Relaxed Flux Balance Analysis: Recon 3

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Reviewer:

Introduction

We consider a biochemical network of m molecular species and n biochemical reactions. The biochemical network is mathematically represented by a stoichiometric matrix $S \in \mathcal{Z}^{m \times n}$. In standard notation, flux balance analysis (FBA) is the linear optimisation problem

$$\min_{v} \quad \rho(v) \equiv c^{T} v$$
s.t. $Sv = b$, $l \le v \le u$,

where $c \in \Re^n$ is a parameter vector that linearly combines one or more reaction fluxes to form what is termed the objective function, and where a $b_i < 0$, or $b_i > 0$, represents some fixed output, or input, of the ith molecular species.

Every FBA solution must satisfy the constraints, independent of any objective chosen to optimise over the set of constraints. It may occur that the constraints on the FBA problem are not all simultaneously feasible, i.e., the system of inequalities is infeasible. This situation might be caused by an incorrectly specified reaction bound or the absence of a reaction from the stoichiometric matrix, such that a nonzero $b \notin \mathcal{R}(S)$. To resolve the infeasiblility, we consider a cardinality optimisation problem that seeks to minimise the number of bounds to relax, the number of fixed outputs to relax, the number of fixed inputs to relax, or a combination of all three, in order to render the problem feasible. The cardinality optimisation problem, termed *relaxed flux balance analysis*, is

$$\min_{\substack{v,r,p,q\\ \text{s.t.}}} \lambda ||r||_0 + \alpha ||p||_0 + \alpha ||q||_0$$

$$\text{s.t.} \quad Sv + r = b$$

$$l - p \le v \le u + q$$

$$p, q, r \ge 0$$

where $p,q \in \mathcal{R}^n$ denote the relaxations of the lower and upper bounds on reaction rates of the reaction rates vector v, and where $r \in \mathcal{R}^m$ denotes a relaxation of the mass balance constraint. Non-negative scalar parameters—and— α can be used to trade off between relaxation of mass balance or bound constraints. A non-negative vector parameter—can be used to prioritise relaxation of one mass balance constraint over another, e.g, to avoid relaxation of a mass balance constraint on a metabolite that is not desired to be exchanged across the boundary of the system. A non-negative vector parameter— α may be used to prioritise relaxation of bounds on some reactions rather than others, e.g., relaxation of bounds on exchange reactions rather than internal reactions. The optimal choice of parameters depends heavily on the biochemical context. A

relaxation of the minimum number of constraints is desirable because ideally one should be able to justify the choice of bounds or choice of metabolites to be exchanged across the boundary of the system by recourse to experimental literature. This task is magnified by the number of constraints proposed to be relaxed.

PROCEDURE: RelaxedFBA applied to Recon3.0model

TIMING: 20 seconds (computation), minutes - days (interpretation)

Recon 3D [brunk recon nodate] is the latest, most comprehensive, manually curated, genome-scale reconstruction of human metabolism. Recon3D is a reconstruction which currently encompasses ~3300 open reading frames, ~8000 unique metabolites, as well as ~12000 biochemical and transport reactions distributed over nine cellular compartments: cytoplasm [c], lysosome [l], nucleus [n], mitochondrion [m], mitochondrial intermembrane space [i], peroxisome [x], extracellular space [e], Golgi apparatus [g], and endoplasmic reticulum [r] [thiele_protocol_2010, brunk_recon_nodate]. Recon3.0model is a flux balance analysis model and the largest stoichiometrically and flux consistent subset of Recon3D. That is, no internal reaction in Recon3.0model is mass imbalanced and furthermore, every internal and every external reaction is admits a non-zero steady state flux. In this example, we take Recon3.0model, set the lower bound on the biomass reaction to require the synthesis of biomass yet close all of the external reactions in the model. The resulting model is therefore infeasible, that is, no steady state flux vector satisfies the steady state constraints and the bound constraints for the resulting flux balance analysis problem, irrespective of the objective coefficients, so we use relaxed flux balance analysis to identify the minimial set of external reaction bounds that are required to be relaxed in order to make biomass synthesis feasible.

Load Recon3.0model, unless it is already loaded into the workspace.

```
global CBTDIR

%Load the model if recon3 is available replace the model name.
modelFileName = 'Recon2.0model.mat';
modelDirectory = getDistributedModelFolder(modelFileName); %Look up the folder for the
modelFileName= [modelDirectory filesep modelFileName]; % Get the full path. Necessary to
model = readCbModel(modelFileName);
modelOrig = model;
```

Identify the exchange reactions and biomass reaction(s) heuristically and close (a subset) of them

```
model = findSExRxnInd(model,size(model.S,1),1);

Found biomass reaction: biomass_reaction
Found biomass reaction: biomass_maintenance
Found biomass reaction: biomass_maintenance_noTrTr
ATP demand reaction is not considered an exchange reaction by default. It should be mass balanced:
DM_atp_c_ h2o[c] + atp[c] -> h[c] + adp[c] + pi[c]

if ~any(model.biomassBool)
    error('Could not heuristically identify a biomass reaction')
end
```

Add a linear objective coefficient corresponding to the biomass reaction

```
model.biomassBool=strcmp(model.rxns,'biomass_reaction');
```

```
model.c(model.biomassBool)=1;
```

Check that biomass production is feasible

```
FBAsolution = optimizeCbModel(model,'max');
if FBAsolution.stat == 1
    disp('Relaxed model is feasible');
    bioMassProductionRate=FBAsolution.x(model.biomassBool);
    fprintf('%g%s\n', bioMassProductionRate, ' is the biomass production rate')
else
    disp('Relaxed model is infeasible');
end
```

```
Relaxed model is feasible 753.336 is the biomass production rate
```

Remove superflous biomass reactions and display the size of the reduced model

```
model = removeRxns(model,{'biomass_maintenance','biomass_maintenance_noTrTr'});
[m,n] = size(model.S);
fprintf('%6s\t%6s\n','#mets','#rxns'); fprintf('%6u\t%6u\t%s\n',m,n,' totals.')

#mets #rxns
5835 10598 totals.
```

First close all exchange reactions, except the biomass reaction

```
model.SIntRxnBool(strcmp(model.rxns,'biomass_reaction'))=0;
model.lb(~model.SIntRxnBool)=0;
model.ub(~model.SIntRxnBool)=0;
```

Now force the biomass reaction to be active

```
model.lb(model.biomassBool) = 1;
model.ub(model.biomassBool) = 10;
```

Check if the model is feasible

```
FBAsolution = optimizeCbModel(model,'max', 0, true);
if FBAsolution.stat == 1
    disp('Model is feasible. Nothing to do.');
    return
else
    disp('Model is infeasible');
end
```

Model is infeasible

Relaxed flux balance analysis is implemented with the function relaxedFBA

```
% [solution] = relaxedFBA(model, relaxOption)
```

The inputs are a COBRA model and an optional parameter vector

```
% INPUTS:
```

```
model:
                     COBRA model structure
응
    relaxOption:
                     Structure containing the relaxation options:
응
 * internalRelax:
  * 0 = do not allow to relax bounds on internal reactions
응
  * 1 = do not allow to relax bounds on internal reactions with finite bounds
   * 2 = allow to relax bounds on all internal reactions
응
응
왕
  * exchangeRelax:
응
                         * 0 = do not allow to relax bounds on exchange reactions
응
                         * 1 = do not allow to relax bounds on exchange reactions of the
응
                         * 2 = allow to relax bounds on all exchange reactions
응
응
                       * steadyStateRelax:
응
                         * 0 = do not allow to relax the steady state constraint <math>S^*v =
응
                         * 1 = allow to relax the steady state constraint S*v = b
응
                       * toBeUnblockedReactions - n x 1 vector indicating the reactions
응
응
                         * toBeUnblockedReactions(i) = 1 : impose v(i) to be positive
응
                         * toBeUnblockedReactions(i) = -1 : impose v(i) to be negative
응
                         * toBeUnblockedReactions(i) = 0 : do not add any constraint
응
응
                       * excludedReactions - n x 1 bool vector indicating the reactions
응
                         * excludedReactions(i) = false : allow to relax bounds on reac
응
                         * excludedReactions(i) = true : do not allow to relax bounds or
응
응
                       st excludedMetabolites - m x 1 bool vector indicating the metabol
응
                         * excludedMetabolites(i) = false : allow to relax steady state
응
                         * excludedMetabolites(i) = true : do not allow to relax steady
응
응
                       * lamda - trade-off parameter of relaxation on steady state cons
응
                       * alpha - trade-off parameter of relaxation on bounds
% Note, excludedReactions and excludedMetabolites override all other relaxation options
```

