1999

p3q2

JGD

Model Answer, Continuous Mathematics, Question 1.

1. The real part of $\mathcal{Z}_3$ is $ac - bd$. Its imaginary part is $ad + bc$.

   [2 marks]

2. Modulus $\|\mathcal{Z}_1\| = \sqrt{a^2 + b^2}$. Modulus $\|\mathcal{Z}_3\| = \sqrt{(ac - bd)^2 + (ad + bc)^2}$.

   [2 marks]

3. Angle $\angle\mathcal{Z}_2 = \tan^{-1}(\frac{d}{c})$.

   [2 marks]

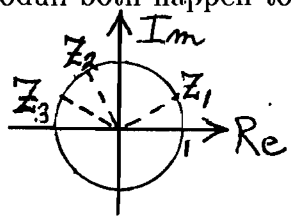
4. In complex polar form, $\mathcal{Z}_1 = \|\mathcal{Z}_1\| \exp(i\angle\mathcal{Z}_1)$.

   [2 marks]

5. As is clear from describing these variables in complex polar form,

   $$\mathcal{Z}_3 = \|\mathcal{Z}_1\| \exp(i\angle\mathcal{Z}_1)\|\mathcal{Z}_2\| \exp(i\angle\mathcal{Z}_2) = \|\mathcal{Z}_1\|\|\mathcal{Z}_2\| \exp[i(\angle\mathcal{Z}_1 + \angle\mathcal{Z}_2)],$$

   and so the angles of the two complex variables just add. This is a rotation in the complex plane. Since the two moduli both happen to equal 1, the modulus of the product $\mathcal{Z}_1\mathcal{Z}_2$ is also $\|\mathcal{Z}_3\| = 1$.

   [4 marks]

6. If $\mathcal{Z} = \exp(2\pi i/5)$ is multiplied by itself five times, its angle is just added to itself five times, producing $\exp(2\pi i) = 1$.

   [2 marks]

7. The real part of $f(x) = \exp(2\pi i\omega x)$ is the function $\cos(2\pi\omega x)$. Its imaginary part is the function $\sin(2\pi\omega x)$.

   [2 marks]

8. If the complex exponential $f(x) = \exp(2\pi i\omega x)$ is operated upon by any linear operator, its functional form cannot change. The most dramatic thing that can happen to it is that it gets multiplied by a complex constant. This means that only its amplitude and phase can be affected. Complex exponentials are the eigenfunctions of linear systems.

   [4 marks]