Probability 2004 Paper 2 Question 4 (FHK)
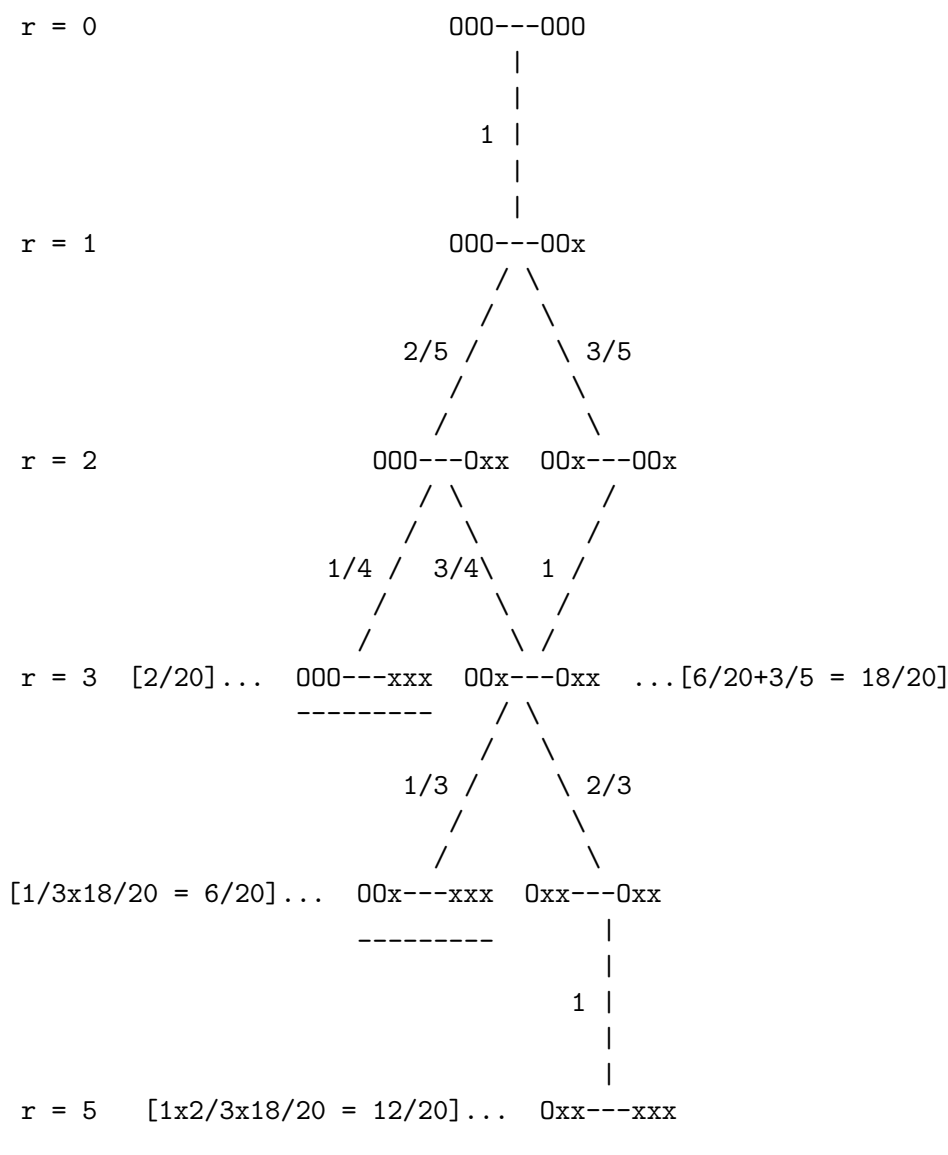
(a) The minimum number of working fans at replacement is 1.
This occurs if, just before the failure that brings out
the maintenance crew, there were two dud fans in each
group. The maximum number of working fans at replacement
is 3. This occurs if the first three failures are all
in the same group. [2 marks]

(b) The event tree can be represented as shown here. A working fan
is shown as O and a failed fan as x. Probabilities are shown
next to the branches of the tree.

```
r = 0                        OOO---OOO
                                 |
                                 |
                             1 |
                                 |
                                 |
r = 1                        OOO---OOx
                                / \
                               /   \
                          2/5 /     \ 3/5
                             /       \
                            /         \
r = 2                   OOO---Oxx  OOx---OOx
                        / \          /
                       /   \        /
                  1/4 / 3/4\   1 /
                     /      \   /
                    /        \ /
r = 3  [2/20]...  OOO---xxx  OOx---Oxx   ...[6/20+3/5 = 18/20]
                  ---------    / \
                              /   \
                         1/3 /     \ 2/3
                            /       \
                           /         \
 [1/3x18/20 = 6/20]...  OOx---xxx  Oxx---Oxx
                        ---------      |
                                       |
                                   1 |
                                       |
                                       |
r = 5   [1x2/3x18/20 = 12/20]...  Oxx---xxx
                                  ---------
```

The three terminal states (when replacement occurs) are shown
underlined (when r is 3, 4 or 5).

Accordingly, the required table is:

r	0	1	2	3	4	5	6
P(X=r)	0	0	0	1/10	3/10	6/10	0

[12 marks]

(c) The Expectation is
$$\frac{0 \times 0 + 1.0 + 2.0 + 3 \times 1 + 4 \times 3 + 5 \times 6}{10} = \frac{45}{10} = \frac{9}{2}$$

$$E(X^2) = \frac{0 \times 0 + 1.0 + 4.0 + 9 \times 1 + 16 \times 3 + 25 \times 6}{10} = \frac{207}{10}$$

The variance is therefore $\frac{207}{10} - \frac{81}{4} = \frac{414 - 405}{20} = \frac{9}{20}$

[6 marks]