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# **OLS** Regression

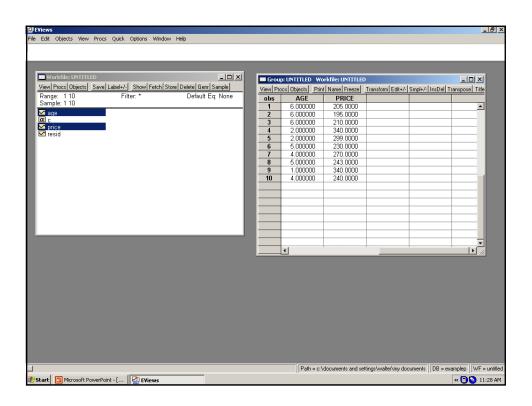
### • Problem

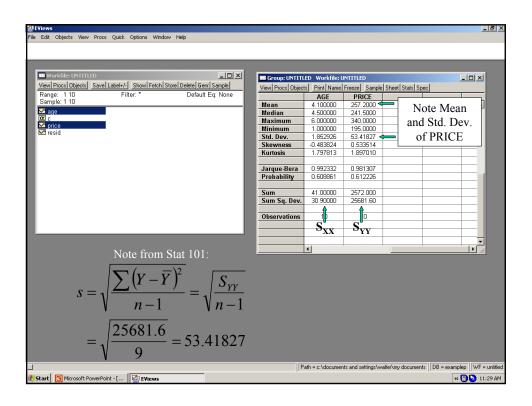
 The Kelley Blue Book provides information on wholesale and retail prices of cars. Following are age and price data for 10 randomly selected Corvettes between 1 and 6 years old. Here, age is in years, and price is in hundreds of dollars.

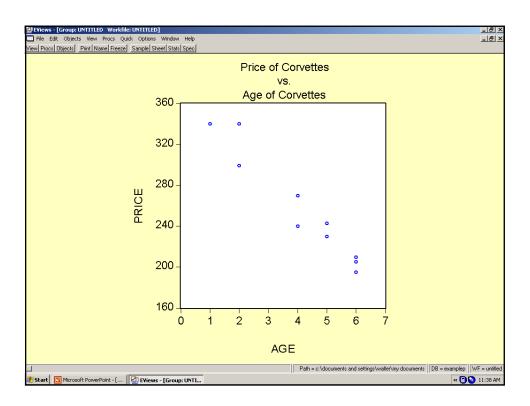
age	6	6	6	2	2	5	4	5	1	4
price	205	195	210	340	299	230	270	243	340	240

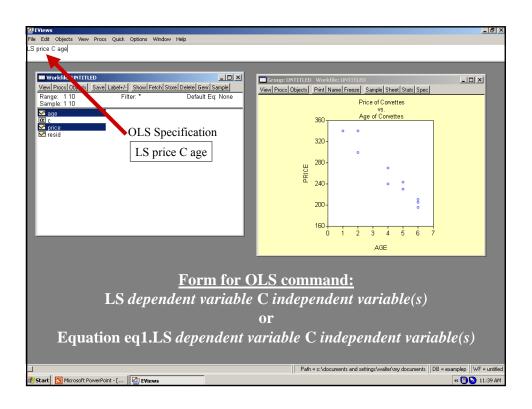
# **OLS Regression**

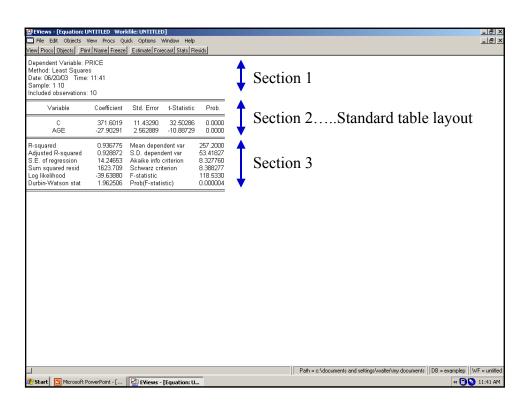
Coding Sheet for Corvette Data						
Variable	Possible Values	Source	Mnemonic			
Age of Corvettes	Years	Kelley Blue Book, various issued	age			
Price of Corvettes	Hundred of Dollars, Nominal	IBID.	price			

