Question 3

a) Logically, how could you (LogicalGoatDiscoveryBot) model this information? Probabilistically, how could you (ProbabilisticGoatDiscoveryBot) model this information? Hint: Consider the statements In A, In B, In C.

1. Logical: I would like to use A, B, and C to represent the goat in which area, if goat is in A, I will use A to represent it.

$$InA+InB+InC = \cup$$

We can describe each situation using the formula before:

$$InA \Rightarrow InA \land \neg InB \land \neg InC,$$
$$InB \Rightarrow InB \land \neg InA \land \neg InC,$$
$$In\ C \Rightarrow In\ C \land \neg In\ A \land \neg In\ B,$$

$$In\ A \land In\ B = \Phi,$$
$$In\ A \land In\ C = \Phi,$$
$$In\ B \land In\ C = \Phi$$

In order to win this game, we can then demonstrate the condition as:

$$(G\ In\ A \land In\ A \land \neg In\ B \land \neg In\ C) \lor (G\ In\ B \land In\ B \land \neg In\ A \land \neg In\ C) \lor (G\ In\ C \land In\ C \land \neg In\ A \land \neg In\ B)$$

G in A mean goat is in A. Since the goat can only be in one location at a time, the condition can be simplified as:

$$(G\ In\ A \land In\ A) \lor (G\ In\ B \land In\ B) \lor (G\ In\ C \land In\ C)$$

Probabilistic: I will assign each area a belief to represent the possibility of the goat is in this area, the initial knowledge base will be:

$$P(A) + P(B) + P(C) = 1.0$$

According to Bayesian formula, the possibility of goat in A can be describe as: The Goat is in A and Goat is not in B and C:

$$P(A) = P(A, \neg B, \neg C) = P(A|\neg B, \neg C) \times P(\neg B|\neg C) \times P(\neg C)$$

$$P(A)=P(A, \neg C, \neg B)=P(A|\neg C,\neg B) \times P(\neg C|\neg B) \times P(\neg B)$$

The possibility of B and C can be described in a similar form.

To win this game, it can be represented as:

P(WIN)=P(InA, GInA)⋁P(InB, GInB)⋁P(InC, GInC)

b) Under the logical formulation, how can you compare the value/results of actions 'Select A', 'Select B', 'Select C'? Is there an obvious choice of best action?

In the logical formula, if there is nothing in our knowledge base, we can't compare the results of actions since we don't know anything about the current situation. Otherwise, if we already know some information, such like if we already know goat is not in A and B (knowledge: not A and not B), we can then infer that goat is in C, and that will be the better action than search A and search B. In this situation, we can get the information that some choices are better than others (best action).

c) Under the probabilistic formulation, how can you compare the value/results of actions 'Select A', 'Select B', 'Select C'? Is there an obvious choice of best action?

To win this game, we can describe the condition of winning when goat is in A:

$$P(In\ A|S_A) = \frac{P(S_A|In\ A)P(In\ A)}{P(S_A)}$$

Since the location of goat is random, the probability can be assigned as:

$$\text{P(In A)} = \text{P(In B)} = \text{P(In C)} = \frac{1}{3}$$

As the possibility of goat in each location is the same, the chance we can find the m in each location will be the same, so the possibility of searching in each location can be the same under this condition.

$$\text{P}(S_A) = \text{P}(S_B) = \text{P}(S_C) = \frac{1}{3}$$

According to the winning formula before, we can then draw a conclusion that under this situation, the winning rate of searching three location is the same. So we can't find a best choice here.

d) Update your logical formulation to reflect the new information

Take the location A as the first searched location, search B or search C is similar to this premise. We can define H(K) to be the CouldBeMoreHelpBot tells you that the goat is not in location K. Thus, if we want to win this game, we would like to search A, and the Bot will tell you that the goat is not in B or C while goat is in A:

$$S(A) \wedge \text{In } A \wedge H(\neg A) = S(A) \wedge \text{In } A \wedge (H(B) \vee H(C))$$

e) Update your logical formulation to reflect the new information

Similar to the premise above, we can define H(K) to be the CouldBeMoreHelpBot, and the winning condition is the same: we would like to search A, and the Bot will tell you that the goat is not in B or C, while goat is in A:

$$P_{win(A)} = P(A, S_A, H(\neg \mathrm{A}))$$

$$P_{win(A)} = P(A|\, S_A, H(\mathrm{B})) + P(A|\, S_A, H(\mathrm{C}))$$

f) Under the logical formulation, how can you compare the value/results of actions 'Re-Select A', 'Re-Select B', 'Re-Select C'? Is there an obvious choice of best action?

First, we represent the initial search as IS, the re-select as RS. And the premise is: first we select A, and the bot tell us H(B) or H(C). In this condition, we can conclude the winning condition below:

The goat is in A: $IS(A) \wedge \text{In } A \wedge H(B) \wedge RS(A)$ or $IS(A) \wedge \text{In } A \wedge H(C) \wedge RS(A)$

The goat is in C: $IS(A) \wedge \text{In } C \wedge H(B) \wedge RS(C)$

The goat is in B: $IS(A) \wedge \text{In } B \wedge H(C) \wedge RS(B)$

In this situation, we can see that the chance of Re-select A could be the best choice because the chance we win seems to be higher. No matter what information the bot will give us, choose A again is reasonable comparing the Re-select B or C since we still have the possibility that the goat is in A.

g) Under the probabilistic formulation, how can you compare the value/results of actions 'Re-Select A', 'Re-Select B', 'Re-Select C'? Is there an obvious choice of best action?

