

# Linear Algebra

# Scalar

# Scalar

1

# Scalar

1

5

# Scalar

1

5

23

# Scalar

1

5

23

42

# Scalar

1

5

23.5

23

42

# Vector

**(0,1,1,2)**



# Vector

**(0,1,1,2)**

**(3,5,8)**

# Vector

**(0,1,1,2)**

**(3,5,8,13)**

**(3,5,8)**

# Tuple

# Tuple

**(0,1,1,2)**

# Tuple

**(0,1,1,2)**

**(0,1,2.1,2.3)**

# Matrix

# Matrix

**(0,1,1,2)**

**(1,5,1,2)**

**(0,1,1,4)**

# Matrix

**(0,1,1,2)**

**(1,5,1,2)**

**(0,1,1,4)**

**(1,5,1,2,7)**

**(0,1,1,4,2)**



# Matrix

**(0,1,1,2)**

**(1,5,1,2)**

**(0,1,1,4)**

**(1,5,1,2,7)**

**(0,1,1,4,2)**

**n - columns**

**m - rows**

# Matrix

**(0,1,1,2)**

**(1,5,1,2)**

**(0,1,1,4)**

**(1,5,1,2,7)**

**(0,1,1,4,2)**

**n - columns**

**m - rows**

**m - by - n**

# Matrix

**3 - by - 4**

**(0,1,1,2)**

**(1,5,1,2)**

**(0,1,1,4)**

**(1,5,1,2,7)**

**(0,1,1,4,2)**

**n - columns**

**m - rows**

**m - by - n**

# Matrix

**3 - by - 4**

