Reg. Lang. & FA 2005 - Paper 2 Question 9 (AMP)

(a) Specification of M :

1 - alphabet of input symbols : $\Sigma_1 \cap \Sigma_2$

1 - set of states : $States_{M_1} \times States_{M_2}$

1 - start state : (s_{M_1}, s_{M_2})

1 - accepting states : $\{(q,q') \mid q \in Accept_{M_1}$ & $q' \in Accept_{M_2}\}$

1 - transitions : $\Delta_M((q,q'),a) = \{(q_1, q_1') \mid q_1 \in \Delta_{M_1}(q,a)$ & $q_1' \in \Delta_{M_2}(q',a)\}$

1 Note that if M_1 & M_2 are deterministic ($\forall a, q \in States_{M_i}\ \exists!\, q'\ \Delta_{M_i}(q,a) = \{q'\}$) then so is M.

Thus $a_0 \ldots a_{n-1} \in L(M)$

iff $(s_{M_1}, s_{M_2}) \xrightarrow{a_0} (q_1, q_1') \rightarrow \cdots \xrightarrow{a_{n-1}} (q_n, q_n') \in Accept_M$ for some $q_1, q_1', \ldots, q_n, q_n'$

iff $(s_{M_1} \xrightarrow{a_0} q_1 \rightarrow \cdots \xrightarrow{a_{n-1}} q_n \in Accept_{M_1}$ for some $q_1, \ldots, q_n)$ & $(s_{M_2} \xrightarrow{a_0} q_1' \rightarrow \cdots \xrightarrow{a_{n-1}} q_n' \in Accept_{M_2}$ for some $q_1', \ldots, q_n')$

3 iff $a_0 \ldots a_{n-1} \in L(M_1)$ & $a_0 \ldots a_{n-1} \in L(M_2)$

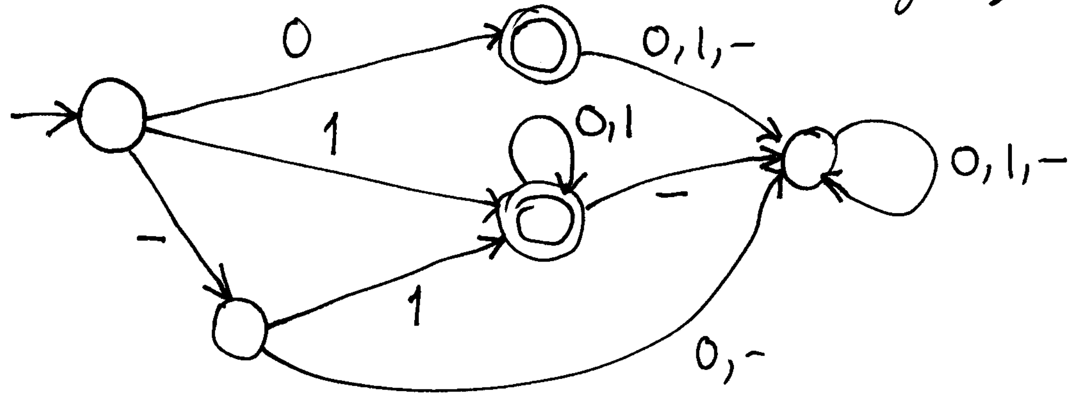
Thus $L(M) = L(M_1) \cap L(M_2) = L_1 \cap L_2$.

1 Note that this holds whether or not M_1 & M_2 are deterministic.

(10)

(b)

(i) P generates all strings in $1\{0,1\}^*$
so N generates all strings in $\{0\} \cup 1\{0,1\}^* \cup -1\{0,1\}^*$
A DFA that accepts this language:



(4)

(ii) Regular expression(s) determining
$\{0\} \cup 1\{0,1\}^* \cup -1\{0,1\}^*$:

$$0 \mid 1(0 \mid 1)^* \mid -1(0 \mid 1)^*$$

(1) (also $0 \mid (\varepsilon \mid -)1(0 \mid 1)^*$, etc.)

(iii) A CFG is regular if it is either left or right linear. Right linearity means every production is either of the form

$x \to uy$ (x, y non-terminals, u a string of terminals)

2 or $x \to u$ (x non-terminal, u a string of terminals)

(Left linearity is defined symmetrically.)

1 The languages generated by regular CFGs is equal to the collection of regular languages (i.e. those accepted by some deterministic finite automaton).

2 The CFG in this question is evidently neither left linear (because of the production $N \to -P$) nor right linear (because of the production $P \to PO$), so is not regular.

5