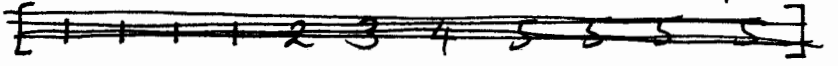
Advanced Graphics 2004 p7q4

(a) (i) $n+1 = 6$, $k = 4$, so 10 equispaced knots

eg. [1 2 3 4 5 6 7 8 9 10]

(ii) $n+1 = 6$, $k = 4$, but now open-uniform, so k repeated knots at each end



[1 1 1 1 2 3 4 4 4 4]

(iii) $n+1 = 6$, $k = 4$, so 10 knots, uniform except that it needs repeated knots to pass through point 3

[1 2 3 4 4 4 5 6 7 8]

(knot 4 is repeated twice, so appears 3 times)

(iv) [1 1 1 1 2 2 2 2]

(b) (i) between the knots B-splines have continuity of all derivatives so C∞ (infinity)

(ii) $k=4 \Rightarrow C_{k-2} = C2$ (second derivative)

(iii) $k=4 \Rightarrow C_{k-2} = C2$ but knot repeats twice so $\Rightarrow C\emptyset$

(c) (i) Butterfly, because new vertex vertices are in the same place as old ones

(ii) Loop - vertex - needs to be modified because there will be a different number of old vertices around it (other than 6), so cannot just use these values (they'll no longer add to 1)

(c) (ii) (cont.)

Loop-edge – does NOT need to be modified because every edge has two triangles and we are only looking at the vertices of those triangles so do not care whether they are extraordinary

Butterfly-vertex – does NOT need to be modified as the new vertex is identical to the old one regardless of whether or not it is extraordinary

Butterfly-edge – does NOT need to be modified because the configuration shown always exists regardless of the valency of the vertices (some people may notice that it's a bit wierd if one of the "8" vertices has valency 3 or 4 – but it is still OK even tho).

(iii) Anything sensible is acceptable for the Loop-vertex case

Two examples: $\frac{10}{10+n}$ in centre $\frac{1}{10+n}$ elsewhere

$\frac{10}{16}$ in centre $\frac{6/n}{16}$ elsewhere

Need a big number in the middle & a small number everywhere else. n is the valency of the extraordinary vertex. The numbers must sum to 1, so if α is the value at the centre and β the value elsewhere, you need: $\alpha + n\beta = 1$

~~(iv) You need to check rules to see if they obey and it gets~~

If people have said that other rules need to be modified, check that they obey the "sums to 1" rule.

2004 p7 q4 Marking Scheme

(a) (i) ten equispaced numbers — 2

(ii) four equal numbers at each end — 1
middle four numbers equispaced — 1

(iii) three equal numbers ~~in positions 4, 5 & 6~~ — 1
- in the fourth, fifth and sixth positions — 1
- numbers 1–4 equispaced and numbers 6–10 equispaced — 1

(iv) eight numbers, first four equal, last four equal — 1

8

(b) (i) ∞ infinity — 1
(ii) 2 second — 1
(iii) $\emptyset$ zero — 1

3

(c) (i) Butterfly — 1

1

(ii) Loop vertex – YES — 1
– Because it no longer matches the template or the numbers don't sum to 1 — 1

Loop edge – NO,* — 1

Butterfly vertex – NO, because it interpolated — 1

Butterfly edge – NO* — 1

* Plus one mark for explaining why the edge rules are "no" (if they've said "yes" to one edge and "no" to the other, they can still get this starred mark for a good explanation of why the "no" is a "no") — 1

6

(iii) A reasonable modification — 1

Whose co-efficients sum to 1 — 1

2

20