P1q2

2000 PR

Discrete Mathematics

Short question

State the conditions for a relation to be a partial order. [3 marks]

A *partition* of a natural number n is a collection of natural numbers (possibly including duplicates and in any order) whose sum is n. Let P_n be the set of partitions of n; for example, $P_4 = \{(4), (3,1), (2,2), (2,1,1), (1,1,1,1)\}$. Order the partitions in P_n as follows: $(a_1, a_2, \ldots a_r) \le (b_1, b_2, \ldots b_s)$ if the (a_i) and (b_j) can be rearranged so that $b_1 = a_1 + a_2 + \ldots a_{k1}$, $b_2 = a_{k1+1} + a_{k1+2} + \ldots + a_{k2}$, $\ldots$ $b_s = a_{km+1} + a_{km+2} + \ldots + a_r$. So $(2,1,1) \le (3,1)$, but $(3,1)$ and $(2,2)$ can not be compared.

Show that $\le$ is a partial order on P_n. [4 marks]

Draw the Hasse diagram for $(P_5, \le)$. [3 marks]

Answer

Reflexive, anti-symmetric and transitive.

Reflexive: $k_i = i$. Anti-symmetric: two partitions must have same number of elements so only one term in each sum. Transitive: substitute one decomposition into the other.