

**Optimising Compilation optcomp8.tex**

Explain the ideas of strictness analysis, including over what languages the ideas are applicable and what transformations are enabled by it. Describe how strictness functions for (a) built-in and (b) user-defined functions are defined, clarifying the similarities and differences. [10 marks]

A language has a user-defined function  $f$  which is defined in terms of built-in functions  $a_1, \dots, a_t$  and possibly recursion. Later, to aid efficiency, an additional function  $a_{t+1}$  is added to the set of system functions, but its effect (semantics) is the same as that of  $f$ . By considering examples similar to those used to show analyses are safe but imprecise, or otherwise, determine a relationship between the strictness functions  $f^\#$  and  $a_{t+1}^\#$ . [5 marks]

It is noted that strictness functions, e.g.

$$\text{cond}^\#(x, y, z) = x \wedge (y \vee z)$$

do not generally use negation in their defining boolean expressions. Show that all strictness functions can be written without negation or find a counter-example. Hint: no computable function  $f$  can have semantics such that there are  $x$  and  $y$  which satisfy

$$f(x, y) = \perp \text{ and } f(x, \perp) \neq \perp.$$

[5 marks]

**Solution Notes**

Part 1: bookwork. Letting  $2 = \{0, 1\}$  then the strictness space for a  $k$ -adic function is  $2^k \rightarrow 2$ .

$$\text{plus}^\#(x, y) = x \wedge y$$

$$\text{cond}^\#(p, x, y) = p \wedge (x \vee y)$$

$f$  is strict in  $i$ th arg if  $f^\#(1, \dots, 1, 0, 1, \dots, 1) = 0$ . If  $f$  is strict in  $i$ th argument it can be implemented using CBV.

Part 2:

$$f^\#(x_1, \dots, x_k) \leq a_{t+1}^\#(x_1, \dots, x_k)$$

Part 3: True since  $f$  is monotonic w.r.t  $\perp \leq x$  then  $f^\#$  is monotonic w.r.t  $0 \leq 1$ .