Week 06 Lectures

Recap on Implementing Selection

Selection = `select * from` *R* `where` *C*

- yields a subset of *R* tuples satisfying condition *C*
- a very important (frequent) operation in relational databases

Types of selection determined by type of condition

- *one*: `select * from` *R* `where id=`*k*
- *pmr*: `select * from` *R* `where age=65`
- *rng*: `select * from` *R* `where age≥18 and age≤21`

Strategies for implementing selection efficiently

- arrangement of tuples in file (e.g. sorting, hashing)
- auxiliary data structures (e.g. indexes, signatures)

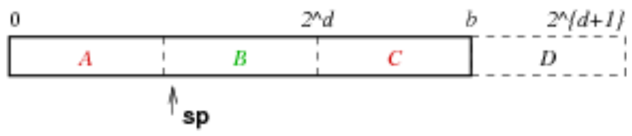
Linear Hashing

File organisation:

- file of primary data blocks
- file of overflow data blocks
- a register called the *split pointer*

Uses systematic method of growing data file ...

- hash function "adapts" to changing address range
- systematic splitting controls length of overflow chains



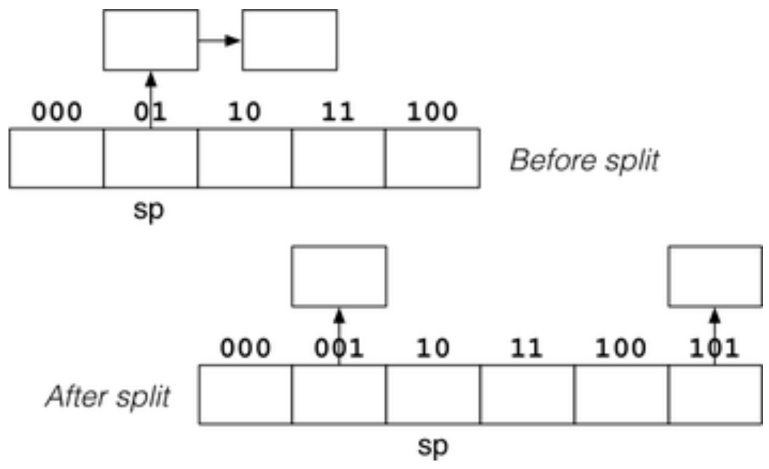
Insertion with Linear Hashing

Abstract view:

```
P = bits(d,hash(key));
if (P < sp) P = bits(d+1,hash(key));
// bucket P = page P + its overflow pages
for each page Q in bucket P {
    if (space in Q) { insert into Q; break; }
}
if (no insertion) {
    add new ovflow page to bucket P
    insert into new page
}
if (need to split) {
    partition tuples from bucket sp
          into buckets sp and sp+2^d
    sp++;
    if (sp == 2^d) { d++; sp = 0; }
}
```

... Insertion with Linear Hashing

Splitting block *sp*=`01`:



... Insertion with Linear Hashing

Splitting algorithm:

```
// partitions tuples between two buckets
newp = sp + 2^d; oldp = sp;
buf = getPage(f,sp);
clear(oldBuf); clear(newBuf);
// start filling data page buffers
for (i = 0; i < nTuples(buf); i++) {
    tup = getTuple(buf,i);
    p = bits(d+1,hash(tup.k));
    if (p == newp)
        addTuple(newBuf,tup);
    else
        addTuple(oldBuf,tup);
}
... remove and re-insert tuples from ovflow chain ...
sp++;
if (sp == 2^d) { d++; sp = 0; }
```

Insertion Cost

If no split required, cost same as for standard hashing:

$Cost_{insert}$ = Best: $1_r + 1_w$, Worst: $(1+max(Ov))_r + 2_w$

If split occurs, incur $Cost_{insert}$ plus cost of splitting:

- read block *sp* (plus all of its overflow blocks)
- write block *sp* (and its new overflow blocks)
- write block $sp+2^d$ (and its new overflow blocks)

On average, $Cost_{split} = (1+Ov)_r + (2+Ov)_w$

Deletion with Linear Hashing

Deletion is similar to ordinary static hash file.

But might wish to contract file when enough tuples removed.

Rationale: *r* shrinks, *b* stays large ⇒ wasted space.

Method: remove last bucket in data file (contracts linearly).

Involves a coalesce procedure which is an inverse split.

Hash Files in PostgreSQL

PostgreSQL uses linear hashing on tables which have been:

```
create index Ix on R using hash (k);
```

Hash file implementation: **`backend/access/hash`**

- **`hashfunc.c`** ... a family of hash functions
- **`hashinsert.c`** ... insert, with overflows
- **`hashpage.c`** ... utilities + splitting
- **`hashsearch.c`** ... iterator for hash files

Based on "A New Hashing Package for Unix", Margo Seltzer, Winter Usenix 1991

... Hash Files in PostgreSQL

PostgreSQL uses a different file organisation ...

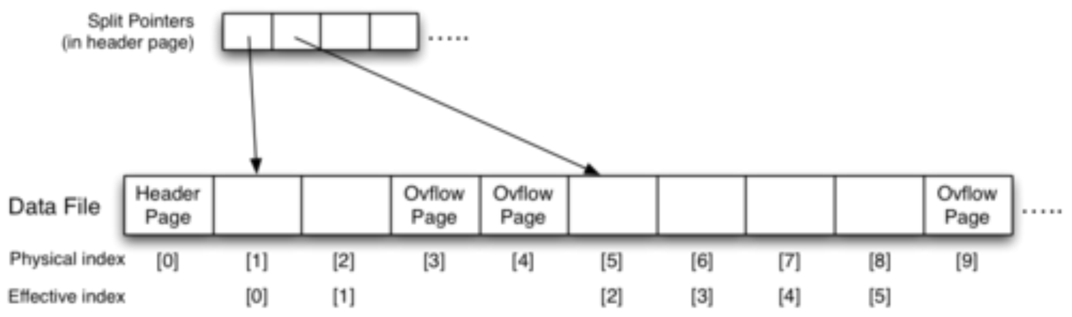
- has a single file containing main and overflow pages
- has *group*s of size 2^n of data pages
- in between groups, arbitrary number of overflow pages
- maintains collection of *group pointers* in header page
- each group pointer indicates start of data page group

Also maintain a split pointer for data pages.

If overflow pages become empty, add to free list and re-use.

... Hash Files in PostgreSQL

PostgreSQL hash file structure:



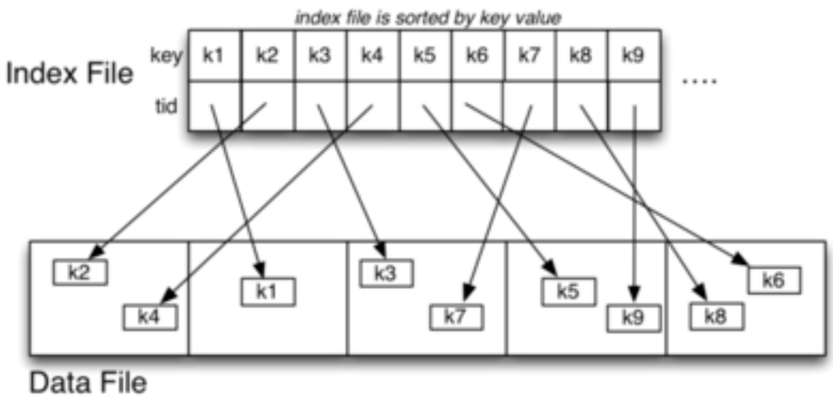
... Hash Files in PostgreSQL

Converting bucket # to page address (adapted from pgsql source):

```
typedef unsigned int Word;
// which page is primary page of bucket B
Word bucket_to_page(Word splits[], B) {
   Word chunk, base, offset;
   chunk = (B<2) ? 0 : lg2(B+1)-1;
   base = splits[chunk];
   offset = (B<2) ? B : B-(1<<chunk);
   return (base + offset);
}
// returns ceil(log_2(n))
int lg2(Word n) {
   int i, v;
   for (i = 0, v = 1; v < n; v <= 1) i++;
   return i;
}
```

Indexing

An index is a table/file of (keyVal,tupleID) pairs, e.g.


index file is sorted by key value
Index File
key
tid
k1 k2 k3 k4 k5 k6 k7 k8 k9
....
k2 k4 k1 k3 k7 k5 k9 k8 k6
Data File

Indexes

A 1-d *index* is based on the value of a single attribute *A*.

