1) SUBGRAPH ISOMORPHISM is in NP as we can build a polynomial time verifier for it. Consider a subgraph of G with mappings between its vertices and vertices of H such that it is possible to verify that H is isomorphic to a subgraph of G in polynomial time. Next we will reduce from CLIQUE to show that SUBGRAPH ISOMORPHISM is NP-hard. Given (G, k) of CLIQUE, we know that a clique of size k exists in G. Consider an instance of SUBGRAPH ISOMORPHISM (G', H) where $G' = G$ and H is a complete graph of size k that is a subgraph of G'. On input (G, H), then (G, H) is an instance of SUBGRAPH ISOMORPHISM if (G, k) is an instance of CLIQUE. This reduction runs in polynomial time. If we have a YES instance of CLIQUE, we know that a connected subgraph of G exists in size k. Since G satisfies CLIQUE and $G' = G$, then a subgraph of size k exists in G' and H is a connected graph of size k and is a subgraph of G'. $\therefore$ G' has a subgraph that is isomorphic to H. Thus yes maps to yes. If we have a YES instance of SUBGRAPH ISOMORPHISM, then we can create a YES instance of CLIQUE. Consider H a complete graph of size k that is a subgraph of G'. Since H is a complete graph of size k we can convert it to a CLIQUE of size k in a given graph $G = G'$. Thus we have created a YES instance (G, k) of CLIQUE. No maps to no. $\therefore$ SUBGRAPH ISOMORPHISM is NP-complete.

2) Given $T \subseteq C$, we can verify that T is a valid cover of U and whether $|T| \leq k$. Specifically, we check that every element $u \in U$ appears in at least one set $S_i \in T$. Since this verification can be done in polynomial time, SET COVER belongs to NP. We now prove that SET COVER is NP-hard by reducing from VERTEX COVER. Let $G = (V, E)$, k be an instance of VERTEX COVER, where G is an undirected graph with vertex set V and edge set E, and k is the maximum allowed size of the vertex cover. We will construct an instance of SET COVER as follows: define the universe $U = E$, the set of elements to be covered corresponds to the edges of G. For each vertex $v \in V$, define a set S_v containing all edges incident to v. Let $C = \{S_v : v \in V\}$ and set the cover size bound to be k. This transformation runs in polynomial time. If G has a vertex cover $V' \subseteq V$ of size at most k. Every edge is contained in at least one of the sets S_u or S_v for $u, v \in V'$. The collection $T = \{S_v : v \in V'\}$ forms a valid set cover of size $\leq k$. $\therefore$ yes maps to yes. If there is a set cover $T \subseteq C$ with $|T| \leq k$, we will define $V' = \{v \in V : S_v \in T\}$. Since T covers $U = E$, every edge $e = (u, v)$ is included in at least one set S_u or S, so at least one of u or v is in V'. $\therefore$ V' is a vertex cover of G with size $\leq k$. $\therefore$ no maps to no. Therefore, we have shown that SET COVER is NP-complete.

3) MINIMUM BISECTION is in NP because given a set $S \subseteq V$, we can verify in polynomial time that $|S| = |V|/2$ and we can count the number of edges crossing the cut to check for at least k edges. We will reduce from MAXCUT, which is NP-complete. Given an instance of MAXCUT with $G = (V, E), k$ then we can reduce as follows: Let G' be the altered version of G such that G' has n more isolated nodes. If there is a cut $S \subseteq V$ in G with M edges crossing it, if $n - |S|$ isolated nodes are added to the old cut, then $\exists$ a bisection of G' with M edges crossing it. Similarly, if $\exists$ a bisection in G' with exactly M edges crossing it, if we remove the isolated nodes, then we have created a cut of G with M edges crossing it. Let E represent the maximum number of parallel edges between two edges in G' such that we let G_1 be the graph with pairs of vertices $u, v \in G'$ where $u \neq v$ with "E minus the number of parallel edges between u and v in G'" numbers of parallel edges. A bisection in G' with x edges has $En^2 - x$ edges crossing it in G_1 and a bisection in G_1 with x edges yields exactly $En^2 - x$ edges crossing G'. Consider G'' to be a graph with $2|V|$ nodes and adding cliques to all of the edges. This reduction produces an instance of MINIMUM BISECTION with $(G'', En^2 + n^2 - k)$ that runs in polynomial time. Consider a cut of G with at least k edges crossing it, then $\exists$ a bisection of G' with at least k edges crossing it and there is also a bisection of G_1 with at most $En^2 - k$ edges crossing it. Since in G'' we have additional n^2 edges from the cliques, then we get $En^2 - k^2 + n^2$ edges crossing it in G''. Thus yes maps to yes. Consider a bisection of G'' with at most $En^2 + n^2 - k$ edges crossing it. We know that the cliques add n^2 edges so the same bisection in G_1 has at most $En^2 - k$ edges crossing it. Thus no maps to no. Therefore we have shown that MINIMUM BISECTION is NP-complete.

