2005 Paper 7 Question 10

Digital Signal Processing (MGK) – Solution Notes

MGK may have available a more up to date version of this answer.

(*a*) (*i*) linear, non-causal, time varying

(*ii*) linear, causal, time invariant

(*iii*) linear, non-causal, time invariant

(*iv*) non-linear, causal, time invariant

(*v*) linear, causal, time varying

(*b*) (*i*) $y_n - y_{n-1} = x_n - x_{n-3} \quad \Longrightarrow \quad y_n = y_{n-1} + x_n - x_{n-3}$

Input:

$$\ldots, x_{-1}, x_0, x_1, \ldots = \ldots, 0, 0, 0, 1, 0, 0, 0, 0, 0, \ldots$$

Output:

$$\ldots, y_{-1}, y_0, y_1, \ldots = \ldots, 0, 0, 0, 1, 1, 1, 0, 0, 0, \ldots$$

$\Longrightarrow$ impulse response = 1, 1, 1

(*ii*) From the impulse response of h, we can see that its defining equation can be rewritten as $y_n = x_n + x_{n-1} + x_{n-2}$. Any sequence with a period of three samples will be turned by h into a constant sequence and all y_n will equal the sum of the three samples from a single period of the input sequence. An example of a sine-wave sequence with a period of three samples is $x_n = \sin\left(\frac{1}{3}2\pi n\right)$, that is $\ldots, 0, \sqrt{3}/2, -\sqrt{3}/2, 0, \sqrt{3}/2, -\sqrt{3}/2, \ldots$

(*iii*)

$$H(z) = \frac{1 - z^{-3}}{1 - z^{-1}} = \frac{z^3(1 - z^{-3})}{z^3(1 - z^{-1})} = \frac{z^3 - 1}{z^2(z - 1)} = \frac{z^2 + z + 1}{z^2} = 1 + z^{-1} + z^{-2}$$

(*iv*) Any z for which

$$H(z) = \frac{z^3 - 1}{z^2(z - 1)} = 0$$

will have to fulfill $z^3 - 1 = 0$ and will therefore have to be one of the three cubic roots of 1: $e^{j\frac{0}{3}2\pi} = 1$, $e^{j\frac{1}{3}2\pi} = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$, and $e^{j\frac{2}{3}2\pi} = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$. The first of these is eliminated by the factor $(z - 1)$ in the denominator, leaving $H\left(e^{j\frac{1}{3}2\pi}\right) = 0$ and $H\left(e^{j\frac{2}{3}2\pi}\right) = 0$.
(Alternative approach: simply apply the quadratic formula to $z^2 + z + 1 = 0$.)

[These questions relate to the course sections on (a) types of discrete systems, (b)(i+ii) linear time-invariant systems, (b)(iii) polynomial representation of filters, and (b)(iv) zeros and poles.]