 mark]

- (ii) If two audio tones fall within the same critical band, the ear is unable to recognize two separate tones and perceives a single tone with the average of their frequency instead. (The human ear has approximately 24 non-overlapping critical bands.) [1 mark]
- (iii) The Bark scale is a non-linear transform of an audible frequency into the number range 0 to 24, such that if two frequencies are less than 1 apart on this scale, they are within the same critical band. [1 mark]
- (d) Weber's law is concerned with how the difference limit, the smallest amplitude change of a stimulus that can be distinguished, depends on the amplitude of the stimulus. (It states that the two are proportional, except for a small correction near the sensation limit.) Steven's law on the other hand is concerned with how the amplitude of a stimulus is perceived in relation to other amplitudes, for example how much must the amplitude raise such that the stimulus is perceived as being twice as strong. (It states a power-law relationship between amplitude and perceived stimulus strength.) [2 marks]

[This question relates to the psychophysics of perception, as discussed in the course section on coding audiovisual signals.]