**(0,1,1,2)**

**(1,5,1,2)**

**(0,1,1,4)**

**2 - by - 5**

**(1,5,1,2,7)**

**(0,1,1,4,2)**

**n - columns**

**m - rows**

**m - by - n**

Diagram illustrating the stress components on a 3D element (cube) in a coordinate system defined by unit vectors  $\mathbf{e}_1$ ,  $\mathbf{e}_2$ , and  $\mathbf{e}_3$ .

The cube is divided into three colored regions (red, orange, yellow) representing different faces. The stress components are labeled as follows:

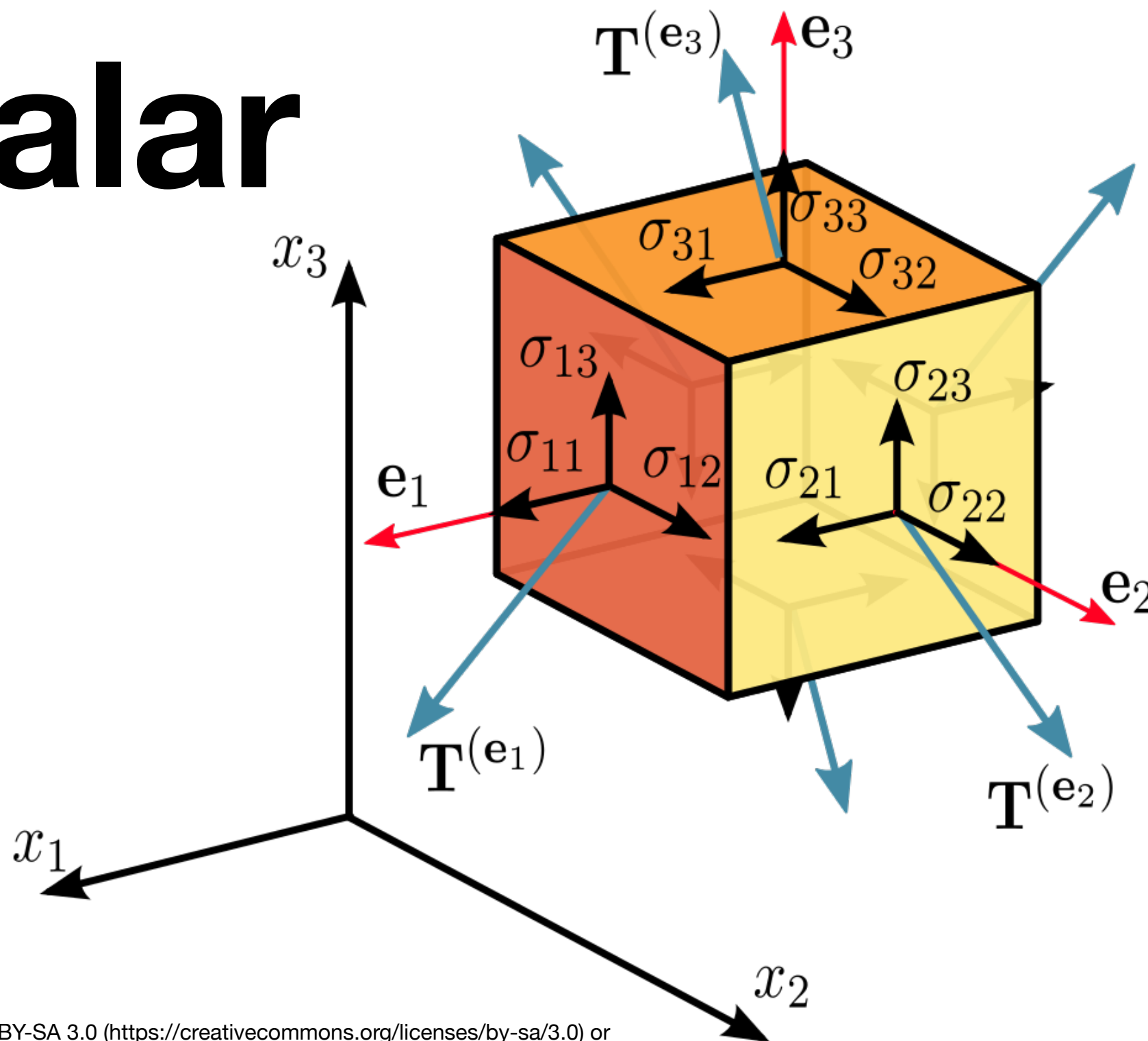
- Red Face (Left):** Normal stress  $\sigma_{11}$ , Shear stress  $\sigma_{12}$  (horizontal), and Shear stress  $\sigma_{13}$  (vertical).
- Orange Face (Top):** Normal stress  $\sigma_{33}$ , Shear stress  $\sigma_{31}$  (horizontal), and Shear stress  $\sigma_{32}$  (vertical).
- Yellow Face (Right):** Normal stress  $\sigma_{22}$ , Shear stress  $\sigma_{21}$  (horizontal), and Shear stress  $\sigma_{23}$  (vertical).

