Math 120A (Differential Geometry)
University of California, Los Angeles

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These are my lecture notes for Math 120A (Differential Geometry), which is taught by Fumiaki Suzuki. The textbook for this class is *Differential Geometry of Curves and Surfaces*, by Kristopher Tapp. Many of the figures I include in these notes are taken from Tapp's book, and will be referenced throughout my notes.

Contents

1   Jan 3, 2022

1.1   What is Differential Geometry?

Differential geometry studies geometry via analysis and linear algebra.

Geometry	Analysis	Linear Algebra
Intuitive	Rigorous	Computable
Curved	$\xrightarrow{\text{tangent space}}$	Linear
Global	Local	

1.2   Parametrized Curves

Example 1.1

A unit circle $S' = \{\vec{x} \text{ in } \mathbb{R}^2 \mid |\vec{x}| = 1\}$
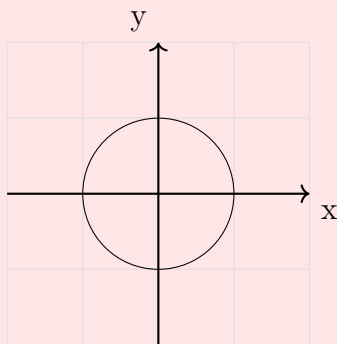
$$\vec{\gamma} : [0, 2\pi) \to \mathbb{R}^2$$
$$t \mapsto (\cos t, \sin t)$$

$\vec{\gamma}[0, 2\pi) = S'$

Definition 1.2 (Parametrized curve and Trace)

A (parametrized) curve is a smooth function $\vec{\gamma} : I \to \mathbb{R}^n$, where I is an interval in $\mathbb{R}$. The image

$$\vec{\gamma}(I) = \{\vec{\gamma}(t) \mid t \in I\}$$

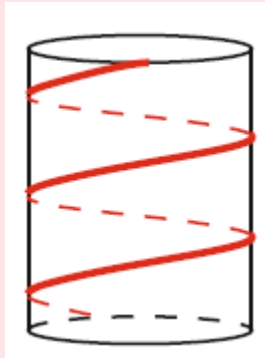is called the trace of $\vec{\gamma}$.

Recall 1.3 An interval is a subset of $\mathbb{R}$ that has one of the following forms:

$$(a, b), [a, b], (a, b], [a, b), (-\infty, b), (-\infty, b], (a, \infty), [a, \infty), (-\infty, \infty) = \mathbb{R}.$$

A function $\vec{\gamma} : I \to \mathbb{R}^n$ is called smooth if $\vec{\gamma}$ is infinitely differentiable, or equivalently, each of the component functions $x_i : I \to \mathbb{R}$ is infinitely differentiable.

Example 1.4

$\vec{\gamma}(t) = (\cos t, \sin t, t)$, $t \in (-\infty, \infty)$ is a curve, called a helix.

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