p6q9 AMRl.

CST IB 2000. Paper 6, q9
Semantics of Programming Languages

Two configurations are bisimilar iff they are related by some bisimulation, R — which is a relation $R \subseteq Config \times Config$ satisfying that whenever $c_1 R c_2$ then

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- $c_1 \xrightarrow{\alpha} c_1' \Rightarrow \exists c_2'.\ c_2 \xrightarrow{\hat{\alpha}} c_2' \ \&\ c_1' R c_2'$

1

- $c_2 \xrightarrow{\alpha} c_2' \Rightarrow \exists c_1'.\ c_1 \xrightarrow{\hat{\alpha}} c_1' \ \&\ c_1' R c_2'$

1 where $\xrightarrow{\hat{\alpha}} \triangleq \begin{cases} \xrightarrow{\tau^*} & \text{if } \alpha = \tau \\ \xrightarrow{\tau^*}\xrightarrow{\alpha}\xrightarrow{\tau^*} & \text{if } \alpha \neq \tau. \end{cases}$

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LTS for communicating processes:

configurations: process expressions with no free integer variables

1 actions: $\alpha ::= \tau \mid c(n) \mid \bar{c}\langle n\rangle$ $\left(\begin{matrix} c \in Chan, \\ n \in \mathbb{Z} \end{matrix}\right)$.

transitions: inductively defined by the following axioms & rules.

1 $(\xrightarrow{in})$ $c(x).P \xrightarrow{c(n)} P[n/x]$

1 $(\xrightarrow{out})$ $\bar{c}\langle E\rangle.P \xrightarrow{\bar{c}\langle n\rangle} P$ if $E \Downarrow n$

1 $(\xrightarrow{+})$
$$\frac{P_i \xrightarrow{\alpha} P_i'}{P_1 + P_2 \xrightarrow{\alpha} P_i'} \qquad (i = 1, 2)$$

1 $(\xrightarrow{par})$
$$\frac{P_1 \xrightarrow{\alpha} P_1'}{P_1 \mid P_2 \xrightarrow{\alpha} P_1' \mid P_2} \quad \text{\& symmetrically}$$

1 $(\xrightarrow{com})$
$$\frac{P_1 \xrightarrow{c(n)} P_1' \qquad P_2 \xrightarrow{\bar{c}\langle n\rangle} P_2'}{P_1 \mid P_2 \xrightarrow{\tau} P_1' \mid P_2'} \quad \text{\& symmetrically}$$

1 $(\xrightarrow{\nu})$
$$\frac{P \xrightarrow{\alpha} P'}{\nu c.P \xrightarrow{\alpha} \nu c.P'} \quad \text{if } \alpha \neq c(n), \bar{c}\langle n\rangle \ (\text{any } n).$$

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(a) <u>Not bisimilar</u>

Write
$$\begin{cases} P \triangleq \bar{c}\langle 1\rangle.(\bar{c}\langle 2\rangle.0 + \bar{c}\langle 3\rangle.0) \\ Q \triangleq (\bar{c}\langle 1\rangle.\bar{c}\langle 2\rangle.0) + (\bar{c}\langle 1\rangle.\bar{c}\langle 3\rangle.0) \end{cases}$$

First note that from the definition of $\rightarrow$, the only possible actions of P, Q and their descendants are:

2

$$\begin{tikzcd}[column sep=small]
 & P \arrow[d, "\bar{c}\langle 1\rangle"] & \\
 & \bar{c}\langle 2\rangle.0 + \bar{c}\langle 3\rangle.0 \arrow[dl, "\bar{c}\langle 2\rangle"'] \arrow[dr, "\bar{c}\langle 3\rangle"] & \\
0 & & 0
\end{tikzcd}
\qquad
\begin{tikzcd}[column sep=small]
 & Q \arrow[dl, "\bar{c}\langle 1\rangle"'] \arrow[dr, "\bar{c}\langle 1\rangle"] & \\
\bar{c}\langle 2\rangle.0 \arrow[d, "\bar{c}\langle 2\rangle"'] & & \bar{c}\langle 3\rangle.0 \arrow[d, "\bar{c}\langle 3\rangle"] \\
0 & & 0
\end{tikzcd}$$

If we had $P \mathrel{R} Q$ for some bisimulation, since $P \xrightarrow{\bar{c}\langle 1\rangle} \bar{c}\langle 2\rangle.0 + \bar{c}\langle 3\rangle.0$, must have

either $(\bar{c}\langle 2\rangle.0 + \bar{c}\langle 3\rangle.0) \,\mathcal{R}\, \bar{c}\langle 2\rangle 0$
or $(\bar{c}\langle 2\rangle.0 + \bar{c}\langle 3\rangle.0) \,\mathcal{R}\, \bar{c}\langle 3\rangle.0$
In either case the LHS can do an action ($\bar{c}\langle 3\rangle$ or $\bar{c}\langle 2\rangle$ respectively) that cannot be matched by the RHS, contradicting the
2 bisimulation property of $\mathcal{R}$. So no such $\mathcal{R}$
4 exists, so P & Q are not bisimilar.

(b) Are bisimilar.
Consider
2 $$\mathcal{R} \stackrel{\text{def}}{=} \{ (P, \nu c.(c(x).0 \mid \bar{c}\langle 1\rangle.P)) \mid c \text{ does not occur in } P \}$$
$$\cup$$
$$\{ (P', \nu c.(0 \mid P')) \mid c \text{ does not occur in } P' \}$$
Suffices to show $\mathcal{R}$ a bisimulation.
This follows from easily verified facts that:

4 (1) only transition of $\nu c.(c(x).0 \mid \bar{c}\langle 1\rangle.P)$ is
• $\xrightarrow{\tau} \nu c.(0 \mid P)$.

(2) if $P \xrightarrow{\alpha} P'$, ~~[illegible]~~ and c does not occur in P, then $\alpha \neq c(n), \bar{c}\langle n\rangle$ and hence
$\nu c.(0 \mid P) \xrightarrow{\alpha} \nu c.(0 \mid P')$.

(3) if $\nu c.(0 \mid P) \xrightarrow{\alpha} Q$ with c not in P, then $\alpha \neq c(n), \bar{c}\langle n\rangle$, $Q = \nu c.(0 \mid P')$ for some P'
6 with $P \xrightarrow{\alpha} P'$.