Regular Languages and Finite Automata 2004
Paper 2 Question 9 (AD)

(a) Prove that if L is a regular language, its complement is also regular. [6 marks]

Bookwork.

Let $M = (Q, \Sigma, \delta, q_0, F)$ be a *deterministic* finite automaton accepting the language L.

Define the automaton $M' = (Q, \Sigma, \delta, q_0, Q \setminus F)$.

We can show that a string w is accepted by M' if, and only if, it is not accepted by M. Hence, the complement of L is regular.

(b) For each of the following languages over the alphabet $\{a, b\}^*$, state whether or not it is regular and justify your answer.

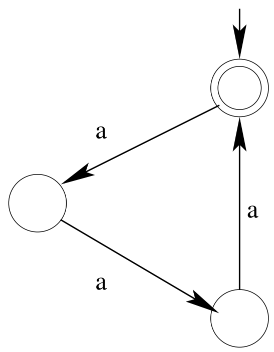
(i) $\{w \mid w \text{ is not a palindrome}\}$

This language is not regular. If it was, then its complement, the language of palindromes would also be regular, by part (a). However, we can show that the set of palindromes is not regular by an application of the pumping lemma.

Suppose $\{w \mid w \text{ is a palindrome}\}$ were regular. Let n be the parameter obtained from the pumping lemma. Consider the string $a^n b^{2n} a^n$, which is a palindrome. By the pumping lemma, there is an $i > 0$ such that $a^{n+i} b^{2n} a^n$ is also in the language but this string is clearly not a palindrome. [5 marks]

(ii) $\{a^k \mid k \text{ is a multiple of } 3\}$

This language is regular. We can construct a three state nondeterministic automaton to recognise it.



[4 marks]

(*iii*) $\{a^k \mid k \text{ is prime}\}$

The language is not regular. The proof is a direct application of the pumping lemma. Suppose the language were regular and let n be the parameter obtained from the pumping lemma. Let p be a prime number with $p > n$. Then, a^p is in the language. By the pumping lemma, there is an i such that a^{p+ik} is in the language for all $k \geq 0$. By choosing $k = p$, we find that the string $a^{p(i+1)}$ is in the language. But, clearly, $p(k + 1)$ is not a prime number. [5 marks]