The traction vectors  $\mathbf{T}(\mathbf{e}_1)$ ,  $\mathbf{T}(\mathbf{e}_2)$ , and  $\mathbf{T}(\mathbf{e}_3)$  are shown as blue arrows originating from the center of the cube. The coordinate axis  $x_2$  is indicated at the bottom left.

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# Tensor

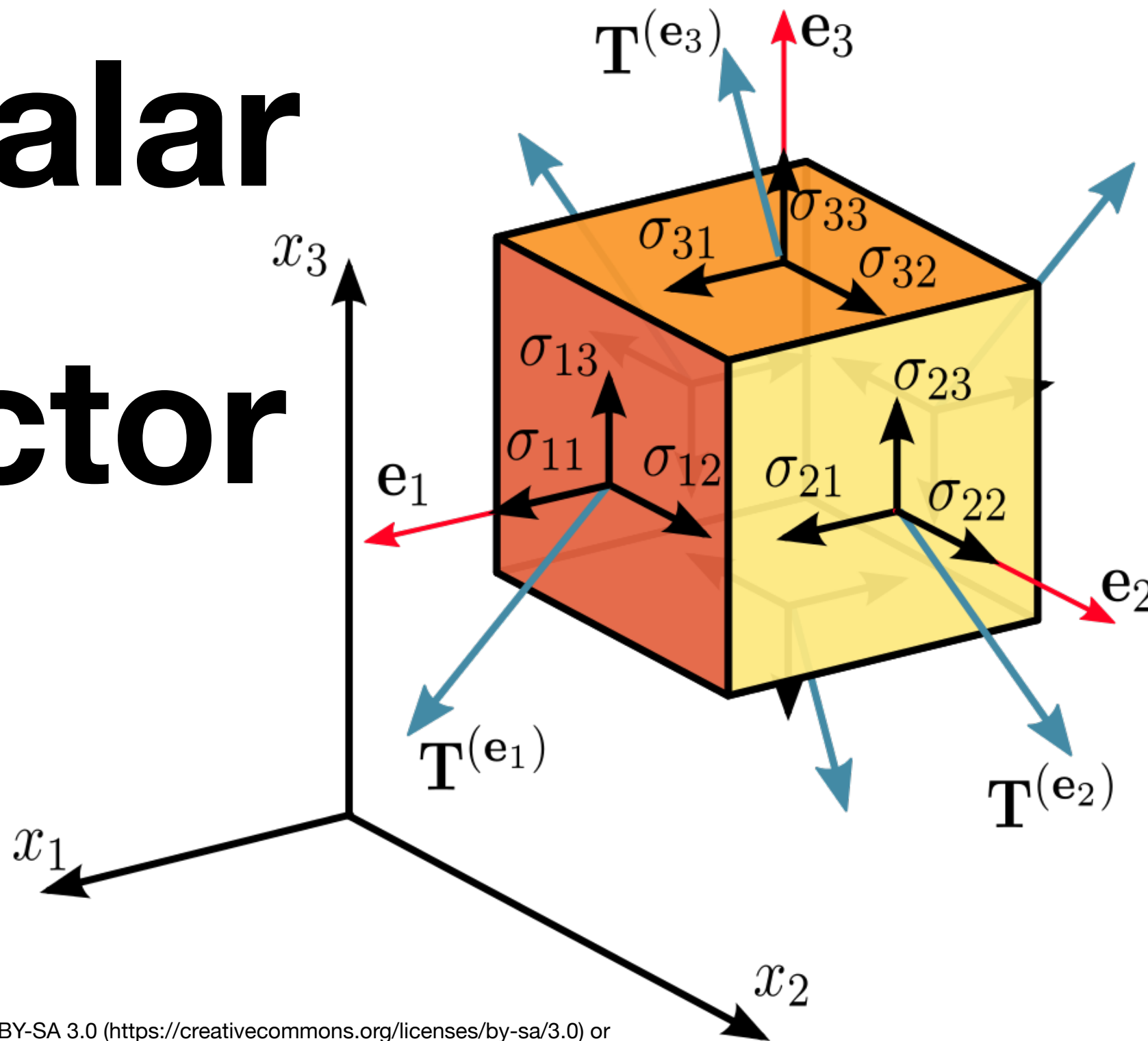
# Scalar



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# Tensor

Scalar  
Vector



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# Tensor



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# Multiplication

# Scalar Multiplication

$$2 * 3 = 6$$

**a3\_m1\_v1\_ex1\_1**

# Scalar Multiplication

$$\begin{bmatrix} 0 \\ 1 \\ 3 \\ 6 \end{bmatrix} * \begin{bmatrix} 2 \\ 7 \\ 13 \\ 20 \end{bmatrix}$$

# Vector Multiplication

$$\begin{bmatrix} 0 \\ 1 \\ 3 \\ 6 \end{bmatrix} * \begin{bmatrix} 2 \\ 7 \\ 13 \\ 20 \end{bmatrix} = 0 * 2$$

# Vector Multiplication

$$\begin{bmatrix} 0 \\ 1 \\ 3 \\ 6 \end{bmatrix} * \begin{bmatrix} 2 \\ 7 \\ 13 \\ 20 \end{bmatrix} = 0 * 2 \quad 1 * 7$$

# Vector Multiplication

$$\begin{bmatrix} 0 \\ 1 \\ 3 \\ 6 \end{bmatrix} * \begin{bmatrix} 2 \\ 7 \\ 13 \\ 20 \end{bmatrix} = 0 * 2 \quad 1 * 7 \quad 3 * 13 \quad 6 * 20$$

# Vector Multiplication

$$\begin{bmatrix} 0 \\ 1 \\ 3 \\ 6 \end{bmatrix} * \begin{bmatrix} 2 \\ 7 \\ 13 \\ 20 \end{bmatrix} = 0 * 2 + 1 * 7 + 3 * 13 + 6 * 20$$



# Vector Multiplication

$$\begin{bmatrix} 0 \\ 1 \\ 3 \\ 6 \end{bmatrix} * \begin{bmatrix} 2 \\ 7 \\ 13 \\ 20 \end{bmatrix} = 0 * 2 + 1 * 7 + 3 * 13 + 6 * 20 = 166$$

# Vector Multiplication

$$\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} = a_1 * x_1 + a_2 * x_2 + a_3 * x_3 + a_4 * x_4$$

