

CST IB 2000. Paper 6, q9

Semantics of Programming Languages

Two configurations are bisimilar iff they are related by some bisimulation, R — which is a relation $R \subseteq \text{Config} \times \text{Config}$ satisfying that whenever $c_1 R c_2$ then

$$\bullet c_1 \xrightarrow{\alpha} c'_1 \Rightarrow \exists c'_2. c_2 \xrightarrow{\hat{\alpha}} c'_2 \text{ \& } c'_1 R c'_2$$

$$\bullet c_2 \xrightarrow{\alpha} c'_2 \Rightarrow \exists c'_1. c_1 \xrightarrow{\hat{\alpha}} c'_1 \text{ \& } c'_1 R c'_2$$

where $\hat{\alpha} \triangleq \begin{cases} \tau^* & \text{if } \alpha = \tau \\ \tau^* \xrightarrow{\alpha} \tau^* & \text{if } \alpha \neq \tau. \end{cases}$

LTS for communicating processes:

configurations: process expressions with no free integer variables

actions: $\alpha ::= \tau \mid c(n) \mid \bar{c}(n) \quad \left(\begin{matrix} c \in \text{Chan}, \\ n \in \mathbb{Z} \end{matrix} \right).$

transitions: inductively defined by the following axioms & rules.

$$(\text{in}) \quad c(x).P \xrightarrow{c(n)} P[n/x]$$

$$(\text{out}) \quad \bar{c}(E).P \xrightarrow{\bar{c}(n)} P \quad \text{if } E \Downarrow n$$

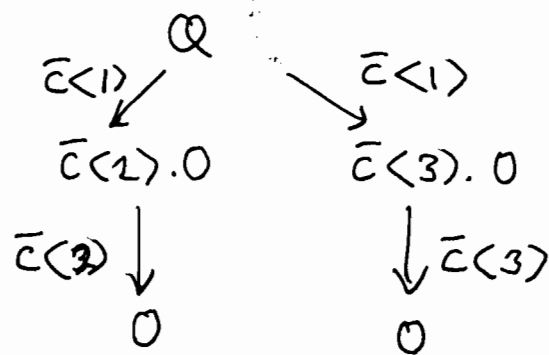
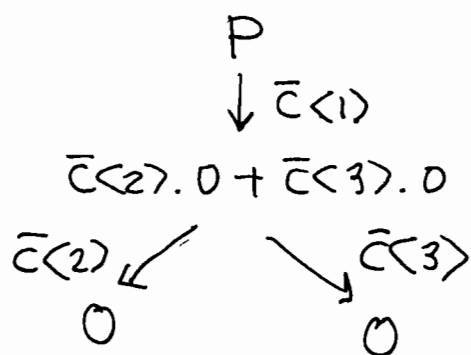
| | | |
|---|----------------|---|
| 1 | $(+)$ | $\frac{P_i \xrightarrow{\alpha} P'_i}{P_1 + P_2 \xrightarrow{\alpha} P'_i} \quad (i=1,2)$ |
| 1 | (par) | $\frac{P_1 \xrightarrow{\alpha} P'_1}{P_1 P_2 \xrightarrow{\alpha} P'_1 P_2} \quad \& \text{ Symmetrically}$ |
| 1 | (com) | $\frac{P_1 \xrightarrow{c(n)} P'_1 \quad P_2 \xrightarrow{\bar{c}(n)} P'_2}{P_1 P_2 \xrightarrow{\tau} P'_1 P'_2} \quad \& \text{ Symmetrically}$ |
| 1 | (v) | $\frac{P \xrightarrow{\alpha} P'}{v.c. P \xrightarrow{\alpha} v.c. P'} \quad \text{if } \alpha \neq c(n), \bar{c}(n) \text{ (any } n).$ |

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(a) Not bisimilar

$$\begin{cases} P \triangleq \bar{c}(1).(\bar{c}(2).0 + \bar{c}(3).0) \\ Q \triangleq (\bar{c}(1).\bar{c}(2).0) + (\bar{c}(1).\bar{c}(3).0) \end{cases}$$

First note that from the definition of \rightarrow , the only possible actions of P, Q and their descendants are:



If we had $P R Q$ for some bisimulation, since $P \xrightarrow{\bar{c}(1)} \bar{c}(2).0 + \bar{c}(3).0$, must have

either $(\bar{c}\langle 2 \rangle.0 + \bar{c}\langle 3 \rangle.0) R \bar{c}\langle 2 \rangle.0$

or $(\bar{c}\langle 2 \rangle.0 + \bar{c}\langle 3 \rangle.0) R \bar{c}\langle 3 \rangle.0$

In either case the LHS can do an action ($\bar{c}\langle 3 \rangle$ or $\bar{c}\langle 2 \rangle$ respectively) that cannot be matched by the RHS, contradicting the bisimulation property of R . So no such R exists, so P & Q are not bisimilar.

(b) Are bisimilar.

Consider

$$R \stackrel{\text{def}}{=} \left\{ (P, \nu c. (c(x).0 \mid \bar{c}\langle 1 \rangle.P)) \mid \begin{array}{l} c \text{ does not} \\ \text{occur in } P \end{array} \right\} \\ \cup \\ \left\{ (P', \nu c. (0 \mid P')) \mid \begin{array}{l} c \text{ does not occur} \\ \text{in } P' \end{array} \right\}$$

Suffices to show R a bisimulation.

This follows from easily verified facts that:

(1) only transition of $\nu c. (c(x).0 \mid \bar{c}\langle 1 \rangle.P)$ is

$$\cdot \xrightarrow{\tau} \nu c. (0 \mid P).$$

(2) if $P \xrightarrow{\alpha} P'$ and c does not occur in P , then $\alpha \neq c(n), \bar{c}\langle n \rangle$ and hence $\nu c. (0 \mid P) \xrightarrow{\alpha} \nu c. (0 \mid P')$.

(3) if $\nu c. (0 \mid P) \xrightarrow{\alpha} Q$ with c not in P , then $\alpha \neq c(n), \bar{c}\langle n \rangle$, $Q = \nu c. (0 \mid P')$ for some P' with $P \xrightarrow{\alpha} P'$.