pSq9
AMP
1.

CST IB 2000. Paper 5, q 9
Semantics of Programming Languages

1 (⇓skip) $\mathrm{skip}, s \Downarrow 0, s$

1 (⇓seq1) $$\frac{C, s \Downarrow 0, s' \quad C', s' \Downarrow n', s''}{(C; C'), s \Downarrow n', s''}$$

1 (⇓seq2) $$\frac{C, s \Downarrow n, s'}{(C; C'), s \Downarrow n, s'} \quad \text{if } n \neq 0$$

1 (⇓if1) $$\frac{C, s \Downarrow n, s'}{(\text{if } B \text{ then } C \text{ else } C'), s \Downarrow n, s'} \quad \text{if } B, s \Downarrow \text{true}$$

1 (⇓if2) $$\frac{C', s \Downarrow n, s'}{(\text{if } B \text{ then } C \text{ else } C'), s \Downarrow n, s'} \quad \text{if } B, s \Downarrow \text{false}$$

1 (⇓ret1) $(\text{if } B \text{ return } n), s \Downarrow n, s \quad \text{if } B, s \Downarrow \text{true}$

1 (⇓ret2) $(\text{if } B \text{ return } n), s \Downarrow 0, s \quad \text{if } B, s \Downarrow \text{false}$

1 (⇓han1) $$\frac{C, s \Downarrow n, s' \quad C', s' \Downarrow n', s''}{(C \text{ handle } n \text{ with } C'), s \Downarrow n', s''}$$

1 (⇓han2) $$\frac{C, s \Downarrow n', s'}{(C \text{ handle } n \text{ with } C'), s \Downarrow n', s'} \quad \text{if } n' \neq n$$

9

1 (a) $C_1 \triangleq$ if true return 0.

Note that $C_1, s \Downarrow n, s'$ holds iff it was deduced from ($\Downarrow$ ret 1) [we assume true, $s \not\Downarrow$ false !] with $n = 0$ and $s' = s$.

1 Similarly skip, $s \Downarrow n, s'$ holds iff $n = 0$ & $s' = s$.

2 Thus $C_1 \cong$ skip.

1 (b) $C_2 \triangleq C$ handle 0 with C'.

Have to prove

(1) $\quad C_2, s \Downarrow n, s' \Rightarrow (C; C'), s \Downarrow n, s'$

(2) $\quad (C; C'), s \Downarrow n, s' \Rightarrow C_2, s \Downarrow n, s'$.

For (1), suppose

(3) $\quad C_2, s \Downarrow n, s'$

holds. Must have been deduced using either ($\Downarrow$han1) or ($\Downarrow$han2).

1 Case ($\Downarrow$ han 1): so we have

(4) $\quad C, s \Downarrow 0, s''$

(5) $\quad C', s'' \Downarrow n, s'$

for some s''. Applying ($\Downarrow$seq1) to (4)&(5) yields

(6) $\quad (C; C'), s \Downarrow n, s'$

1 Case ($\Downarrow$ han2): so we have

(7) $\quad C, s \Downarrow n, s'$ with $n \neq 0$

Applying ($\Downarrow$seq2) to (7) yields (6).

So in either case (3) $\Rightarrow$ (6), as required for (1).

For (2), suppose (6) holds. It was deduced using either ($\Downarrow$seq1) or ($\Downarrow$seq2).

Case ($\Downarrow$seq1): so (4) & (5) hold; and applying ($\Downarrow$han1) to them, we get (3).

Case ($\Downarrow$seq2): so (7) holds; and applying ($\Downarrow$han2) we get (3) again.

Thus (6) $\Rightarrow$ (3), as required for (2).

All in all, we have proved $C_2 \cong$ ~~[illegible]~~ $C;C'$.

3

(c) Choose some integer $m \neq 0$ which does not occur in C' and define

$$C_3 \triangleq ((\text{if } B \text{ return } m) \text{ handle } 0 \text{ with } C') \text{ handle } m \text{ with } C$$

We have to prove (8) $\Leftrightarrow$ (9) where

(8) $C_3, s \Downarrow n, s'$

(9) $(\text{if } B \text{ then } C \text{ else } C'), s \Downarrow n, s'$.

But

$$(8) \Leftrightarrow \begin{cases} \underline{\text{either}} \begin{cases} ((\text{if } B \text{ return } m) \text{ handle } 0 \text{ with } C'), s \Downarrow m, s'' \\ \& \quad C, s'' \Downarrow n, s' \qquad (\text{some } s'') \end{cases} \\ \underline{\text{or}} \quad ((\text{if } B \text{ return } m) \text{ handle } 0 \text{ with } C'), s \Downarrow n, s' \\ \qquad \text{with } n \neq m \end{cases}$$

$$\overset{\text{by (b)}}{\Longleftrightarrow} \begin{cases} \underline{\text{either}}\ (10)\ (\text{if } B \text{ return } m); C', s \Downarrow m, s'' \ (\text{some } s'') \\ \quad (11)\ \& \ C, s'' \Downarrow n, s' \\ \underline{\text{or}}\ (12)\ (\text{if } B \text{ return } m); C', s \Downarrow n, s' \ \& \ n \neq m. \end{cases}$$

Since m does not occur in C' & $m \neq 0$

(10) $\Leftrightarrow$ $B, s \Downarrow true$ & $s'' = s$

(12) $\Leftrightarrow$ $B, s \Downarrow false$ & $C', s \Downarrow n, s'$

Hence

(8) $\Leftrightarrow$ $\begin{cases} \underline{\text{either}} \quad B, s \Downarrow true \ \& \ C, s \Downarrow n, s' \\ \underline{\text{or}} \quad B, s \Downarrow false \ \& \ C', s \Downarrow n, s' \end{cases}$

$\Leftrightarrow$ (9) by ($\Downarrow$if1) & ($\Downarrow$if 2).

$\square$

4

6