

SOLUTION NOTES

Foundations of Functional Programming 2002 Paper 5 Question 10 (ACN)

Syllabus Part B item 1, clause 2.

- (a) 3 rules: $[x]x = S\ K\ K$, $[x]c = K\ c$ (c free of x), $[x]f\ g = S\ [x]f\ [x]g$. Now map $\text{fn } x \Rightarrow A$ to $[x]A$. Can optimise $[x](c\ x) = c$ if c is free of x .
- (b) $I = S\ K\ K$ is a base case
- (c) $[x]f\ (g\ x) = S\ (K\ f)\ g$. Now do $[g]$ and get just $S\ (K\ f)$. Finally $[f]\ S\ (K\ f) = S\ (K\ S)\ K$.
- (d) $[y]f\ y\ x = S\ f\ (K\ x)$. Now do $[x]$. I will make my life easy and write this as $B\ (S\ f)\ K$. I could write $S\ (K\ S)\ K$ for B if I needed. Finally $[f]$ gives $S\ (B\ B\ S)\ (K\ K)$. Again can expand B to get proper result.
- (e) $[y]y\ ((x\ x)y) = S\ I\ (B\ (x\ x)\ I)$. Now try $[x]$ and get $B\ (S\ I)\ (C\ (B\ B\ (S\ I\ I))\ I)$. This is a big mess! Let me note that A comes from 1995p5Q10 and $(A\ A)$ is a fixed point operator. I allocate only 4 marks to this bit so that people can pick up good marks on the earlier easier sections and only the real experts can finish all the way.