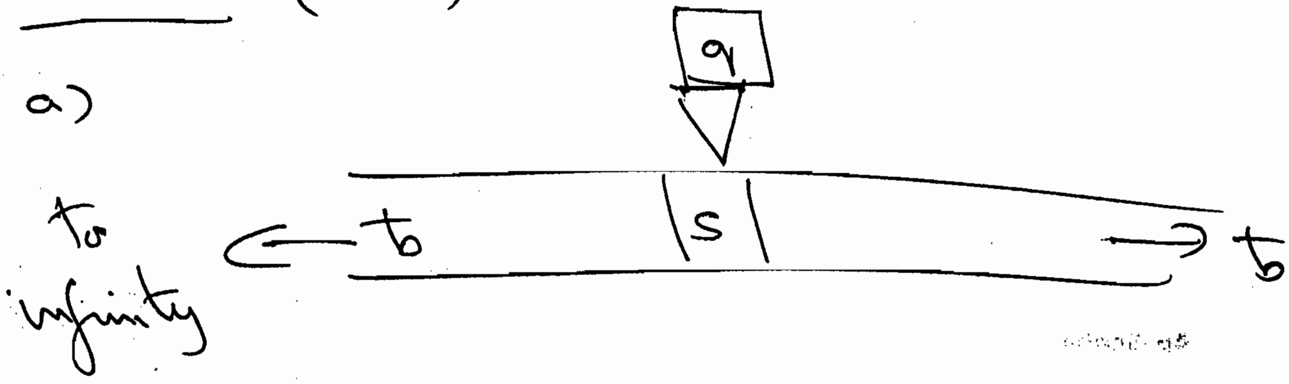
Solution (3, 10)

a)



A Turing machine is characterised by a finite state machine together with an infinite linear tape on which may be written the symbols of a finite alphabet S. Initially all but a finite number of the squares of the tape contain a special blank symbol. The machine is started in a specified initial state $q \in Q$, on a particular square of the tape. The action is deterministic and sequential; the process of computation depends solely on the current

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a) state $q \in Q$ and the symbol $s \in S$ on the current square: the machine has three deterministic response functions.

i) it enters a new state $Q(q,s)$;

ii) it overwrites the current symbol s with replacement symbol $R(q,s)$;

iii) it moves one square on the tape in direction $D(q,s)$ ($=$ L or R).

A possible value for the new state is the HALT state. The machine terminates, and the result of the computation is the final contents of the tape.

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b) A configuration of a Turing m/c records the current machine state $q \in Q$ together with a tape description. Typically we record the tape state by identifying the symbols $s \in S$ with digits $0, 1 \ldots (k-1)$ on scale k, ensuring that 0 represents b. We may now identify the tape state with a triple (s, m, n), where s is the current symbol value and integers m, n are the values of the left and right half-tapes in the natural representation in scale k.

c) To identify a Turing machine computation we must identify the quintuplet description

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and also the initial tape state. Assume given a pairing function $Z(x,y)$, say $[x,y]$. If we identify the states as say $0, 1, \ldots (n-1)$, we may let 0 be the initial state, 1 the unique HALTing state. Then each quintuplet may be written as (say) $[[q,s],[[q',s'],d]]$. The complete machine logic can be represented by a stack of quintuplets, coded by a single natural number e. The initial tape can be represented by $t = [s,[m,n]]$. Finally the pair code $c = [e,t]$ describes the complete computation.

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d) Suppose we could compute the maximum distance moved, say $l = l(c)$, as a function of the specification of the computation. The maximum number of possible configurations that the machine's tape could take is therefore bounded by k^{2l+1}, hence there is a total of at most $N . k^{2l+1} \times (2l+1)$ for the possible head positions! possible Turing machine configurations. Hence no halting computation could require more than $N . k^{2l+1}$ steps, since otherwise it must enter the same configuration twice, and therefore loop.

That would allow us to solve the HALTing problem by direct simulation.

hence $l(c)$ CANNOT be computable.