Some possible properties of *A*:

- may be used to sort data file (or may be sorted on some other field)
- values may be unique (or there may be multiple instances)

Taxonomy of index types, based on properties of index attribute:

primary	index on unique field, may be sorted on *A*
clustering	index on non-unique field, file sorted on *A*
secondary	file *not* sorted on *A*

A given table may have indexes on several attributes.

... Indexes

Indexes themselves may be structured in several ways:

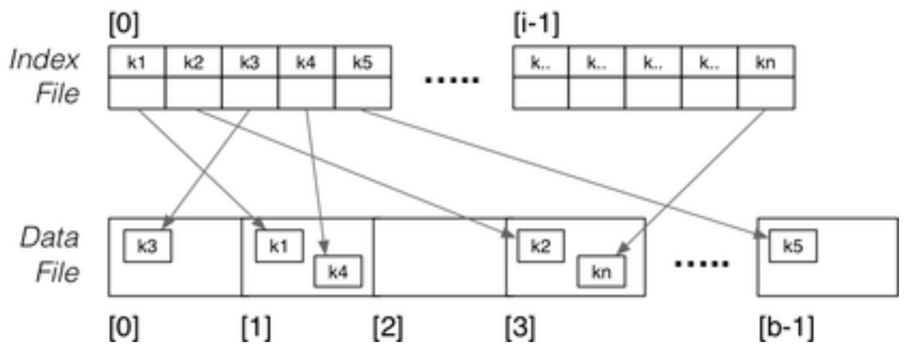
dense	every tuple is referenced by an entry in the index file
sparse	only some tuples are referenced by index file entries
single-level	tuples are accessed directly from the index file
multi-level	may need to access several index pages to reach tuple

Index file has total *i* pages (where typically $i \ll b$)

Index file has page capacity c_i (where typically $c_i \gg c$)

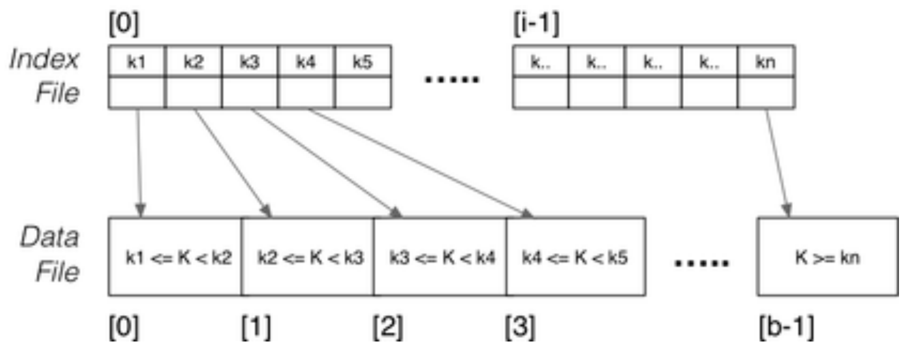
Dense index: $i = ceil(r/c_i)$ Sparse index: $i = ceil(b/c_i)$

Dense Primary Index



Data file unsorted; one index entry for each tuple

Sparse Primary Index



Data file sorted; one index entry for each page

Exercise 1: Index Storage Overheads

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Consider a relation with the following storage parameters:

- *B = 8192,   R = 128,   r = 100000*
- header in data pages: 256 bytes
- key is integer, data file is sorted on key
- index entries (keyVal,tupleID): 8 bytes
- header in index pages: 32 bytes

How many pages are needed to hold a dense index?

How many pages are needed to hold a sparse index?

Selection with Primary Index

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For *one* queries:

```
ix = binary search index for entry with key K
if nothing found { return NotFound }
b = getPage(pageOf(ix.tid))
t = getTuple(b,offsetOf(ix.tid))
   -- may require reading overflow pages
return t
```

Worst case: read $log_2 i$ index pages + read $1+Ov$ data pages.

Thus, $Cost_{one,prim} = log_2 i + 1 + Ov$

Assume: index pages are same size as data pages ⇒ same reading cost

... Selection with Primary Index

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For *range* queries on primary key:

- use index search to find lower bound
- read index sequentially until reach upper bound
- accumulate set of buckets to be examined
- examine each bucket in turn to check for matches

For *pmr* queries involving primary key:

- search as if performing *one* query.

For queries not involving primary key, index gives no help.

Exercise 2: Selection with Primary Index

Consider a range query like

```
select * from R where a between 10 and 30;
```

Give a detailed algorithm for solving such range queries

- assume table is indexed on attribute `a`
- assume file is *not* sorted on `a`
- assume existence of `Set` data type:
  `s=empty(); insert(s, n); foreach elems(s)`
- assume "the usual" operations on relations:
  `r = openRelation(name,mode); b=nPages(r); file(r)`
- assume "the usual" operations on pages:
  `buf=getPage(f,pid); foreach tuples(buf); pid = next(buf)`

Insertion with Primary Index

Overview:

```
insert tuple into page P
find location for new entry in index file
   // could check whether it already exists
insert new index entry (k,tid) into index file
   // tid = tupleID = (P + offset within page)
```

Problem: order of index entries must be maintained

- need to avoid overflow pages in index
- so we need to reorganise index file

On average, this requires us to read/write half of index file.

$Cost_{insert,prim} \; = \; (log_2 i)_r + i/2.(1_r + 1_w) + (1+Ov)_r + (1+\delta)_w$

Deletion with Primary Index

Overview:

```
find tuple using index
mark tuple as deleted
delete index entry for tuple
```

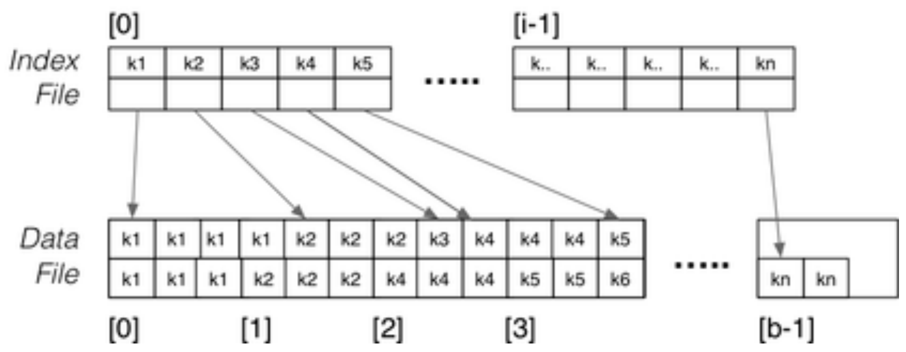
If we delete index entries by marking ...

- $Cost_{delete,prim} \; = \; (log_2 \, i + 1 + Ov)_r + 2_w$

If we delete index entry by index file reorganisation ...

- $Cost_{delete,prim} \; = \; (log_2 \, i + 1 + Ov)_r + i/2.(1_r + 1_w) + 1_w$

Clustering Index



Data file sorted; one index entry for each key value

... Clustering Index

Index on non-unique ordering attribute A_c.

Usually a sparse index; one pointer to first tuple containing value.

Assists with:

- *range* queries on A_c (find lower bound, then scan data)
- *pmr* queries involving A_c (search index for specified value)

Insertions are expensive: rearrange index file and data file.

Deletions relatively cheap (similar to primary index).