Do not allow to relax bounds on any internal reaction

```
relaxOption.internalRelax = 0;
```

Allow to relax bounds on all exchange reactions

```
relaxOption.exchangeRelax = 2;
```

Do not allow to relax the steady state constraint $S^*v = b$

```
relaxOption.steadyStateRelax = 0;
```

Set the tolerance to distinguish between zero and non-zero flux

```
feasTol = getCobraSolverParams('LP', 'feasTol');
relaxOption.epsilon = feasTol/100;%*100;
```

Set the trade-off parameter for relaxation of bounds (advanced user). A larger value of gamma will

```
relaxOption.gamma = 10;
```

Set the trade-off parameter for relaxation on steady state constraint (advanced user)

```
relaxOption.lambda = 10;
```

Call the relaxedFBA function, deal the solution, and set small values to zero

```
tic;
solution = relaxedFBA(model,relaxOption);
timeTaken=toc;
[v,r,p,q] = deal(solution.v,solution.r,solution.p,solution.q);
if 0
    p(p<relaxOption.epsilon) = 0;%lower bound relaxation
    q(q<relaxOption.epsilon) = 0;%upper bound relaxation
    r(r<relaxOption.epsilon) = 0;%steady state constraint relaxation
end</pre>
```

The output is a solution structure with a 'stat' field reporting the solver status and a set of fields matching the relaxation of constraints given in the mathematical formulation of the relaxed flux balance problem above.

```
% OUTPUT:
응
                     Structure containing the following fields:
     solution:
응
                        * stat - status
응
                          * 1 = Solution found
응
                          * 0 = Infeasible
응
                          * -1 = Invalid input
응
                        * r - relaxation on steady state constraints S*v = b
응
                        * p - relaxation on lower bound of reactions
                        * q - relaxation on upper bound of reactions
응
응
                        * v - reaction rate
```

Summarise the proposed relaxation solution

```
if solution.stat == 1
    dispCutoff=relaxOption.epsilon;
    fprintf('%s\n',['Relaxed flux balance analysis problem solved in ' num2str(timeTaked fprintf('%u%s\n',nnz(r),' steady state constraints relaxed');

fprintf('%u%s\n',nnz(abs(p)>dispCutoff & ~abs(q)>dispCutoff & model.SIntRxnBool),' fprintf('%u%s\n',nnz(abs(q)>dispCutoff & ~abs(p)>dispCutoff & model.SIntRxnBool),' fprintf('%u%s\n',nnz(abs(p)>dispCutoff & abs(q)>dispCutoff & model.SIntRxnBool),' fprintf('%u%s\n',nnz(abs(p)>dispCutoff & ~abs(q)>dispCutoff & ~model.SIntRxnBool), fprintf('%u%s\n',nnz(abs(q)>dispCutoff & ~abs(p)>dispCutoff & ~model.SIntRxnBool), fprintf('%u%s\n',nnz(abs(p)>dispCutoff & abs(q)>dispCutoff & ~model.SIntRxnBool),' fprintf('%u%s\n',nnz(abs(p)>dispCutoff & abs(q)>dispCutoff & ~model.SIntRxnBool),' fprintf('%u%s\n',nnz(abs(p)>dispCutoff | abs(q)>dispCutoff & ~model.SIntRxnBool),' fprintf('%u%s\n',nnz(abs(p)>dispCutoff & ~model.SIntRxnBool),' fprintf('%u%s\n',nnz(abs(p
```

```
minLB = min(-max(model.ub), min(model.lb));
intRxnFiniteBound = ((model.ub < maxUB) & (model.lb > minLB));
fprintf('%u%s\n',nnz(abs(p)>dispCutoff & intRxnFiniteBound),' finite lower bounds in fprintf('%u%s\n',nnz(abs(q)>dispCutoff & intRxnFiniteBound),' finite upper bounds in exRxn00 = ((model.ub == 0) & (model.lb == 0));
fprintf('%u%s\n',nnz(abs(p)>dispCutoff & exRxn00),' lower bounds relaxed on fixed in fprintf('%u%s\n',nnz(abs(q)>dispCutoff & exRxn00),' upper bounds relaxed on fixed in the disp('relaxedFBA problem infeasible, check relaxOption fields');
end
```

```
Relaxed flux balance analysis problem solved in 47.6492 seconds.

0 steady state constraints relaxed

0 internal only lower bounds relaxed

0 internal lower and upper bounds relaxed

497 external only lower bounds relaxed

498 external only upper bounds relaxed

107 external lower and upper bounds relaxed

1102 external lower or upper bounds relaxed

604 finite lower bounds relaxed

605 finite upper bounds relaxed

606 lower bounds relaxed on fixed reactions (lb=ub=0)

605 upper bounds relaxed on fixed reactions (lb=ub=0)
```

TROUBLESHOOTING

Given an infeasible problem,

$$Sv = b,$$

$$l \le v \le u,$$

the relaxed flux balance analysis problem

$$\min_{\substack{v,r,p,q\\ \text{s.t.}}} \lambda ||r||_0 + \gamma ||p||_0 + \gamma ||q||_0$$

$$\text{s.t.} \quad Sv + r = b$$

$$l - p \le v \le u + q$$

$$p, q, r \ge 0$$

will always find a solution. However, relaxedFBA offers the user the option to disallow relaxation of some of the constraints. If too many constraints are not allowed to be relaxed, then relaxedFBA will report an infeasible problem. The fields of relaxOption should be reviewed. For example, if relaxation of steady state constraints is not alllowed, yet b is nonzero and not in the range of the stoichiometric matrix, then the relaxedFBA problem will be infeasible. To allow the relaxation of the steady state constraint, S*v = b, then use

```
%relaxOption.steadyStateRelax = 1;
```

If relaxedFBA does return a solution, but it is not biochemcially realistic, then again review the fields of relaxOption, to allow or disallow relaxation of certain constraints. For example, to specifically disallow relaxation of the bounds on reaction with model.rxns abbreviation 'myReaction', use

```
%relaxOption.excludedReactions=false(n,1);
```

```
%relaxOption.excludedReactions(strcmp(model.rxns,'myReaction'))=1;
```

To specifically disallow relaxation of the steady state constraint on a molecualr species with model.mets abbreviation 'myMetabolite', then use:

```
%relaxOption.excludedMetabolite=false(m,1);
%relaxOption.excludedMetabolite(strcmp(model.mets,'myMetabolite'))=1;
```

