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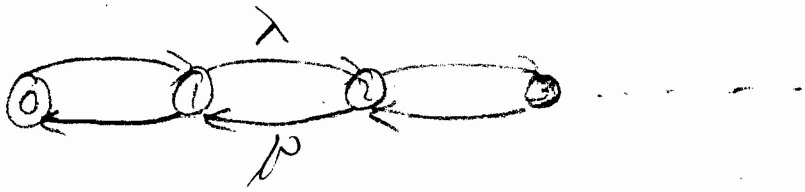
Computer Systems Modelling Paper 7 Q8 RJG
[RELATES TO M/M/1 queue]

2002

(a) [4 marks]



States 0, 1, 2, ... corresponding to number of customers present

Transition $i \to i+1$ at arrival rate λ if customer arrives

$i = 0, 1, 2, \ldots$

Transition $i \to i-1$ at departure rate μ if customer departs

$i = 1, 2, \ldots$

The state diagram shows the states and / the possible transitions that can take place in the Markov chain model of the M/M/1 queue. The state diagram shows that the Markov chain is a birth-death process so that detailed balance equations can be used.

(b) [2 marks]

Condition for existence of steady-state equilibrium is

$$\rho = \frac{\lambda}{\mu} < 1$$

traffic intensity

(c)

Detailed balance equations:

$$\mu p_k = \lambda p_{k-1}$$

$$p_k = \frac{\lambda}{\mu} p_{k-1}$$

$$= \frac{\lambda}{\mu} \cdot \frac{\lambda}{\mu} p_{k-2}$$

$$= \left(\frac{\lambda}{\mu}\right)^k p_0$$

$$= \rho^k p_0 \qquad \text{where } \rho = \frac{\lambda}{\mu} < 1 \text{ for stability.}$$

So, $p_0\left(1 + \sum_{k=1}^{\infty} \rho^k\right) = 1$ normalisation condition

So,

$$p_0 = \frac{1}{1+\sum_{k=1}^{\infty}\rho^k}$$

$$= \frac{1}{1+\rho\sum_{k=0}^{\infty}\rho^k}$$

$$= \frac{1}{1+\rho\left(\frac{1}{1-\rho}\right)} \qquad \text{sum of G.P.}$$

$$= (1-\rho)$$

[2 marks]

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↑ traffic intensity

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for stability

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sum of G.P.

$$= \frac{1}{1 + \rho\left(\frac{1}{1-\rho}\right)}$$

$$= (1-\rho)$$

[8 marks]

(e) Suppose there are K sources connected

then $\lambda = 2K$ items/second and $1/\mu = 0.025$ second

So, $\rho = \frac{\lambda}{\mu} = 0.05K$

So to meet criteria (i) require $\rho < 1$

So $0.05K < 1$

$\Rightarrow K < 20$ sources

The average time spent in the system (from (d)) is $\frac{1}{\mu - \lambda}$

So, require $\frac{1}{40 - 2K} \leq 0.1$ to

meet criteria (ii) i.e.

$K \leq 15$

Hence (ii) is a stronger criteria than (i).