MODEL ANSWER

Information Theory and Coding 2002 Paper 9 Question 10 (JGD)

(Subject areas: Error correcting codes. Signals and spectra. Zero-crossings.)

A.
A Hamming Code transmits 7 bits in order to encode reliably 4 data bits; the 3 syndrome bits are added to guarantee detection and correction of 1 erroneous bit in any such block of 7 bits transmitted. Thus the maximum rate of information transmission is 4/7ths of a bit per bit.

[2 marks]

Syndromes are constructed by taking the Exclusive-OR of three different subsets of 4 bits from the 7 bits in a block. This Boolean operation is performed before transmission. Upon reception, if the three syndrome bits are all 0, then there was no error; otherwise they identify which bit was corrupted.

[2 marks]

B.

1-A. Example: a Gaussian function, whose Fourier transform is also Gaussian.
2-C. Example: a sinusoid, whose Fourier transform is two discrete spikes.
3-B. Example: a delta function, whose Fourier transform is a complex exponential.
4-D. Example: a comb sampling function, whose Fourier Transform is also a comb function.

[8 + 4 = 12 marks in total]

C.
1. The zero-crossings in a two- (or higher-) dimensional signal, such as an image, are not denumerable. 2. The extension of the one-octave bandlimiting constraint to the Fourier plane does not seem to be possible in an isotropic manner. If applied isotropically (i.e. a one-octave annulus centred on the origin of the Fourier plane), then in fact both the vertical and horizontal frequencies are each low-pass, not bandpass. But if applied in a bandpass manner to each of the four quadrants, then the different orientations in the image are treated differently (anisotropically).

[4 marks]