Even if the set of relaxations are properly set, in a boolean sense, tweaking of the DCA card trade off parameters can help narrow down to a biochemically realistic solution, by iterating between the biochemical literature and the numerical results from relaxedFBA after tweaking the parameters. This flexibility is provided for the expert user. See relaxFBA_cappedL1.m. A standard set of advanced parameters are:

```
%relaxOption.nbMaxIteration = 1000; %max number of iterations of the cappedL1 problem
                           %trade-off parameter of 10 part of v
%relaxOption.gamma0 = 0;
                            %trade-off parameter of 11 part of v
%relaxOption.gamma1 = 0;
%relaxOption.lambda0 = 10;
                            %trade-off parameter of 10 part of r
%relaxOption.lambda1 = 0;
                          %trade-off parameter of 11 part of r
%relaxOption.alpha0 = 10;
                            %trade-off parameter of 10 part of p and q
%relaxOption.alpha1 = 0;
                            %trade-off parameter of 11 part of p and q
%relaxOption.theta
                    = 2;
                            %parameter of capped 11 approximation
```

ANTICIPATED RESULTS

relaxedFBA will return a set of steady state constraints, lower bounds, and upper bounds, that are required to be relaxed to ensure that the FBA problem is feasible. It is necessary to analyse the solution biochemically, to see if it makes sense to relax the suggested constraints. The following code will report a summary of the results.

```
if solution.stat == 1
    printFlag=0;
    lineChangeFlag=0;
    if 1
        dispCutoffLower=relaxOption.epsilon;
        dispCutoffUpper=inf;
        %useful for numerical debugging
        dispCutoffLower=-10;
        dispCutoffUpper=10;
    end
   if any(r)
        fprintf('\n%s\n','Steady state constraints relaxed');
        for i=1:m
            if abs(r(i))>dispCutoffLower && abs(r(i))<dispCutoffUpper</pre>
                fprintf('%s\n',model.mets{i});
            end
        end
   else
       fprintf('\n%s\n','No steady state constraints relaxed');
   end
    if any(p)
        fprintf('%s\n','Lower bounds relaxed');
        for j=1:n
```

```
if abs(p(j))>dispCutoffLower && abs(p(j))<dispCutoffUpper && p(j)~=0</pre>
             rxnAbbrList=model.rxns(j);
             fprintf('\%6g\t\%s',p(j),formulas\{1\});
          end
      end
   else
      fprintf('\n%s\n','No lower bounds relaxed');
   end
   if any(q)
      fprintf('\n%s\n','Upper bounds relaxed');
      for j=1:n
          if abs(q(j))>dispCutoffLower && abs(q(j))<dispCutoffUpper && q(j)~=0</pre>
             rxnAbbrList=model.rxns(j);
             fprintf('\%6g\t\%s',q(j),formulas\{1\});
          end
      end
   else
      fprintf('\n%s\n','No upper bounds relaxed');
   end
end
No steady state constraints relaxed
```

Lower bounds relaxed 1000 datp[m] -> 1000 datp[n] 1000 dctp[n] -> 1000 dgtp[n] 1000 dttp[n] -> 1000 ethamp[r] -> gpi_sig[r] -> 1000 1000 mem2emgacpail_prot_hs[r] -> 1000 Ser_Gly_Ala_X_Gly[1] -> 1000 4nph[e] -> 1000 5adtststerone[e] -> 1000 7dhf[e] -> 1000 7thf[e] -> 1000 adp[e] -> adprbp[e] -> 1000 1000 adrn1[e] -> 1000 ala_D[e] -> aqcobal[e] -> 1000 1000 arachd[e] -> ascb_L[e] -> 1000 1000 atp[e] -> bilglcur[e] -> 1000 1000 biocyt[e] -> 1000 cholate[e] 1000 chsterol[e] 1000 chtn[e] -> cmp[e] -> 1000 1000 crmp_hs[e] -> crn[e] -> 1000 crtsl[e] -> 1000 1000 cspg_c[e] -> 1000 dag_hs[e] -> 1000 dheas[e] -> 1000 dlnlcg[e] ->

```
0.15
     eicostet[e] ->
       estrones[e] ->
1000
     gbside_hs[e] ->
gchola[e] ->
1000
1000
     gluala[e] ->
1000
1000
     glygn2[e] ->
1000
     glygn4[e] ->
1000
     gthrd[e] ->
1000
     gtp[e] ->
1000 h2o2[e] ->
1000 ha[e] ->
1000 hdcea[e] ->
1000 i[e] ->
1000 idp[e] ->
1000 imp[e] ->
1000 ksi[e] ->
1000 Lcystin[e] ->
1000 leuktrA4[e] ->
1000 leuktrF4[e] ->
1000 lnlc[e] ->
1000 lnlnca[e] ->
     nadp[e] ->
1000
    nadp[e] ->
ncam[e] ->
o2s[e] ->
ocdca[e] ->
ocdcea[e] ->
pe_hs[e] ->
peplys[e] ->
1000
1000
1000
1000
1000
1000
1000
1000 prostgd2[e] ->
1000 prostgel[e] ->
1000 prostgf2[e] ->
1000 ps_hs[e] ->
1000 ptdca[e] ->
1000 retinol[e] ->
1000 s2l2fn2m2masn[e] ->
1000 spc_hs[e] ->
1000 sph1p[e] ->
1000 sphslp[e] ->
1000 strch1[e] ->
1000 strch2[e] ->
1000 strdnc[e] ->
1000 tag_hs[e] ->
     tchola[e] ->
1000
     thf[e] ->
1000
     thym[e] ->
triodthy[e]
ttdca[e] ->
utp[e] ->
1000
1000
       triodthy[e] ->
1000
1000
     vacc[e] ->
1000
     whhdca[e] ->
1000
1000 xoltri27[e] ->
1000 xylt[e] ->
1000 pre_prot[r] ->
1000 4abutn[e] ->
1000 ctp[e] ->
1000 dgmp[e] ->
1000 dha[e] ->
1000 dttp[e] ->
1000 fad[e] ->
1000 fald[e] ->
1000 HC00250[e] ->
1000
     HC01361[e] ->
     HC01446[e] ->
1000
```

```
1000
      cpppg1[e] ->
1000
      itp[e] ->
      udpg[e] ->
1000
1000
      HC00955[e] ->
1000
      C02470[e] ->
     HC00822[e] ->
1000
     HC02193[e] ->
1000
1000 HC02195[e] ->
1000 HC02196[e] ->
1000 HC02191[e] ->
1000 HC02194[e] ->
1000 HC02203[e] ->
1000 HC02217[e] ->
1000 malcoa[e] ->
1000 arachcoa[e] ->
1000 CE4722[e] ->
1000 CE4723[e] ->
1000 CE4724[e] ->
1000 CE2839[e] ->
1000 CE2838[e] ->
1000
    23cump[e] ->
    CE5788[e] ->
1000
     CE5798[e] ->
1000
     CE5787[e] ->
1000
     CE5791[e] ->
1000
     CE5867[e] ->
CE4633[e] ->
1000
1000
1000
     CE5854[e] ->
     udpgal[e] ->
1000
1000
    CE0074[e] ->
1000 CE5853[e] ->
20.6508 h2o[c] + 20.7045 atp[c] + 0.38587 glu_L[c] + 0.35261 asp_L[c] + 0.036117 gtp[c] + 0.5056
1000 c101coa[c] ->
1000 doco13ac[e] ->
1000 octdececoa[c] ->
1000 tetdec2coa[c] ->
1000 tetdece1coa[c] ->
1000 5HPET[r] ->
1000 taur[c] ->
1000 pe_hs[r] ->
1000 pmtcoa[r] ->
1000 alaala[e] ->
1000
     bglc[e] ->
    glygly[e] ->
1000
1000
      gum[e] ->
1000
      leugly[e] ->
     pect[e] ->
1000
     psyl[e] ->
1000
     slfcys[e] ->
1000
     dpcoa[e] ->
1000
1000
     oh1[e] ->
1000 q10[e] ->
1000 Lcystin[c] ->
1000 ncam[c] ->
1000 pnto_R[c] ->
1000 34hpp[e] ->
1000 3mob[e] ->
1000 3mop[e] ->
1000 4mop[e] ->
1000 aicar[e] ->
1000 cbasp[e] ->
1000 2pg[e] ->
1000
    5hoxindoa[e] ->
```