(Note: can't mark index entry for value *X* until all *X* tuples are deleted)

Secondary Index

Generally, dense index on non-unique attribute A_s

- data file is not ordered on attribute A_s
- index file *is* ordered on attribute A_s

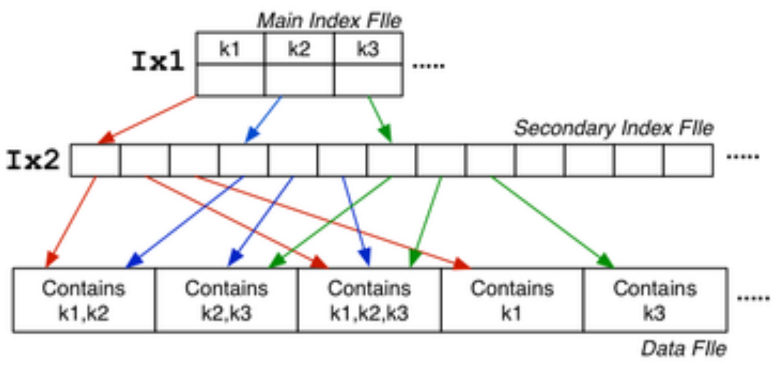
Problem: multiple tuples with same value for A_s.

A solution:

- dense index (`Ix2`) containing just `TupleId`'s
- sparse index (`Ix1`) on dense index containing *(key,offset)* pairs

Each *offset* references an entry in `Ix2`

... Secondary Index



$Cost_{pmr} = Cost_{range} = (log_2 i + a_{q_2} + b_q \cdot (1 + Ov))$
$Cost_{range} = (log_2 i + a_{q_1} + a_{q_2} + b_q \cdot (1 + Ov))$

Insertion/Deletion with Secondary Index

Insertion:

- each insert requires three files to be updated
- potentially costly rearrangement of index files

Deletion:

- use mark-style (tombstone) deletion for data tuples
- `Ix2` entries: can always mark as "deleted"
- `Ix1` entries: mark only after removing last instance for *k* in `Ix2`
- periodic "vacuum" to reduce storage overhead if many deletions

---

Multi-level Indexes

Above Secondary Index used two index files to speed up search

- by keeping the initial index search relatively quick
- `Ix1` small (depends on number of unique key values)
- `Ix2` larger (depends on amount of repetition of keys)
- typically, $b_{Ix1} \ll b_{Ix2}$

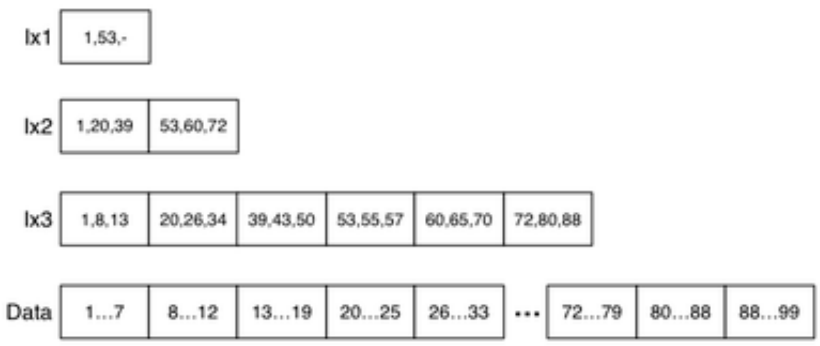
Could improve further by

- making `Ix1` sparse, since `Ix2` is guaranteed to be ordered
- in this case, $b_{Ix1} = ceil(b_{Ix2} / c_i)$
- if `Ix1` becomes too large, add `Ix3` and make `Ix2` sparse
- if data file ordered on key, could make `Ix3` sparse

Ultimately, reduce top-level of index hierarchy to one page.

... Multi-level Indexes

Example data file with three-levels of index:



Assume: not primary key, $c = 100$, $c_i = 3$

Select with Multi-level Index

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For *one* query on indexed key field:

```
I = top level index page
for level = 1 to d {
    read index page I
    search index page for J'th entry
        where index[J].key <= K < index[J+1].key
    if J=0 { return NotFound }
    I = index[J].page
}
-- I is now address of data page
search page I and its overflow pages
```

Read d index blocks and $1+Ov$ data blocks.

Thus, $Cost_{one,mli} = (d + 1 + Ov)_r$

(Note that $d = ceil(log_{c_i} r)$ and c_i is large because index entries are small)

B-Trees

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B-trees are MSTs with the properties:

- they are updated so as to remain balanced
- each node has at least $(n-1)/2$ entries in it
- each tree node occupies an entire disk page

B-tree insertion and deletion methods

- are moderately complicated to describe
- can be implemented very efficiently

Advantages of B-trees over general MSTs

- better storage utilisation (around 2/3 full)
- better worst case performance (shallower)

... B-Trees

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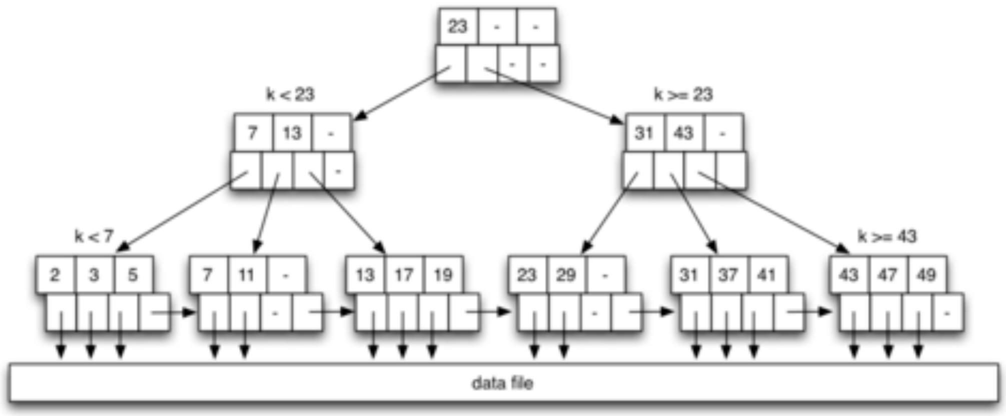
Example B-tree (depth=3, n=3):



(Note that nodes are pages, with potential for large branching factor, e.g. *n=500*)

B-Tree Depth

Depth depends on effective branching factor (i.e. how full nodes are).

Simulation studies show typical B-tree nodes are 69% full.

Gives load $L_i = 0.69 \times c_i$ and depth of tree ~ $ceil(log_{L_i} r)$.

Example: $c_i=128$, $L_i=88$

Level	#nodes	#keys
root	1	87
1	88	7656
2	7744	673728
3	681472	59288064

Note: c_i is generally larger than 128 for a real B-tree.

Insertion into B-Trees

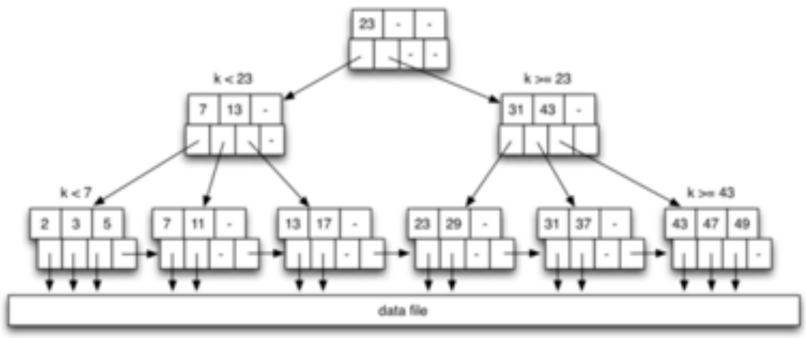
Overview of the method:

1. find leaf node and position in node where entry would be stored
2. if node is not full, insert entry into appropriate spot
3. if node is full, split node into two half-full nodes
   and promote middle element to parent
4. if parent full, split and promote

Note: if duplicates not allowed and key is found, may stop after step 1.

