Microeconomic Origins of Macroeconomic Tail Risks[†]

By Daron Acemoglu, Asuman Ozdaglar, and Alireza Tahbaz-Salehi*

Using a multisector general equilibrium model, we show that the interplay of idiosyncratic microeconomic shocks and sectoral heterogeneity results in systematic departures in the likelihood of large economic downturns relative to what is implied by the normal distribution. Such departures can emerge even though GDP fluctuations are approximately normally distributed away from the tails, highlighting the different nature of large economic downturns from regular business-cycle fluctuations. We further demonstrate the special role of input-output linkages in generating tail comovements, whereby large recessions involve not only significant GDP contractions, but also large simultaneous declines across a wide range of industries. (*JEL* D57, E16, E23, E32)

Most empirical studies in macroeconomics approximate the deviations of aggregate economic variables from their trends with a normal distribution. Besides analytical tractability, this approach has a natural justification: since most macro variables, such as GDP, are obtained from combining more disaggregated ones, it is reasonable to expect that a central limit theorem-type result should imply normality. As an implicit corollary to this argument, most of the literature treats the standard deviations of aggregate variables as sufficient statistics for measuring aggregate economic fluctuations.

A closer look, however, suggests that the normal distribution does not provide a good approximation to the distribution of aggregate variables at the tails. This can be seen from panels A and B of Figure 1, which depict the quantile-quantile (Q-Q) plots of the US postwar quarterly GDP growth and HP-detrended log GDP
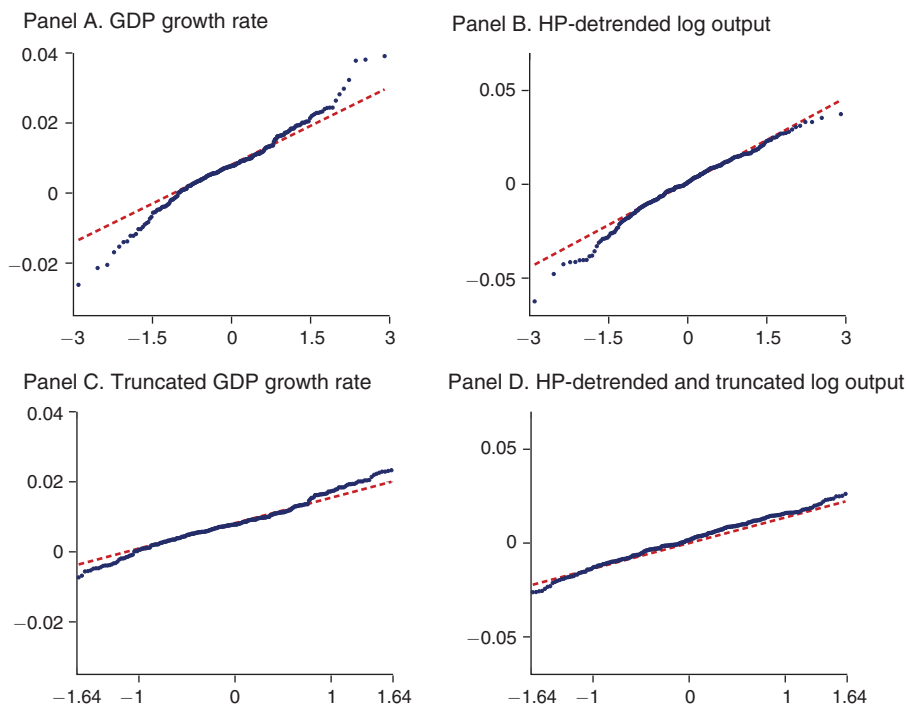
FIGURE 1. QUANTILE-QUANTILE PLOTS OF POSTWAR (1947:I–2015:I) US OUTPUT FLUCTUATIONS

