2004 11

Semantics of Programming Languages – Paper 5 Question ~~2?~~ (PMS)

Solution Notes (a) and (b) are bookwork, depending mostly on Section 2 and Section 5.2 of the Notes, though internal consistency should make it possible to work many of the details out on the fly rather than remember them. Some minor variants in the semantics would be acceptable. (c) could cover several alternatives (one wouldn't expect all of these): an assignment that returns the value assigned instead of a unit; assignment with right-to-left instead of left-to-right evaluation order; a restriction of what can be stored to base types; store locations that are default-initialized to some particular values – though here one would have to say what those values are for all types, and this is not a good design; storing expressions rather than values – mismatches the CBV semantics.

(*a*)

$$v \;::=\; n \mid \mathbf{skip} \mid \mathbf{fn}\; x{:}T \Rightarrow e \mid \ell$$

(*b*) Type environments Γ are finite partial functions from variables to types and from locations to types of the form T **ref**; stores s are finite partial functions from locations to values.

$$\text{(ref)}\quad \frac{\Gamma \vdash e{:}T}{\Gamma \vdash \mathsf{ref}\, e : T\ \mathsf{ref}}$$

$$\text{(assign)}\quad \frac{\Gamma \vdash e_1{:}T\ \mathsf{ref} \qquad \Gamma \vdash e_2{:}T}{\Gamma \vdash e_1 := e_2{:}\mathsf{unit}}$$

$$\text{(deref)}\quad \frac{\Gamma \vdash e{:}T\ \mathsf{ref}}{\Gamma \vdash\, !e{:}T}$$

$$\text{(loc)}\quad \frac{\Gamma(\ell) = T\ \mathsf{ref}}{\Gamma \vdash \ell{:}T\ \mathsf{ref}}$$

$$\text{(ref1)}\quad \langle\, \mathsf{ref}\, v, s\rangle \longrightarrow \langle \ell, s + \{\ell \mapsto v\}\rangle \quad \ell \notin \mathrm{dom}(s)$$

$$\text{(ref2)}\quad \frac{\langle e, s\rangle \longrightarrow \langle e', s'\rangle}{\langle\, \mathsf{ref}\, e, s\rangle \longrightarrow \langle\, \mathsf{ref}\, e', s'\rangle}$$

(deref1) $\langle !\ell, s\rangle \longrightarrow \langle v, s\rangle \quad$ if $\ell \in \mathrm{dom}(s)$ and $s(\ell) = v$

(deref2) $$\frac{\langle e, s\rangle \longrightarrow \langle e', s'\rangle}{\langle !e, s\rangle \longrightarrow \langle !e', s'\rangle}$$

(assign1) $\langle \ell := v, s\rangle \longrightarrow \langle \mathbf{skip}, s + \{\ell \mapsto v\}\rangle \quad$ if $\ell \in \mathrm{dom}(s)$

(assign2) $$\frac{\langle e, s\rangle \longrightarrow \langle e', s'\rangle}{\langle \ell := e, s\rangle \longrightarrow \langle \ell := e', s'\rangle}$$

(assign3) $$\frac{\langle e, s\rangle \longrightarrow \langle e', s'\rangle}{\langle e := e_2, s\rangle \longrightarrow \langle e' := e_2, s'\rangle}$$