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Discrete mathematics – Question 2

What does it mean for a partial order to be well founded? [3 marks]

Given two well ordered, partially ordered sets $(A, \leq_A)$ and $(B, \leq_B)$, define the *lexicographic order* on $A \times B$ and show that it is well founded. [5 marks]

Two elements x and y of a partially ordered set are said to be *separated* if for all $k \geq 1$ there is a sequence of elements $z_1, z_2, \ldots z_k$ with $x < z_1 < z_2 < \ldots < z_k < y$.

Give an example of a well founded, partially ordered set that contains infinitely many pairs of separated elements. [5 marks]

Prove that no well ordered, partially ordered set has every pair of elements separated. [7 marks]

Answer

Every infinite descending sequence of elements is ultimately constant.

$(a_1, b_1) \leq (a_2, b_2) \Leftrightarrow (a_1 <_A a_2) \vee ((a_1 = a_2) \wedge (b_1 \leq_B b_2))$. Bookwork.

$N \times N$ with the lexicographic order: $(1,1)$ is separated from $(2,1)$ which is separated from $(3,1)$ and so on.

Take any pair of elements x and y. Wlog $x < y$. x and y are separated, so find z_1 with $x < z_1 < y$. Now x and z_1 are separated, so find z_2 with $x < z_2 < z_1$. Hence form an infinite descending sequence.