Notes: The vertical and horizontal axes correspond, respectively, to the quantiles of the sample data and the reference probability distribution. Panels A and B depict, respectively, the Q-Q plots for the GDP growth rate and HP-detrended log output against the normal distribution. Panels C and D depict the Q-Q plots of the two datasets after removing data points that are more than 1.645 standard deviations away from their respective means against a similarly truncated normal distribution.

against the standard normal distribution. The deviations of both samples' quantiles from the standard normal's reveal that the normal distribution significantly underestimates the frequency of large economic downturns.[1] Interestingly, however, once such large deviations are excluded, GDP fluctuations are well approximated by a normal distribution. This is illustrated by panels C and D of Figure 1, which depict the Q-Q plots after excluding quarters in which GDP growth and detrended log GDP are more than 1.645 standard deviations away from their respective means.[2] Both

---

[1] Formal goodness-of-fit tests confirm these observations. We test the distributions of US GDP growth and HP-detrended log GDP between 1947:I and 2015:I against the normal distribution using Anderson-Darling, Kolmogorov-Smirnov, and Cramér-von Mises tests, with p-values computed using the parametric bootstrap (Stute, Manteiga, and Quindimil 1992). All three tests reject normality of the two time series at the 1 percent significance level. We also verified that the tests continue to reject normality even after filtering the two time series for time-varying volatility using a GARCH(1,1) model.

The departure of the distribution of aggregate economic variables from normality in the United States and other OECD economies has also been documented by several prior studies, including Lee et al. (1998); Canning et al. (1998); Christiano (2007); Fagiolo, Napoletano, and Roventini (2008); Cúrdia, Del Negro, and Greenwald (2014); and Ascari, Fagiolo, and Roventini (2015).

[2] The truncation level of 1.645 standard deviations is chosen to correspond to the interdecile range of the normal distribution, and thus excludes the top and bottom 5 percent of data points of a normally distributed sample.

graphs now indicate a close correspondence between the two truncated samples and a similarly truncated normal distribution.[3]

In this paper, we argue that macroeconomic tail risks, such as the ones documented in panels A and B of Figure 1, can have their origins in idiosyncratic microeconomic shocks to disaggregated sectors. We demonstrate that sufficiently high levels of sectoral heterogeneity can lead to systematic departures in the frequency of large economic downturns from what is implied by the normal distribution. Crucially, macroeconomic tail risks can coexist with approximately normally distributed fluctuations away from the tails, consistent with the pattern of postwar US GDP fluctuations documented in panels C and D of Figure 1. Taken together, these results illustrate that the microeconomic underpinnings of macroeconomic tail risks can be distinct from those of regular business-cycle fluctuations.

We develop these ideas in the context of a model economy comprised of n competitive sectors that are linked to one another via input-output linkages and are subject to idiosyncratic productivity shocks. Using an argument similar to those of Hulten (1978) and Gabaix (2011), we first show that aggregate output depends on the distribution of microeconomic shocks as well as the empirical distribution of (sectoral) *Domar weights*, defined as sectoral sales divided by GDP. We also establish that the empirical distribution of Domar weights is in turn determined by the extent of heterogeneity in (i) the weights households place on the consumption of each sector's output (which we refer to as *primitive heterogeneity*); and (ii) the sectors' role as input-suppliers to one another (which we refer to as *network heterogeneity*).

Using this characterization, we investigate whether microeconomic shocks can translate into significant macroeconomic tail risks, defined as systematic departures in the frequency of large economic downturns from what is predicted by the normal distribution.[4] Our main result establishes that macroeconomic tail risks can emerge if two conditions are satisfied. First, microeconomic shocks themselves need to exhibit some minimal degree of tail risk relative to the normal distribution (e.g., by having exponential tails), as aggregating normally distributed shocks always results in normally distributed GDP fluctuations.[5] Second, the economy needs to exhibit sufficient levels of *sectoral dominance*, in the sense that the most dominant disaggregated sectors (i.e., those with the largest Domar weights) ought to be sufficiently large relative to the variation in the importance of all sectors. This condition guarantees that tail risks present at the micro level do not wash out after aggregation. We then demonstrate that macroeconomic tail risks can emerge even if the central limit theorem implies that, in a pattern consistent with Figure 1, fluctuations are normally distributed away from the tails.

