

Solution notes

Data Structures and Algorithms 2005 (MR) Paper 5 Question 1, Paper 12 Question 1

Warshall's and Floyd's algorithms are covered in detail in the course.

- a) $M_{ij}=1$ (or true) iff there is an edge from $v_i \rightarrow v_j$
- b) Bookwork plus proof by induction based on a statements of the meaning of the matrix after processing column k .
- c) Also book work, and not difficult.
- d) This is NOT bookwork and requires some thought. One way is to copy the elements of M into an integer matrix and apply Floyd's algorithm to obtain the minimum costs matrix. Then fill in each entry R_{ij} with k where $i \rightarrow k$ is an edge and $C_{kj}=C_{ij}-1$. After cost of $O(n*n*n)$ the cost of this final stage is no worse and can probably be done in $O(n*n)$. A better alternative is: for all i,j initialise C_{ij} to 1 if M_{ij} otherwise $\maxint/2$, and initialise R_{ij} to j if M_{ij} is true and zero otherwise, and add the assignment $R_{ij} := R_{ik}$ at the point in Floyd's algorithm updates C_{ij} with a smaller value. Ie

```
for i= 1 to n for j = 1 to n do
{ if  $M_{ij}$  then  $C_{ij}, R_{ij} := 1, j$ 
    else  $C_{ij}, R_{ij} := \maxint/2, 0$ 
}
```

```
for k = 1 to n for i = 1 to n for j = 1 to n do
    if  $C_{ij} > C_{ik}+C_{kj}$  do
    {  $C_{ij} := C_{ik}+C_{kj}$ 
       $R_{ij} := R_{ik}$ 
    }
```