P8v16
RJG (1)

Computer Systems Modelling 2004
[Interarrival times, random distributions]

(a) $f_Y(y)$ is a density, therefore

$$1 = \int_0^\infty f_Y(y)\,dy = \int_0^\infty C \cdot y \cdot f_X(y)\,dy$$

$$= C \int_0^\infty y \cdot f_X(y)\,dy$$

$$= C \cdot E(X) = C\mu$$

$$\therefore \quad C = \frac{1}{\mu}$$

(b) Average interarrival time as seen by randomly arriving customer is

$$\int_0^\infty y f_Y(y)\,dy = \int_0^\infty y \frac{1}{\mu} y f_X(y)\,dy$$

$$= \frac{1}{\mu}\int_0^\infty y^2 f_X(y)\,dy$$

$$= \frac{E(X^2)}{\mu} = \frac{\sigma^2+\mu^2}{\mu}$$

So, average waiting time is $\frac{\sigma^2+\mu^2}{2\mu}$ being one half that of the average interval.

(c)

(i) X deterministic

$\mu = 10 \, , \; \sigma^2 = 0$

average waiting time $= \dfrac{10^2 + 0^2}{2.10} = 5$

(ii) X exponential

$\mu = 10 \, , \;$ variance $= 10^2$

average waiting time $= \dfrac{10^2 + 10^2}{2.10} = 10$

(as expected given memoryless property of exponential distribution)

(iii) X general distribution with

$\mu = 10, \; \sigma^2 = 500$

then average waiting time $= \dfrac{10^2 + 500}{2.10}$

$= \dfrac{600}{20}$

$= 30$