---

[3] We perform the same three goodness-of-fit tests in footnote 1 to test the two truncated samples against a similarly truncated normal distribution, with p-values once again computed via the parametric bootstrap. All tests fail to reject the null hypothesis at the 5 percent level.

[4] Formally, we measure the extent of macroeconomic tail risks by the likelihood of a τ standard deviation decline in log GDP relative to the likelihood of a similar decline under the normal distribution over a sequence of economies in which both the number of sectors (n) and the size of the deviation (τ) grow to infinity. The justification for these choices is provided in Section II.

[5] Estimates for the tail behavior of five-factor total factor productivity (TFP) growth rate for 459 four-digit manufacturing industries in the NBER-CES Manufacturing Industry Database suggests that the tails are indeed much closer to exponential than normal. See footnote 19 for more details.

Our result that high levels of sectoral dominance transform microeconomic shocks into sizable macroeconomic tail risks is related to the findings of Gabaix (2011) and Acemoglu et al. (2012), who show that microeconomic shocks can lead to aggregate volatility (measured by the standard deviation of GDP) if some sectors are much larger than others or play much more important roles as input-suppliers in the economy. However, the role played by heterogeneity in Domar weights in generating tail risks is distinct from its role in creating aggregate volatility. Indeed, we show that structural changes in an economy can simultaneously reduce aggregate volatility while increasing macroeconomic tail risks, in a manner reminiscent of the experience of the US economy over the last several decades, where the likelihood of large economic downturns may have increased behind the façade of the Great Moderation.

Our main results show that the distribution of microeconomic shocks and the Domar weights in the economy serve as sufficient statistics for the likelihood of large economic downturns. Hence, two economies with identical Domar weights exhibit equal levels of macroeconomic tail risks, regardless of the extent of network and primitive heterogeneity. Nevertheless, we also establish that economic downturns that arise due to the presence of the two types of heterogeneity are meaningfully different in nature. In an economy with no network heterogeneity—where Domar weights simply reflect the differential importance of disaggregated goods in household preferences—large economic downturns arise as a consequence of contractions in sectors with high Domar weights, while other sectors are, on average, in a normal state. In contrast, large economic downturns that arise from the interplay of microeconomic shocks and network heterogeneity display *tail comovements*: they involve not only very large drops in GDP, but also significant simultaneous contractions across a wide range of sectors within the economy.[6] We formalize this argument by showing that a more interconnected economy exhibits more tail comovements relative to an economy with identical Domar weights, but with only primitive heterogeneity.

As our next result, we characterize the extent of macroeconomic tail risks in the presence of heavy-tailed microeconomic shocks (e.g., shocks with Pareto tails). Using this characterization, we demonstrate that sufficient levels of sectoral dominance can translate light-tailed (such as exponential) shocks into macroeconomic tail risks that could have only emerged with heavy-tailed shocks in the absence of sectoral heterogeneity.[7]

We then undertake a simple quantitative exercise to further illustrate our main results. Assuming that microeconomic shocks have exponential tails—chosen in a way that is consistent with GDP volatility observed in the US data—we find that the empirical distribution of Domar weights in the US economy is capable of generating departures from the normal distribution similar to the patterns documented

---

[6] Indeed, shipments data for 459 manufacturing industries from the NBER Manufacturing Productivity Database during the period 1958–2009 are suggestive of significant levels of tail comovements. We find that a two standard deviation annual decline in GDP, which takes place twice in our sample, in 1973 and 2008, is associated with a two standard deviation decline in 10.46 percent and 13.73 percent of manufacturing industries, respectively. These numbers are clearly much larger than what one would have expected to observe had shipments been independently distributed across different industries (see Section VI). They are also much larger than the average in the rest of the sample (2.81 percent).

