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NAD

2001

Continuous maths papers 3 & 10 – Solution Notes

We know that: $\frac{d}{dx}\sin x = \cos x$

$\frac{d}{dx}\cos x = -\sin x$

$\frac{d}{dx}e^x = e^x$

and: $\sin 0 = 0$

$\cos 0 = 1$

$e^0 = 1$

Therefore:

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$

$e^{i\phi}$, by MacLaurin is:

$$e^{i\phi} = 1 + i\phi - \frac{\phi^2}{2!} - \frac{i\phi^3}{3!} + \frac{\phi^4}{4!} + \frac{i\phi^5}{5!} - \frac{\phi^6}{6!} - \frac{i\phi^7}{7!} + \dots$$

$$= \left(1 - \frac{\phi^2}{2!} + \frac{\phi^4}{4!} - \frac{\phi^6}{6!} + \dots\right) + i\left(\phi - \frac{\phi^3}{3!} + \frac{\phi^5}{5!} - \frac{\phi^7}{7!} + \dots\right)$$

$$= \cos\phi + i\sin\phi$$

$$\mathcal{B}(\nu) = \int_{-\infty}^{\infty} b(x)\, e^{-i2\pi\nu x}\, dx$$

$$= \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-i2\pi\nu x}\, dx$$

$$= \frac{1}{-i2\pi\nu} e^{-i2\pi\nu x} \Big|_{-\frac{1}{2}}^{\frac{1}{2}}$$

$$= \frac{1}{-i2\pi\nu}\left(e^{-i\pi\nu} - e^{i\pi\nu}\right)$$

$$B(\nu) = \frac{1}{-i2\pi\nu}\left(\cos\pi\nu - i\sin\pi\nu - \cos\pi\nu - i\sin\pi\nu\right)$$

$$= \frac{\sin\pi\nu}{\pi\nu}$$

If $t(x) = b(x) * b(x)$

then $T(\nu) = B(\nu) \times B(\nu)$

$$= \frac{\sin^2\pi\nu}{(\pi\nu)^2}$$