Semantics of Programming Languages

2003

pbq 11

PMS

Q2 Solution

This question is based on the lectures on records and subtyping. Part (a) is bookwork; parts (b) and (c) require understanding of the subtype system.

(a)

$$\text{(s-refl)}\ \dfrac{}{T <: T}$$

$$\text{(s-trans)}\ \dfrac{T <: T' \qquad T' <: T''}{T <: T''}$$

$$\text{(s-fn)}\ \dfrac{T_1' <: T_1 \qquad T_2 <: T_2'}{T_1 \to T_2 <: T_1' \to T_2'}$$

(s-record-width)

$$\{lab_1 : T_1, .., lab_k : T_k, lab_{k+1} : T_{k+1}, .., lab_{k+k'} : T_{k+k'}\}$$

$$<:$$

$$\{lab_1 : T_1, .., lab_k : T_k\}$$

$$\text{(s-record-depth)}\ \dfrac{T_1 <: T_1' \quad .. \quad T_k <: T_k'}{\{lab_1 : T_1, .., lab_k : T_k\} <: \{lab_1 : T_1', .., lab_k : T_k'\}}$$

(s-record-order)

$$\dfrac{\pi \text{ a permutation of } 1, .., k}{\{lab_1 : T_1, .., lab_k : T_k\} <: \{lab_{\pi(1)} : T_{\pi(1)}, .., lab_{\pi(k)} : T_{\pi(k)}\}}$$

[7]

(b) 1. One derivation goes as follows (there are others, for example using subsumption in the function body).

Let ∇ be

$$\dfrac{\dfrac{\overline{\mathsf{x{:}\{p{:}real\} \vdash x{:}\{p{:}real\}}}}{\mathsf{x{:}\{p{:}real\} \vdash \#px{:}real}}}{\mathsf{\{\} \vdash (\mathbf{fn}\ x{:}\{p{:}real\} \Rightarrow \#px){:}\{p{:}real\} \to real}}$$

in

$$\dfrac{\nabla \qquad \dfrac{\dfrac{\overline{\mathsf{\{\} \vdash 1{:}int}} \qquad \overline{\mathsf{\{\} \vdash \mathbf{true}{:}bool}}}{\mathsf{\{\} \vdash \{p = 1, q = \mathbf{true}\}{:}\{p{:}int, q{:}bool\}}} \qquad \mathsf{\{p{:}int, q{:}bool\} <: \{p{:}real\}}}{\mathsf{\{\} \vdash \{p = 1, q = \mathbf{true}\}{:}\{p{:}real\}}}}{\mathsf{\{\} \vdash (\mathbf{fn}\ x{:}\{p{:}int\} \Rightarrow \#px)\{p = 1, q = \mathbf{true}\}{:}real}}$$

where the subtype relationship is derived by:

$$\text{(tran)}\ \dfrac{\text{(rec-width)}\ \dfrac{}{\mathsf{\{p{:}int, q{:}bool\} <: \{p{:}int\}}} \qquad \text{(rec-depth)}\ \dfrac{\text{(num)}\ \dfrac{}{\mathsf{int <: real}}}{\mathsf{\{p{:}int\} <: \{p{:}real\}}}}{\mathsf{\{p{:}int, q{:}bool\} <: \{p{:}real\}}}$$

[5]

2. Not typable, as in the body of (**fn** x:{q:bool} ⇒ #p x) the x is not known to have a p field.[3]

3. Not typable, as the function expects an argument of type T = {r:{p:int, q:bool}} but is given a value only of type {r:{p:int}} – which is not a subtype of T. [3]

(c) real → int, real → real, int → int, int → real.[2]