SOLUTION NOTES

Complexity Theory 2002 Paper 5 Question 12 (AD)

(*a*) A is *polynomial-time reducible* to B if there is a function f computable in polynomial time such that $f(x) \in B$ if, and only if, $x \in A$.

(*b*) (*i*) True. To determine that a graph G is in this class, we need to establish that for every pair u, v of vertices, v is reachable from u. There are fewer than n^2 such pairs, and the reachability problem is solvable in time n^2.

(*ii*) Unknown. The problem of determining whether a graph has a Hamiltonian cycle has been shown to be NP-complete. Thus, it is polynomial time decidable if, and only if, P=NP, which is a well-known open question.

(*iii*) Unknown. Since the class P is closed under complementation, if this problem were polynomial time decidable, its complement would also be. However, its complement is the problem of deciding whether a graph is 3-colourable, which is known to be NP-complete. Hence, this problem is decidable in polynomial time if, and only if, P=NP.

(*iv*) True. **Hamilton** is NP-complete, which means that every problem in NP is polynomial time reducible to it. **Connect** is in P and therefore also in NP. Therefore, **Connect** is reducible to **Hamilton**.

(*v*) Unknown. Since **Hamilton** is NP-complete, if it were reducible to **non-3-colour**, every problem in NP would also be reducible to **non-3-colour**, by composition of reductions. However, since **non-3-colour** is in co-NP, this can only happen if NP=co-NP, which is an open problem.

(*vi*) Unknown. **non-3-colour**, being the complement of an NP-complete problem, is complete for the complexity class co-NP. If it were reducible to a problem in P, such as **Connect**, then every problem in co-NP would be in P. Since P is closed under complementation, this is equivalent to the statement that P=NP.