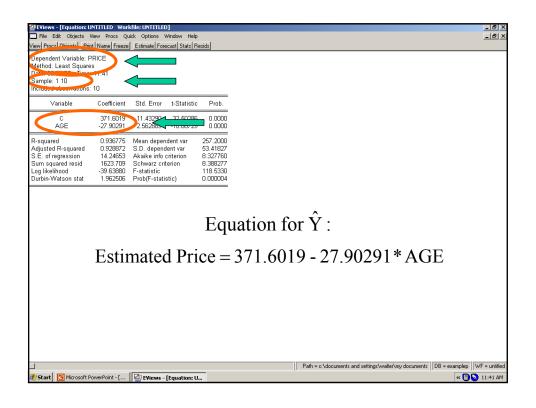


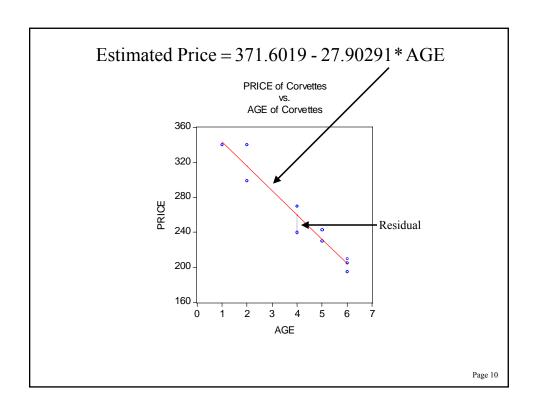


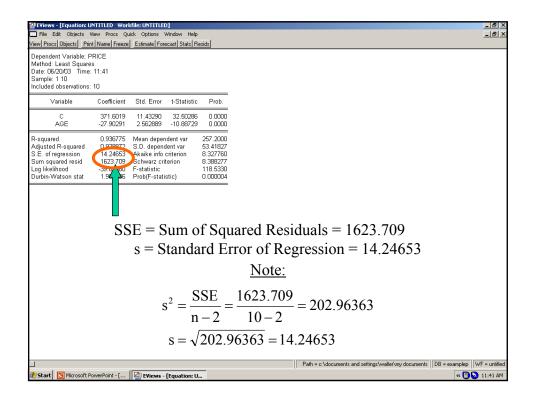


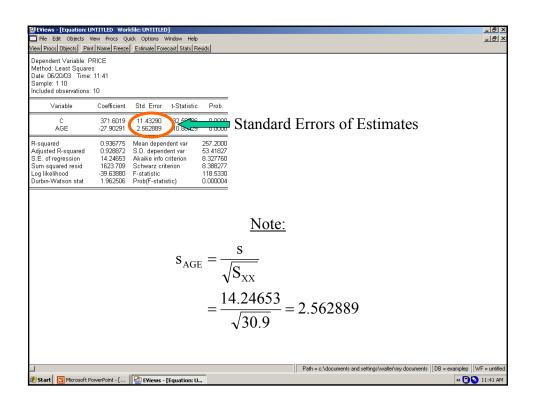


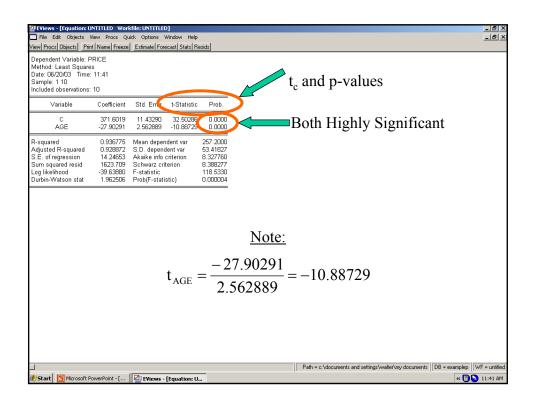


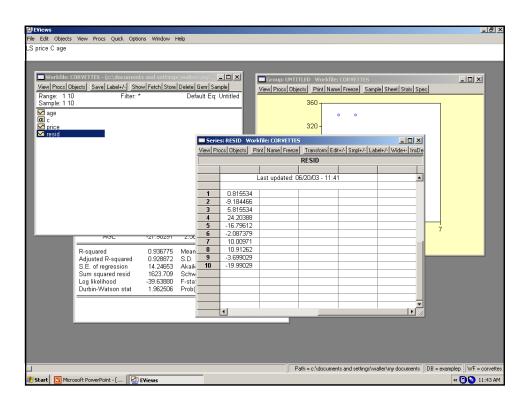


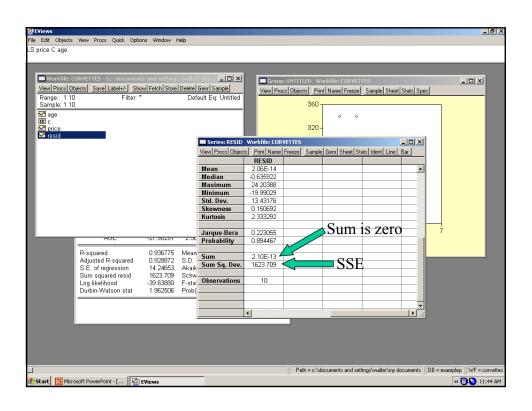


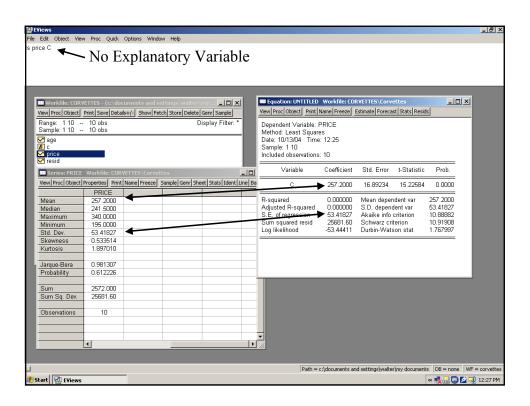


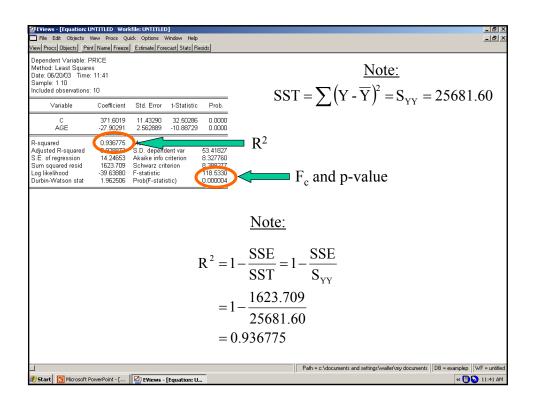


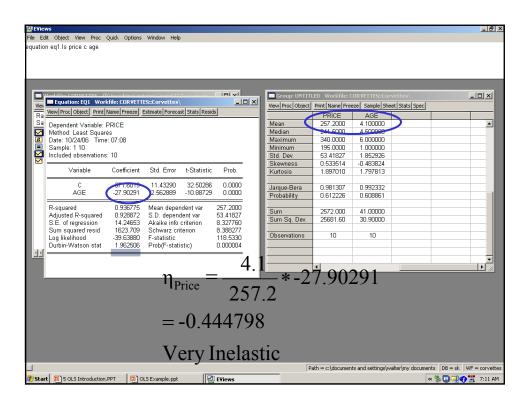












# **OLS** Regression

#### **Elasticity Summary Table**

Variable	Estimate	Mean	Elasticity	Classification
Price	-27.90291	4.1	-0.444798	Inelastic

**Interpretation**: Price of a corvette is inelastic with respect to the age of the corvette so that a 1% increase in age decreases the price by only 0.4%. Other factors are at play regarding the lower prices, but age is certainly a major factor as evidenced by the R<sup>2</sup> of 0.94.

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Other Examples
Problem 1: CAPM

- Problem
  - Estimate  $\beta$ , the systematic risk, for CAPM
    - CAPM is

$$E(r_{i} \mid \Omega_{t}) = r_{f} + \beta_{i} [E(r_{m} \mid \Omega_{t}) - r_{f}]$$

• An empirical version is  $\widetilde{r}_{i} = \alpha + \beta_{i} \widetilde{r}_{m} + \epsilon_{i}$  $\epsilon \sim N(0, \sigma^{2})$ 

$$\widetilde{\mathbf{r}}_{\mathbf{i}} = \mathbf{r}_{\mathbf{i}} - \mathbf{r}_{\mathbf{f}}$$

$$\widetilde{r}_{m} = r_{m} - r_{f}$$

$$\alpha = 0$$

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### Problem 1: CAPM

- beta is key
  - Each security has a beta
  - Each portfolio has a beta
    - Portfolio beta is weighted average of individual betas

• beta is proportionality factor

$$\beta_{i} = \frac{E(r_{i} \mid \Omega_{t}) - r_{f}}{E(r_{m} \mid \Omega_{t}) - r_{f}}$$

- Excess returns for security over riskless return is proportional to excess returns in market over riskless returns
- Excess is extra return to compensate for risk for not holding the market portfolio
  - Premium on security is proportional to premium on the market portfolio

