SOLUTION NOTES

Complexity Theory 2001 Paper 6 Question 12 (AD)

(b) Suppose M is a non-deterministic machine recognising a language L in time $f(n)$. The computation of M can be simulated by a deterministic machine in space $f(n)$. The simulating machine M' has a tape of length $f(n)$, each cell of which holds a number between 1 and d, where d is the maximum number of branches from any state in M. This tape encodes the possible non-deterministic choices that M might take. For each value on this tape, a simulation of M is run. Since this take $f(n)$ time, it requires no more than $f(n)$ space. Thus, the total amount of space used is $2f(n)$, which is $O(f(n))$.

[4 marks]

(b) Suppose M is a machine accepting a language L using work space $f(n)$. If q is the number of states of the machine M, and d is the size of the tape alphabet, the number of distinct configurations of the machine is bounded by $qnd^{f(n)}$. For some constant e, this is the same as $e^{\log n+f(n)}$. The machine M can now be simulated by a deterministic machine that constructs the configuration graph of M, that is the directed graph which contains these configurations as nodes, and has an edge from a to b if the machine can reach configuration b from a in one step. The simulating machine now solves the graph reachability problem for this machine. Since graph reachability can be done in quadratic time, this gives an upper bound of $e^{2(\log n+f(n))}$. By setting the constant $c = e^2$, this is seen to be $c^{(\log n+f(n))}$.

[6 marks]

(b) NL $\subseteq$ P, by the second simulation. This is because if $f(n) = O(\log n)$, then $c^{(\log n+f(n))}$ is bounded by a polynomial in n.

co-NL $\subseteq$ P by a similar argument, combined with the fact that P is closed under complementation.

NP $\subseteq$ PSPACE by the first simulation, since it shows that a polynomial bound on the time of a nondeterministic machine translates into a polynomial bound on the space of a deterministic machine. [4 marks]

(d) If reachability is in co-NL, then we can also establish that NL $\subseteq$ co-NL and therefore also co-NL $\subseteq$ NL. This is because we can reduce any problem in NL to the reachability problem by an argument similar to the second simulation above. The reduction can be verified to be in logarithmic space. So, combining the reduction with the fact that reachability is in co-NL, we have that every problem in NL is in co-NL. For the second inclusion, note that since the complement of every language in NL is in co-NL and vice-versa, the inclusion relations between the two classes are symmetric.

[6 marks]