

# Specification and Verification I 2001

## SV1.1: Solution Notes

What is the difference between partial and total correctness. [4 marks]

A partial correctness specification  $\{P\}C\{Q\}$  requires that the postcondition  $Q$  hold *only if* the execution of the command  $C$  terminates when started in a state satisfying  $P$ .

A total correctness specification  $[P]C[Q]$  requires that the execution of the command  $C$  terminates when started in a state satisfying  $P$ , and also that the postcondition  $Q$  holds in the final state.

Why is the assignment axiom more problematical for total correctness than for partial correctness. [4 marks]

An assignment  $X := E$  may not terminate if  $E$  contains errors (like division by zero) or non-terminating function calls. The assignment axiom for total correctness is only valid if assignments terminate, so this is a problematical assumption.

State the WHILE-rule for total correctness. [4 marks]

$$\frac{\vdash [P \wedge S \wedge (E = n)] C [P \wedge (E < n)], \quad \vdash P \wedge S \Rightarrow E \geq 0}{\vdash [P] \text{ WHILE } S \text{ DO } C [P \wedge \neg S]}$$

where  $n$  is a ghost (or auxiliary) variable

What needs to be added to the method of verification conditions to make it work for total correctness. [4 marks]

All WHILE-commands need to be annotated with variants, then the verification conditions need to be adjusted so that they are based on the rule for total correctness.

Explain how a total correctness specification  $[P]C[Q]$  can be embedded as a term  $\text{TotalSpec } p \ c \ q$  in higher order logic. [4 marks]

$[P]C[Q]$  embeds as  $\text{TotalSpec } [P] \ [C] \ [Q]$ , where  $[P]$ ,  $[C]$  and  $[Q]$  are the translations of  $P$ ,  $C$  and  $Q$  to predicates, respectively, and  $\text{TotalSpec}$  is defined by

$$\text{TotalSpec } p \ c \ q == \forall s. p \ s \Rightarrow \exists s'. c(s, s') \wedge q \ s'$$