Q1.1.1 Butterworth Lowpass

$$0.89 \le |H(e^{j\omega})| \le 1 \quad , \quad 0 \le \omega \le 0.2\pi$$
$$|H(e^{j\omega})| \le 0.18 \quad , \quad 0.6\pi \le \omega \le \pi$$

Assume that $T_d = 2$ implying $\Omega = \tan\left(\frac{\omega}{2}\right)$

Prewarp the edge frequencies:

$$\Omega_1 = \tan\left(\frac{0.2\pi}{2}\right) = \tan(0.1\pi) \quad ; \quad \Omega_2 = \tan\left(\frac{0.6\pi}{2}\right) = \tan(0.3\pi)$$

Determine CT transfer function:

$$|H_c(j\Omega)|^2 = \frac{1}{1 + \left(\frac{\Omega}{\Omega_c}\right)^{2N}}$$

$$0.89^2 = \frac{1}{1 + \left(\frac{\tan(0.1\pi)}{\Omega_c}\right)^{2N}}$$

$$\left(\frac{\tan(0.1\pi)}{\Omega_c}\right)^{2N} = \left(\frac{1}{0.89^2}\right) - 1 \qquad ①$$

$$0.18^2 = \frac{1}{1+\left(\frac{\tan(0.3\pi)}{\Omega_c}\right)^{2N}}$$

$$\left(\frac{\tan(0.3\pi)}{\Omega_c}\right)^{2N} = \left(\frac{1}{0.18^2}\right) - 1 \qquad ②$$

$$\frac{①}{②} \qquad \left[\frac{\tan(0.1\pi)}{\tan(0.3\pi)}\right]^{2N} = 8.788\times10^{-3}$$

$$2N = 3.2795$$

$$N = 1.6397 \Rightarrow N = 2$$

Ex 1.1.4 Butterworth filter of order 4 and cut off frequency @ 1.5 kHz. Also needs a 40dB attenuation at 3kHz

Assume $T_d = 1$ so $\Omega = \omega$

We want the attenuation to be 40dB at 3kHz with a cutoff of 1kHz

Converting dB to linear :

$$40 = 20 Log(x_1)$$
$$x_1 = 10^{-4}$$

$$10^{-4} = \frac{1}{1 + \left(\frac{3\times10^3\times2\pi}{1.5\times10^3\times2\pi}\right)^{2N}}$$

$$10^{-4} = \frac{1}{1 + 2^{2N}}$$

$$2^{2N} = \frac{1}{10^{-4}} - 1$$

$$N = \frac{Ln\left(\frac{1}{10^{-4}} - 1\right)}{2Ln(2)} = 6.64$$

$$\Rightarrow 7$$

We should then expect 14 poles evenly spread around a circle of radius $2\pi \times 3000$ at angles of θ_k:

$$\theta_k = \frac{(N+1+2k)\pi}{2N} = \frac{2(4+k)\pi}{7}$$

for $k = 0, 1, \dots, 13$