p84, 12
JGD

Computer Vision, Question 1. 2000

Explain the Bayesian approach to solving problems in computer vision. Explain the notion of an "Inverse Problem," and how computer vision can be regarded thereby in a formal sense as "inverse graphics." Write down Bayes' rule in general form, and explain the interpretation of its terms as:

- probability of the image, given the object;
- probability of the object, given the image.

What is the role of the "prior?"

Discuss and illustrate the Bayesian approach in terms of 3D surface reconstruction, given the reflectance data in an image.

[20 marks]

Model Answer, Computer Vision, Question 1.

The Bayesian approach to solving problems in computer vision evaluates the image evidence (E) for the hypothesis (H) that there is a particular object or property in the world, by relating the conditional probability $p(H|E)$ to the conditional probability $p(E|H)$. These two quantities are interpretable as:

$p(H|E) \equiv$ the likelihood that hypothesis H is true, given the image evidence E. (This is what we wish to evaluate, and it is therefore called the "posterior probability.").

$p(E|H) \equiv$ the likelihood that <u>if</u> hypothesis H were true, the image would contain particular evidence E. (Calculating this quantity is based upon our scientific knowledge of the image formation process, e.g. that a certain set of surface properties and illumination conditions would result in a certain image being formed as a result.)

Because of the inverse conditionalising in the two cases, problems that are formulated in this way can be described as "Inverse Problems." We need to figure out the state of the world that would produce the image that we have. (This question might not have a unique answer, nor a stable one.)

Computer vision is a form of "inverse graphics" because graphics seeks to create images from configurations and properties of model worlds, whereas vision must infer the configurations and properties of the world, given the available graphical information in the image.

In order to relate the above two conditional probabilities $p(H|E)$ and $p(E|H)$, we may use

Bayes' Theorem if we can also estimate the unconditional probabilities $p(E)$ and $p(H)$:

$$p(H|E) = \frac{p(E|H)p(H)}{p(E)}$$

$p(E) \equiv$ the unconditional likelihood that the evidence E would be found in images anyway, regardless of whether or not hypothesis H is true. Thus if $p(E)$ is large (e.g. images contain some bright regions no matter what), then this reduces our confidence in inferring any particular hypothesis H as a result of observing E.

$p(H) \equiv$ the priors: our *a priori* assumptions about the likelihood of the hypothesis in the first place. If H is extremely improbable, then we should be more reluctant to conclude it; we should demand stronger evidence to support it.

Priors are a way of incorporating our assumptions and prior knowledge about the world, into solving the problem.

In 3D surface reconstruction from reflectance data in an image, the 3D surface reconstruction is the H having the most likely $p(H|E)$ given the data E. Some hypotheses can be rejected because they are so improbable, e.g. $p(H)$ is too small in the above equation because our prior assumption is that bits of the landscape cannot be suspended in thin air above the ground. The informativeness of different kinds of evidence E depends on how likely it is in all images anyway, given by $p(E)$. So, in using Bayesian inference to solve inverse problems, our aim is to maximize $p(H|E)$: to find the most likely hypothesis (surface reconstruction) that explains the image data (reflectance).