1000

cholp[e] ->

```
1000
         cyst_L[e] ->
1000
        dmgly[e] ->
      g3pc[e] ->
1000
         gudac[e] ->
1000
      gudac[e] ->
hcys_L[e] ->
1000
1000
      icit[e] ->
      pep[e] ->
1000
1000 xtsn[e] ->
1000
      3pg[e] ->
1000 udpglcur[e] ->
1000 nicrnt[e] ->
1000 orot5p[e] ->
1000 glyc3p[e] ->
1000 acrn[e] ->
1000 pcrn[e] ->
1000 lneldccrn[e] ->
1000 odecrn[e] ->
1000 stcrn[e] ->
1000 pmtcrn[e] ->
1000 hdcecrn[e] ->
1000 15HPET[e] ->
      3mhis[e] ->
1000
      5HPET[e] ->
7dhchsterol[e] ->
1000
1000
      aclys[e] ->
adpoh[e] ->
amet[e] ->
biliverd[e] ->
C02356[e] ->
1000
1000
1000
1000
1000
1000
      CE0955[e] ->
1000 CE1556[e] ->
1000 CE2176[e] ->
1000 CE7082[e] ->
1000 forglu[e] ->
1000 HC00900[e] ->
1000 hmcr[e] ->
1000 lnlccrn[e] ->
1000 lthstrl[e] ->
1000 mev_R[e] ->
1000 pel2_hs[e] ->
1000 pel3_hs[e] ->
1000 pe15_hs[e] ->
1000 pel5_hs[e] ->
1000 pel61_hs[e] ->
1000 pe224_hs[e] ->
1000 pe226_hs[e] ->
1000 pedh203_hs[e] ->
1000 pelinl_hs[e] ->
1000 peole_hs[e] ->
1000 pepalm_hs[e] ->
1000 peste_hs[e] ->
1000 saccrp L[e] ->
      saccrp_L[e] ->
1000
1000 xolest205_hs[e]
1000 3moxtyr[e] -> 1000 5aop[e] ->
1000 alltn[e] ->
1000 CE2510[e] ->
1000 ddca[e] ->
1000 glyc_R[e] ->
1000 Lcyst[e] ->
1000 oaa[e] ->
1000 ttdcea[e] ->
1000 bgly[e] ->
      retinal[e] ->
1000
1000
      maltttr[e] ->
```

```
1000
       progly[e] ->
1000
       dhbpt[e] ->
1000
       alaargcys[e]
1000
       alaasnleu[e]
1000
       alahisala[e]
     alalysthr[e]
1000
     argalaala[e] ->
1000
     argalaphe[e] ->
1000
     argalathr[e] ->
1000
     argarglys[e] ->
1000
1000
    argargmet[e] ->
1000 argcysgly[e] ->
1000 argcysser[e] ->
1000
    argglupro[e] ->
1000
    argleuphe[e] ->
1000
    argphearg[e] ->
1000
    argpromet[e] ->
1000
     argserser[e] ->
     argtyrval[e] ->
1000
     argvalcys[e] ->
1000
1000
       argvaltrp[e] ->
1000
       asnmetpro[e] ->
       asnpheasp[e] ->
1000
       asnphecys[e] ->
1000
       asntyrgly[e] ->
1000
1000
       asntyrphe[e] ->
1000
       aspalaarg[e]
                   ->
1000
       aspasnglu[e] ->
1000
       aspglupro[e]
1000
       aspglutrp[e]
                   ->
1000
       asphiscys[e] ->
1000
       asplysglu[e] ->
1000 aspmetasp[e] ->
1000
       aspvalasn[e] ->
1000 cysasnmet[e] ->
1000 cysaspphe[e] ->
1000 cysglnmet[e] ->
1000 cysglutrp[e] ->
1000
    cysleuthr[e] ->
1000
    cystyrasn[e] ->
1000
       glnasngln[e] ->
1000
       glnlyslys[e] ->
1000
       glnproglu[e]
                   ->
1000
       glntrpglu[e]
                   ->
1000
       glntyrleu[e] ->
1000
       gluargleu[e]
                   ->
1000
       gluasnleu[e] ->
1000
       glumethis[e] ->
1000
       gluthr[e] ->
1000
       gluthrlys[e] ->
     glutrpala[e] ->
1000
     glyhisasn[e] ->
1000
     glyhislys[e] ->
1000
1000
     glylysphe[e] ->
1000
     glytyrlys[e] ->
1000
     glyvalhis[e] ->
1000
     hisarqcys[e] ->
1000
     hisargser[e] ->
1000
     hiscyscys[e] ->
1000
     hisqlnala[e] ->
1000
     hisqluqln[e] ->
1000
     hisqlylys[e] ->
1000
     hislysala[e] ->
1000
     hislysglu[e] ->
```

```
1000
        hislysile[e] ->
1000
        hislysval[e]
1000
        hismetgln[e]
1000
        hisphearg[e]
      histrphis[e]
1000
      ileargile[e]
1000
                      ->
      ileasnhis[e] ->
1000
1000
      ileglyarg[e] ->
      ileprolys[e] ->
1000
      ileserarg[e] ->
1000
1000
     iletrptyr[e] ->
1000 leualaarg[e] ->
1000 leuasplys[e] ->
1000
     leusertrp[e] ->
1000
     lyscyshis[e] ->
1000
     lysglnphe[e] ->
1000
     lyslyslys[e] ->
1000
      lyspheile[e] ->
1000
      lystyrile[e] ->
1000
      lysvalphe[e] ->
1000
      lysvaltrp[e] ->
     lysvaltrp[e] ->
metargleu[e] ->
metasntyr[e] ->
metglntyr[e] ->
metglyarg[e] ->
metmetile[e] ->
metphearg[e] ->
mettrpphe[e] ->
pheasn[e] ->
1000
1000
1000
1000
1000
1000
1000
1000
      pheasp[e] ->
1000
      pheglnphe[e] ->
1000
1000 pheleu[e] ->
1000 pheleuasp[e] ->
1000 pheleuhis[e] ->
1000 phelysala[e] ->
1000 phelyspro[e] ->
1000 phepheasn[e] ->
1000 phephethr[e] ->
1000 pheproarg[e] ->
1000 phesertrp[e] ->
1000 phethrlys[e] ->
1000 phetrpleu[e] ->
1000
      phetyr[e] ->
1000 phetyrgln[e] ->
1000 phetyrlys[e] ->
1000 proargcys[e] ->
1000 proasncys[e] ->
      proglulys[e] ->
prophe[e] ->
1000
1000
      proproarg[e] ->
1000
      propropro[e] ->
1000
      provalgln[e] ->
1000
1000 serargala[e] ->
1000 serargtrp[e] ->
1000 sercysarg[e] ->
1000 serlyshis[e] ->
1000 serphelys[e] ->
1000 thrglnglu[e] ->
1000 thrilearg[e] ->
1000 thrmetarg[e] ->
1000 thrphearq[e] ->
1000
     thrserarg[e] ->
1000
      thrtyrmet[e] ->
1000
        trpgluleu[e] ->
```

```
1000
       trpglupro[e] ->
1000
       trpglutyr[e] ->
     trpglyphe[e] ->
trpglyval[e] ->
trpilelys[e] ->
1000
1000
1000
     trpiletrp[e] ->
1000
     trpleuval[e] ->
1000
1000
     trpmetval[e] ->
     trpphe[e] ->
1000
1000 trpproval[e] ->
1000 trpsertyr[e] ->
1000 trpthrglu[e] ->
1000 trpthrile[e] ->
1000 trpvalasp[e] ->
1000 tyrala[e] ->
1000 tyralaphe[e] ->
1000 tyrargglu[e] ->
1000 tyrargser[e] ->
1000 tyrcysgly[e] ->
     tyrcysthr[e] ->
1000
     tyrglu[e] ->
1000
     tyrphetyr[e] ->
1000
     tyrvalmet[e] ->
1000
     valarggly[e] ->
1000
     valarggly[e] ->
valhisasn[e] ->
valleuphe[e] ->
vallystyr[e] ->
valphearg[e] ->
valprotrp[e] ->
1000
1000
1000
1000
1000
     valserarg[e] ->
1000
1000 valtrpphe[e] ->
1000 valtrpval[e] ->
1000 trpglyasp[e] ->
1000 hxa[e] ->
1000 Lhcystin[e] ->
1000 pe_hs[c] ->
1000 akg[c] ->
1000 bandmt[c] ->
1000 for[c] ->
1000 mil4p[c] ->
1000 pchol_hs[c] ->
1000 C02712[c] ->
1000 C02528[c] ->
1000 HC02191[c] ->
1000 HC02192[c] ->
     HC02192[c] ->
HC02197[c] ->
HC02198[c] ->
HC02220[c] ->
Tyr_ggn[c] ->
c226coa[c] ->
1000
1000
1000
1000
1000