[7] Fama (1963) and Ibragimov and Walden (2007) observe that the presence of extremely heavy-tailed shocks with infinite variances leads to the breakdown of the central limit theorem. Our results, in contrast, are about the (arguably more subtle phenomenon of) emergence of macroeconomic tail risks in the absence of heavy-tailed micro or macro shocks.

in Figure 1. We then demonstrate that the US economy exhibits high enough levels of network heterogeneity to create significant levels of macroeconomic tail risks. Finally, we show that the pattern of input-output linkages in the US data can generate tail comovements, highlighting the importance of intersectoral linkages in shaping the nature of business-cycle fluctuations.

Related Literature.—Our paper belongs to the small literature that focuses on large economic downturns. A number of papers, including Cole and Ohanian (1999, 2002) and Kehoe and Prescott (2002), have used the neoclassical growth framework to study Great Depression-type events in the United States and other countries. More recently, there has been a growing emphasis on deep Keynesian recessions due to liquidity traps and the zero lower bound on nominal interest rates (Christiano, Eichenbaum, and Rebelo 2011; Eggertsson and Krugman 2012; Eggertsson and Mehrotra 2015). Relatedly, Christiano, Eichenbaum, and Trabandt (2015) argue that financial frictions can account for the key features of the recent economic crisis. Though our paper shares with this literature the emphasis on large economic downturns, both the focus and the underlying economic mechanisms are substantially different.

This line of work is also related to the literature on *rare disasters*, such as Rietz (1988); Barro (2016); Gabaix (2012); Nakamura et al. (2013); and Farhi and Gabaix (2016), which argues that the possibility of rare but extreme disasters is an important determinant of risk premia in asset markets. Gourio (2012) studies a real business-cycle model with a small risk of economic disaster. This literature, however, treats the frequency and the severity of such rare disasters as exogenous. In contrast, we not only provide a possible explanation for the endogenous emergence of macroeconomic tail risks, but also characterize how the distributional properties of micro shocks coupled with input-output linkages in the economy shape the likelihood and depth of large economic downturns.

Our paper is most closely connected to and builds on the literature that studies the microeconomic origins of economic fluctuations. As already mentioned, Gabaix (2011) argues that if the firm size distribution is sufficiently heavy tailed (in the sense that the largest firms contribute disproportionately to GDP), firm-level idiosyncratic shocks may translate into aggregate fluctuations. Acemoglu et al. (2012) show that the propagation of microeconomic shocks over input-output linkages can result in aggregate volatility. On the empirical side, Carvalho and Gabaix (2013) explore whether changes in the sectoral composition of the postwar US economy can account for the Great Moderation and its unwinding, while Foerster, Sarte, and Watson (2011) and Atalay (2015) study the relative importance of aggregate and sectoral shocks in aggregate economic fluctuations. Complementing these studies, di Giovanni, Levchenko, and Mejean (2014) use a database covering the universe of French firms and document that firm-level shocks contribute significantly to aggregate volatility, while Carvalho et al. (2015) and Acemoglu, Akcigit, and Kerr (2016) provide firm and sectoral-level evidence for the transmission of shocks over input-output linkages.[8]

---

[8] Other studies in this literature include Jovanovic (1987); Durlauf (1993); Horvath (1998, 2000); Dupor (1999); Carvalho (2010); and Burlon (2012). For a survey of this literature, see Carvalho (2014). Also see Acemoglu, Ozdaglar, and Tahbaz-Salehi (2016) for a unified, reduced-form framework for the study of how network interactions can function as a mechanism for propagation and amplification of shocks.

Even though the current paper has much in common with the studies mentioned above, it also features major differences from the rest of the literature. First, rather than focusing on the standard deviation of GDP as a notion of aggregate fluctuations, we study the determinants of macroeconomic tail risks, which, to the best of our knowledge, is new. Second and more importantly, this shift in focus leads to a novel set of economic insights by establishing that the extent of macroeconomic tail risks is determined by the interplay between the shape of the distribution of microeconomic shocks and the heterogeneity in Domar weights (as captured by our notion of sectoral dominance), a result with no counterpart in the previous literature.

