1999 Comm. Autom. and the Pi Calculus. Question B, Solution notes

An abstraction is of form $(x_1, \cdots, x_n)P$; it represents the process P rendered parametric on the names $x_1, \cdots x_n$. A concretion is $(\nu\vec{x})(\vec{y}, P)$ where $\vec{x} \subseteq \vec{y}$; it represents an output datum $\vec{y}$ paired with a continuation process P. A commitment has three forms:

$P \xrightarrow{\tau} P'$ representing a communication within P

$P \xrightarrow{z} (\vec{x})P'$ represents the input of a datum (which will replace $\vec{x}$) along channel z.

$P \xrightarrow{\bar{z}} \nu\vec{x}(\vec{y}, P')$ representing the output of datum $\vec{y}$ along z.

Strong bisimulation S: a relation such that if $P \xrightarrow{\alpha} A$ and $(P,Q) \in S$ then $Q \xrightarrow{\alpha} B$ with $(A,B) \in S$, and the same condition for S^{-1}.

(SEE NEXT PAGE FOR SECOND PART) [5 MARKS]

$$P \sim in(y).(\nu\, mid\, m)\big(C\langle y, in, mid, \ell, m\rangle \mid C\langle x, mid, out, m, r\rangle\big)$$
$$+ \overline{out}\langle x\rangle.(\nu\, mid\, m)\big(B\langle in, mid, \ell, m\rangle \mid B\langle mid, out, m, r\rangle\big)$$
$$+ \tau.C\langle x, in, out, \ell, r\rangle.$$

The justification is in terms of the three commitments which P can make, justified by the rules of commitment.

[6 MARKS]

If P is changed by making $in = out$, then there is an extra commitment due to a communication between C outputting on $\overline{in}$ and B inputting on in:

$$P = new\ mid, m\ \big((in(y).\, C\langle y, in, mid, \ell, m\rangle + \ldots) \mid \overline{in}\langle x\rangle.B\langle mid, in, m, r\rangle + \ldots$$
$$\xrightarrow{\tau} new\ mid, m\ \big(C\langle x, in, mid, \ell, m\rangle \mid B\langle mid, in, m, r\rangle\big)$$
$$\xrightarrow{\tau} P \text{ again.}$$

Thus the datum x can cycle indefinitely between the pair of cells.

[5 MARKS]

(ii)

Let $P = (\nu x)\bar{x}\langle y\rangle$, $Q = (\nu x)\bar{y}\langle x\rangle$.

Then (a) $Q \not\equiv P$, $Q \not\equiv 0$, $Q \not\sim P$, $Q \not\sim 0$ since Q has the commitment $Q \xrightarrow{\bar{y}} \nu x(x, 0)$ and the other two have no commitments.

(b) $P \sim 0$, since neither has any commitment.

(c) $P \not\equiv 0$, since none of the rules of structural congruence allow any preactions $x(z)$ or $\bar{x}\langle y\rangle$ to be added or eliminated.

[4 MARKS]