**a3\_m1\_v1\_ex1\_2**

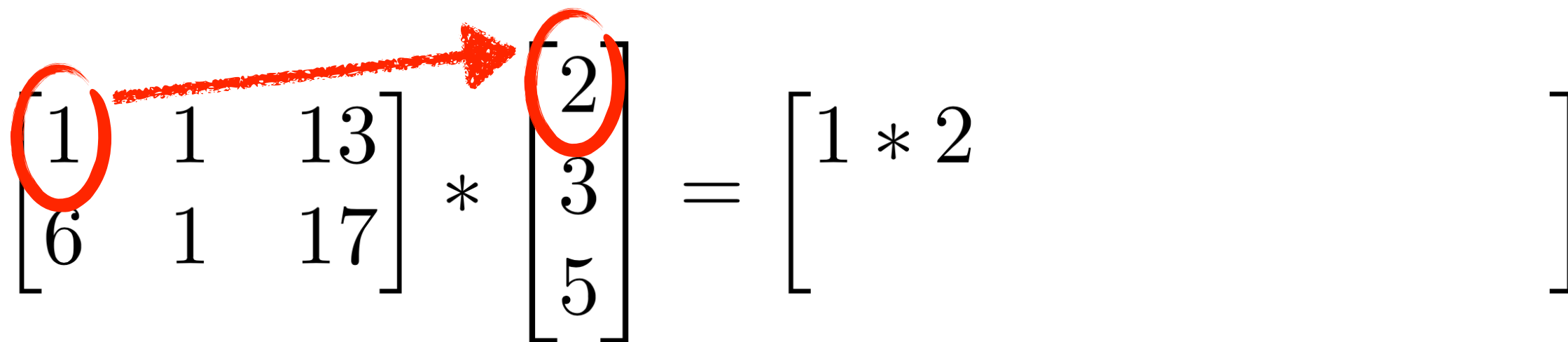
# Vector Matrix Multiplication

$$\begin{bmatrix} 1 & 1 & 13 \\ 6 & 1 & 17 \end{bmatrix} * \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$$

# Vector Matrix Multiplication

$$\begin{bmatrix} 1 & 1 & 13 \\ 6 & 1 & 17 \end{bmatrix} * \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 * 2 \\ \phantom{1 * 2} \end{bmatrix}$$

# Vector Matrix Multiplication


$$\begin{bmatrix} 1 & 1 & 13 \\ 6 & 1 & 17 \end{bmatrix} * \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 * 2 & & \\ & & \\ & & \end{bmatrix}$$

# Vector Matrix Multiplication

$$\begin{bmatrix} 1 & 1 & 13 \\ 6 & 1 & 17 \end{bmatrix} * \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 * 2 \\ \phantom{1 * 2} \\ \phantom{1 * 2} \end{bmatrix}$$

# Vector Matrix Multiplication

$$\begin{bmatrix} 1 & 13 \\ 6 & 17 \end{bmatrix} * \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 * 2 & 1 * 3 \\ 6 * 2 & 6 * 3 \end{bmatrix}$$



# Vector Matrix Multiplication

$$\begin{bmatrix} 1 & 13 & 17 \\ 6 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 * 2 & 13 * 3 & 17 * 5 \\ 6 * 2 & 1 * 3 & 1 * 5 \end{bmatrix}$$

# Vector Matrix Multiplication

The diagram illustrates the multiplication of a row vector by a column vector. On the left, a row vector  $\begin{bmatrix} 1 & 1 & 13 \\ 6 & 1 & 17 \end{bmatrix}$  is shown with its second row  $[6 \ 1 \ 17]$  circled in red. An orange arrow with a '\*' symbol points to a column vector  $\begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$ , which is also circled in red. A second orange arrow with an '=' symbol points to the resulting row vector  $\begin{bmatrix} 1 * 2 & 1 * 3 & 13 * 5 \\ 6 * 2 & 1 * 3 & 17 * 5 \end{bmatrix}$ . The second row of the result,  $[6 * 2 \ 1 * 3 \ 17 * 5]$ , is circled in red.

$$\begin{bmatrix} 1 & 1 & 13 \\ 6 & 1 & 17 \end{bmatrix} * \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 * 2 & 1 * 3 & 13 * 5 \\ 6 * 2 & 1 * 3 & 17 * 5 \end{bmatrix}$$

# Vector Matrix Multiplication

$$\begin{bmatrix} 1 & 1 & 13 \\ 6 & 1 & 17 \end{bmatrix} * \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 * 2 + 1 * 3 + 13 * 5 \\ 6 * 2 + 1 * 3 + 17 * 5 \end{bmatrix}$$

# Vector Matrix Multiplication

$$\begin{bmatrix} 1 & 1 & 13 \\ 6 & 1 & 17 \end{bmatrix} * \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 * 2 + 1 * 3 + 13 * 5 \\ 6 * 2 + 1 * 3 + 17 * 5 \end{bmatrix} = \begin{bmatrix} 70 \\ 100 \end{bmatrix}$$

# Vector Matrix Multiplication

$$\begin{bmatrix} a & b & c \\ \vdots & & \\ d & e & f \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a * x + b * y + c * z \\ \vdots \\ d * x + e * y + f * z \end{bmatrix}$$

**a3\_m1\_v1\_ex1\_3**

# Deep Feedforward Networks