2003 p8q14
GW

Den. Sem. Qu 1.

(1.) Soundness:

If $t \Downarrow v$, then $[\![t]\!] = [\![v]\!]$

~~2.~~ Adequacy:

If $[\![t]\!] = [\![v]\!]$ for t a closed term of ground type <u>bool</u> or <u>int</u> and v a value of the same ground type, then $t \Downarrow v$ according to the operational semantics.

3. Full abstraction:

Two terms are contextually equivalent iff they have the same denotation.

Terms t_1, t_2 of the same type are contextually equivalent iff for all ground contexts $\mathcal{C}[-]$ (i.e. context of type <u>bool</u> or <u>int</u>) $\mathcal{C}[t_1] \Downarrow v$ iff $\mathcal{C}[t_2] \Downarrow v$.

(2) Let $f : D \to D$ be a continuous fn. on a domain D. Let $Q(x)$, $x \in D$, be an admissible property. Then,

$$Q(\mathrm{fix}(f))$$

if $\forall x \in D.\ Q(x) \Rightarrow Q(f(x))$.

(3) We require

(i) $h(\perp) = \perp$

(ii) $S = \{(x,y) \in D \times E \mid h(x) = y\}$ chain closed.

(i) is clear as h is strict.

(ii) Suppose $(x_0, y_0) \sqsubseteq \cdots \sqsubseteq (x_n, y_n) \sqsubseteq \cdots$ and $(x_n, y_n) \in S$ all n. Then,

$$h\left(\bigsqcup_n x_n\right) = \bigsqcup_n h(x_n) \qquad (h \text{ cts})$$

$$= \bigsqcup_n y_n$$

$\therefore \left(\bigsqcup_n x_n, \bigsqcup_n y_n\right) \in S$. I.e. S is chain-closed.

(4) Use fixed pt. induction with inclusive property $Q(x,y) \Leftrightarrow_{def} h(x) = y$.

$$(fix(f), fix(g)) = fix(\varphi)$$

where $\varphi(x,y) = (f(x), g(y))$.

By f.p.i. it suffices to show

$$Q(x,y) \Rightarrow Q(\varphi(x,y))$$

ie $Q(x,y) \Rightarrow Q(f(x), g(y))$

for all $x \in D, y \in E$. But

$$h(x) = y \Rightarrow \cancel{g}\, h(x) = \cancel{g}(y)$$
$$\Rightarrow \cancel{gh(x)}\, h(f(x)) = g(y)$$

as $h \circ f = g \circ h$. $\boxtimes$