Solution notes (a) and (b) are bookwork, from Sections 2 and 6 of the notes, reminding the candidate what semantic equivalence means for L1. (c) requires some thought about each example.

(a)

(if1) $\langle\textbf{if true then } e_2 \textbf{ else } e_3, s\rangle \longrightarrow \langle e_2, \ s\rangle$

(if2) $\langle\textbf{if false then } e_2 \textit{ else } e_3, s\rangle \longrightarrow \langle e_3, \ s\rangle$

(if3) $$\frac{\langle e_1, s\rangle \longrightarrow \langle e_1', s'\rangle}{\langle\textbf{if } e_1 \textbf{ then } e_2 \textbf{ else } e_3, s\rangle \longrightarrow \langle\textbf{if } e_1' \textbf{ then } e_2 \textbf{ else } e_3, s'\rangle}$$

(while)
$\langle\textbf{while } e_1 \textbf{ do } e_2, s\rangle \longrightarrow \langle\textbf{if } e_1 \textbf{ then } (e_2; \textbf{ while } e_1 \textbf{ do } e_2) \textbf{ else skip}, \ s\rangle$

(b)

Define $e_1 \simeq_\Gamma^T e_2$ to hold iff forall s such that $\text{dom}(\Gamma) \subseteq \text{dom}(s)$, we have $\Gamma \vdash e_1 : T$, $\Gamma \vdash e_2 : T$, and either

1. $\langle e_1, s\rangle \longrightarrow^\omega$ and $\langle e_2, s\rangle \longrightarrow^\omega$, or

2. for some v, s' we have $\langle e_1, \ s\rangle \longrightarrow^* \langle v, s'\rangle$ and $\langle e_2, s\rangle \longrightarrow^* \langle v, s'\rangle$.

(c)

(i) l does not occur in e. This ensures that the behaviour of e is not affected by prefixing with an assignment to l and also that the final value stored in l is not affected by executing e.

(ii)   there is no location depended upon by one of e and e_1 and semantically affected by the other, ensuring that the two commute.

(iii) true.

(iv) e_2 does not write l. This ensures that l holds the same value at the beginning of each corresponding iteration in the two expressions.