Similar with the former question, we can conclude the winning condition as follow (when we initial select A):

$$P(win) = P(\mathrm{IS(A), In\ A, H(B), RS(A)}) + \mathrm{P(IS(A), In\ A, H(B), RS(C))} + \mathrm{P(IS(A), In\ A, H(C), RS(B))} + \mathrm{P(IS(A), In\ A, H(C), RS(A))}$$

From the formula above, we can simplify the winning condition into three different events: goat is in A, goat is in B and goat is in C. To make the representation simpler, we assume that the bot will give us the information that goat is not in B. The other situation can be represented in a similar way:

$$P\big(In\ A, IS(A), H(B)\big) = P(In\ A|IS(A), H(B))$$

According to the Bayesian formula, we can get:

$$P\big(In\ A\big|IS(A), H(B)\big) = \frac{P(H(B), IS(A)|In\ A)P(In\ A)}{P(IS(A), H(B))}$$

Since the bot already know that goat is in A, and H(B)=H(C), we can infer that $P(H(B), IS(A)|In\ A) = \frac{1}{2}$. P(In A)=$\frac{1}{3}$. Thus, we can get:

$$P\big(IS(A), H(B)\big) = P(H(B), IS(A)|In\ A)P(In\ A) + P(IS(A), H(B)/In\ B)P(In\ B) + P(IS(A), H(B)/In\ C)P(In\ C)$$

As the bot already told us that the goat is not in B. So P(IS(A), H(B)|In B)=0. P(IS(A), H(B)|In C) = 1. Since the In C is condition now, and we already picked A, so The bot will always point to B, and the goat won't be in B. Thus, $P(IS(A), H(B) = \frac{1}{2}$.

Now we can get the possibility of goat in A:

$$P\big(In\ A\big|IS(A), H(B)\big) = \frac{1}{3}$$

Similarly, we can have the$P\big(In\ C\big|IS(A), H(B)\big) = \frac{2}{3}$. Since bot already tell us that goat is not in B, $P(In\ B|IS(A), H(B)) = 0$.

Thus, we can tell that if given that the initial select is A, and bot show us that goat is not in B, the possibility of goat being A is $\frac{1}{3}$ and goat being C is $\frac{2}{3}$.

Based on the conclusion we got above, we now say that the best choice can be found. If we initial select A and bot says that the goat is not in B, Re-select C will be better choice than Re-select A.

h) Under the logical formulation, having initially selected location A, should you stick with location A or change? Justify your choice.

As we discussed in f), re-select A could be the best choice.

i) Under the probabilistic formulation, having initially selected location A, should you stick with location A or change? Justify your choice.

As we discussed in g), the original choice always has smaller chance than the other one. Thus, in this situation, we need to change our selection.

j) Who is more successful in their mission, LogicalGoatDiscoveryBot or ProbabilisticGoatDiscoveryBot? Justify your answer.

The ProbabilisticGoatDiscoveryBot did a more successful job. It not only gives us guidance of making choices, also it shows us the exact probability of doing each action. Then, the probabilistic approach can quantify the probability of each action based on the knowledge base, which can be implemented and understood in a programming way, this would help us with the limit of the human brain when the problem is getting more and more complicated.

Bonus: You initially select location A. Suppose that you know CouldBeMoreHelpfulBot is biased, in the following way: if CBMHBot has a choice between telling you B and C, then CBMHBot tells you B with probability p, and C with probability $1 - \text{p}$. CBMHBot tells you that the goat is not in location B. What is the utility of sticking with your initial selection? What is the utility of switching to C? What is the rational choice, and does it depend on p?

Here, we can take advantage of the formula we already discussed in g).

$$P(win) = P(\mathrm{IS(A), In\ A, H(B), RS(A)}) + \mathrm{P(IS(A), In\ A, H(B), RS(C))} + \mathrm{P(IS(A), In\ A, H(C), RS(B))} \\ + \mathrm{P(IS(A), In\ A, H(C), RS(A))}$$

Since we've already assumed that we will initial select A:

$$P\big(In\ A, IS(A), H(B)\big) = P(In\ A|IS(A), H(B))$$

$$P\big(In\ A, IS(A), H(C)\big) = P(In\ A|IS(A), H(C))$$

According to the Bayesian formula, we can get:

$$P\left(In\ A\middle|IS(A),H(B)\right)=\frac{P(H(B),IS(A)|In\ A)P(In\ A)}{P(IS(A),H(B))}$$

$$P\left(In\ A\middle|IS(A),H(C)\right)=\frac{P(H(C),IS(A)|In\ A)P(In\ A)}{P(IS(A),H(C))}$$

From the condition we can get:

$$P(H(B),IS(A)|In\ A)=p.$$

$$P(H(C),IS(A)|In\ A)=1-p.$$

And $P(In\ A)=\frac{1}{3}$, and we can get this equation:

$$P\big(IS(A),H(B)\big)=P(H(B),IS(A)|In\ A)P(In\ A)+P(IS(A),H(B)|In\ B)P(In\ B)+P(IS(A),H(B)|In\ C)P(In\ C)$$

As bot already told us goat is definitely not in B, $P\big(IS(A),H(B)\big)=\frac{p+1}{3}$.

Now we can get the possibility of goat in A:

$$P\left(In\ A\middle|IS(A),H(B)\right)=\frac{p}{p+1}$$

$$p(In\ C|IS(A),H(B)=1-\frac{p}{p+1}=\frac{1}{p+1}$$

Since we already know that 0<p<1, So Re-select C will be a better choice.