INFO THEORY - PAPER 7, 2003 q 8

(a) Data which is amenable to data compression that is LOSSY are primarily those data which are intended for visual or aural presentation to humans. This is because the human perceptual systems have limitations, thus, above a certain threshold, humans cannot distinguish between different sampling and quantisation levels.

There are a range of mechanisms used to perform lossy compression. The guiding principle is to transform the data to a space in which perceptually important information is captured in as few components as possible. The can be represented with many quantisation levels while other components are represented by few (or no) quantisation levels.

Students may refer to particular techniques such as aural masking & DCT/DWT coding but the general principles outlined above are sufficient to gain the majority of the marks.

(b) Let L_i be the average symbol code length

$$L_1 = 2(p_a + p_b + p_c + p_d) \qquad [\text{where } p_a = p(a) \text{ etc}]$$
$$L_2 = p_a + 2p_b + 3(p_c + p_d)$$

We want to know when $L_2 < L_1$

$$\Rightarrow p_a + 2p_b + 3(p_c + p_d) < 2p_a + 2p_b + 2(p_c + p_d)$$
$$\Rightarrow \quad p_c + p_d < p_a$$

Q.E.D.

(c) for an alphabet $\mathbb{A}$ of m symbols, the length of each symbol in the binary code is $l(x)$, $x \in \mathbb{A}$

By the Kraft-McMillan inequality, for a binary prefix code:

$$\sum_{x \in \mathbb{A}} 2^{-l(x)} \leq 1$$

The average symbol length is $\sum_{x \in \mathbb{A}} p(x) . l(x)$

where $p(x) = \frac{1}{m}$ is the probability that symbol x occurs

Construct a probability distribution $q(x) = 2^{-l(x)}$

Average symbol length is now $\sum_{x \in \mathbb{A}} p(x) . \log_2 \frac{1}{q(x)}$

Kullback Leibler distance is:

$$D(p\|q) = \underbrace{\sum_{x \in \mathbb{A}} p(x) . \log_2 \frac{1}{q(x)}}_{\text{average symbol length}} - \underbrace{\sum_{x \in \mathbb{A}} p(x) . \log_2 \frac{1}{p(x)}}$$

$$= \sum_{x \in \mathbb{A}} \frac{1}{m} \cdot \log_2 m$$

$$= \log_2 m \quad \text{as } |\mathbb{A}| = m$$

By Information Inequality Theorem

$$D(p\|q) \geq 0 \quad \forall p, q$$

So, average symbol length is greater than or equal to $\log_2 m$.

Q.E.D

This question tests these parts of the syllabus

(a) asks for an overview of Markus Kuhn's three lectures

(b) tests basic understanding of probability & an ability to think beyond the basic teaching in the course

(c) requires the students to bring together two theorems from Ch 2 (Information Inequality Theorem) and Ch 5 (Kraft McMillan Inequality) of the textbook

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MARKING SCHEME

(a) types of data — 2
why they are amenable — 2
description of overall principles of the mechanisms — 2
either specific examples or more detail of the overall principles — 2
8

(b) $p_c + p_d < p_a$ — 2
working — 4
6

(c) (there are at least two ways to do this.)
marks will be given for correct working — 4
as well as for a complete solution — 2
6

20