A a) $(A \times B) = \{(a,b) \mid a \in A, b \in B\}$

where $(a_1, b_1) = (a_2, b_2)$ iff $a_1 = a_2$

and $b_1 = b_2$.

b) R is a relation between A and B

iff $R \subseteq (A \times B)$ is any subset. Hence the set of such relations is $\mathcal{P}(A \times B)$, the power set

c) $\Delta_A = \{(a,a) \mid a \in A\}$. IDENTITY

$S^{-1} = \{(b,a) \in (B \times A) \mid (a,b) \in S\}$, a relation between B and A.

$S \circ T = \{(a,c) \in (A \times C) \mid \exists\, b \in B$ s.t. $(a,b) \in S$ and $(b,c) \in T\}$,

a relation between A and C.

$\therefore (S \circ T)^{-1} = \{(c,a) \in (C \times A) \mid \exists\, b \in B$ s.t. $(a,b) \in S$ and $(b,c) \in T\}$

A ctd) $= \{ (c,a) \in (C \times A) \mid \exists\, b \in B \text{ s.t. } (c,b) \in T^{-1} \text{ and } (b,a) \in S^{-1} \}$

$= T^{-1} \circ S^{-1}$ as required

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i) $(f^{-1} \circ f) \subseteq \Delta_B$.

ii) $R \supseteq \Delta_A$ iii) $R^{-1} \subseteq R$ (or $R^{-1} = R$)

iv) $(R \circ R) \subseteq R$.

Let $Q = f \circ f^{-1}$. Then $Q^{-1} = (f \circ f^{-1})^{-1}$
$= (f^{-1})^{-1} \circ f^{-1}$
$= f \circ f^{-1}$.

$(Q \circ Q) = (f \circ f^{-1}) \circ (f \circ f^{-1})$

$= f \circ (f^{-1} \circ f) \circ f^{-1}$

by i) $\subseteq f \circ \Delta_B \circ f^{-1} = (f \circ f^{-1}) = Q$

A ctd) Hence Q is certainly symmetric and transitive (i.e. a PARTIAL equiv^{nce} rel^{n})

Q is reflexive iff f is TOTAL.

There's a lot of bookwork here, but the final part is almost certainly unfamiliar, and harder than I've made it look. The question is probably a bit less demanding than ones set recently, but that's a good thing.