CST IB Semantics of Programming Languages
2002, Paper 5, question 9

(a)

1 (1) $$\frac{}{\langle s, n\rangle \Downarrow \langle s, n\rangle}$$

1 (2) $$\frac{x \in dom(s) \ \& \ s(x) = n}{\langle s, x\rangle \Downarrow \langle s, n\rangle}$$

1 (3) $$\frac{x \in dom(s) \ \& \ s(x) = n}{\langle s, x{+}{+}\rangle \Downarrow \langle s[x \mapsto n+1], n\rangle}$$

(where $s[x \mapsto n+1]$ maps x to $n+1$ & otherwise acts like s)

1 (4) $$\frac{x \in dom(s) \ \& \ s(x) = n}{\langle s, {+}{+}x\rangle \Downarrow \langle s[x \mapsto n+1], n+1\rangle}$$

1 (5) $$\frac{\begin{cases}\langle s, e_1\rangle \Downarrow \langle s', n_1\rangle \\ \langle s', e_2\rangle \Downarrow \langle s'', n_2\rangle \\ n = n_1 + n_2\end{cases}}{\langle s, e_1 + e_2\rangle \Downarrow \langle s'', n\rangle}$$

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(b)

1 (6) $$\frac{\langle s, e\rangle \Downarrow \langle s, n\rangle}{\langle s, x = e\rangle \Downarrow s[x \mapsto n]}$$

2 (7) $$\frac{\begin{cases}\langle s, e\rangle \Downarrow \langle s', n\rangle \\ x \in dom(s') \ \& \ s'(x) = n' \\ n'' = n' + n\end{cases}}{\langle s, x \mathrel{+}= e\rangle \Downarrow s'[x \mapsto n'']}$$

1 $$(8)\ \frac{\langle s, c_1\rangle \Downarrow s' \quad \langle s', c_2\rangle \Downarrow s''}{\langle s, c_1; c_2\rangle \Downarrow s''}$$

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(c)
Expressions e_1 and e_2 are semantically equivalent, written $e_1 \approx e_2$, if & only if for all states s, s' and all integers n

2 $$\langle s, e_1\rangle \Downarrow \langle s', n\rangle \text{ iff } \langle s, e_2\rangle \Downarrow \langle s', n\rangle$$

Commands c_1 and c_2 are semantically equivalent, written $c_1 \approx c_2$, if & only if for all states s, s'

3 1 $$\langle s, c_1\rangle \Downarrow s' \text{ iff } \langle s, c_2\rangle \Downarrow s'$$

(d)

1 (i) Yes, $++x \approx x{+}{+} + 1$.

1 <u>Proof</u>: if

$$(9)\qquad \langle s, ++x\rangle \Downarrow \langle s', n'\rangle$$

then this must have been proved by applying rule (4), so

$$x \in dom(s),\ s(x) = n \text{ say},$$
$$s' = s[x \mapsto n+1], \text{ and } n' = n+1.$$

Then by rule (3), $\langle s, x{+}{+}\rangle \Downarrow \langle s', n\rangle$, so by rules (1) and (5) we have

(10) $\langle s, x{+}{+} + 1\rangle \Downarrow \langle s', n'\rangle$.

Conversely if (10) holds, it must have been deduced by applying rule (5) to

(11) $\langle s, x{+}{+}\rangle \Downarrow \langle s_1, n_1\rangle$
(12) $\langle s_1, 1\rangle \Downarrow \langle s', n_2\rangle$
$\Big\}$ for some s_1, n_1, n_2

with $n_1 + n_2 = n'$. Now (11) (resp. (12)) must have been deduced from (3) (resp. (1)), so

$$\begin{aligned} &x \in dom(s) \\ &n_1 = s(x) \\ &s_1 = s[x \mapsto n_1 + 1] \\ &n_2 = 1 \;\&\; s' = s_1 \end{aligned}$$

and hence $n' = n_1 + n_2 = n_1 + 1$. Hence by rule (4), (9) holds.

② Thus (9) $\Leftrightarrow$ (10) for all s, s', n': hence $++x \approx x{+}{+} + 1$.

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1 (ii) No, $(x = ++x) \not\approx (x = x{+}{+})$.

For example

$\langle [x \mapsto 0], x = ++x\rangle \Downarrow [x \mapsto 1]$

whereas

② 1 $\langle [x \mapsto 0], x = x{+}{+}\rangle \Downarrow [x \mapsto 0]$.

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1 (iii) Yes, $(x = ++x) \approx (x += 1)$.

1 Proof: if

(13) $\langle s, x = ++x\rangle \Downarrow s'$

then this was deduced from (4) & then (6), so
$x \in \mathrm{dom}(s)$, $s(x) = n$ say, and $s' = s[x \mapsto n+1]$.
Hence by (1) & (7)

(14) $\quad \langle s, x += 1 \rangle \Downarrow s'$

Conversely, if (14) holds, it was deduced from (1) & (7) and then by (4) & (6), (13) holds.

2 Thus (13) $\Leftrightarrow$ (14) for all s, s': so $(x = ++x) \approx (x += 1)$.

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1 (iv) No, in general $(x += e) \not\approx (x = x + e)$.
For example, take $e = x{+}{+}$. Then

1 $\langle [x \mapsto 0], x += e \rangle \Downarrow [x \mapsto 1]$

whereas

2 $\langle [x \mapsto 0], x = x + e \rangle \Downarrow [x \mapsto 0]$.

To which parts of the lecture course does this question refer?

Parts (a) & (b) require a knowledge of lectures 3 & 4, but are not exactly the same as any example given there.

Part (c) is a definition from Lecture 5 (applied to this unfamiliar language)

Part (d) requires problem-solving ability based on experience gained from lectures 5 & 6.