1999

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PR

Discrete Mathematics

Long question 2

Let Ω be a universal set and define a relation between subsets A, B $\subseteq \Omega$ by $A \cong B \Leftrightarrow \exists$ a bijection $f: A \to B$. Prove carefully that $\cong$ is an equivalence relation. [6 marks]

What does it mean to say that a set is *countable*? [2 marks]

State without proof the Schröder-Bernstein theorem concerning the existence of a bijection between two sets. [2 marks]

Show that the integers and the rational numbers are countable but that the real numbers are uncountable. [6 marks]

An ML program consists of a finite sequence of characters drawn from a finite alphabet. Show that the set of ML programs is countable. [4 marks]

Answer

Reflexive by considering identity function. Symmetric since inverse of bijection is a bijection. Transistive because composition of bijections is a bijection.

A is countable if $A \cong N$, the natural numbers, (or if A is finite).

Given injections $A \to B$ and $B \to A$, $\exists$ a bijection $A \to B$.

$z \to 2z + 1$ if $z > 0$, $-2z$ otherwise. $a/b \to 2^a5^b$ if $a > 0$, $3^{-a}5^b$ otherwise and use S-B. Show P(N) uncountable by contradiction, construct injection $P(N) \to R$ by $\{a_i\} \to \Sigma\ 10^{-a_i}$ and use S-B for contradiction.

Let A_n be the programs of length n and so finite. Countable union of finite sets is countable.