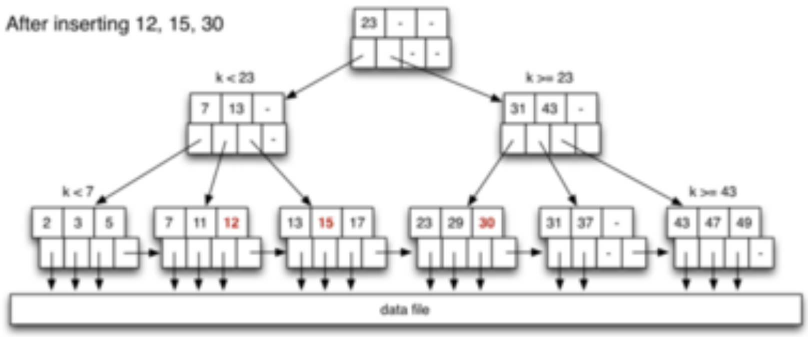
Example: B-tree Insertion

Starting from this tree:

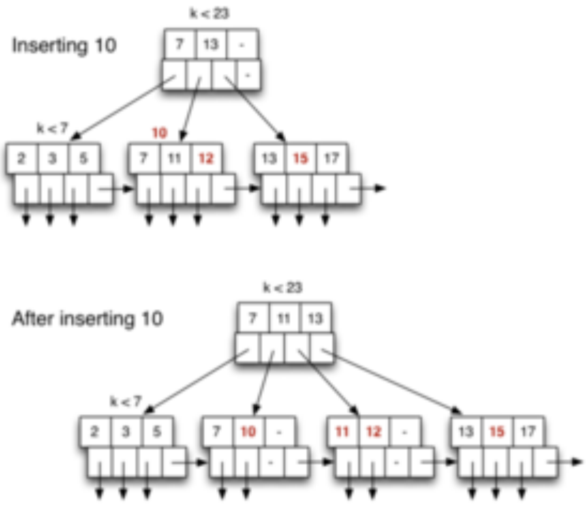


insert the following keys in the given order `12 15 30 10`

... Example: B-tree Insertion



... Example: B-tree Insertion



B-Tree Insertion Cost

Insertion cost = $Cost_{treeSearch} + Cost_{treeInsert} + Cost_{dataInsert}$

Best case: write one page (most of time)

- traverse from root to leaf
- read/write data page, write updated leaf

$Cost_{insert} = D_r + 1_w + 1_r + 1_w$

Common case: *3* node writes (rearrange 2 leaves + parent)

- traverse from root to leaf, holding nodes in buffer
- read/write data page
- update/write leaf, parent and sibling

$Cost_{insert} = D_r + 3_w + 1_r + 1_w$

... B-Tree Insertion Cost

Worst case: *2D-1* node writes (propagate to root)

- traverse from root to leaf, holding nodes in buffers
- read/write data page
- update/write leaf, parent and sibling
- repeat previous step *D-1* times

$Cost_{insert} = D_r + (2D-1)_w + 1_r + 1_w$

Selection with B-Trees

For *one* queries:

```
N = B-tree root node
while (N is not a leaf node)
   N = scanToFindChild(N,K)
TupleID = scanToFindEntry(N,K)
access tuple t using TupleID from N
```

$Cost_{one} = (D + 1)_r$

For *range* queries (assume sorted on index attribute):

```
search index to find leaf node for Lo
for each leaf node entry until Hi found {
        access tuple t using TupleId from entry
}
```

$Cost_{range} = (D + b_i + b_q)_r$

B-trees in PostgreSQL

PostgreSQL implements Lehman/Yao-style B-trees.

A variant that works effectively in high-concurrency environments.

B-tree implementation: **`backend/access/nbtree`**

- **`nbtree.c`** ... interface functions (for iterators)
- **`nbtsearch.c`** ... traverse index to find key value

- **nbtinsert.c** ... add new entry to B-tree index

... B-trees in PostgreSQL

Interface functions for B-trees

```
// build Btree index on relation
Datum btbuild(rel,index,...)
// insert index entry into Btree
Datum btinsert(rel,key,tupleid,index,...)
// start scan on Btree index
Datum btbeginscan(rel,key,scandesc,...)
// get next tuple in a scan
Datum btgettuple(scandesc,scandir,...)
// close down a scan
Datum btendscan(scandesc)
```

N-dimensional Selection

N-dimensional Queries

Have looked at one-dimensional queries, e.g.

```
select * from R where a = K
select * from R where a between Lo and Hi
```

and *heaps*, *hashing*, *indexing* as ways of efficient implementation.

Now consider techniques for efficient *multi-dimensional* queries.

Compared to 1-d queries, multi-dimensional queries

- typically produce fewer results
- require us to consider more information
- require more effort to produce results

Operations for Nd Select

N-dimensional select queries = condition on ≥*1* attributes.

- *pmr* = partial-match retrieval (equality tests), e.g.

  ```
  select * from Employees
  where  job = 'Manager' and gender = 'M';
  ```

- *space* = tuple-space queries (range tests), e.g.

  ```
  select * from Employees
  where 20 ≤ age ≤ 50 and 40K ≤ salary ≤ 60K
  ```

N-d Selection via Heaps

Heap files can handle *pmr* or *space* using standard method:

```
// select * from R where C
r = openRelation("R",READ);
for (p = 0; p < nPages(r); p++) {
    buf = getPage(file(r), p);
    for (i = 0; i < nTuples(buf); i++) {
        t = getTuple(buf,i);
        if (matches(t,C))
            add t to result set
    }
}
```

$Cost_{pmr} = Cost_{space} = b$

N-d Selection via Multiple Indexes

DBMSs already support building multiple indexes on a table.

Which indexes to build depends on which queries are asked.

```
create table R (a int, b int, c int);
create index Rax on R (a);
create index Rbx on R (b);
create index Rcx on R (c);
create index Rabx on R (a,b);
create index Racx on R (a,c);
create index Rbcx on R (b,c);
create index Rallx on R (a,b,c);
```

But more indexes ⇒ space + update overheads.

N-d Queries and Indexes

Generalised view of *pmr* and *space* queries:

```
select * from R
where  a1 op1 C1 and ... and an opn Cn
```

pmr : all op_i are equality tests. *space* : some op_i are range tests.

Possible approaches to handling such queries ...

1. use index on one a_i to reduce tuple tests
2. use indexes on all a_i, and intersect answer sets

... N-d Queries and Indexes

If using just *one* of several indexes, *which one* to use?

```
select * from R
where  a1 op1 C1 and ... and an opn Cn
```

The one with best *selectivity* for $a_i\ op_i\ C_i$ (i.e. fewest matches)

Factors determining selectivity of $a_i\ op_i\ C_i$

- assume uniform distribution of values in *dom(a_i)*
- equality test on primary key gives at most one match
- equality test on larger *dom(a_i)* ⇒ less matches
- range test over large part of *dom(a_i)* ⇒ many matches

... N-d Queries and Indexes

Implementing selection using *one of several* indices:

```
// Query: select * from R where $a_1op_1C_1$ and ... and $a_nop_nC_n$
// choose $a_i$ with best selectivity
TupleIDs = IndexLookup(R,$a_i$,$op_i$,$C_i$)
// gives { $tid_1$, $tid_2$, ...} for tuples satisfying $a_iop_iC_i$
PageIDs = { }
foreach tid in TupleIDs
   { PageIDs = PageIDs ∪ {pageOf(tid)} }

// PageIDs = a set of $b_{q_{ix}}$ page numbers
...
```

Cost = $Cost_{index} + b_{q_{ix}}$ (some pages do not contain answers, $b_{q_{ix}} > b_q$)

DBMSs typically maintain statistics to assist with determining selectivity

---

... N-d Queries and Indexes

Implementing selection using *multiple* indices:

```
// Query: select * from R where $a_1op_1C_1$ and ... and $a_nop_nC_n$
// assumes an index on at least $a_i$
TupleIDs = IndexLookup(R,$a_1$,$op_1$,$C_1$)
foreach attribute $a_i$ with an index {
   tids = IndexLookup(R,$a_i$,$op_i$,$C_i$)
   TupleIDs = TupleIDs ∩ tids
}
PageIDs = { }
foreach tid in TupleIDs
   { PageIDs = PageIDs ∪ {pageOf(tid)} }
// PageIDs = a set of $b_q$ page numbers
...
```

Cost = $k.Cost_{index} + b_q$ (assuming indexes on *k* of *n* attrs)

Exercise 3: One vs Multiple Indices

Consider a relation with *r* = 100,000, *B* = 4K, defined as:

```
create table Students (
    id       integer primary key,
    name     char(10), -- simplified
    gender   char(1)   -- 'm' or 'f',
    birthday date      -- 1980 .. 2000
);
... and a query on this relation ...
```

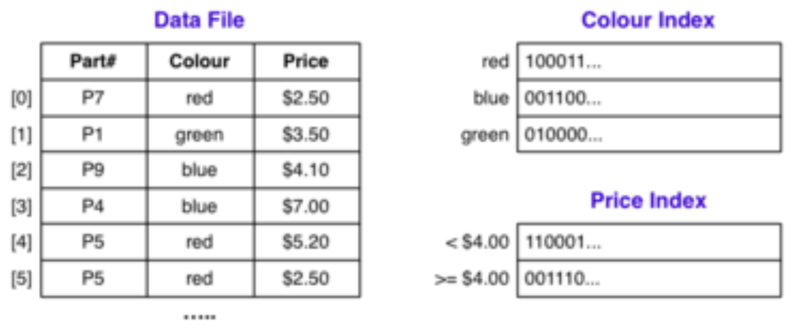
```
select * from Students
where  gender='m' and birthday='YYYY-02-29'
```

which has a B-tree index on each attribute ...

