P8q10
P12q10
MROD

Numerical Analysis II

Acceptable Answer A

$$\sum_{i=1}^{n} (\xi_i - \xi_{i-1}) f(x_i)$$

is a Riemann sum over $[a, b]$ if $a = \xi_0 < \xi_1 < \dots < \xi_n = b$ such that $x_i \in [\xi_{i-1}, \xi_i]$. The mesh norm is $\Delta\xi = \max_i |\xi_i - \xi_{i-1}|$.

[4 marks]

If $a = \xi_0 = -1$, $b = \xi_n = +1$ the sum of the weights must be 2.

(i) $\xi_1 = -0.8$, $\xi_2 = 0$, $\xi_3 = +0.8$ $\Rightarrow$ Riemann sum

(ii) $\xi_1 = -0.2$, $\xi_2 = 0$, $\xi_3 = +0.2$ $\Rightarrow$ Riemann sum

(iii) $\xi_1 = -0.3$, $\xi_2 = 0$ but $x_2 = -0.4 \notin [-0.3, 0]$
so not a Riemann sum

(iv) weights do not add up to 2
so not a Riemann sum

(v) $\xi_1 = -0.7$, $\xi_2 = +0.3$ $\Rightarrow$ Riemann sum

[5 marks]

$$(n \times R) f = \frac{b-a}{2n} \sum_{i=1}^{n} \sum_{j=1}^{m} w_j f(x_{ij})$$

where x_{ij} is the jth abscissa of the ith subinterval. Reverse summations, then take limits

$$\lim_{n\to\infty} (n \times R) f = \frac{1}{2} \sum_{j=1}^{m} w_j \lim_{n\to\infty} \left\{ \frac{b-a}{n} \sum_{i=1}^{n} f(x_{ij}) \right\}$$

$$= \frac{1}{2} \int_a^b f(x)\,dx \sum_{j=1}^{m} w_j$$

since the sum over i is a Riemann sum and therefore converges. Since $\sum_{j=1}^{m} w_j = R.1 = 2$ the proof is complete.

[6 marks]

PTO.

All rules except (iv) converge in composite form since the only requirement is that constants are integrated exactly.

[2 marks]

The rule

$$-0.5\, f(-1) + 1.5\, f(-0.4) + 1.5\, f(+0.4) - 0.5\, f(+1)$$

integrates constants exactly, so it does converge in composite form. However it is not a Riemann sum rule, and the use of negative weights makes loss of significance likely especially when used in composite form. [3 marks]