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### Problem 1: CAPM

• If

$$\beta_{i} = \frac{E(r_{i} \mid \Omega_{t}) - r_{f}}{E(r_{m} \mid \Omega_{t}) - r_{f}} > 1 \Rightarrow Premium_{i} > Premium_{m}$$

then asset i is viewed as riskier than the market

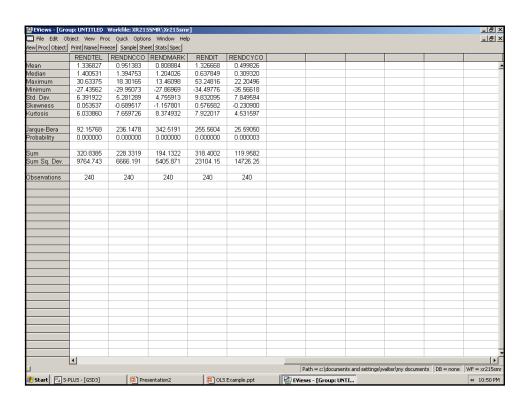
beta	Interpretation	<b>Example</b>
1	Moves With Market	Conglomerates (AT&T)
>1	More Volatile	Companies Sensitive To Macro Events (Autos)
0 <beta<1< td=""><td>Less Volatile</td><td>Mildly Sensitive To Macro Events (Electric Utilities)</td></beta<1<>	Less Volatile	Mildly Sensitive To Macro Events (Electric Utilities)
<0	Move Counter To Market	Goldmining Stocks
		Page 25

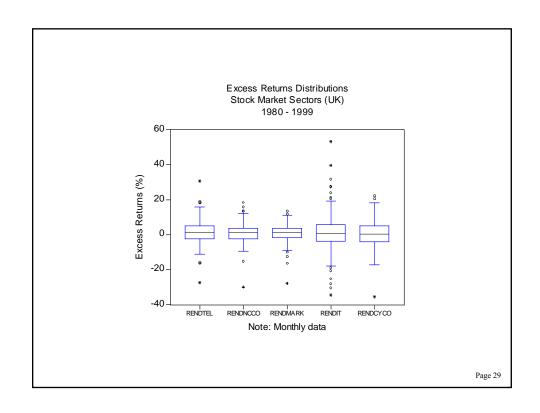
Coding Sheet for CAPM Data						
Variable	Possible Values	Source	Mnemonic			
Excess returns for an overall stock market index of the total market in the UK.  Monthly, 1/80-12/99.	Percentages*	DataStream (2000)	rendmark			
Excess returns on an index of 104 stocks in the cyclical consumer goods sector in the UK. Monthly, 1/80-12/99.	Percentages*	IBID.	rendcyco			
Excess returns on an index of 104 stocks in the noncyclical consumer goods sector in the UK. Monthly, 1/80-12/99.	Percentages*	IBID.	rendncco			
Excess returns on an index of 104 stocks in the information tech sector in the UK.  Monthly, 1/80-12/99.	Percentages*	IBID.	rendit			
Excess returns on an index of 104 stocks in the telcom sector in the UK. Monthly, 1/80-12/99.	Percentages*	IBID.	rendtel			

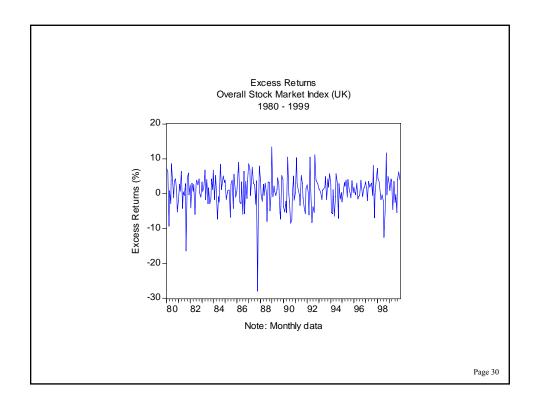
\*Note: See calculations note.

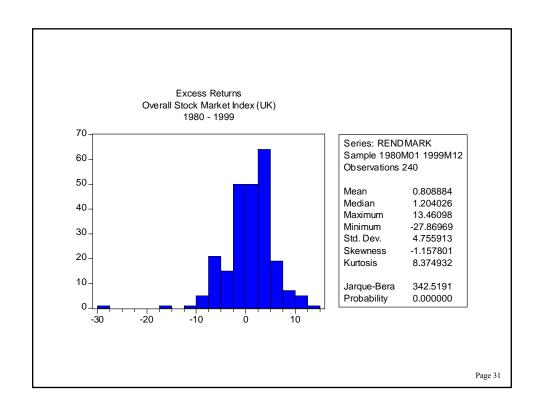
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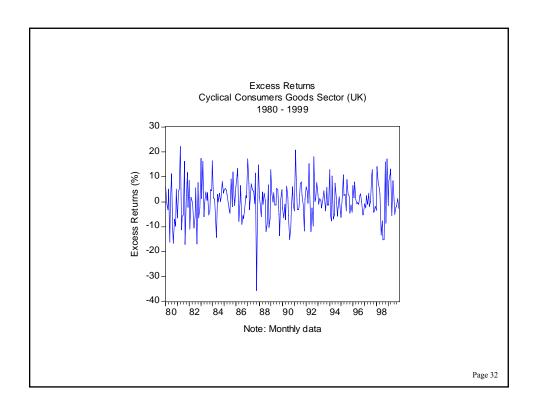
- Excess returns are calculated as follows
  - Let  $p_i$  be the closing price of the index at the last trading day in month i and let  $r_i$  be the onemonth interest rate at the start of month i. Then the return  $v_i$  of the index is  $v_i = (p_i p_{i-1})/p_{i-1}$  and the excess return is  $v_i r_i$ . The reported numbers are  $100(v_i r_i)$ .