- describe the selectivity of each attribute
- estimate the cost of answering using one index
- estimate the cost of answering using both indices

Bitmap Indexes

Alternative index structure, focussing on sets of tuples:



Data File

	Part#	Colour	Price
[0]	P7	red	$2.50
[1]	P1	green	$3.50
[2]	P9	blue	$4.10
[3]	P4	blue	$7.00
[4]	P5	red	$5.20
[5]	P5	red	$2.50
	.....		

Colour Index

red	100011...
blue	001100...
green	010000...

Price Index

< $4.00	110001...
>= $4.00	001110...

Index contains bit-strings of *r* bits, one for each value/range

---

... Bitmap Indexes

Answering queries using bitmap index:

```
Matches = AllOnes(r)
foreach attribute A with index {
   // select ith bit-string for attribute A
   // based on value associated with A in WHERE
   Matches = Matches & Bitmaps[A][i]
}
// Matches contains 1-bit for each matching tuple
foreach i in 0..r {
   if (Matches[i] == 0) continue;
   t = fetchTuple(i)
   Results = Results ∪ {t}
}
```

... Bitmap Indexes

Storage costs for bitmap indexes:

- one bitmap for each value/range for each indexed attribute
- each bitmap has length *ceil(r/8)* bytes
- e.g. with 50K records and 8KB pages, bitmap fits in one page

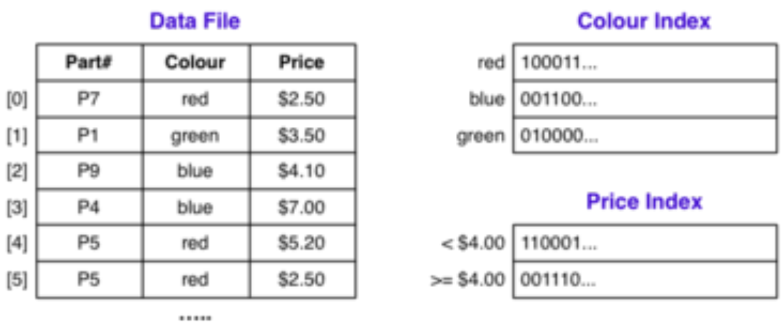
Query execution costs for bitmap indexes:

- read one bitmap for each indexed attribute in query
- perform bitwise AND on bitmaps (in memory)
- read pages containing matching tuples

Note: bitmaps could index pages (shorter bitmaps, more comparisons)

Exercise 4: Bitmap Index

Using the following file structure:



Data File

	Part#	Colour	Price
[0]	P7	red	$2.50
[1]	P1	green	$3.50
[2]	P9	blue	$4.10
[3]	P4	blue	$7.00
[4]	P5	red	$5.20
[5]	P5	red	$2.50

.....

Colour Index

red	100011...
blue	001100...
green	010000...

Price Index

< $4.00	110001...
>= $4.00	001110...

Show how the following queries would be answered:

```
select * from Parts
where colour='red' and price < 4.00

select * from Parts
where colour='green' or colour ='blue'
```

Hashing for N-d Selection

Hashing and *pmr*

For a *pmr* query like

select * from R where a_1 = C_1 and ... and a_n = C_n

- if one a_i is the hash key, query is very efficient
- if no a_i is the hash key, need to use linear scan

Can be alleviated using *multi-attribute hashing* (*mah*)

- form a composite hash value involving all attributes
- at query time, some components of composite hash are known
  (allows us to limit the number of data pages which need to be checked)

MA.hashing works in conjunction with any dynamic hash scheme.

... Hashing and *pmr*

Multi-attribute hashing parameters:

- file size = $b = 2^d$ pages $\Rightarrow$ use d-bit hash values
- relation has n attributes: $a_1, a_2, \ldots a_n$
- attribute a_i has hash function h_i

- attribute a_i contributes d_i bits (to the combined hash value)
- total bits $d = \sum_{i=1}^{n} d_i$
- a *choice vector* (*cv*) specifies for all *k* ...
  bit *j* from $h_i(a_i)$ contributes bit *k* in combined hash value

MA.Hashing Example

Consider relation `Deposit(branch,acctNo,name,amount)`

Assume a small data file with 8 main data pages (plus overflows).

Hash parameters: $d=3$ $d_1=1$ $d_2=1$ $d_3=1$ $d_4=0$

Note that we ignore the `amount` attribute ($d_4=0$)

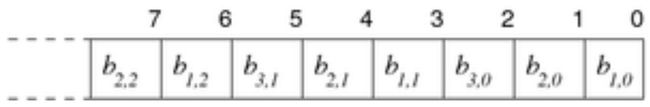
Assumes that nobody will want to ask queries like

```
select * from Deposit where amount=533
```

Choice vector is designed taking expected queries into account.

... MA.Hashing Example

Choice vector:


7 6 5 4 3 2 1 0
$b_{2,2}$ $b_{1,2}$ $b_{3,1}$ $b_{2,1}$ $b_{1,1}$ $b_{3,0}$ $b_{2,0}$ $b_{1,0}$

This choice vector tells us:

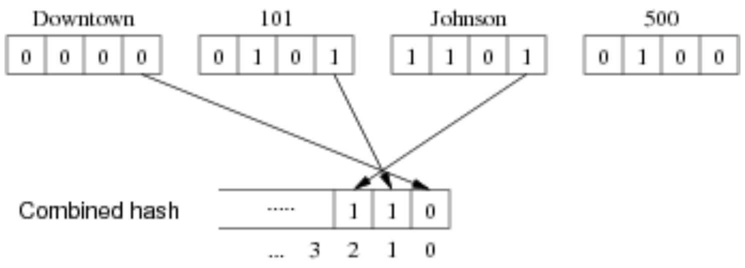
- bit 0 in hash comes from bit 0 of $hash_1(a_1)$ ($b_{1,0}$)
- bit 1 in hash comes from bit 0 of $hash_2(a_2)$ ($b_{2,0}$)
- bit 2 in hash comes from bit 0 of $hash_3(a_3)$ ($b_{3,0}$)
- bit 3 in hash comes from bit 1 of $hash_1(a_1)$ ($b_{1,1}$)
- etc. etc. etc. (up to as many bits of hashing as required, e.g. 32)

... MA.Hashing Example

Consider the tuple:

branch	acctNo	name	amount
Downtown	101	Johnston	512

Hash value (page address) is computed by:



MA.Hashing Hash Functions

Auxiliary definitions:

```
#define MaxHashSize 32
typedef unsigned int HashVal;

// extracts i'th bit from hash value
#define bit(i,h) (((h) & (1 << (i))) >> (i))

// choice vector elems
typedef struct { int attr, int bit } CVelem;
typedef CVelem ChoiceVec[MaxHashSize];

// hash function for individual attributes
HashVal hash1(Tuple t, int i) { ... }
```

... MA.Hashing Hash Functions

Produce combined *d*-bit hash value for tuple *t*:

```
HashVal hash(Tuple t, ChoiceVec cv, int d)
{
    HashVal h[nAttr(t)+1];  // hash for each attr
    HashVal res = 0, oneBit;
    int     i, a, b;

    for (i = 1; i <= nAttr(t); i++)
        h[i] = hash1(t,i);
    for (i = 0; i < d; i++) {
        a = cv[i].attr;
        b = cv[i].bit;
        oneBit = bit(b, h[a]);
        res = res | (oneBit << i);
    }
    return res;
}
```

