GW

2002 Discrete mathematics (10 marks question) p1q2

(a) A well-founded relation on a set A is a relation $\prec \subseteq A \times A$ s.t. there are no infinite descending chains

$$\cdots \prec a_n \prec \cdots \prec a_2 \prec a_1 \prec a_0$$

(b) Suppose $S \subseteq A$ and $S \neq \emptyset$. Pick $a_0 \in S$. ~~Either a_0 is $\prec$-minimal or not~~.
If a_0 is minimal, we have a suitable m.
Otherwise pick $a_1 \prec a_0$ with $a_1 \in S$.
If a_1 is minimal, we have a suitable m.
Otherwise continuing in this fashion we either find a $\prec$-minimal element in S or produce an infinite descending chain — impossible as $\prec$ is well-founded.

(c) Suppose there were a string such that

$$au = ub.$$

There is then one of minimum length. Wlog. assume u has minimum length.
But then

(*) $$au = avb \quad \text{for some } v.$$

where

$$u = av,$$ ~~and $u = vb$~~ But also, cancelling the "a", in (*) $u = vb$.

$$\therefore \quad av = vb$$

for some ~~smaller~~ string v strictly smaller than u.
— a contradiction.