RJG

2002

Computer Systems Modelling   Paper 8 q 14

[Relates to M/G/1 queue]

(a) M/G/1   [2 marks]

arrival rate λ inter-arrival distribution is exponential mean λ^{-1}

service time general,   mean service time μ^{-1}
coefficient of variation, C

For non-saturation of the server we require

traffic intensity, $\rho = \frac{\lambda}{\mu} < 1$

[4 marks]

(b) The Pollaczek–Khintchine formula is

$$\bar{N} = \rho + \frac{\rho^2(1+C^2)}{2(1-\rho)}$$

where $\bar{N}$ is the steady-state average number of customers present, ρ is the traffic intensity and C^2 is the squared coefficient of variation.

(c) Edward : exponential mean 0.9 min } [8 marks

$c^2 = 1 \quad \mu^{-1} = 0.9$ min

Ursula : uniform on 0.8 to 1.2 mins

mean $\mu^{-1} = 1.0$ 1.0 min

$$c^2 = \frac{\frac{1}{12}(1.2-0.8)^2}{(1.0)^2}$$ variance of uniform

$$= \frac{(0.4)^2}{12} = \frac{0.16}{12} = 0.01\dot{3}\ldots$$

Arrival rate 50 per hour

So $\lambda = 50$ per hour $= 50/60$ per min

(i)

Edward : traffic intensity, $\rho = \frac{\lambda}{\mu} = \frac{50}{60} \times 0.9 = 0.75$

So Edward copes as $\rho < 1$

Ursula : traffic intensity, $\rho = \frac{\lambda}{\mu} = \frac{50}{60} \times 1.0$

$= 0.8\dot{3}$

So Ursula copes as $\rho < 1$.

(ii) Edward :

$$\bar{N} = \rho + \frac{\rho^2(1+c^2)}{2(1-\rho)} = 0.75 + 2.25 = 3$$

Ursula :

$$\bar{N} = \rho + \frac{\rho^2(1+c^2)}{2(1-\rho)} = \frac{5}{6} + \frac{\left(\frac{5}{6}\right)^2\left(1 + \frac{0.16}{12}\right)}{2(1/6)}$$

$$= \frac{5}{6} + 2.11 = 2.94$$

So Ursula is prefered

(d) [3 marks]

Arrivals, exponential rate $\lambda = 50$ per hour

Edward's service times exponential, mean 0.9 min

Ursula's service times, uniform on [0.8, 1.2]

Exponential, parameter λ
so distribution function is

$$F(x) = 1 - e^{-\lambda x}$$

Let RHS be uniform [0, 1] ie $U_1, U_2, \ldots$

then
$$U_i = 1 - e^{-\lambda x_i}$$
$$1 - U_i = e^{-\lambda x_i}$$
$$\ln(1 - U_i) = -\lambda x_i$$
$$x_i = -\frac{1}{\lambda} \ln(1 - U_i)$$

(or just $x_i = -\frac{1}{\lambda} \ln U_i$)

are pseudo-random exponentials with parameter λ, covers case of arrivals and Edward's service times

For uniform (a, b) distribution function is

$$F(x) = (x-a)/(b-a)$$

So if RHS is Uniform $[0,1]$ ie $U_1, U_2, \ldots$

then $U_i = (x_i - a)/(b-a)$

So,

$$x_i = (b-a)\,U_i + a$$

$a = 0.8$, $b = 1.2$ $\quad (b-a) = 0.4$

So $x_i = 0.4\,U_i + 0.8$

are uniform on $[0.8, 1.2]$

[3 marks]

Analytic: Advantages:
quick, precise answers for given system

Disadvantages:
unclear how robust answers are to distributional assumptions

Simulation: Advantages:
very flexible, easy to replace assumptions

Disadvantages,

Slow to formulate and run
Produces estimates with statistical ~~[illegible]~~ error