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JGD

Model Answer: Information Theory and Coding, Question 1. 2001

1.
The upper bound on the joint entropy $H(X_1, X_2, ..., X_n)$ of all the random variables is:

$$H(X_1, X_2, ..., X_n) \leq \sum_{i=1}^{n} H(X_i)$$

[2 marks]

This upper bound is reached only in the case that all the random variables are independent. [2 marks]

The lower bound on the joint entropy $H(X_1, X_2, ..., X_n)$ is the smallest of their individual entropies:

$$H(X_1, X_2, ..., X_n) \geq H(X_j)$$

[2 marks]

This lower bound is reached if all the random variables are some deterministic function or mapping of each other, for example if they are all identical, because then after one of them is known, there is no uncertainty about any of the other variables. [2 marks]

2.
The Kolmogorov algorithmic complexity K of a string of data is defined as the length of the shortest binary program that can generate the string. Thus the data's Kolmogorov complexity is its "Minimal Description Length."

[2 marks]

The expected relationship between the Kolmogorov complexity K of a set of data, and its Shannon entropy H, is that approximately $K \approx H$. [2 marks]

Because fractals can be generated by extremely short programs, namely iterations of a mapping, such patterns have Kolmogorov complexity of nearly $K \approx 0$. [2 marks]

3.
The information capacity C of any tiny portion $\Delta\omega$ of this noisy channel's total frequency band, near frequency ω where the signal-to-noise ratio happens to be $SNR(\omega)$, is:

$$C = \Delta\omega \log_2(1 + SNR(\omega))$$

in bits/second. Integrating over all of these small $\Delta\omega$ bands in the available range from ω_1 to ω_2, the total capacity in bits/second of this variable-SNR channel is therefore:

$$C = \int_{\omega 1}^{\omega 2} \log_2(1 + SNR(\omega)) d\omega$$

[6 marks]