Denotation Semantics Qu 1

P8q15
GW

Computer Science Tripos Part II 2005

Paper 8 Question 15

GW — Denotational Semantics

(a) Bookwork from Den. Sem. notes P.29

(b) $$h(\mathit{if}(b,d,d')) = \begin{cases} h(d) & \text{if } b=\mathit{true} \\ h(d') & \text{if } b=\mathit{false} \\ h(\bot) & \text{if } b=\bot \end{cases}$$

$$= \begin{cases} h(d) & \text{if } b=\mathit{true} \\ h(d') & \text{if } b=\mathit{false} \\ \bot & \text{if } b=\bot \end{cases} \quad \text{as } h \text{ is strict}$$

$$= \mathit{if}(b, h(d), h(d'))$$

(c) ~~Qtg~~ $Q(\bot)$ as:

lhs $h(\bot(x,y)) = h(\bot) = \bot$ as h is strict

rhs $\bot(x, h(y)) = \bot$

Suppose $g_0 \sqsubseteq \cdots \sqsubseteq g_n \sqsubseteq \cdots$ is a chain

s.t. $Q(g_n)$ all n., i.e.

$$h(g_n(x,y)) = g_n(x, h(y)) \text{ all } x,y$$

Then,

$$h((\bigsqcup_n g_n)(x,y)) = h(\bigsqcup_n g_n(x,y)) \quad \text{lubs of fns. got pointwise}$$

$$= \bigsqcup_n h(g_n(x,y)) \quad \text{by cty of } h$$

$$= \bigsqcup_n g_n(x, h(y))$$ by hypothesis

$$= \Big(\bigsqcup_n g_n\Big)(x, h(y))$$ as lubs of fns are got pointwise.

(d) Its required to show that

$Q(g) \Rightarrow Q(\varphi(g))$ where

$$\varphi(g) =_{def} \lambda x, y.\ \mathit{if}\,(p(x), y, h(g(k(x), y))).$$

Assume $Q(g)$. Let $x, y \in D$.

$$h(\varphi(g)(x,y)) = h(\mathit{if}\,(p(x), y, h\,g(k(x), y)))$$

$$= \mathit{if}\,(p(x), h(y), h\,h\,g(k(x), y))$$ by (b)

$$= \mathit{if}\,(p(x), h(y), h(g(k(x), h(y))))$$ by induction hypothesis

$$= \varphi(g)(x, h(y))$$

Thus $Q(\varphi(g))$. $\boxtimes$

The Den. Sem. notes are available from

www.cl.cam.ac.uk/~gw104