CST IB 2001, paper 6, question 9
Semantics of Programming Languages AMP

1 (a) Let $\to^*$ denote the reflexive-transitive closure of a relation $\to$. Then for all LC configurations $\langle P, s\rangle$ (P a phrase & s a state) and all terminal configurations

1 $\langle V, s'\rangle$ (so that $V ::= n \mid true \mid false \mid skip$), it is the case that

$$\langle P, s\rangle \Downarrow \langle V, s'\rangle$$

1 holds iff

$$\langle P, s\rangle \to^* \langle V, s'\rangle$$

iff

1 $$\langle P\cdot nil, nil, s\rangle \to^* \begin{cases} \langle nil, V\cdot nil, s'\rangle & \text{if } P \text{ is an integer or boolean phrase} \\ \langle nil, nil, s'\rangle & \text{if } P \text{ is a command} \end{cases}$$

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(b) LC phrases P_1 & P_2 are <u>semantically equivalent</u>, written $P_1 \cong P_2$, if & only if for all states s and all terminal configurations $\langle V, s'\rangle$ (see above)

2 $\langle P_1, s\rangle \Downarrow \langle V, s'\rangle$ holds iff $\langle P_2, s\rangle \Downarrow \langle V, s'\rangle$ does.

1 $\cong$ is an equivalence relation:

- $P \cong P$
- $P \cong Q \Rightarrow Q \cong P$
- $P \cong Q$ & $Q \cong R \Rightarrow P \cong R$

and a congruence for LC in the sense that

1 $$P \cong Q \Rightarrow C[P] \cong C[Q]$$

where $C[P]$ is any LC phrase containing
1 an occurrence of P and $C[Q]$ is the phrase with that occurrence replaced by Q.

E.g. $C_1 \cong C_2 \Rightarrow (\text{while } B \text{ do } C_1) \cong (\text{while } B \text{ do } C_2)$

$B_1 \cong B_2 \Rightarrow (\text{while } B_1 \text{ do } C) \cong (\text{while } B_2 \text{ do } C)$

5 etc., etc.

(c) The call-by-name rule:

1 $$\frac{\langle M_1, s\rangle \Downarrow \langle \lambda x. M_1', s'\rangle \qquad \langle M_1'[M_2/x], s'\rangle \Downarrow \langle V, s''\rangle}{\langle M_1 M_2, s\rangle \Downarrow \langle V, s''\rangle}$$

The call-by-value rule:

2 $$\frac{\langle M_1, s\rangle \Downarrow \langle \lambda x. M_1', s'\rangle \qquad \langle M_2, s'\rangle \Downarrow \langle V_2, s''\rangle \qquad \langle M_1'[V_2/x], s''\rangle \Downarrow \langle V, s'''\rangle}{\langle M_1 M_2, s\rangle \Downarrow \langle V, s'''\rangle}$$

1 where $M[M'/x]$ denotes the result of <u>substituting</u> the (closed) expression M' for all <u>free</u> occurrences of x in M.

The evaluation relations for LFP

$\Downarrow_n$ based on call-by-name

$\Downarrow_v$ based on call-by-value

1 are incomparible, i.e. in general

5 $\langle M, s\rangle \Downarrow_n \langle V, s'\rangle \;\underset{\not\Leftarrow}{\not\Rightarrow}\; \langle M, s\rangle \Downarrow_v \langle V, s'\rangle$.

(d) Two configurations c_1 & c_2 in a labelled transition system $(Config, Act, \rightarrow)$ are <u>bisimilar</u>, $c_1 \approx c_2$, iff $c_1 \mathcal{R} c_2$ holds for

1 some <u>bisimulation relation</u> $\mathcal{R}$ — this is a binary relation $\mathcal{R} \subseteq Config \times Config$ satisfying for all $c_1, c_2 \in Config$ that $c_1 \mathcal{R} c_2$ implies

for all $\alpha \in Act$, $c_1' \in Config$,

$$c_1 \xrightarrow{\alpha} c_1' \Rightarrow \exists c_2' \,(c_2 \xrightarrow{\hat{\alpha}} c_2' \;\&\; c_1' \mathcal{R} c_2')$$

2 and for all $\alpha \in Act$, $c_2' \in Config$

$$c_2 \xrightarrow{\alpha} c_2' \Rightarrow \exists c_1' \,(c_1 \xrightarrow{\hat{\alpha}} c_1' \;\&\; c_1' \mathcal{R} c_2')$$

Here $\xrightarrow{\hat{\alpha}}$ is defined to be

2 $$\begin{cases} (\xrightarrow{\tau})^* & \text{if } \alpha = \tau \\ (\xrightarrow{\tau})^*(\xrightarrow{\alpha})(\xrightarrow{\tau})^* & \text{if } \alpha \neq \tau \end{cases}$$

5 where $(\;)^*$ is reflexive-transitive closure.

(e) Rules for parallel composition:

1 $$\frac{P_1 \xrightarrow{\alpha} P_1'}{P_1 | P_2 \xrightarrow{\alpha} P_1' | P_2}$$ & Symmetrically

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1 $$\frac{P_1 \xrightarrow{c(n)} P_1' \quad P_2 \xrightarrow{\bar{c}\langle n \rangle} P_2'}{P_1 | P_2 \xrightarrow{\tau} P_1' | P_2'}$$ & symmetrically

1 where $\alpha ::= \tau \mid c(n) \mid \bar{c}\langle n \rangle$ ranges over LCP actions.

Rule for restriction:

1 $$\frac{P \xrightarrow{\alpha} P'}{\nu c.\, P \xrightarrow{\alpha} \nu c.\, P'}$$ if $\alpha \neq c(n), \bar{c}\langle n \rangle$ any $n \in \mathbb{Z}$.

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