Reg. Langs & Finite Automata 2005 Paper 2 4-mark q10(AMP)

1 Since $\begin{cases} L(r(sr)^*) = \bigcup_{n \geq 0} L(r(sr)^n) \\ L((rs)^* r) = \bigcup_{n \geq 0} L((rs)^n r) \end{cases}$

it suffices to show $(\forall n \geq 0)\ L(r(sr)^n) = L((rs)^n r)$

1 and we can do this <u>by induction on n</u>:

1 Base case $n = 0$:

$L(r(sr)^0) = L(r\varepsilon) = L(r) = L(\varepsilon r) = L((rs)^0 r)$. ✓

1 Induction step:

Suppose $L(r(sr)^n) = L((rs)^n r)$; then

$$\begin{aligned} L(r(sr)^{n+1}) &= L(r(sr)^n(sr)) \\ &= L(r(sr)^n)\,L(sr) \\ &= L((rs)^n r)\,L(sr) \quad \text{by induction hypothesis} \\ &= L((rs)^n (rs) r) \\ &= L((rs)^{n+1} r). \quad ✓ \end{aligned}$$

(Less formal arguments are acceptable, if the $n=0$ case is treated convincingly.)

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