1999

p9 q 11

JGD

Model Answer, Information Theory and Coding, Question 2.

1. The family of continuous signals having maximum entropy per variance (or power level) are Gaussian signals. Their probability density function for excursions x around a mean value μ, when the power level (or variance) is σ^2, is:

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}$$

[4 marks]

2. The Fourier power spectrum of this class of signals is flat, or white. Hence these signals correspond to "white noise." The distribution of spectral energy has uniform probability over all possible frequencies, and therefore this continuous distribution has maximum entropy.

[4 marks]

3. An error-correcting Hamming code with a 7 bit block size uses 3 bits for error correction and 4 bits for data transmission. It would fail to correct errors that affected more than one bit in a block of 7; but in the example given, with $p = 0.001$ for a single bit error in a block of 7, the probability of two bits being corrupted in a block would be about 1 in a million.

[4 marks]

by 3W

4. The channel capacity C would be reduced a~~bout three-fol~~d if the noise power level increased eight-fold. This is because the channel capacity depends upon the $\log_2$ of the signal-to-noise ratio.

[4 marks]

5. In an efficient compression scheme, there would be few correlations in the compressed representations of the images. Compression depends upon decorrelation. An efficient scheme would have low entropy; Shannon's Source Coding Theorem tells us a coding rate R as measured in bits per pixel can be found that is nearly as small as the entropy of the image representation. The compression factor can be estimated as the ratio of this entropy to the entropy of the uncompressed image (i.e. of its pixel histogram).

[4 marks]