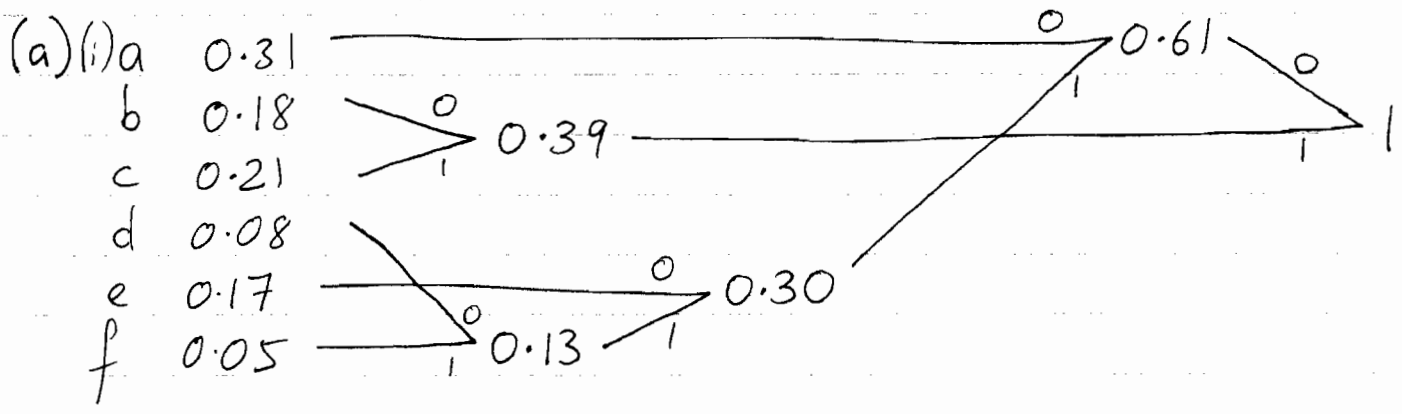
INFO THEORY - paper 8, 2003 q. 10 NAD

(a)(i)



a	00	2 bits
b	10	2 bits
c	11	2 bits
d	0110	4 bits
e	010	3 bits
f	0111	4 bits

(ii) average bits per symbol $= (0.31 + 0.18 + 0.21) \times 2$
$+ 0.17 \times 3$
$+ (0.08 + 0.05) \times 4$
$= 1.40 + 0.51 + 0.52$
$= 2.43$

this is greater than the entropy
(it cannot be less than because this is impossible;
it cannot be equal because the probabilities are not all of the form $\frac{1}{2^n}$, $n \in \mathbb{Z}^+$)

(b)(i)

$8+8+8+24+12+4 = 64$

$\frac{8}{64} = \frac{1}{8}$
$\frac{12}{64} = \frac{3}{16}$
$\frac{4}{64} = \frac{1}{16}$
$\frac{24}{64} = \frac{3}{8}$

Y \ X	2	3	4	
C	1/8	3/8	1/16	9/16
M	1/8	1/8	3/16	7/16
	1/4	1/2	1/4	

$H(X) = 2 \times \frac{1}{4} \log 4 + \frac{1}{2} \log 2$
$= \frac{1}{2} \times 2 + \frac{1}{2} \times 1$
$= 3/2 = 1.500$

$H(Y) = \frac{9}{16} \log \frac{16}{9} + \frac{7}{16} \log \frac{16}{7}$
$= 0.467 + 0.522$
$= 0.989$

$H(X,Y) = 3 \times \frac{1}{8} \log 8 + \frac{3}{8} \log \frac{8}{3} + \frac{3}{16} \log \frac{16}{3} + \frac{1}{16} \log 16$
$= 3 \times 0.375 + 0.531 + 0.453 + 0.250$
$= 2.359$ (or 2.358 if you add then round to 3dp)

$H(x|y) = H(X,Y) - H(Y)$
$= 1.370$ (or 1.369)

$H(y|x) = H(X,Y) - H(X)$
$= 0.859$ (or 0.858)

$I(X;Y) = H(X) + H(Y) - H(X,Y)$
$= 0.130$ (or 0.129)

(ii) There are several possible answers. The three which require the least re-calculation are:

(A) both rows identical
(B) rows multiplicatively related in the ration 9/16 : 7/16
(C) one row all zeros.

(A)

8	16	8
8	16	8

1/8	1/4	1/8
1/8	1/4	1/8

$$H(X,Y) = 4 \times \tfrac{1}{8} \log 8 + 2 \times \tfrac{1}{4} \log 4$$
$$= \tfrac{1}{2} \times 3 + \tfrac{1}{2} \times 2$$
$$= 2.5$$

(B)

9	18	9
7	14	7

			9/16
			7/16
1/4	1/2	1/4	

$H(X,Y) = H(X) + H(Y)$ which are the same as in part (i)

$\therefore = 1.500 + 0.989$
$= 2.489$

(C)

16	32	16
0	0	0

			1
			0
1/4	1/2	1/4	

$H(X,Y) = H(X) + H(Y)$ where $H(X)$ is as in part (i) and $H(Y) = 0$ by inspection

$= 1.500$

Any answer of this form is acceptable

x	2x	x
16-x	32-2x	16-x

$x \in \{0, 1, 2, \ldots, 16\}$

but only $x \in \{0, 7, 8, 9, 16\}$ allows for easy calculation of $H(X,Y)$

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Anyone who has paid attention in the course can do very well on this question BUT parts (ii) of both (a) and (b) can be done much more rapidly by a student who understood the materially thoroughly. Indeed (b)(ii) will be beyond a student who has not understood but has merely memorised.

(a) tests parts of Ch 5 & an understanding of Ch 2
(b) tests the core relationships in information theory

MARKING SCHEME

(a)(i) correct method 3
a correct code 2
5

(ii) correct method 1
correct answer (2·43) 1
that it is GREATER THAN (not = nor <) 1
3

(b)(i) $H(X)$, $H(Y)$, $H(X,Y)$, $H(X|Y)$, $H(Y|X)$, $I(X;Y)$ - 1 mark each 6
correct method for calculating $H(a)$ 1
" " " " $H(a|b)$ 1
" " " " $I(a;b)$ 1
9

(ii) a correct table 2
a correct value of $H(X,Y)$ for the table 1
3

20