MODEL ANSWER

Computer Vision 2002 Paper 8 Question 13 (JGD)

(Subject area: Edge Detection and primary visual operations.)

1.
The operator is a convolution. Image $I(x, y)$ is being filtered by the Laplacian of a Gaussian to emphasize edges of a certain scale. [2 marks]

2.
The zeroes of the equation, the points where $g(x, y) = 0$, correspond to edges (at any angle) within the image $I(x, y)$. Thus this operator serves as an isotropic (non orientation-selective) edge detector. [3 marks]

3.
Parameter σ determines the scale of image analysis at which edges are detected. If its value were increased, there would be fewer edges detected, i.e. fewer zeroes of $g(x, y)$, but also fewer false edge detections related to spurious noise. [3 marks]

4.
In the 2D Fourier domain, the operator is a bandpass filter whose centre frequency is determined by σ. Low frequencies are attenuated, and also high frequencies are attenuated, but middle frequencies (determined by the value of σ) are emphasized. However, all orientations are treated equivalently: the operator is isotropic. [4 marks]

5.
The operation would be much easier to implement via Fourier methods, because convolution is achieved by the simple multiplication of the Fourier transforms of the two functions being convolved. (In the case in question, these are the image and the Laplacian of a Gaussian filter.) In contrast, image-domain convolution requires a double integral to be computed in order to evaluate $g(x, y)$ for each point (x, y). But a Fourier cost is the requirement first to compute the Fourier transform of the image, and then to compute the inverse Fourier transform of the result after the multiplication, in order to recover the desired $g(x, y)$ function. The computational complexity (and speed) of using Fourier methods becomes favourable for convolution kernels larger than about 5 x 5. [4 marks]

6. By application of the 2D Differentiation Theorem:

$$G(u, v) = -4\pi^2(u^2 + v^2)\ e^{-(u^2+v^2)\sigma^2} F(u, v)$$

[4 marks]