University of Bourgogne

France

Advanced Image Analysis Project Wavelet Transform and its applications for Image Decomposition, Reconstruction and Denoising

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Submitted by :
Mr. Nayee Muddin Khan
DOUSAI

Submitted to :
Dr. Philippe Carre

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1 Objective

The main objective of the report is to implement the multi-level wavelet transform and multi-level inverse wavelet transform from the scratch by using initial low pass filter with Daubechies D4 filter. The next step is to analyze the image denoising performance of the wavelet transform for soft and hard thresholding.

2 Wavelet Transform

Wavelet as a subject is highly interdisciplinary and it draws in crucial ways on ideas from the outside world. The working of wavelet in image processing is analogous to the working of human eyes. Wavelet transform is an efficient algorithm to represent different types of images. It has been developed to allow some temporal or spatial information. The basic idea for developing wavelet transform is to localize the family of functions in both time and frequency. By using discrete wave transform the wavelets are sampled discretely.

The principle of the wavelet decomposition is to transform the original given raw image into several components with single low-resolution component called “approximation” and the other components called “details” as shown in figure 1

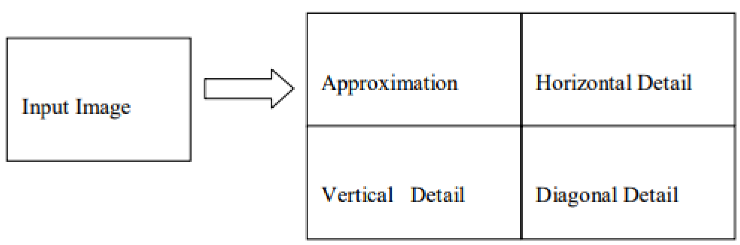

Figure 1: Wavelet decomposition of an 2D Image

The images are considered to by M columns and N rows,for every wavelet decomposition the horizontal data is filtered and the approximation and details are filtered on columns. At every level four sub images are obtained, the approximation, the horizontal detail, the vertical detail and the diagonal detail. For the next level of decomposition we will consider the approximation sub-image and it goes on for every higher level of decomposition. The multi-level wavelet decomposition is labelled in figure 2

In the below sections we have explained about the multi-level wavelet transform and Inverse multi-level wavelet transform. Then we will discuss about the implementation of the image denoising of the wavelet transform with respect to different examples based on mean, variance and thresholding.

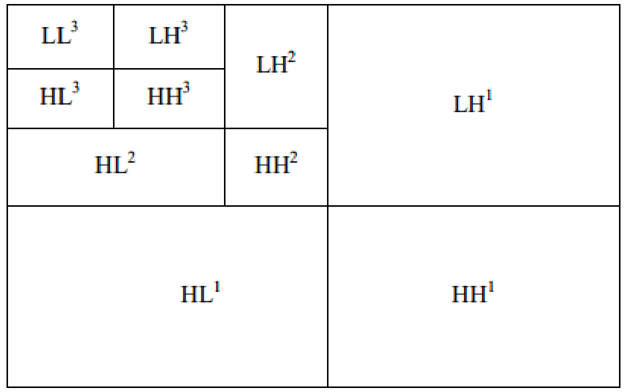

Figure 2: Multi-Level Wavelet decomposition of an 2D Image

2.1 Multi-Level Wavelet Transfrom

Multi-level wavelet transform is used for the decomposition of the image based on the some filtering operations. To perform wavelet decomposition for the image, we will implement Daubechies D4 filter as the initial low pass filter. The steps for the decomposition of the image is explained as below steps:

1. Calculate the high pass filter from the given low pass filter by doing
   g = h.*power(-1*ones(1,length(h)),(0:length(h)-1));
2. Perform low pass row-wise filtering to the image and store the result in m1 matrix
3. Perform high pass row-wise filtering to the image and store the result in m2 matrix
4. Now we will do downsampling for both m1 and m2 matrices on columns and concatenate the downsampled matrices in a new matrix rm.
5. For the next level, we will perform column-wise low pass filtering on the resulted above matrix rm and the store the results on a new matrix m1 again.
6. perform column-wise high pass filtering on the resulted above matrix rm and the store the results on a new matrix m2.
7. we will do downsampling again for both new m1 and m2 matrices on rows and concatenate the downsampled matrices in a new matrix cm.
8. This is the final result matrix which consists of three coefficients as horizontal, vertical and diagonal.

The image decomposition is implemented on 256 X 256 lena image with different levels and the results are as followed from the below figure 3 and the detailed code is explained in *multiwaveletdecomposition.m*

Figure 3: Wavelet decomposition for lena image, From Left: Original Image, Level 1 wavelet decomposition and Level 2 wavelet decomposition