Our work is also related to Lee et al. (1998); Canning et al. (1998); Fagiolo, Napoletano, and Roventini (2008); Cúrdia, Del Negro, and Greenwald (2014); and Ascari, Fagiolo, and Roventini (2015), who document that the normal distribution does not provide a good approximation to many macroeconomic variables in OECD countries. Similarly, Atalay and Drautzburg (2015) find substantial cross-industry differences in the departures of employment growth rates from the normal distribution, and compute the contribution of the independent component of industry-specific productivity shocks to the skewness and kurtosis of aggregate variables.

Finally, our paper is linked to the growing literature that focuses on the role of power laws and large deviations in various contexts. For example, Gabaix et al. (2003, 2006) provide a theory of excess stock market volatility in which market movements are due to trades by very large institutional investors, while Kelly and Jiang (2014) investigate the effects of time-varying extreme events in asset markets.

Outline of the Paper.—The rest of the paper is organized as follows. We present our benchmark model in Section I. In Section II, we formally define our notion of tail risks. Our main results are presented in Section III, where we show that the severity of macroeconomic tail risks is determined by the interaction between the nature of heterogeneity in the economy's Domar weights and the distribution of microeconomic shocks. We present our results on tail comovements and the extent of tail risks in the presence of heavy-tailed microeconomic shocks in Sections IV and V, respectively. Section VI contains our quantitative exercises. We provide a dynamic variant of our model in Section VII and conclude the paper in Section VIII. All proofs and some additional mathematical details are provided in the Appendix.

I. Model

In this section, we present a multisector model that forms the basis of our analysis. The model is a static variant of the model of Long and Plosser (1983), which is also analyzed by Acemoglu et al. (2012).

Consider a static economy consisting of n competitive sectors denoted by $\{1, 2, \ldots, n\}$, each producing a distinct product. Each product can be either consumed or used as input for production of other goods. Firms in each sector employ Cobb-Douglas production technologies with constant returns to scale to transform labor and inputs from other sectors into final products. In particular,

$$(1) \qquad x_i = \Xi_i \zeta_i l_i^{1-\mu} \left(\prod_{j=1}^{n} x_{ij}^{a_{ij}} \right)^{\mu},$$

where x_i is the output of sector i, Ξ_i is a Hicks-neutral productivity shock, l_i is the amount of labor hired by firms in sector i, x_{ij} is the amount of good j used for production of good i, $\mu \in [0, 1)$ is the share of material goods in production, and $\zeta_i > 0$ is some normalization constant.[9] The exponent $a_{ij} \geq 0$ in (1) represents the share of good j in the production technology of good i. A larger a_{ij} means that good j is more important in producing i, whereas $a_{ij} = 0$ means that good j is not a required input for i's production technology. Constant returns to scale implies $\sum_{j=1}^{n} a_{ij} = 1$ for all i. We summarize the intersectoral input-output linkages with matrix $\mathbf{A} = [a_{ij}]$, which we refer to as the economy's *input-output matrix*.

We assume that microeconomic shocks, $\epsilon_i = \log(\Xi_i)$, are i.i.d. across sectors; are symmetrically distributed around the origin with full support over $\mathbb{R}$; and have a finite standard deviation, which we normalize to 1. Throughout, we denote the common cumulative distribution function (CDF) of ϵ_is by F.

The economy is also populated by a representative household, who supplies one unit of labor inelastically and has logarithmic preferences over the n goods given by

$$u(c_1, \ldots, c_n) = \sum_{i=1}^{n} \beta_i \log(c_i),$$

where c_i is the amount of good i consumed and $\beta_i > 0$ is i's share in the household's utility function, normalized such that $\sum_{i=1}^{n} \beta_i = 1$.

