* 1. For a given node , Define an indicator variable for whether there is an edge between . The edge exists as long as at least one triangle is created with and another node. Then .

Apply the binomial theorem and substitute :

Take out the first two terms of the summation:

so, for large , the summation goes to 0:

* 1. For large , it is very unlikely for placed triangles to overlap. If no triangles overlap, then each node must have and even number of edges and if is odd. If is even, we can express it as where is also the number of triangles involving a given node . This becomes a binomial distribution where , which is the maximum value for .For large , we can approximate this using a poisson distribution with . Then . Finally,
  2. As before, we use the assumption that for large , triangles have near- probability of overlapping. There are possible triangles, each occurring with probability . On average, there will be triangles. For clustering, these are counted times, for each node in the triangle. Then the numerator is . For the number of possible triangles, we count triangles plus the number of incomplete triangles. For a given node with two triangles and , we have four incomplete triangles by taking along with combinations of . From part (a), we know that each node has on average triangles. Then we have open triangles for each of the pairs of triangles. If we add across all nodes, we have . However this double counts open triangles, so in total there are open triangles. Then .

1. We can approximate the in-degree distribution using the power law with . Then .
   1. 30. This is the average in-degree of all nodes. This is equal to the average out-degree of all nodes, which is 30.
   2. Here we can use the closed form solution for the indegree distribution:

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* 1. Here we can use the CCDF of the power-law approximation:

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* 1. Suppose we add a new node, . for all existing nodes . The expected number of new edges to degree- nodes is:

After adding the new node, the expected number of degree- nodes is:

Proof by induction:

Suppose

QED.

1. See 3\_4.ipynb
   1. .

* 1. Symmetry is exhibited by 2,3 and by 6,7
     1. 1 -
     2. 2/3 -
     3. 4 -
     4. 5 -
     5. 6/7 -
     6. 8 -



* 1. represents the probability of making a group between groups . To promote assortativity, we want to be large and to be small. Then we can add a constraint that . This ensures that groups will always be more likely to have edges with themselves than with other groups.

* 1. , We can model the number of common neighbors as a binomial distribution with . Then the mean is .
  2. . For a specific triangle, we can model its occurrence with an indicator variable . Thus . There are triangles that include a given node . WLOG, suppose that are these triangles. Then we want
  3. Define an indicator variable for the k-clique. as we must have an edge between every pair of nodes in the clique. There are possible cliques so .
  4. Define an indicator variable for the k-star. The k-star must have edges from the center to the spokes and missing edges between any two spokes. Then . There are possible stars so the expected number of stars is .