1. By definition of odd,

* + 1. The whole graph is a 3-core as every node has edges to 3 nodes
    2. There is no 3-core. Starting with the whole graph, we must remove the bottom node as it connects to only 2 nodes. Then the bottom corners connect to only 2 nodes and must be removed. This repeats until there are no nodes left. Thus there is no set of nodes that is a 3-core.
  1. SCCs:

Reflexive Pairs: {AB}

Triangles: {ACD}, {EFH}, {FHI}

We can combine SCCs if there are edges to and from nodes in each SCC: {EFHI}, {ABCD}

A diagram of a diagram

Description automatically generated

* 1. , , ,

A diagram of a triangle with black circles and letters

Description automatically generated

* 1. 1 = left, 2 = right

* 1. Every pair of nodes has one shortest path that either start/ends in the center or goes through the center. Thus
  2. The diameter is 1 because any pair of nodes must have an edge between them (by def. of clique) which is also the shortest path.
  3. The diameter is (so for the square case). The length of the geodesic path between any two nodes is the Manhattan distance between them. The furthest nodes are the opposite corners which must travel edges in each dimension.
  4. In the first step, there are reachable nodes. For each subsequent step, every reachable node can move to new nodes. Then at the end, there are nodes. To travel between two nodes, we must move down to the common ancestor and then back up to the second node. At the worst case, the common ancestor Is the center node. Then the diameter is , where is the number of branches outwards.

Diameter =

* + 1. CWE. A clique has a constant diameter.
    2. Not CWE. The diameter of the lattice increases linearly with .
    3. CWE. The math in (c) reduces to scaling since the terms are constant. The number of nodes increases exponentially as the diameter increases linearly. Thus if the nodes increases linearly, the diameter is increasing logarithmically.

1. Here’s a log-log plot of the degree counts vs degree with a linear regression (calculated by google sheets). The slope of the best-fit line is . Then .

A graph with a line graph

Description automatically generated

1. Let group 1 have nodes and group 2 have nodes.

* 1. By definition of even, . So

so this point is a local (and global) maximum for the modularity

* 1. T
  2. T
  3. T
  4. T
  5. T
  6. T
  7. T