SOLUTION NOTES

Complexity Theory 2003 Paper 6 Question 11 (AD)

1. An algorithm to decide k-clique could work by examining, for each set of k vertices of G whether they form a clique. If G contains n vertices, there are fewer than n^k such sets. Checking whether such a set forms a clique requires looking up fewer than k^2 pairs of vertices and checking whether there is an edge. Thus, the algorithm runs in time $O(k^2 n^k)$. Since k is fixed, this is polynomial time.

2. The proof proceeds by giving a reduction of the problem 3SAT to the problem IND. This reduction is involved but covered in detail in the lectures. This is then composed with the easy reduction from IND to Clique which simply takes a graph and complements it.

3. The factorisation function, which maps a number n to its prime factors is in the function class FNP of witness functions for languages in NP. It can be shown that if P=NP then there is a polynomial time algorithm for each problem in FNP. A proof of this fact, using binary search algorithms, is given in the lecture.