Exercise 5: Multi-attribute Hashing

Compute the hash value for the tuple

```
('John Smith','BSc(CompSci)',1990,99.5)
```

where $d=6$, $d_1=3$, $d_2=2$, $d_3=1$, and

- *cv = <(1,0), (1,1), (2,0), (3,0), (1,2), (2,1), (3,1), (1,3), ...>*
- $hash_1$(`'John Smith'`) = `...0101010110110100`
- $hash_2$(`'BSc(CompSci)'`) = `...1011111101101111`
- $hash_3$(`1990`) = `...0001001011000000`

Queries with MA.Hashing

In a partial match query:

- values of some attributes are known
- values of other attributes are unknown

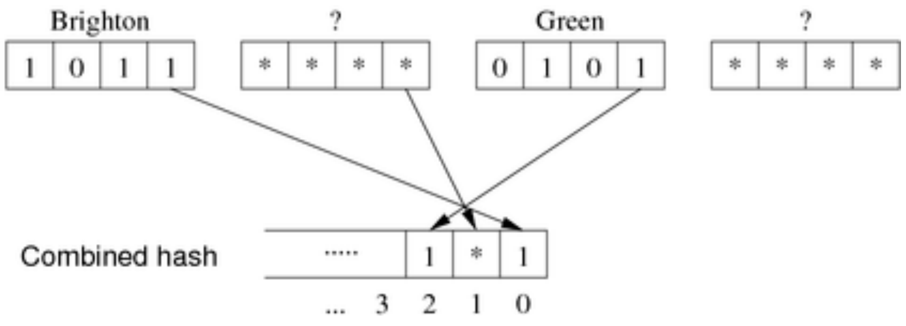
E.g.

```
select  amount
from    Deposit
where   branch = 'Brighton' and name = 'Green'
```

for which we use the shorthand `(Brighton, ?, Green, ?)`

... Queries with MA.Hashing

In composite hash for query, values for some bits are unknown:

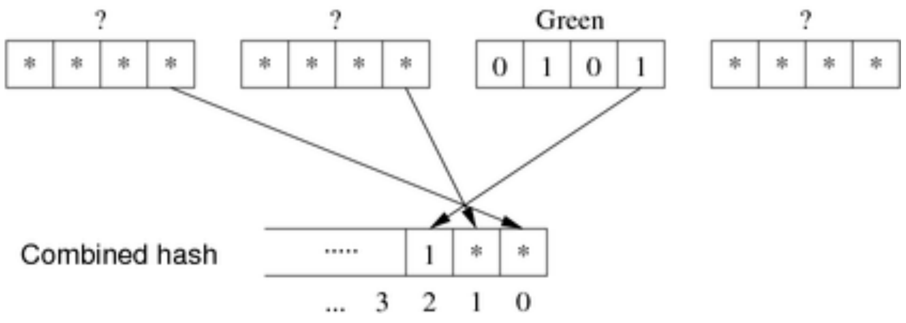


What this tells us: any matching tuples *must* be in pages `101`, `111`

... Queries with MA.Hashing

Consider the query:

```
select amount from Deposit where name = 'Green'
```



Need to check pages: `100`, `101`, `110`, `111`.

Exercise 6: Partial hash values in MAH

Given the following:

- d=6, b=2^6, CV = <(0,0),(0,1),(1,0),(2,0),(1,1),(0,2), ...>
- hash (a) = ...00101101001101
- hash (b) = ...00101101001101
- hash (c) = ...00101101001101

What are the query hashes for each of the following:

- (a,b,c), (?,b,c), (a,?,?), (?,?,?)

MA.Hashing Query Algorithm

```
// Builds the partial hash value (e.g. 10*0*1)
// Treats query like tuple with some attr values missing
nstars = 0;
for each attribute i in query Q {
    if (hasValue(Q,i)) {
        set d[i] bits in composite hash
            using choice vector and hash(Q,i)
    } else {
        set d[i] *'s in composite hash
            using choice vector
        nstars++;
    }
}
...
```

... MA.Hashing Query Algorithm

```
...
// Use the partial hash to find candidate pages

r = openRelation("R",READ);
for (i = 0; i < 2**nstars; i++) {
    P = composite hash
    replace *'s in P
        using i and choice vector
    Buf = readPage(file(r), P);
    for each tuple T in Buf {
        if (T satisfies pmr query)
            add T to results
    }
}
```

Exercise 7: Representing Stars

Our hash values are bit-strings (e.g. `0100101110101`)

MA.Hashing introduces a third value (`*` = unknown)

How could we represent "bit"-strings like `01011*1*0**010`?

Exercise 8: MA.Hashing Query Cost

Consider `R(x,y,z)` using multi-attribute hashing where

$d = 9$ $d_x = 5$ $d_y = 3$ $d_z = 1$

How many buckets are accessed in answering each query?

1. `select * from R where x = 4 and y = 2 and z = 1`
2. `select * from R where x = 5 and y = 3`
3. `select * from R where y = 99`

4. select * from R where z = 23
5. select * from R where x > 5

Query Cost for MA.Hashing

Multi-attribute hashing handles a range of query types, e.g.

```
select * from R where a=1
select * from R where d=2
select * from R where b=3 and c=4
select * from R where a=5 and b=6 and c=7
```

A relation with *n* attributes has 2^n different query types.

Different query types have different costs (different no. of *'s)

Query distribution gives probability p_Q of asking each query type *Q*.

... Query Cost for MA.Hashing

For a relation `R(a,b,c,d)` ...

```
select * from R where a=1
-- has 1 specified attribute (a)
-- has 3 unspecified attributes (b,c,d)

select * from R where b=5 and d=2
-- has 2 specified attributes (b,d)
-- has 2 unspecified attributes (a,c)

select * from R
where a=1 and b=5 and c=3 and d=2
-- has 4 specified attributes (a,b,c,d)
-- has 0 unspecified attributes
```

... Query Cost for MA.Hashing

Consider a query of type *Q* with *m* attributes unspecified.

Each unspecified A_i contributes d_i *'s.

Total number of *'s is $s = \Sigma_{i \notin Q} d_i$.

⇒ Number of pages to read is $2^s = \Pi_{i \notin Q} 2^{d_i}$.

Ignoring overflows, $Cost(Q) = 2^s$ (where *s* is determined by *Q*)

Including overflows, $Cost(Q) = 2^s(1+Ov)$

... Query Cost for MA.Hashing

Min query cost occurs when all attributes are used in query

$Min\ Cost_{pmr} = 1$

Max query cost occurs when no attributes are specified

$Max\ Cost_{pmr} = 2^d = b$

Average cost is given by weighted sum over all query types:

$Avg\ Cost_{pmr} = \Sigma_Q\, p_Q\, \Pi_{i \notin Q}\, 2^{d_i}$

Aim to minimise the weighted average query cost over possible query types

---

Optimising MA.Hashing Cost

For a given application, useful to minimise $Cost_{pmr}$.

Can be achieved by choosing appropriate values for d_i (*cv*)

Heuristics:

- distribution of query types (more bits to frequently used attributes)
- size of attribute domain (≤ #bits to represent all values in domain)
- discriminatory power (more bits to highly discriminating attributes)

Trade-off: making query type Q_j more efficient makes Q_k less efficient.

This is a combinatorial optimisation problem, and can be handled by standard optimisation techniques e.g. simulated annealing.