The competitive equilibrium of this economy is defined in the usual way: it consists of a collection of prices and quantities such that (i) the representative household maximizes her utility; (ii) the representative firm in each sector maximizes its profits while taking the prices and the wage as given; and (iii) all markets clear.

Throughout the paper, we refer to the logarithm of value added in the economy as *aggregate output*, and we denote it by y. Our first result provides a convenient characterization of aggregate output as a function of microeconomic shocks and the technology and preference parameters.

PROPOSITION 1: *The aggregate output of the economy is given by*

$$(2) \qquad\qquad y = \log(\text{GDP}) = \sum_{i=1}^{n} v_i \epsilon_i,$$

where

$$(3) \qquad\qquad v_i = \frac{p_i x_i}{\text{GDP}} = \sum_{j=1}^{n} \beta_j \ell_{ji}$$

and ℓ_{ji} is the (j, i) element of the economy's Leontief inverse $\mathbf{L} = (\mathbf{I} - \mu\mathbf{A})^{-1}$.

This result is related to Hulten (1978) and Gabaix (2011), who show that in a competitive economy with constant returns to scale technologies, aggregate output is a linear combination of sectoral-level productivity shocks, with coefficients v_i

---

[9] In what follows, we set $\zeta_i = (1 - \mu)^{-(1-\mu)} \prod_{j=1}^{n} (\mu a_{ij})^{-\mu a_{ij}}$. This choice simplifies our key expressions without any bearing on our results.

given by sectors' *Domar weights* (defined as sectoral sales divided by GDP: Domar 1961; Carvalho and Gabaix 2013). In addition, Proposition 1 establishes that with Cobb-Douglas preferences and technologies, these weights take a particularly simple form: the Domar weight of each sector depends only on the preference shares, $(\beta_1, \ldots, \beta_n)$, and the corresponding column of the economy's Leontief inverse, which measures that sector's importance as an input-supplier to other sectors in the economy.

The heterogeneity in Domar weights plays a central role in our analysis. Equation (3) provides a clear decomposition of this heterogeneity in terms of the structural parameters of the economy. At one extreme, corresponding to an economy with no input-output linkages (i.e., $\mu = 0$), the heterogeneity in Domar weights simply reflects differences in preference shares: $v_i = \beta_i$ for all sectors i. We refer to this source of variation in Domar weights as *primitive heterogeneity*.[10] At the other extreme, corresponding to an economy with identical β_is, the heterogeneity in v_is reflects differences in the roles of different sectors as input-suppliers to the rest of the economy (as in Acemoglu et al. 2012); we refer to this source of variation in Domar weights as *network heterogeneity*. In general, the empirical distribution of Domar weights is determined by the combination of primitive and network heterogeneity.

Finally, we define a *simple economy* as an economy with symmetric preferences (i.e., $\beta_i = 1/n$) and no input-output linkages (i.e., $\mu = 0$). As such, a simple economy exhibits neither primitive nor network heterogeneity. This, in turn, implies that in this economy all sectors have identical Domar weights and aggregate output is the unweighted average of microeconomic shocks: $y = (1/n)\sum_{i=1}^{n} \epsilon_i$.

II. Tail Risks

The main focus of this article is on whether idiosyncratic, microeconomic shocks to disaggregated sectors can lead to the emergence of macroeconomic tail risks. In this section, we first provide a formal definition of tail risks and explain the motivation for our choice. We then argue that to formally capture whether macroeconomic tail risks can originate from microeconomic shocks, one needs to focus on the extent of tail risks in a sequence of economies in which the number of sectors grows.

