ADVANCED GRAPHICS 2003, PAPER 7 Q 4

(a) (i) Ray: eye point, E, 3D point
viewing direction, D, 3D vector
$P(t) = E + tD$ where $t \in \mathbb{R}$, $0 \le t$

Sphere: centre, C, 3D point
radius, r, $r \in \mathbb{R}$, $0 \le r$

(ii) a point on the ray is defined as $P(t) = E + tD$
a point on the sphere is defined as $|C - P| = r$

The ray equation is put into the sphere equation:

$$|C - E - tD| = r$$

$$\Rightarrow (C - E - tD)^2 = r^2$$

$$\Rightarrow C\cdot C - 2C\cdot E + E\cdot E - 2t\,C\cdot D + 2t\,E\cdot D + t^2 D\cdot D = r^2$$

$$\Rightarrow t^2(D\cdot D) + t(2(E-C)\cdot D) + ((C-E)^2 - r^2) = 0$$

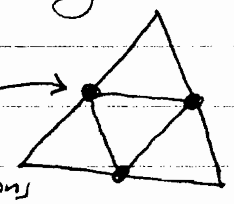
$$\Rightarrow t^2 a + tb + c = 0$$ with appropriate equations for a, b, c

Algorithm: given the parameters E, D, C and r
calculate a, b, c as above
solve $t^2 a + tb + c = 0$ for t (e.g. $t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$)
if no real values of t then return $\langle$NO INTERSECTION$\rangle$
otherwise there will be two values for t, call them t_1 and t_2, sort so that $t_1 \le t_2$
if $(t_1 \le t_2 < 0)$ return $\langle$NO INTERSECTION$\rangle$
if $(0 \le t_1 \le t_2)$ return $\langle P(t_1) = E + t_1 D;\ N = P(t_1) - C\rangle$
if $(t_1 < 0 \le t_2)$ return $\langle P(t_2) = E + t_2 D;\ N = C - P(t_2)\rangle$

(b) there are two ways to do this

① start with a regular tetrahedron, octahedron or icosahedron, ~~To g~~ with vertices on the sphere.
A subdivision step splits each triangle into four, quadrupling the total number of triangles.

After a split, the new vertices are pushed out along a ray from the sphere's centre so that they lie on the sphere itself.

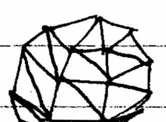


As the number of triangles only quadruples at each step, it is obvious that there exists an n for which:

$$D/10 < N_0 . 4^n < 10D \quad \forall D \geq N_0$$

$$N_0 \in \{4, 8, 20\}$$

② the easier method is to evenly subdivides the sphere into m zones along lines of latitude and n along lines of longitude, connecting triangles appropriately across the quadrilateral-like divisions which result

e.g.  It is easy to see that there will be

$$2(n-2).m \text{ triangles}$$

Ideally $m \approx 2n$. ~~and~~ $n-2 \simeq n$ so we are generating about m^2 triangles. Set $m = \lfloor \sqrt{D} \rfloor$ to get the desired number.

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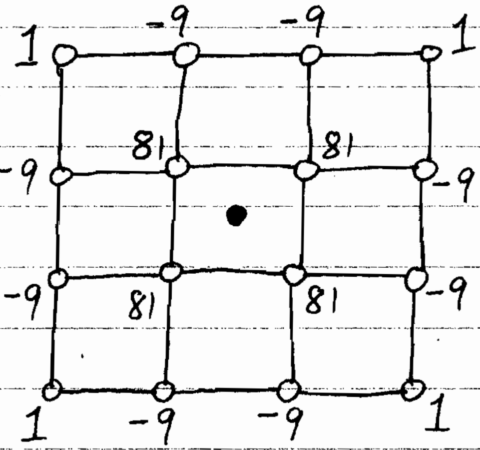
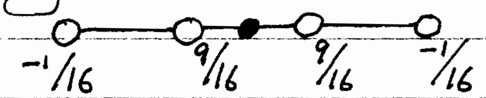
(c) (i) face



all divided by 256

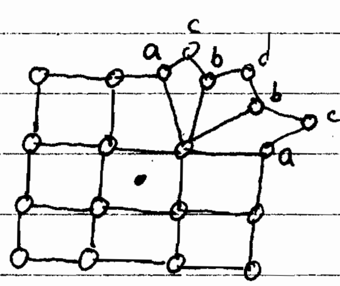
edge



vertex • 1

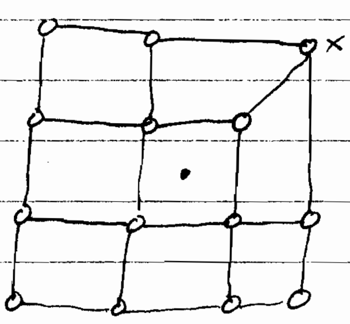
(ii) around extraordinary vertices you get issues with creating new edge & face vertices because it is no longer clear which vertices should be put into the mixture & with what proportions.

e.g.



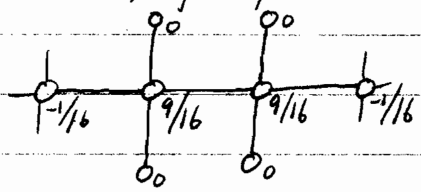
In the conventional situation there would be two vertices with weight $\frac{-9}{256}$ & one with weight $\frac{1}{256}$. How do we split this between the a, b, c & d vertices? Should other vertices also change their weights?

e.g.



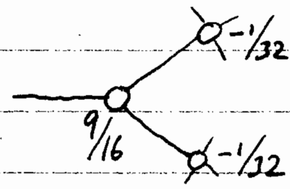
This is a valid arrangement but what weight should be given to vertex x?

For the specific problem of edge vertices, here is one idea:

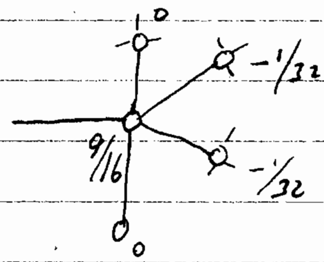


the standard case.

Vertex of valency 3:



Vertex of valency 5:



Vertices of higher valency: zero weight to two "closest" to the edge $-\frac{1}{16(v-3)}$ where v is valency, to rest.

The critical point is that the $-1/16$ is split somehow between ~~each~~ the vertices.

For even valencies, it could just be given to the central one.

Most reasonably sensible & justified answers would get full marks.

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This question covers the ray tracing, converting primitives to polygons, and subdivision parts of the course requiring knowledge of two of the eight lectures.

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Marking Scheme

(a) (i)	eye point + direction vector	$\frac{1}{2}+\frac{1}{2} = 1$
	centre + radius	$\frac{1}{2}+\frac{1}{2} = 1$
		2
(ii)	correct equations for ray & sphere	1
	correct equation for t ($at^2+bt+c=0$)	1
	no real values $\Rightarrow$ no intersection	½
	no positive values $\Rightarrow$ no intersection	½
	correct normal vector	1
	correct intersection point	1
		5
(b)	a workable method	3
	explanation of how to get within $\frac{D}{10} \cdots 10D$	1
		4
(c)(i)	face	2
	edge	2
	vertex	1
		5
(ii)	don't know what weight to assign each vertex	2
	a sensible idea for coping with edges	1
	... that is justified	1
		4
		20