     chol[c] ->
1000
     cholate[c] ->
1000
1000 coa[c] ->
     crvnc[c] ->
1000
1000 gchola[c] ->
1000 glygn2[c] ->
1000 hdca[c] ->
1000 lnlccoa[c] ->
1000 retfa[c] ->
1000 retinol[c] ->
1000 tchola[c] ->
1000
     tdechola[c] ->
1000
     thmpp[c] ->
1000
     thmtp[c] ->
```

```
tmndnccoa[c] ->
1000
1000
       vitd3[c] ->
     dhcholestanate[
thcholstoic[c]
xo17ah3[c] ->
1000
       dhcholestanate[c] ->
1000
       thcholstoic[c] ->
1000
     xol7aone[c] ->
1000
1000
       7klitchol[c] ->
     dchac[c] ->
1000
1000 CE1273[c] ->
1000 2obut[e] ->
1000 acac[e] ->
1000 but[e] ->
1000 cgly[e] ->
1000 co2[e] ->
1000 cytd[e] ->
1000 dgsn[e] ->
1000 din[e] ->
1000 duri[e] ->
1000 fe3[e] ->
1000 fum[e] ->
    glyleu[e] ->
glyphe[e] ->
glypro[e] ->
h[e] ->
1000
1000
1000
1000
     h2o[e] ->
1000
     ins[e] ->
lac_L[e]
1000
1000
       lac_L[e] ->
     lys_L[e] ->
1000
     na1[e] ->
1000
     o2[e] ->
1000
1000 orn[e] ->
1000 ppi[e] ->
1000 pro_L[e] ->
1000 ser_L[e] ->
1000 so4[e] ->
1000 thymd[e] ->
1000 urea[e] ->
1000 cys_L[e] ->
1000 his_L[e] ->
1000 thr_L[e] ->
1000 gln_L[e] ->
1000 phe_L[e] ->
1000 arg_L[e] ->
     nac[e] ->
1000
     cit[e] ->
etha[e] -:
1000
      etha[e] ->
1000
     fol[e] ->
glyc[e] ->
malt[e] ->
1000
1000
1000
1000
     malttr[e] ->
1000
     rib_D[e] ->
1000 trp_L[e] ->
1000 xyl_D[e] ->
1000 34dhpha[e] ->
1000 ppa[e] ->
1000 tre[e] ->
1000 lcts[e] ->
1000 ade[e] ->
1000 etoh[e] ->
1000 phpyr[e] ->
1000 2h3mv[e] ->
1000 2hiv[e] ->
1000 sucsal[e] ->
     3ityr_L[e] ->
1000
```

```
35diotyr[e] ->
1000
1000
       13_cis_retn[e] ->
1000
       CE1617[e] ->
1000
       34dhoxmand[e]
     CE5643[e] ->
1000
     n8aspmd[e] ->
1000
     13dampp[e] ->
1000
1000
    12ppd_R[e] ->
1000
    xylu_L[e] ->
1000 xylu_D[e] ->
1000 CE0737[e] ->
1000 hdd2crn[e] ->
1000 mlthf[e] ->
1000 sphgn[e] ->
1000 coke[e] ->
1000 hdl_hs[e] ->
1000 HC00005[e] ->
1000 CE2172[e] ->
1000 CE5629[e] ->
1000 gd3_hs[e] ->
    gluside_hs[e] ->
gm3_hs[e] ->
cmpacna[e] ->
34dhpac[c] ->
1000
1000
1000
1000
     ts3[c] ->
1000
     gd3_hs[1] ->
k[g] ->
1000
1000
     na1[r] ->
1000
     pail_hs[e] ->
1000
1000
     CE1243[e] ->
1000 CE5026[e] ->
1000 CE1261[e] ->
1000 gdlb_hs[e] ->
1000 nadh[e] ->
1000 sbt_D[e] ->
1000 12dhchol[c] ->
1000 3dhcdchol[c] ->
1000 3dhchol[c] ->
1000 3dhdchol[c] ->
1000 3dhlchol[c] ->
1000 7dhcdchol[c] ->
1000 7dhchol[c] ->
1000 ca3s[c] ->
     coprost[c] ->
1000
     dca3s[c] ->
1000
     gca3s[c] ->
gcdca3s[c] ->
1000
1000
1000
     gdca3s[c] ->
       gudca3s[c] ->
1000
     gudca3s[c] ->
hyochol[c] ->
1000
1000
     icdchol[c] ->
1000
     isochol[c] ->
1000 lca3s[c] ->
1000 tca3s[c] ->
1000 tcdca3s[c] ->
1000 tdca3s[c] ->
1000 thyochol[c] ->
1000 tudca3s[c] ->
1000 uchol[c] ->
1000 udca3s[c] ->
1000 hyochol[e] ->
1000 amlccs[e] ->
1000
     amlcsa[e] ->
1000
     am9csa[e] ->
```

```
csa[e] ->
 1000
 1000
       fvs[e] ->
 1000
       glz[e] ->
        lvst[e] ->
 1000
      mhglz[e] ->
 1000
      nfd[e] ->
 1000
      nfdoh[e] ->
 1000
 1000 ptvstlac[e] ->
 1000 pvs[e] ->
 1000 tlacfvs[e] ->
 1000 tmdm1[e] ->
 1000 tripvs[e] ->
 1000 C13856[e] ->
 1000 M02956[e] ->
 1000 M00241[e] ->
 1000 M00008[e] ->
 0.05 \quad M00017[e] \rightarrow
 1000 M00019[e] ->
 0.15 M00117[e] ->
 1000 M01197[e] ->
      M01207[e] ->
 0.05
      M01235[e] ->
 1000
      M01238[e] ->
 1000
 1000
      h2co3[e] ->
 1000
      M02837[e] ->
his_L[c] ->
 1000
 1000
      ile_L[c] ->
 1000
      leu_L[c] ->
 1000
 1000
      lys_L[c] ->
 1000 met_L[c] ->
 1000 phe_L[c] ->
 1000 thr_L[c] ->
 1000 trp_L[c] ->
 1000 val_L[c] ->
 1000 ala_L[c] ->
 1000 arg_L[c] ->
 1000 asn_L[c] ->
 1000 asp_L[c] ->
 1000 cys_L[c] ->
 1000 gln_L[c] ->
 1000 glu_L[c] ->
      pro_L[c] ->
ser L[c] ->
 1000
        ser_L[c] ->
 1000
      tyr_L[c] ->
 1000
      gly[c] ->
 1000
        4abut[1] ->
 1000
      CE5026[c] ->
4glu56dihdind
dopa[c] ->
 1000
 1000
        4glu56dihdind[c] ->
 1000
 1000
      srtn[c] ->
      adrnl[c] ->
 1000
 1000 ach[c] ->
 1000 hista[c] ->
 1000 nrpphr[c] ->
 1000 Lkynr[c] ->
 1000 btn[m] ->
Upper bounds relaxed
 1000 13_cis_retn[n] ->
 1000 datp[m] ->
 1000 dgpi_prot_hs[r] ->
 1000 dgtp[m] ->
 1000
      dttp[m] ->
      melanin[c] ->
 1000
```

```
1000
      mem2emgacpail_prot_hs[r] ->
1000
        10fthf[e] ->
      10fthf[e] ->
10fthf5glu[e]
10fthf6glu[e]
13_cis_retnglc
2hb[e] ->
1000
        10fthf5glu[e] ->
1000
1000
        13_cis_retnglc[e] ->
1000
1000 34dhoxpeg[e] ->
1000 34dhphe[e] ->
1000 5adtststerones[e] ->
1000 5dhf[e] ->
1000 5mthf[e] ->
1000 5thf[e] ->
1000 6dhf[e] ->
1000 6thf[e] ->
1000 abt[e] ->
1000 acetone[e] ->
1000 ach[e] ->
0.45 adrn[e] ->
1000 amp[e] ->
0.1 arach[e] ->
1000 arachd[e] ->
1000 bhb[e] ->
1000 bhb[e] ->
1000 bilirub[e] ->
0.5 clpnd[e] ->
1000 crtstrn[e] ->
1000 crvnc[e] ->
1000 dag_hs[e] ->
1000 dhdascb[e] ->
1000 dhf[e] ->
0.2 dlnlcg[e] ->
0.2 dlnlcg[e] ->
1000 dopa[e] ->
1000 elaid[e] ->
1000 estradiol[e] ->
1000 fuc_L[e] ->
1000 glyc_S[e] ->
1000 glygn5[e] ->
1000 gmp[e] ->
1000 gsn[e] ->
1000 hco3[e] ->
1000 hdca[e] ->
1000 hdcea[e] ->
0.2 hexc[e] ->
1000 hista[e] ->
1000 hpdca[e] ->
1000 inost[e] ->
      ksi_deg1[e] ->
lac_D[e] ->
leuktrC4[e] ->
leuktrD4[e] ->
leuktrE4[e] ->
1000
1000
1000
1000
1000
1000 lgnc[e] ->
1000 lneldc[e] ->
1000 lnlncg[e] ->
1000 lpchol_hs[e] ->
1000 mag_hs[e] ->
1000 meoh[e] ->
1000 mercplaccys[e] ->
1000 mthgxl[e] ->
1000 nad[e] ->
1000 nrpphr[e] ->
0.1 nrvnc[e] ->
1000 oagd3_hs[e] ->
1000 ocdca[e] ->
1000 pchol_hs[e] ->
```

```
1000
     pglyc_hs[e] ->
1000
       prostge2[e] ->
1000
       rbt[e] ->
     rbt[e] -> retfa[e] ->
1000
1000
       retn[e] ->
     Rtotal[e] ->
1000
     Rtotal2[e] ->
1000
     Rtotal3[e] ->
1000
1000 s212n2m2masn[e] ->
1000 sl_L[e] ->
