Paper 10 (comment) 2004 p10 q8

JKMM

This is VERY MUCH BOOKWORK. Even the two riders at the end are lifted from the notes — as far as I can tell the class don't try them, so this is an experiment.

Do we need to reiterate $|A| = a$, $|B| = b$ at (*)? I hope it's totally obvious

ack; ~~?~~

SOLUTION

Cartesian product $A \times B = \{(a,b) \mid a \in A, b \in B\}$, where $(a_1,b_1) = (a_2,b_2)$ IFF $a_1 = a_2$ and $b_1 = b_2$.

Disjoint union $A + B = (\{1\} \times A) \cup (\{2\} \times B)$, (or similarly with any other suitable pair of flags).

DON'T need discussion of $in_1(A)$ etc.

$$f^{-1} = \{(b,a) \in (B \times A) \mid (a,b) \in f\} \subseteq (B \times A)$$

Paper 10 SOLUTION ctd).

$$f \circ g = \{ (a,c) \mid \exists b \in B \text{ s.t. } (a,b) \in f \text{ and } (b,c) \in g \} \subseteq (A \times C).$$

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f must be everywhere defined (TOTAL):

$\forall a \in A, \ \exists \ b \in B$ st $(a,b) \in f$.

f must be single-valued:

$(a,b_1) \in f, \ (a,b_2) \in f \implies b_1 = b_2$.

$|A \times B| = a.b, \quad |A+B| = a+b, \quad |A \to B| = b^a.$

If f, f^{-1} are BOTH functions:

a) f everywhere defined, f^{-1} single-valued

$\implies a \leq b$

b) f single-valued, f^{-1} everywhere defined

$\implies b \leq a$.

Paper 10 SOLUTION ctd)

Notation's a problem in both parts – really, anything clear and sensible goes.

i) given $f \in A \to (B \times C)$, define

$\phi(f) = (g_1, g_2) \in (A \to B) \times (A \to C)$, WHERE –

$g_1(a) = (f(a))_1$, $g_2(a) = (f(a))_2$.

ϕ is evidently a BIJECTION (proof!).

ii) given $h \in (A+B) \to C$, define

$\psi(h) = (k_1, k_2) \in (A \to C) \times (B \to C)$, WHERE

$k_1(a) = h(1,a)$, $k_2(b) = h(2,b)$.

Again, it is relatively easy to show that ψ is a BIJECTION.

If A, B, C are all FINITE, cardinalities a, b, c:

In i), $|lhs| = (bc)^a$, $|rhs| = b^a \cdot c^a$.

In ii), $|lhs| = c^{a+b}$, $|rhs| = c^a \cdot c^b$

OK!