4a) PARTITION is in NP because given $T \subseteq \{1, 2, \ldots n\}$ we can verify that $\sum_{i \in T} a_i = \sum_{i \notin T} a_i$ in polynomial time. We will show that PARTITION is NP-hard by reduction from SUBSET SUM. Consider the instance $(a_1, a_2, \ldots, a_n, B)$ of SUBSET SUM. This reduction produces an instance of PARTITION: $a_1, a_2, \ldots a_n, a_{n+1} = \sum_i a_i + 1 - B, a_{n+2} = \sum_i a_i + 1 - (\sum_i a_i - B)$ which runs in polynomial time. If we have a YES instance of SUBSET SUM, we will show that it produces a YES instance of PARTITION. Consider a subset $T \subseteq \{1, 2, \ldots n\}$ where $\sum_{i \in T} a_i = B$. Thus $\sum_{i \notin T} a_i = \sum_i a_i - B$ and $a_{n+1} + \sum_{i \in T} a_i = a_{n+2} + \sum_{a \notin T} a_i$, which is clearly able to be partitioned. $\therefore$ yes maps to yes. If we have a YES instance of PARTITION, then we also have a YES instance of SUBSET SUM. Consider the partition T. a_{n+1} and a_{n+2} can't be in the same partition because the sum of the integers would be greater than $\sum_i a_i$ and the sum of the integers in the other part would be at most $\sum_i a_i$. $\longrightarrow$

4a) Thus $\Sigma_{i \in T} a_i = \Sigma_{i \notin T} a_i$. If a_{n+1} is in the first partition then $T - a_{n+1}$ is a subset that sums to B, and if B is the second part then $\overline{T} - a_{n+1}$ is a subset that sums to B. Thus, the set $(a_1, a_2, \dots a_n, B)$ is a YES instance of SUBSET SUM and a YES instance of PARTITION indicates a YES instance of SUBSET SUM. Thus no maps to no. Therefore PARTITION is NP-complete.

b) KNAPSACK is NP because given a subset of the n elements we can verify that the sum of the values is at least V and takes at most C in polynomial time. We will reduce from SUBSET SUM to prove KNAPSACK is NP-hard. Given an instance of SUBSET SUM of $(a_1, a_2 \dots a_n)$ we can reduce as follows. Let cost of item i be c_i where $c_i = a_i$ and the value v_i of the item is also a_i. Consider $V = C = B$ | our reduction runs in polynomial time. If we started with a YES instance of SUBSET SUM, we will show that we also have a YES instance of KNAPSACK. Consider the subset $T \subseteq \{a_1, a_2, \dots a_n\}$ where $\Sigma_{a \in T} a = B$. Then the element in T in KNAPSACK has cost B and value B | the reduction of KNAPSACK is a YES instance. ∴ yes maps to yes. If the reduction produces a YES instance of KNAPSACK, we show that we also have a YES instance of SUBSET SUM. Consider $T \subseteq \{a_1, a_2, \dots a_n\}$ be in KNAPSACK | the total value of the items is at least V and cost is at most C | $\Sigma_{a \in T} a \geq V$ and $\Sigma_{a \in T} a \leq C$ | $\Sigma_{a \in T} a = B$. Thus, the instance $(a_1, a_2, \dots a_n, B)$ is a YES instance of SUBSET SUM. ∴ no maps to no. Therefore KNAPSACK is NP-complete.