1000 tchola[e] ->
1000 thmtp[e] ->
1000 thyox_L[e] ->
1000 tmndnc[e] ->
1000 tststerone[e] ->
1000 tsul[e] ->
1000 udp[e] ->
1000 ump[e] ->
1000 urate[e] ->
     vitd3[e] ->
1000
     whtststerone[e] ->
1000
     xolest_hs[e] ->
1000
     xolest_lis[e] ->
xolest2_hs[e] ->
acmana[e] ->
ahdt[e] ->
ctp[e] ->
dgtp[e] ->
1000
1000
1000
1000
1000
     dtmp[e] ->
1000
     g1p[e] ->
1000
     isomal[e] ->
1000
     HC01104[e] ->
1000
1000 HC01444[e] ->
1000 HC01577[e] ->
1000 HC01609[e] ->
1000 HC01700[e] ->
1000 orot[e] ->
1000 prpp[e] ->
1000 so3[e] ->
1000 prostgh2[e] ->
1000 prostgi2[e] ->
1000 HC00004[e] ->
1000
     HC00822[e] ->
1000
     HC02192[e] ->
     HC02193[e] ->
1000
     HC02220[e] ->
1000
     HC02220[e] ->
HC02197[e] ->
HC02198[e] ->
HC02187[e] ->
HC02202[e] ->
HC02204[e] ->
1000
1000
1000
1000
1000
     coa[e] ->
1000
     CE2250[e] ->
1000
     CE1943[e] ->
1000
1000
     CE2915[e] ->
1000 CE2916[e] ->
1000 CE2917[e] ->
1000 malthp[e] ->
1000 CE2839[e] ->
1000 3ump[e] ->
1000 CE5786[e] ->
1000
     CE5789[e] ->
1000
     CE5797[e] ->
1000
     CE5868[e] ->
1000
     CE5869[e] ->
```

```
1000
        CE4881[e] ->
        CE1926[e] ->
1000
1000 crm_hs[e] ->
1000 galside_hs[e] ->
1000 CE1925[e] ->
1000 3bcrn[e] ->
1000 3hdececrn[e] ->
1000 3octdeccrn[e] -> 1000 3octdecelcrn[e] ->
1000 c101crn[e] ->
1000 c10crn[e] ->
1000 c4dc[e] ->
1000 c51crn[e] ->
1000 c8crn[e] ->
0.25 docosac[e] ->
1000 tetdec2crn[e] ->
1000 tetdece1crn[e] ->
1000 4abut[n] ->
1000 dchac[e] ->
1000 dchac[e] ->
1000 glgchlo[e] ->
1000 gltcho[e] ->
1000 gumgchol[e] ->
1000 gumtchol[e] ->
1000 pectintchol[e] ->
1000 psylchol[e] ->
1000 psyltchol[e] ->
1000 tdechola[e] ->
1000 fmn[e] ->
1000 pan4p[e] ->
1000 glmtchol[e] ->
1000 fmn[e] ->
1000 5HPET[c] ->
1000 Lcystin[c] ->
1000 fol[c] ->
1000 ncam[c] ->
1000 ahcys[e] ->
1000 cholp[e] ->
1000 cyst_L[e] ->
1000 dcmp[e] ->
1000 ethamp[e] ->
1000 glyald[e] ->
1000 icit[e] ->
1000 L2aadp[e] ->
1000 L2aadp[e] ->
1000 Lkynr[e] ->
1000 xmp[e] ->
1000 hLkynr[e] ->
1000 nicrnt[e] ->
1000 argsuc[e] ->
1000 pcrn[e] ->
1000 lneldccrn[e] ->
1000 stcrn[e] ->
1000 3mtp[e] ->
1000 3mtp[e] ->
1000 2oxoadp[e] ->
1000 34hpl[e] ->
1000 3hpp[e] ->
1000 3uib[e] ->
1000 56dura[e] ->
1000 acgly[e] ->
1000 acthr_L[e] ->
1000 C02712[e] ->
1000 C05957[e] ->
1000 C06314[e] ->
       C06315[e] ->
1000
```

```
1000
       C11695[e] ->
       CE1273[e] ->
1000
       CE1556[e] ->
1000
       CE2176[e] ->
1000
       CE5304[e] ->
1000
     CE6031[e] ->
1000
     didecaeth[e] ->
1000
1000
     diholineth[e] ->
1000
     docohxeth[e] ->
     docteteth[e] ->
1000
1000
     elaidcrn[e] ->
1000 hepdeceth[e] ->
1000 hexdeceeth[e] ->
    hexdiac[e] ->
1000
1000 hxcoa[e] ->
1000 lineth[e] ->
1000 lnlccrn[e] ->
1000
    milp_D[e] ->
1000
     Nacasp[e] ->
1000
     oleth[e] ->
     pailste_hs[e] ->
1000
       pchol2palm_hs[e] ->
1000
1000
       pcholn261_hs[e] ->
1000
       pcholn28_hs[e] ->
       pcholn281_hs[e] ->
1000
1000
       pmeth[e] ->
1000
       sphmyln180241_hs[e] ->
1000
       sphmyln18114 hs[e] ->
1000
       sphmyln18115_hs[e]
1000
       sphmyln18116_hs[e] ->
1000
       sphmyln181161_hs[e] ->
1000
       sphmyln18117_hs[e] ->
1000
       sphmyln18118_hs[e] ->
1000
       sphmyln181181_hs[e] ->
1000
       sphmyln18120_hs[e] ->
1000
       sphmyln181201_hs[e] ->
1000
       sphmyln18121_hs[e] ->
1000
       sphmyln18122_hs[e] ->
1000
       sphmyln181221_hs[e] ->
       sphmyln18123_hs[e] ->
1000
1000
       sphmyln1824_hs[e] ->
1000
       sphmyln1825_hs[e] ->
1000
       steeth[e] ->
1000
       tmlys[e] ->
1000
       trideceth[e] ->
1000
       xolest183_hs[e] ->
1000
       abt_D[e] ->
       glyc2p[e] -> glyclt[e] ->
1000
1000
     phlac[e] ->
1000
     pser_L[e] ->
1000
     bz[e] ->
1000
     mepi[e] ->
1000
1000
     1mncam[e] ->
1000
     progly[e] ->
1000
    thbpt[e] ->
1000 itp[n] ->
1000
    alaargcys[e] ->
1000
    alaarggly[e] ->
1000 alaqlylys[e] ->
1000 alahisala[e] ->
1000
     argalaphe[e] ->
1000
     argarg[e] ->
1000
     argarglys[e] ->
```

```
1000
       argcysgly[e] ->
1000
       argcysser[e]
                   ->
     arggluglu[e]
argglygly[e]
arghisthr[e]
1000
1000
1000
     arglysasp[e]
1000
                    ->
     argprothr[e] ->
1000
1000
     argserser[e] ->
     argvalcys[e] ->
1000
     asnasnarg[e] ->
1000
1000 asncyscys[e] ->
1000 asnphecys[e] ->
1000 asntyrthr[e] ->
1000
     aspasnglu[e] ->
1000
     aspglu[e] ->
1000
     asphispro[e] ->
1000
     asplysglu[e] ->
1000
     asplyshis[e] ->
     aspprolys[e] ->
1000
     cysasnmet[e] ->
1000
     cysaspphe[e] ->
1000
     cyscys[e] ->
1000
     cyscys[e] ->
cysglnmet[e] ->
cysgluhis[e] ->
cysglutrp[e] ->
cyssermet[e] ->
1000
1000
1000
1000
     glnasngln[e] ->
1000
     glnhishis[e] ->
1000
     glnhislys[e] ->
1000
     glnlyslys[e] ->
1000
     glnlystrp[e] ->
1000
1000
     gluglu[e] ->
1000 gluilelys[e] ->
1000 gluleu[e] ->
1000 glumet[e] ->
1000 glumethis[e] ->
1000 glutrpala[e] ->
1000 glylyscys[e] ->
1000
     glylysphe[e] ->
1000
     glyvalhis[e] ->
1000
     hisasp[e] ->
1000
     hiscyscys[e] ->
1000
     hisglu[e] ->
1000
     hishislys[e] ->
     hislysthr[e] ->
1000
1000
       hismet[e] ->
1000
       hisprolys[e] ->
       histrphis[e] ->
1000
1000
       ileargile[e] ->
1000
       ileasp[e] ->
     ileglnglu[e] ->
1000
     ileglyarg[e] ->
1000
     ileserarg[e] ->
1000
     leuasnasp[e] ->
1000
     leuleutrp[e] ->
1000
1000
     leupro[e] ->
1000
     leuproarg[e] ->
1000
     leutrp[e] ->
1000
     leutrparg[e] ->
1000
     leutyrtyr[e] ->
1000
     leuval[e] ->
1000
     lysargleu[e] ->
1000
     lysgluglu[e] ->
1000
       lyslyslys[e] ->
```

```
1000
        lyspheile[e] ->
1000
       lystrparg[e] ->
1000
