p64 1
MR

4 Data structures and algorithms 2004

(a) Describe and efficient algorithm to determine whether two finite line segments in a plane intersect. You may assume the end points of each line are given as x-y coordinates. [8 marks]

(b) Describe in detail an efficient algorithm to find the convex hull of as set of points lying on a plane. Show that the complexity of the Graham scan used in the algorithm is $O(n)$ and that the algorithm as a whole has complexity $O(n \log n)$. [8 marks]

(c) Describe and discuss how if is possible to eliminate many of the points before the convex hull algorithm is entered. [4 marks]

ANSWER NOTES:

(a) Bookwork

Lines are (x1,y1)-(x2,y2) and (X1,Y1)-(X2,Y2)

(X1,Y1) and (X2,Y2) are on different sides of (x1,y1)-(x2,y2)
((x2-x1)*(X1-x1) - (Y1-y1)*(y2-y1)) *
((x2-x1)*(X2-x1) - (Y2-y1)*(y2-y1)) < 0

plus similar expression for
(x1,y1) and (x2,y2) are on different sides of (X1,Y1)-(X2,Y2)

(b) Bookwork.

Find lowest leftmost point. Sort other points by increasing angle.
Cost O(n log n) Use abs(dy)/(abs(dx)+abs(dy)) trick for angles. Use
correction for dx<0. Don't need to correct for dx>0, dy<0 since dy>=0.
Apply Graham scan, cost O(n). Total cost O(n log n)

(c) First remove all points inside rectangle consisting of point with
max and min x and max and min y. Plus other directions possibly eg max
x+y. Can be done in O(n) time. May remove a lot of points. Of course,
no improvement if all points are on the convex hull.