1999

P1q2
PR

Discrete Mathematics

Short question

Let $M_n = 2^n - 1$ be the n^{th} Mersenne number.

Show that M_n can only be prime if n is. [5 marks]

Let $\Delta_m = m(m+1)/2$ be the m^{th} triangular number and recall that a perfect number is one equal to the sum of its factors (including 1 but excluding the number itself).

Suppose that $p = M_n$ is prime. Show that Δ_p is a perfect number. [5 marks]

Answer

If n = a.b with a, b > 1, then $2^n - 1 = (2^b - 1)(2^{n-b} + 2^{n-2b} + 2^{n-3b} + \dots 2^{n-ab})$.

$\Delta_p = p.2^{n-1}$ and so has proper factors $1, 2, 2^2, 2^3, \dots, 2^{n-1}, p, 2p, 2^2p, 2^3p, \dots, 2^{n-2}p$ whose sum is $2^n-1 + (2^{n-1}-1)p = 2^{n-1}p = \Delta_p$.