       lysvalphe[e]
1000
       methislys[e]
      metmetile[e]
1000
     pheasnmet[e] ->
1000
     pheglnphe[e] ->
1000
     phelysala[e] ->
1000
     phephe[e] ->
1000
     phepheasn[e] ->
1000
1000 phephethr[e] ->
1000 phesertrp[e] ->
1000 proargasp[e] ->
1000 procys[e] ->
1000 proglnpro[e] ->
1000 prohis[e] ->
1000 prohistyr[e] ->
1000
     proleuarg[e] ->
1000 prolyspro[e] ->
     proproarg[e] ->
1000
    proproarg[e] ->
propropro[e] ->
protrplys[e] ->
protrpthr[e] ->
serargala[e] ->
sercysarg[e] ->
serglyglu[e] ->
serphelys[e] ->
thrargtyr[e] ->
thrasptyr[e] ->
1000
1000
1000
1000
1000
1000
1000
1000
1000
     thrasntyr[e] ->
1000
1000 thrglntyr[e] ->
1000 thrhishis[e] ->
1000 thrthrarg[e] ->
1000 trpalapro[e] ->
1000 trpargala[e] ->
1000 trpaspasp[e] ->
1000 trpglngln[e] ->
1000 trpglugly[e] ->
1000 trpglyleu[e] ->
1000 trpglyval[e] ->
1000
     trphismet[e] ->
1000
     trpiletrp[e] ->
1000
       trplys[e] ->
1000
       trpmetarg[e] ->
1000
       trpprogly[e] ->
1000
                     ->
       trpproleu[e]
1000
       trpthrtyr[e]
                     ->
1000
       trptyrgln[e]
                     ->
1000
       trptyrtyr[e]
                     ->
1000
       tyralaphe[e] ->
1000
     tyrasparg[e] ->
1000
     tyrcysgly[e] ->
     tyrleuarg[e] ->
1000
     tyrthr[e] ->
1000
1000
     tyrtrpphe[e] ->
1000
     tyrtyr[e] ->
1000 tyrvalmet[e] ->
1000 valarggly[e] ->
1000 valleuphe[e] ->
1000 valphearg[e] ->
1000
     valserarg[e] ->
1000
     valtrpphe[e] ->
1000
     valtrpval[e] ->
1000
      valval[e] ->
```

```
1000
      homoval[e] ->
1000
      pe_hs[c] ->
1000 adprbp[c] ->
1000 mi145p[c] ->
1000 band[c] ->
1000 acgal[e] ->
1000 acnam[e] ->
1000 HC00342[e] ->
1000 pa_hs[e] ->
1000 CE2934[e] ->
1000 HC02191[c] ->
1000 HC02193[c] ->
1000 HC02194[c] ->
1000 HC02195[c] ->
1000 HC02196[c] ->
1000 Tyr_ggn[c] ->
1000 btn[c] ->
1000 coa[c] ->
1000 fad[c] ->
1000 lnlc[c] ->
1000 lnlccoa[c] ->
1000 nad[c] ->
     nad[c] ->
odecoa[c] ->
pmtcoa[c] ->
retinol[c] ->
stcoa[c] ->
tag_hs[c] ->
thf[c] ->
tmndnc[c] ->
dxtrn[e] ->
1000
1000
1000
1000
1000
1000
1000
1000 dxtrn[e] ->
1000 dhcholestanate[e] ->
1000 thcholstoic[e] ->
1000 xol7ah3[e] ->
1000 xol7ah3[c] ->
1000 xol7aone[e] ->
1000 7klitchol[e] ->
1000 glcn[e] ->
1000 acac[e] ->
1000 adn[e] ->
1000 akg[e] ->
1000 asn_L[e] ->
1000 asp_L[e] ->
1000 co2[e] ->
     dad_2[e] ->
1000
     dcyt[e] ->
1000
     fe2[e] ->
1000
     gal[e] ->
1000
      glu_L[e] ->
glyb[e] ->
1000
1000
      glypro[e] ->
1000
     ile_L[e] ->
1000
1000 ins[e] ->
1000 k[e] ->
1000 lac_L[e] ->
1000 leu_L[e] ->
1000 mal_L[e] ->
1000 met_L[e] ->
1000 no2[e] ->
1000 pi[e] ->
1000 pro_L[e] ->
1000 ser_L[e] ->
1000 succ[e] ->
1000 urea[e] ->
     uri[e] ->
1000
```

```
val_L[e] ->
1000
1000
       pnto_R[e] ->
     gly[e] -> cys_L[e] ->
1000
1000
     ala_L[e] ->
1000
     thr_L[e] ->
1000
     gln_L[e] ->
1000
1000
     phe_L[e] ->
1000
     tyr_L[e] ->
1000 for[e] ->
1000 nh4[e] ->
1000 ac[e] ->
1000 acgam[e] ->
1000 cit[e] ->
1000 drib[e] ->
1000 fru[e] ->
1000 galt[e] ->
1000 glcr[e] ->
1000
    glcur[e] ->
     glyc[e] ->
1000
    hxan[e] ->
1000
     malt[e] ->
1000
     ptrc[e] ->
spmd[e] ->
trp_L[e] ->
1000
1000
1000
     ura[e] -> xan[e] ->
1000
1000
     ppa[e] ->
pyr[e] ->
1000
1000
1000
     btn[e] ->
1000 ade[e] ->
1000 acald[e] ->
1000 gua[e] ->
1000 4abut[e] ->
1000 taur[e] ->
1000 phpyr[e] ->
1000 CE4970[e] ->
1000 CE4968[e] ->
1000 vanillac[e] ->
1000 2m3hvac[e] ->
1000 3h3mglt[e] ->
1000
    3mglutac[e] ->
     3mglutr[e] ->
1000
     mvlac[e] ->
1000
     ethmalac[e] ->
methsucc[e] ->
4ohbut[e] ->
1000
1000
1000
     agm[e] ->
T4hcinnm[e]
egme[e] ->
1000
1000
       T4hcinnm[e] ->
1000
     HC02020[e] ->
1000
1000 chsterols[e] ->
1000 CE1401[e] ->
1000 melatn[e] ->
1000 CE4890[e] ->
1000 C09642[e] ->
1000 mhista[e] ->
1000 ppbng[e] ->
1000 sphings[e] ->
1000 CE1918[e] ->
1000 aact[e] ->
1000
    Nlaspmd[e] ->
1000
     fdp[e] ->
1000
     ldl_hs[e] ->
```

```
HC00006[e] ->
1000
1000
       HC00007[e] ->
     HC00008[e] ->
1000
1000
     gluside_hs[e] ->
1000
     34dhpe[e] ->
1000
     sph1p[n] ->
1000
1000 sphs1p[n] ->
1000 gd3_hs[g] ->
1000 na1[g] ->
1000 na1[c] ->
1000 retn[n] ->
1000 Ser_Gly_Ala_X_Gly[r] ->
1000 5cysgly34dhphe[e] ->
1000 galgluside_hs[e] ->
1000 mem2emgacpail_prot_hs[e] ->
1000 qlc_D[e] ->
1000 cdca24g[c] ->
1000 cdca3g[c] ->
1000 dca24g[c] ->
1000 hca24g[c] ->
     hca6g[c] ->
1000
     hyochol[c] ->
1000
     lca24g[c] ->
1000
     lca3g[c] ->
1000
     12dhchol[e] ->
3dhcdchol[e] ->
3dhchol[e] ->
1000
1000
1000
1000
1000 3dhlchol[e] ->
1000 7dhcdchol[e] ->
1000 7dhchol[e] ->
1000 ca3s[e] ->
1000 coprost[e] ->
1000 dca3s[e] ->
1000 gca3s[e] ->
1000 gcdca3s[e] ->
1000 gdca3s[e] ->
1000 gudca3s[e] ->
1000 icdchol[e] ->
1000 isochol[e] ->
1000 lca3s[e] ->
1000 tca3s[e] ->
1000 tcdca3s[e] ->
     tdca3s[e] ->
1000
     thyochol[e] ->
tudca3s[e] ->
uchol[e] ->
udca3s[e] ->
1000
1000
1000
1000
1000
     3ispvs[e] -> 56dhpvs[e] ->
       3ispvs[e] ->
1000
1000 6epvs[e] ->
1000 6melvst[e] ->
1000 amlaccs[e] ->
1000 amlalcs[e] ->
1000 am4n9cs[e] ->
1000 am4ncs[e] ->
1000 crglz[e] ->
1000 deoxfvs[e] ->
1000 dhqlz[e] ->
1000 dspvs[e] ->
1000 nfdlac[e] ->
1000 nfdnpy[e] ->
1000
     ptvst[e] ->
```

```
1000
      thrfvs[e] ->
1000
      tmdm5[e] ->
1000
      M01807[e]
1000
      M00503[e]
1000
      M01820[e]
1000
      M00510[e] ->
     M00003[e] ->
1000
     M00010[e] ->
1000
     M02457[e] ->
0.2
0.05
    M03045[e] ->
1000 M02561[e] ->
1000 M01111[e] ->
1000 M01966[e] ->
1000 M01989[e] ->
1000 gpi_sig[e] ->
1000 glu_L[c] ->
1000 tyr_L[c] ->
1000 CE4888[c] ->
1000
    4abut[c] ->
    kynate[c] ->
1000
    tym[c] ->
1000
    cb12[m] ->
1000
1000
      protein[c] ->
```