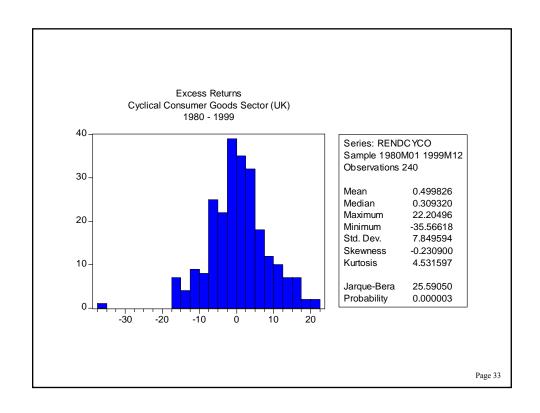


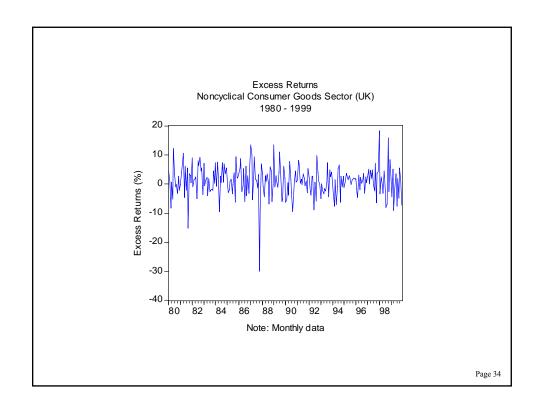


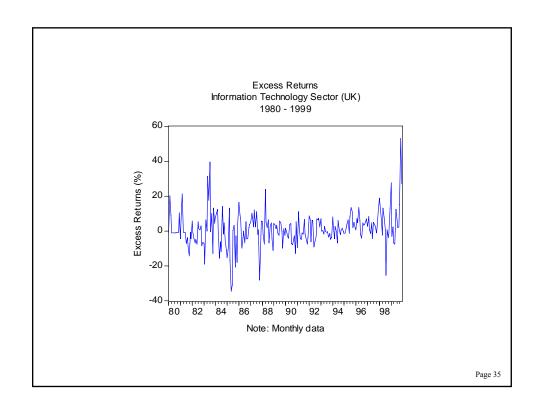


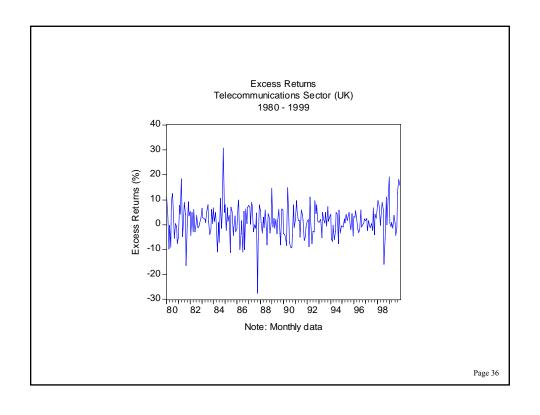


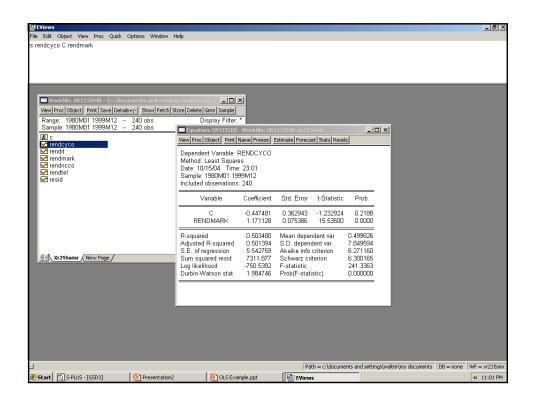












### • Interpretation

- The intercept is highly insignificant suggesting that the line goes through the origin
- The slope is highly significant and positive at the 0.0 level indicating that the market rate of return (excess returns) have a positive effect on the returns for the cyclical consumer goods sector in the UK
- The R<sup>2</sup> is 0.50 suggesting that 50% of the variation in returns in the cyclical consumer goods sector is accounted for by the market excess returns
- The model is significantly different from the naïve model as indicated by the p-value for F

