p7q10

MJCG

1999

Specification & Verification 2: Solution Notes to Question 1

With explicit clock edges (event timescale):

```
 Sum(clk,a,r)  =
   r(0) = 0 /\ !t. ~r(t+1) = ((~(clk t) /\ clk(t+1)) => a(t)+r(t) | r(t))
```

At the cycle timescale:

```
 AbsSum(a,r)  =  (r 0 = 0) /\ !n. ~r(n+1) = a(n)+r(n)
```

The expected relationship is that if the event timescale is temporally abstracted at rising clock edges, and the clock is live (infinitely many edges) then the result of the abstraction is the cycle model, i.e. something like:

```
   Inf(clk) /\ Sum(clk,a,r)
   ==>
   AbsSum(a when (posedge clk), r when (posedge clk))
```

where:

```
    Inf clk            = "clk is never eventually constant"
    posedge clk 0      = F
    posedge clk (t+1)  = ~(clk t) /\ clk(t+1)
    (s when p)(n)      = "value of s when p is true for the nth time"
```

There are subtleties in such temporal abstraction (e.g. whether one samples at the beginning or edge of the event). If the candidate gave full definitions and a proof of an abstraction relationship (unlikely) then I'd certainly give full marks. However, full marks can also be got from an intelligent discussion showing a good grasp of the issues, even if technical details are not perfect.

Finally, the question asks one to formalise and prove an easy property.

Good marks will be given if the candidate does something sensible.
There is no particular solution in mind

For example, to prove that if:

```
 AbsSum(a,r)  =  (r 0 = 0) /\ !n. ~r(n+1) = a(n)+r(n)
```

then:

```
 AbsSum(a,r) ==> !n. r(n) = Sum(a,n)
```

where:

```
   Sum(a,0) = 0    /\    Sum(a,n+1) = a(n) + Sum(a,n)
```

Assume AbsSum(a,r) and prove !n. r(n) = Sum(a,n) by induction on n.

Basis. n=0:

```
AbsSum(a,r) = (r 0 = 0) /\ !n. ~r(n+1) = a(n)+r(n)
            ==> r(0) = 0 = Sum(a,0)
```

Step. Assume r(n) = Sum(a,n):

```
AbsSum(a,r) ==> (r(n+1) = a(n)+r(n))
            =   (r(n+1) = a(n)+Sum(a,n))
            =   (r(n+1) = a(n)+Sum(a,n))
            =   (r(n+1) = Sum(a,n+1))
```

This proof is pretty trivial, so I'll expect it to be done beautifully
for full marks!