Generate a relaxed model and test if it is feasible.

```
if solution.stat == 1
    modelRelaxed=model;
    delta=0;%can be used for debugging, in case more relaxation is necessary
    modelRelaxed.lb = model.lb - p - delta;
    modelRelaxed.ub = model.ub + q + delta;
    modelRelaxed.b = model.b - r;

FBAsolution = optimizeCbModel(modelRelaxed,'max', 0, true);
    if FBAsolution.stat == 1
        disp('Relaxed model is feasible');
    else
        disp('Relaxed model is infeasible');
        solutionRelaxed = relaxedFBA(modelRelaxed,relaxOption);
    end
end
```

Relaxed model is feasible

EXPECTED RESULTS

The relaxed model should be feasible. Indicated by 'Relaxed model is feasible'

TROUBLESHOOTING

If the relaxed model is not feasible. If not, there could be a numerical issue due to the numerical tolerance of the linear optimisation solutions or due to the numerical tolerance on the relaxedFBA algorithm, both of which are by default set to the feasibility tolerance for the currently installed solver (typically 1e-6 for a double precision

solver like Gurobi). If problems persist, examine the numerical properties of the constraints, esp wrt scaling, or try the dqqMinos solver.

%changeCobraSolver('dqqMinos','LP')

REFERENCES

Fleming, R.M.T., et al., Cardinality optimisation in constraint-based modelling: Application to Recon 3D (submitted), 2017.

Brunk, E. et al. Recon 3D: A resource enabling a three-dimensional view of gene variation in human metabolism. (submitted) 2017.