A. *Defining Tail Risks*

We start by proposing a measure of tail risks for a random variable z by comparing its (left) tail behavior to that of a normally distributed random variable with the same standard deviation. More specifically, consider the probability that z deviates by at least τ standard deviations from its mean relative to the probability of an identical deviation under the standard normal distribution:

$$(4) \qquad r_z(\tau) = \frac{\log \Pr(z < E[z] - \tau \cdot \text{stdev}(z))}{\log \Phi(-\tau)},$$

---

[10] We use the term primitive heterogeneity as opposed to preference heterogeneity since, in general (with non-Cobb-Douglas technologies), differences in other *primitives* (such as average sectoral productivities) play a role similar to the β_is in the determination of the Domar weights.

where τ is a positive constant and Φ denotes the CDF of the standard normal. This ratio, which is always positive, has a natural interpretation: $r_z(\tau) < 1$ if and only if the probability of a τ standard deviation decline in z is greater than the corresponding probability under the normal distribution. Moreover, a smaller $r_z(\tau)$ means that a τ standard deviation contraction in z relative to a normally distributed random variable is more likely. The next definition introduces our notion of tail risk in terms of $r_z(\tau)$.

DEFINITION 1: *Random variable z exhibits tail risks* (*relative to the normal distribution*) *if* $\lim_{\tau\to\infty} r_z(\tau) = 0.$

Thus, z exhibits tail risks if, for any $\rho > 1$, there exists a large enough T such that for all $\tau > T$ the probability that z exhibits a τ standard deviation contraction below its mean is at least ρ times larger than the corresponding probability under the normal distribution. This definition therefore provides a natural notion for deviations from the normal distribution at the tails.[11]

We remark that, even though related, whether a random variable exhibits tail risks in the sense of Definition 1 is distinct from whether it has a light- or a heavy-tailed distribution, as our concept is defined in comparison to the tail of the normal distribution. To be more concrete, we follow Foss, Korshunov, and Zachary (2011) and say z is *light-tailed* if $E[\exp(bz)] < \infty$ for some $b > 0$. Otherwise, we say z is *heavy tailed*. In this sense, any heavy-tailed random variable exhibits tail risks, but not all random variables with tail risks are necessarily heavy tailed. For example, even though a random variable with a symmetric exponential distribution has light tails, it exhibits tail risks relative to the normal distribution in the sense of Definition 1. For most of the paper, we focus on microeconomic shocks with light tails, though in Section V, we consider the implications of heavy-tailed microeconomic shocks as well.

With the concepts above in hand, we can now define macroeconomic tail risks for our economy from Section I. We define the *aggregate economy's* τ-*tail ratio* analogously to (4), as the probability that aggregate output deviates by at least τ standard deviations from its mean relative to the probability of an equally large deviation under the standard normal:

$$R(\tau) = \frac{\log \Pr(y < -\tau\sigma)}{\log \Phi(-\tau)},$$

where $y = \log(\text{GDP})$ is the economy's aggregate output, characterized in (2); $\sigma = \text{stdev}(y)$ denotes the economy's *aggregate volatility*; and for notational convenience, instead of $r_y(\tau)$ as in equation (4), we use $R(\tau)$ to denote the tail ratio of aggregate output.[12] We now use the following counterpart to Definition 1 to formalize our notion of macroeconomic tail risks.

DEFINITION 2:   *The   economy   exhibits   macroeconomic   tail   risks   if* $\lim_{\tau\to\infty} R(\tau) = 0.$

---

[11] Even though Definition 1 focuses on the left tail of the distribution, one can define an identical concept for the distribution's right tail.

[12] Note that Proposition 1, alongside the assumption that microeconomic shocks have a symmetric distribution around the origin, guarantees that (i) $Ey = 0$ and (ii) aggregate output's distribution is symmetric around the origin. As a result, it is immaterial whether we focus on the left or the right tail of the distribution.

One key feature of our notion of macroeconomic tail risks is that, by construction, it does not reflect differences in the magnitude of aggregate volatility, as it compares the likelihood of large deviations relative to a normally distributed random variable of the same standard deviation. Hence, even though increasing the standard deviation of sectoral shocks increases the economy's aggregate volatility, it does not impact the extent of macroeconomic tail risks.