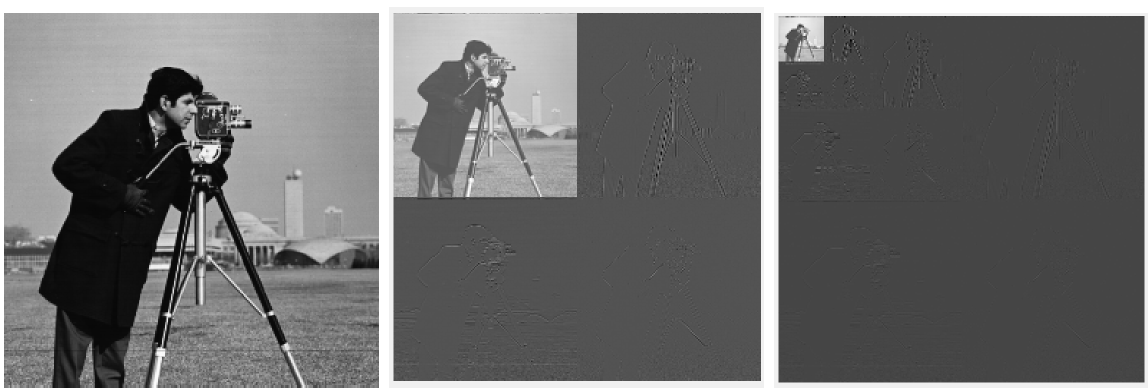

Figure 4: Wavelet decomposition for Cameraman image, From Left: Original Image, Level 1 wavelet decomposition and Level 3 wavelet decomposition

2.2 Multi-Level Inverse Wavelet Transfrom

Multi-level inverse wavelet transform is used for reconstruction of the image. It is implemented by taking the wavelet coefficients from a previous decomposition matrix cm and performing the inverse wavelet transform. The steps for the reconstruction of the image is explained as below steps:

1. we will take the top coefficients from the last result matrix as one single matrix and apply upsampling on rows and then flip the values.
2. Perform low pass filtering in column-wise and store the result in a matrix 1
3. Now take the vertical and diagonal coefficients from the below of the resultant matrix and perform upsampling on rows and then flip the values.
4. Perform high pass filtering in column-wise and store the result in the same matrix 2

5. Add both the matrices and flip them
6. Now take the left part of the flipped matrix, Perform upsampling in column-wise and flip it
7. Perform low pass filtering in row-wise and save the result in a matrix 1
8. Take the right part of the flipped matrix as before, Perform upsampling in column-wise and flip it
9. Perform high pass filtering in row-wise and save the result in the matrix 2
10. Add the results from both the matrices 1 and 2 and flip them which will result the reconstructed image

The image decomposition is implemented on 256 X 256 lena image with different levels and the results are as followed from the below figure 5 and the detailed code is explained in *multiwaveletreconstruction.m*

Figure 5: Wavelet reconstruction for lena image, Left: Original Image, Right: Level 5 wavelet reconstruction image

2.3 Image Denoising

For Image denoising we will add white gaussian noise for the lena image with zero mean and the variance from 2 to 20. We will compute the transforms and set the madnitude with whose value is great than 3times variance. We can even try the larger value. By using wavelet decomposition we will get the wavelet transform of the image. Wavelet reconstruction is used to check the limit of threshold declared and do the inverse wavelet transform for hard and soft thresolding. Hard and soft thresholding are expressed by the following equations:

$$T^{hard}\left(d_{l,k}\right)=\begin{cases} d_{j,k} \text{ if } \left|d_{l,k}\right| \geq \lambda \\ 0 \text{ if } \left|d_{l,k}\right| < \lambda \end{cases} \qquad T^{soft}\left(d_{j,k}\right)=\begin{cases} sign\left(d_{j,k}\right)\left(\left|d_{j,k}\right|-\lambda\right) \text{if } \left|d_{j,k}\right| \geq \lambda \\ 0 \text{ if } \left|d_{j,k}\right| < \lambda \end{cases}$$

As we apply the threshold limit we will vary the hard and soft thresolding with respective to the variance. The results are mentioned in the below figures.

Figure 6: Original Lena Image and added Gaussian White Noise image

Figure 7: Image denoising with Mean = 0,Variance= 10 and Threshold = 3Variance, From Left: Noise Image, Hard Thresholding and Soft Thresholding

Figure 8: Image denoising with Mean = 0,Variance= 15 and Threshold = 3Variance, From Left: Noise Image, Hard Thresholding and Soft Thresholding

Figure 9: Image denoising with Mean = 0,Variance= 15 and Threshold = 5Variance, From Left: Noise Image, Hard Thresholding and Soft Thresholding

Figure 10: Image denoising with Mean = 0,Variance= 20 and Threshold = 5Variance, From Left: Noise Image, Hard Thresholding and Soft Thresholding

Figure 11: Image denoising with Mean = 0,Variance= 20 and Threshold = 15Variance, From Left: Noise Image, Hard Thresholding and Soft Thresholding

Figure 12: Image denoising with Mean = 0,Variance= 20 and Threshold = 10Variance, From Left: Noise Image, Hard Thresholding and Soft Thresholding

Figure 13: Image denoising with Mean = 0,Variance= 10 and Threshold = 15Variance, From Left: Noise Image, Hard Thresholding and Soft Thresholding

3 Conclusion

We can conclude that we have implemented and tested Multi-Level Wavelet Transform and Multi-Level Inverse Wavelet Transform for an gray scale lena Image. We have also analyze the performance of the image denoising by changing the values of threshold and Variance. The results are observed and evaluated in the other sections for both hard and soft thresholding.