Solution notes

Artificial Intelligence II 2005 – Paper 9 Question 8 (SBH)

This relates to the section of the course devoted to probabilistic reasoning through time.

(a) In a Markov process $\Pr(S_t|S_{0:t-1}) = \Pr(S_t|S_{t-1})$.

The transition model is $\Pr(S_t|S_{t-1})$ and the sensor model is $\Pr(E_t|S_t)$.

(b) Filtering means computing $\Pr(S_t|e_{1:t})$, prediction means computing $\Pr(S_{t+T}|e_{1:t}), T > 0$, smoothing means computing $\Pr(S_T|e_{1:t}), 0 \leq T < t$.

(c) Say you know $\Pr(S_{t-1}|e_{1:t-1})$. Then

$$\Pr(S_t|e_{1:t}) = \Pr(S_t|e_t, e_{1:t-1})$$

and by Bayes theorm

$$\Pr(S_t|e_t, e_{1:t-1}) = c\Pr(e_t|S_t, e_{1:t-1})\Pr(S_t|e_{1:t-1})$$

The first probability on the right hand side is just the sensor model. To obtain the second probability

$$\Pr(S_t|e_{1:t-1}) = \sum_{s_{t-1}} \Pr(S_t, s_{t-1}|e_{1:t-1}) = \sum_{s_{t-1}} \Pr(S_t|s_{t-1}, e_{1:t-1})\Pr(s_{t-1}|e_{1:t-1})$$

Here the first probability reduces to the transition model using the independence assumptions and the second probability is the result of the earlier filtering step.

(d) It has a single discrete state variable with values $s_1, \ldots, s_n$.

(e) Define the matrix $\mathbf{S}$ having elements $S_{i,j} = \Pr(S_{t+1} = s_j|S_t = s_i)$ and the diagonal matrix $\mathbf{E}_T$ with ith diagonal element $\Pr(e_T|S_T = s_i)$. Define $f_{1:t}$ as the column vector with ith element $\Pr(S_t = s_i|e_{1:t})$ which in this context represents the result of filtering at a given time. It is a straightforward exercise in matrix manipulation to show that

$$f_{1:t+1} = c\mathbf{E}_{t+1}\mathbf{S}^T f_{1:t}$$