# • Interpretation

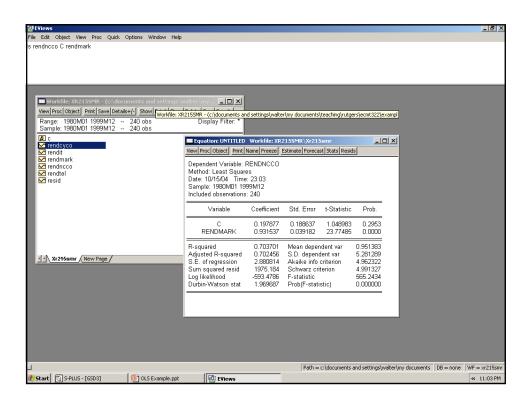
- Since this is a CAPM, the slope has the interpretation of systematic risk
  - Since it is greater than 1.0, this indicates that the cyclical consumer goods sector is riskier that the market as a whole
  - When the market rises, the excess returns in this sector will rise more than the market

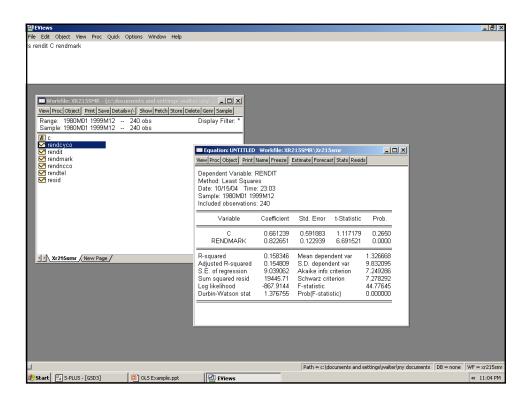
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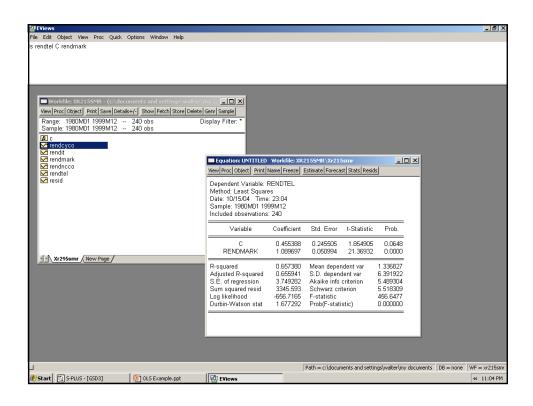
Problem 1: CAPM

Model Portfolio

Model Politiono				
	Model			
Intercept	-0.477			
	(0.2188)			
RendMark	1.17*			
	(0.0000)			
$\mathbb{R}^2$	0.5035			
F <sub>C</sub> 241.3363				
	(0.0000)			
Notes: p-value in parenthese; *=significant				