MA.Hashing Cost Example

Consider a table with four attributes:

(branch, account, name, amount) (abbreviated to *(br,ac,nm,amt)*)

Possible query types, and likelihood of each:

Query type	**Cost**	p_Q
(?, ?, ?, ?)	8	0
(*br*, ?, ?, ?)	4	0.25
(?, *ac*, ?, ?)	4	0
(*br*, *ac*, ?, ?)	2	0
(?, ?, *nm*, ?)	4	0
(*br*, ?, *nm*, ?)	2	0
(?, *ac*, *nm*, ?)	2	0.25
(*br*, *ac*, *nm*, ?)	1	0
(?, ?, ?, *amt*)	8	0
(*br*, ?, ?, *amt*)	4	0
(?, *ac*, ?, *amt*)	4	0
(*br*, *ac*, ?, *amt*)	2	0
(?, ?, *nm*, *amt*)	4	0
(*br*, ?, *nm*, *amt*)	2	0.5

(?, *ac*, *nm*, *amt*)	2	0
(*br*, *ac*, *nm*, *amt*)	1	0

Cost values are based on choice vector $(d_{br} = d_{ac} = d_{nm} = 1)$
p_Q values can be determined by observation of DB use.

... MA.Hashing Cost Example

Consider $r=10^6$, $N_r=100$, $b=10^4$, $d=14$.

Attribute *br* occurs in 0.5+0.25 used query types
⇒ allocate many bits to it e.g. $d_1=6$.

Attribute *nm* occurs in 0.5+0.25 of queries
⇒ allocate many bits to it e.g. $d_3=4$.

Attribute *amt* occurs in 0.5 of queries
⇒ allocate less bits to it e.g. $d_4=2$.

Attribute *ac* occurs in 0.25 of queries
⇒ allocate least bits to it e.g. $d_2=2$.

... MA.Hashing Cost Example

With bits distributed as: $d_1=6, d_2=2, d_3=4, d_4=2$

Query type	**Cost**	p_Q
(*br*, ?, ?, ?)	$2^8 = 256$	0.25
(?, *ac*, *nm*, ?)	$2^8 = 256$	0.25
(*br*, ?, *nm*, *amt*)	$2^2 = 4$	0.5

$Cost \ = \ 0.5 \times 2^2 + 0.25 \times 2^8 + 0.25 \times 2^8 \ = \ 130$

Exercise 9: MA.Hashing Design

Consider relation `Person(name,gender,age)` with *b=32* and ...

p_Q	Query Type *Q*

0.5	`select name from Person`
	`where gender=X and age=Y`

0.25	`select age from Person`
	`where name=X`

0.25	`select name from Person`
	`where gender=X`

Assume that all other query types have $p_Q=0$.

Design a choice vector to minimise average selection cost.

Tree Indexes for N-d Selection

Multi-dimensional Tree Indexes

Over the last 20 years, from a range of problem areas

- different multi-d tree index schemes have been proposed
- varying primarily in how they partition tuple-space

Consider three popular schemes: kd-trees, Quad-trees, R-trees.

Example data for multi-d trees is based on the following relation:

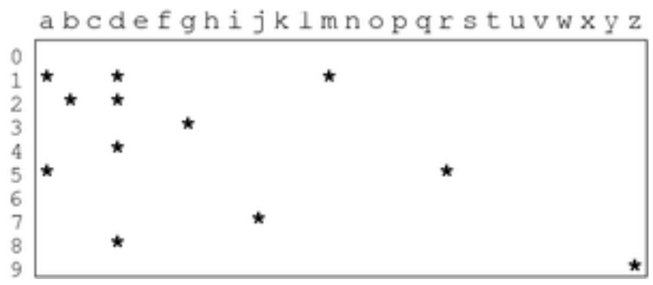
```
create table Rel (
    X char(1) check (X between 'a' and 'z'),
    Y integer check (Y between 0 and 9)
);
```

... Multi-dimensional Tree Indexes

Example tuples:

```
Rel('a',1)  Rel('a',5)  Rel('b',2)  Rel('d',1)
Rel('d',2)  Rel('d',4)  Rel('d',8)  Rel('g',3)
Rel('j',7)  Rel('m',1)  Rel('r',5)  Rel('z',9)
```

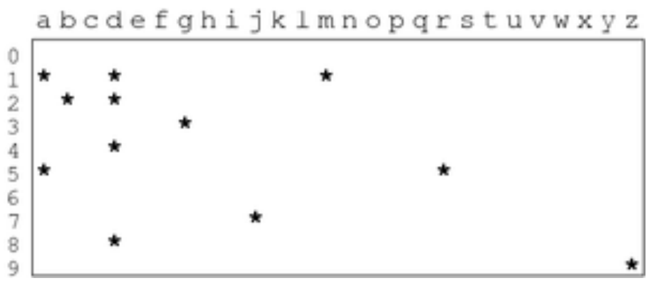
The tuple-space for the above tuples:



Exercise 10: Query Types and Tuple Space

Which part of the tuple-space does each query represent?

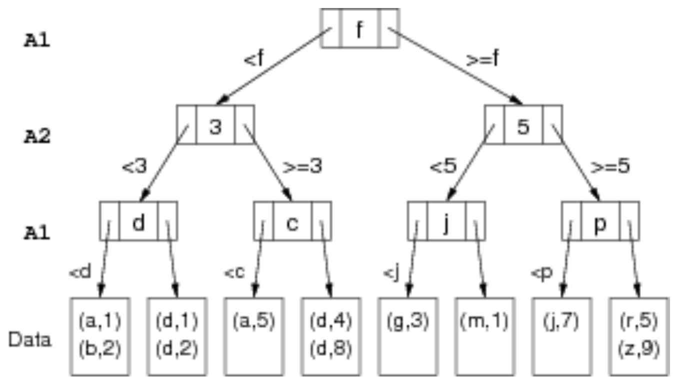
```
Q1: select * from Rel where X = 'd' and Y = 4
Q2: select * from Rel where 'j' < X ≤ 'r'
Q3: select * from Rel where X > 'm' and Y > 4
Q4: select * from Rel where 'k' ≤ X ≤ 'p' and 3 ≤ Y ≤ 6
```



kd-Trees

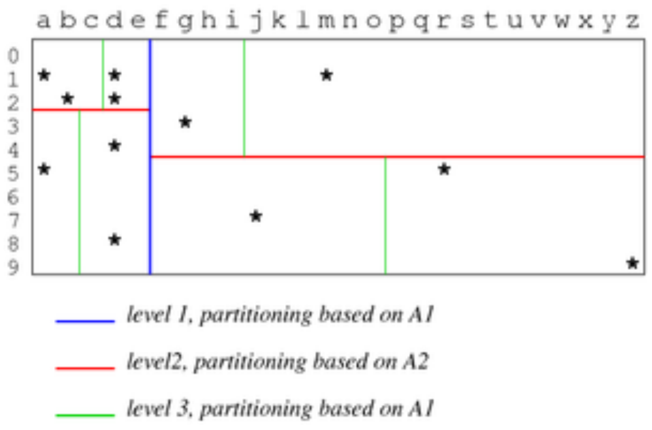
kd-trees are multi-way search trees where

- each level of the tree partitions on a different attribute
- each node contains *n-1* key values, pointers to *n* subtrees



... kd-Trees

How this tree partitions the tuple space:

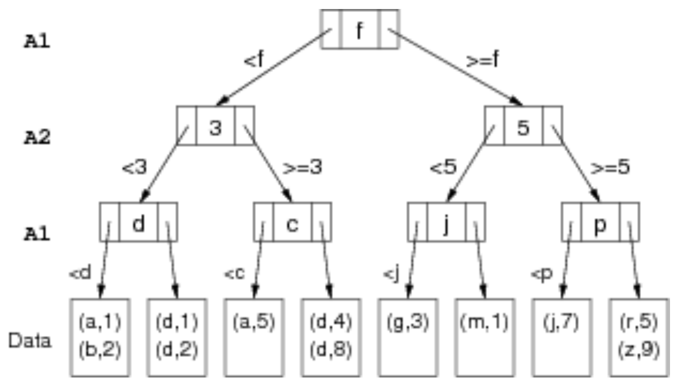


Searching in kd-Trees

```
// Started by Search(Q, R, 0, kdTreeRoot)
Search(Query Q, Relation R, Level L, Node N)
{
   if (isDataPage(N)) {
      Buf = getPage(fileOf(R),idOf(N))
      check Buf for matching tuples
   } else {
      a = attrLev[L]
      if (!hasValue(Q,a))
         nextNodes = all children of N
      else {
         val = getAttr(Q,a)
         nextNodes = find(N,Q,a,val)
      }
      for each C in nextNodes
         Search(Q, R, L+1, C)
}  }
```

Exercise 11: Searching in kd-Trees

Using the following kd-tree index



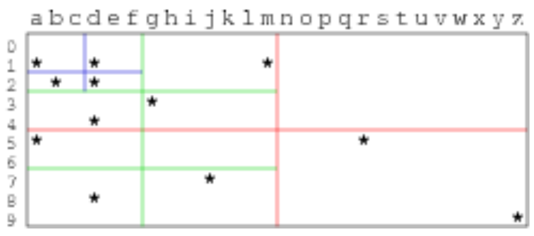
Answer the queries `(m,1), (a,?), (?,1), (?,?)`

Quad Trees

Quad trees use regular, disjoint partitioning of tuple space.