# Other Examples Problem 2: Bank Wages

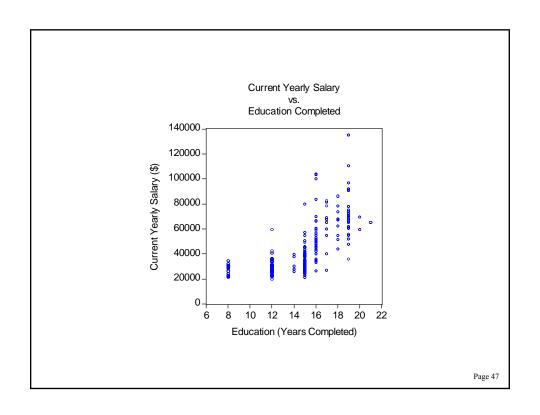
# Problem 2: Bank Wages

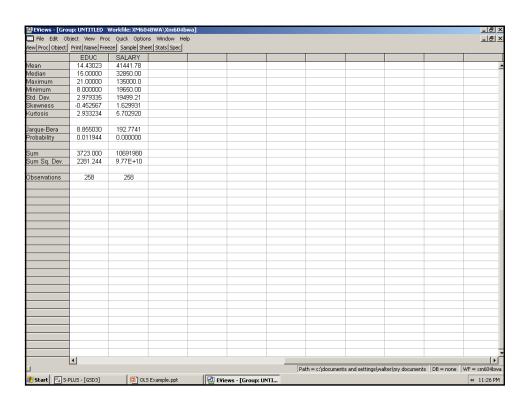
### • Problem

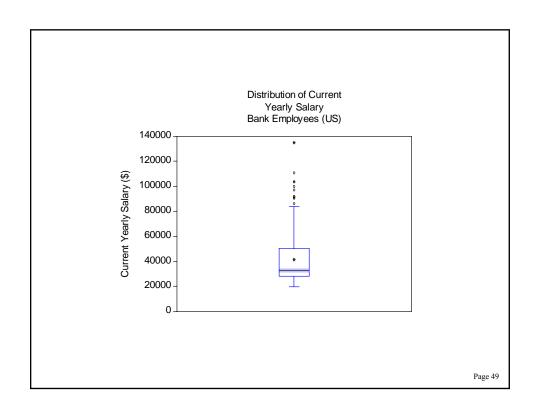
 Find the relationship between the salary of employees at a major US bank and their years of education completed

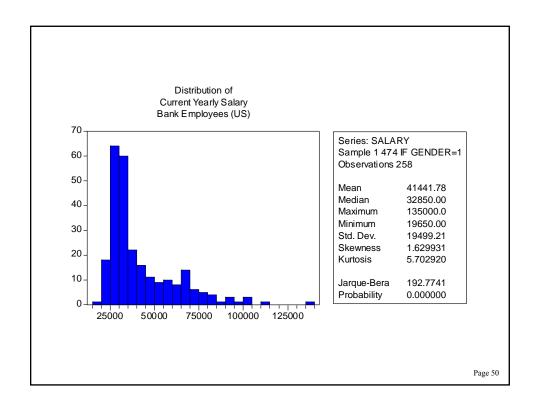
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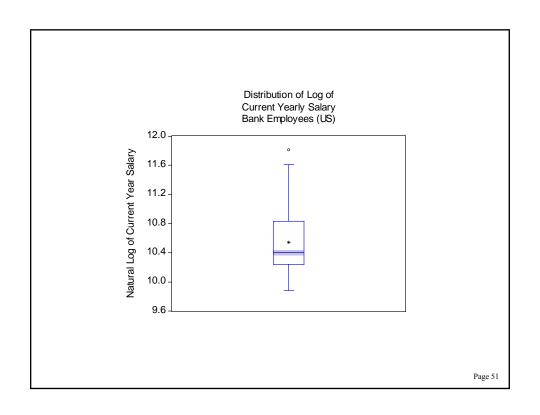
Coding Sheet for Bank Salary Data							
Variable	Possible Values	Source	Mnemonic				
Current yearly salary of bank employees in US	Nominal dollars	SPSS, version 10 (2000)	salary				
Number of years of education completed	Years	IBID.	educ				
Natural log of salary	logs	Calculated as natural log of salary	logsalary				

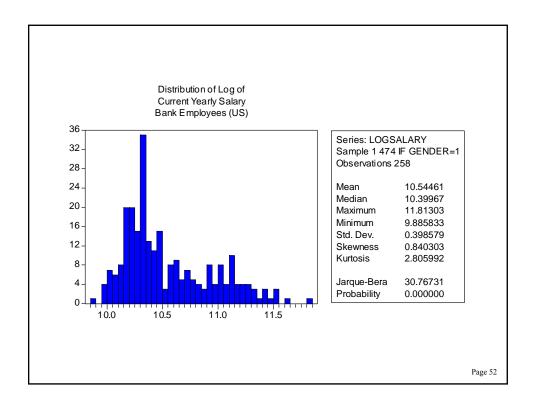












### Problem 2: Bank Wages

· Our model is

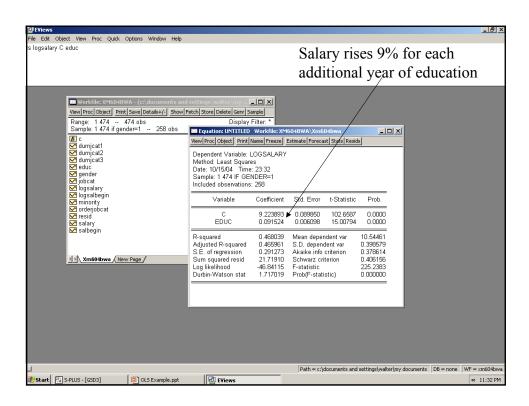
$$Salary_{_{i}}=Ae^{\beta_{l}Educ_{_{i}}}e^{\epsilon_{i}}$$

or

$$ln(SALARY_i) = \beta_0 + \beta_1 EDUC_i + \varepsilon_i$$

• Interpret the slope parameter as the percentage increase in salary (S) due to one additional year of education

$$\frac{d\ln(S)}{dx} = \frac{dS/S}{dx}$$



### Problem 2: Bank Wages

# • Interpretation

- The intercept and slope parameters are highly significant at the 0.0 level
  - This indicates that education has a significant, positive effect on salary
    - The higher the level of education, the higher the salary
  - For each extra 1 year of education, salary rises 9%
- Almost 49% of the variation in (log) salary is accounted for by education
- The model is significantly different from the naïve model as indicated by the p-value for F

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Problem 2: Bank Wages

Model Portfolio

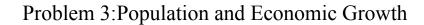
Model Portiolio				
	Model			
Intercept	9.22*			
	(0.0000)			
Educ	0.092*			
	(0.0000)			
$\mathbb{R}^2$	0.4680			
F <sub>C</sub> 225.2383				
	(0.0000)			
Notes: p-value in parenthese; *=significant				

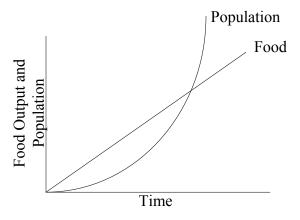
# Other Examples Problem 3: Population and Economic Growth

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### Problem 3:Population and Economic Growth

- Population growth is an important factor for long-term economic growth
  - Malthus assumed (correctly) that population will grow geometrically (e.g., 1, 3, 9, 27, 81,...) while the food supply will increase arithmetically (e.g., 10, 10, 30, 40 ...)
    - Population would double every 24 years
  - As a result of the faster growth of population, population growth would overtake food supply growth – but he didn't specify when





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# Problem 3:Population and Economic Growth

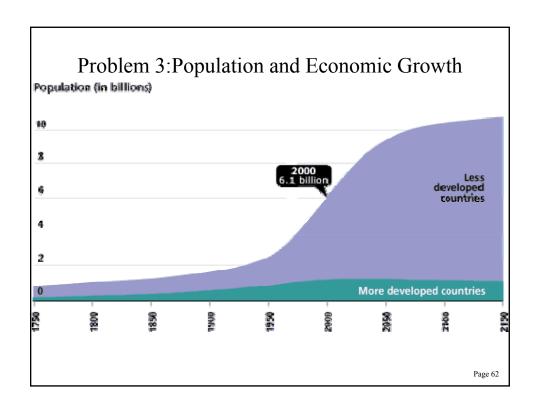
- Digression on compound growth
  - Compounding

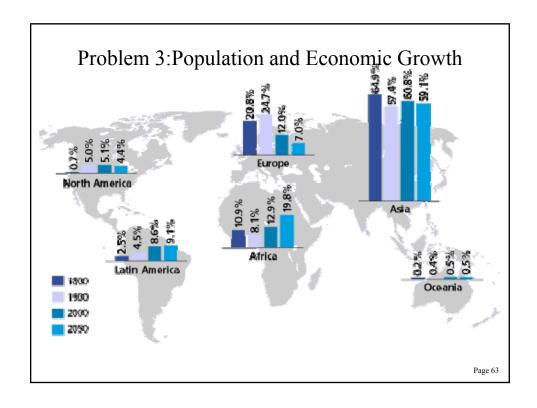
$$FV_t = PV_0(1+g)^t$$

- Time to double
  - What is the implied population growth rate, *g*, for Malthus?
  - How long to double at 1.9%?

# Problem 3:Population and Economic Growth

- Curent population projections
  - The Year 2000 population growth rate, g, was about 1.4%
    - World population in 2000 was about 6.1Billion people
    - Absolute growth: 85.4 Million in one year





# Problem 3:Population and Economic Growth

- The future
  - There is some hope ahead
    - The UN forecasts a drop in fertility in developing countries
    - Developed countries will grow by less than 1% annually
    - Predicted world's population in 2300
      - Old: 12 BillionNew: 9 Billion

### Problem 3:Population and Economic Growth

- Malthus had checks on population growth
  - Contraceptives
  - Abortion
  - Self-restraint
  - Wars
- Malthus wrote right at the time of the Industrial and Green Revolutions, so he could not see their implications

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### Problem 3:Population and Economic Growth

- Data from the World Development Indicators, World Bank (2004)
  - Calculated average annual growth for GDP and population

$$g_i = \left(\frac{\text{Value in } 2003_i}{\text{Value in } 1963_i}\right)^{\frac{1}{40}} - 1, i = 1, ..., 107 \text{ countries}$$

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Coding Sheet for Growth Data						
Variable	Possible Values	Source	Mnemonic			
Real GDP in country (1995 \$US), 1963 and 2003	Real dollars	World Development Indicators (World Bank, 2004)	gdp63, gdp03			
Average annual growth in GDP	Decimal values	Calculated	g			
Population in country, 1963 and 2003	Count	World Development Indicators (World Bank, 2004)	pop63, pop03			
Average annual growth in population	Decimal values	Calculated	p			