- for *2d*, partition space into quadrants (NW, NE, SW, SE)
- each quadrant can be further subdivided into four, etc.

Example:



... Quad Trees

Basis for the partitioning:

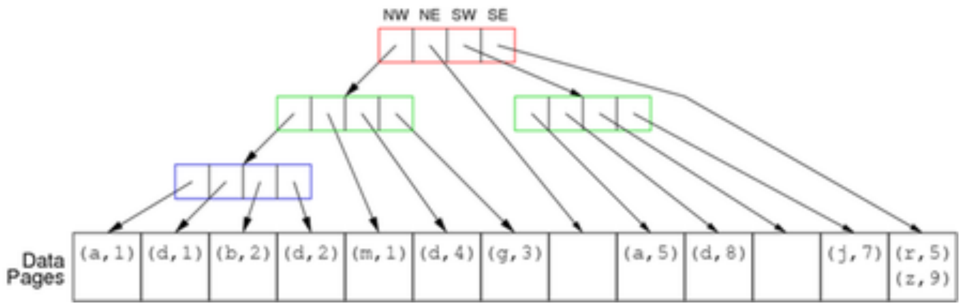
- a quadrant that has no sub-partitions is a *leaf quadrant*
- each leaf quadrant maps to a single data page
- subdivide until points in each quadrant fit into one data page

- ideal: same number of points in each leaf quadrant (balanced)
- point density varies over space
  ⇒ different regions require different levels of partitioning
- this means that the tree is not necessarily balanced

Note: effective for *d≤5*, ok for *6≤d≤10*, ineffective for *d>10*

... Quad Trees

The previous partitioning gives this tree structure, e.g.

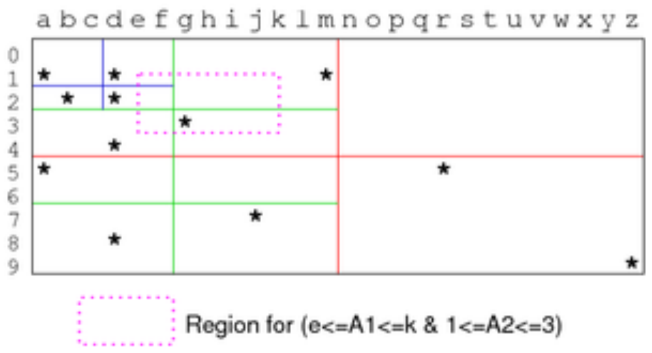


In this and following examples, we give coords of top-left,bottom-right of a region

---

Searching in Quad-tree

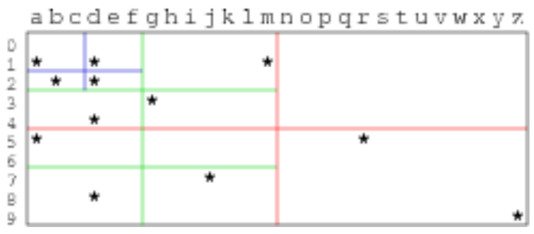
Space query example:


a b c d e f g h i j k l m n o p q r s t u v w x y z
0 1 2 3 4 5 6 7 8 9
Region for (e<=A1<=k & 1<=A2<=3)

Need to traverse: red(NW), green(NW,NE,SW,SE), blue(NE,SE).

Exercise 12: Searching in Quad-trees

Using the following quad-tree index



Answer the queries `(m,1), (a,?), (?,1), (?,?)`

R-Trees

R-trees use a flexible, overlapping partitioning of tuple space.

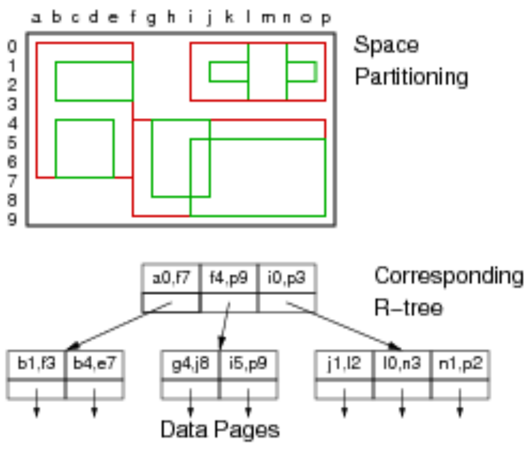
- each node in the tree represents a *k*d hypercube
- its children represent (possibly overlapping) subregions
- the child regions do not need to cover the entire parent region

Overlap and partial cover means:

- can optimize space partitioning wrt data distribution
- so that there are similar numbers of points in each region

Aim: height-balanced, partly-full index pages (cf. B-tree)

... R-Trees



Insertion into R-tree

Insertion of an object *R* occurs as follows:

- start at root, look for children that completely contain *R*
- if no child completely contains *R*, *choose one* of the children and expand its boundaries so that it does contain *R*
- if several children contain *R*, *choose one* and proceed to child
- repeat above containment search in children of current node
- once we reach data page, insert *R* if there is room
- if no room in data page, replace by two data pages
- *partition* existing objects between two data pages
- update node pointing to data pages
  (may cause B-tree-like propagation of node changes up into tree)

Note that *R* may be a point or a polygon.

Query with R-trees

Designed to handle *space* queries and "where-am-I" queries.

"Where-am-I" query: find all regions containing a given point *P*:

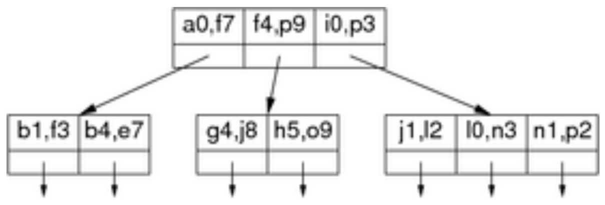
- start at root, select all children whose subregions contain *P*
- if there are zero such regions, search finishes with *P* not found
- otherwise, recursively search within node for each subregion
- once we reach a leaf, we know that region contains *P*

Space (region) queries are handled in a similar way

- we traverse down any path that intersects the query region

Exercise 13: Query with R-trees

Using the following R-tree:



Show how the following queries would be answered:

```
Q1: select * from Rel where X='a' and Y=4
Q2: select * from Rel where X='i' and Y=6
Q3: select * from Rel where 'c'≤X≤'j' and Y=5
Q4: select * from Rel where X='c'
```

Note: can view unknown value `X=?` as range *min*(`X`) ≤ `X` ≤ *max*(`X`)

Multi-d Trees in PostgreSQL

Up to version 8.2, PostgreSQL had R-tree implementation

Superseded by *GiST* = Generalized Search Trees

GiST indexes parameterise: data type, searching, splitting

- via seven user-defined functions (e.g. `picksplit()`)

GiST trees have the following structural constraints:

- every node is at least fraction *f* full (e.g. 0.5)
- the root node has at least two children (unless also a leaf)
- all leaves appear at the same level

Details: src/backend/access/gist

Costs of Search in Multi-d Trees

Difficult to determine cost precisely.

Best case: *pmr* query where all attributes have known values

- in kd-trees and quad-trees, follow single tree path
- cost is equal to depth *D* of tree
- in R-trees, may follow several paths (overlapping partitions)

Typical case: some attributes are unknown or defined by range

- need to visit multiple sub-trees
- how many depends on: range, choice-points in tree nodes

Produced: 30 Aug 2018