We further note that even though measures such as kurtosis—frequently invoked to measure deviations from normality (Fagiolo, Napoletano, and Roventini 2008; Atalay and Drautzburg 2015)—are informative about the likelihood of large deviations, they are also affected by regular fluctuations in aggregate output, making them potentially unsuitable as measures of tail risk. In contrast, the notion of tail risk introduced in Definition 2 depends only on the distribution of aggregate output far away from the mean. The following example highlights the distinction between our notion of tail risk and kurtosis.

Example 1: Consider an economy in which aggregate output y has the following distribution: with probability $p > 0$, it has a symmetric exponential distribution with mean zero and variance σ^2, whereas with probability $1 - p$, it is uniformly distributed with the same mean and variance. It is easy to verify that the excess kurtosis of aggregate output, defined as $\kappa_y = E[y^4]/E^2[y^2] - 3$, satisfies

$$(5) \qquad \kappa_y = (1 - p)\kappa_{uni} + p\kappa_{exp},$$

where $\kappa_{uni} < 0$ and $\kappa_{exp} > 0$ are, respectively, the excess kurtoses of the uniform and exponential distributions.[13] Therefore, for small enough values of p, aggregate output exhibits a smaller kurtosis relative to that of the normal distribution. This is despite the fact that, for all values of $p > 0$, the likelihood that y exhibits a large enough deviation is greater than what is predicted by the normal distribution. In contrast, Definition 2 adequately captures the presence of this type of tail risk: for any $p > 0$, there exists a large enough τ such that $R(\tau)$ is arbitrarily close to zero.

An argument similar to the example above readily shows that any normalized moment of aggregate output satisfies a relationship identical to (5), and as a result is similarly inadequate as a measure of tail risk.

B. *Micro-Originated Macroeconomic Tail Risks*

Definition 2 formally defines macroeconomic tail risk in a given economy, regardless of its origins. However, what we are interested in is whether such tail risks can emerge as a consequence of idiosyncratic shocks to disaggregated sectors. In what follows, we argue that to meaningfully represent whether macroeconomic tail risks can originate from microeconomic shocks, one needs to focus on the extent of tail risks in *large economies*, formally represented as a sequence of economies where

---

[13] Excess kurtosis is defined as the difference between the kurtosis of a random variable and that of a normally distributed random variable. Therefore, the excess kurtosis of the normal distribution is zero, whereas the excess kurtoses of uniform and symmetric exponential distributions are equal to $-6/5$ and 3, respectively.

the number of sectors grows, i.e., where $n \rightarrow \infty$. This increase in the number of sectors should be interpreted as focusing on finer and finer levels of disaggregation of the same economy.[14]

The key observation is that in any economy that consists of finitely many sectors, idiosyncratic microeconomic shocks do not fully wash out, and as a result, would have some macroeconomic impact. Put differently, even in the presence of independent sectoral-level shocks $(\epsilon_1, \ldots, \epsilon_n)$, the economy as a whole is subject to some residual level of aggregate uncertainty, irrespective of how large n is. The following result formalizes this idea.

PROPOSITION 2: *If microeconomic shocks exhibit tail risks, then any economy consisting of finitely many sectors exhibits macroeconomic tail risks.*

This observation shows that to asses whether microeconomic shocks lead to the emergence of macroeconomic tail risks in a meaningful fashion, one has to focus on a sequence of economies with $n \rightarrow \infty$ and measure how the extent of tail risks decreases along this sequence. This is indeed the strategy adopted by Gabaix (2011) and Acemoglu et al. (2012), who study whether microeconomic shocks can lead to nontrivial levels of aggregate volatility (and is implicit in Lucas' 1977 famous argument that micro shocks should be irrelevant at the aggregate level).

Yet, this strategy raises another technical issue. As highlighted by Definition 2, our notion of tail risks entails studying the deviations of aggregate output from its mean as $\tau \rightarrow \infty$. This means that the order in which τ and n are taken to infinity becomes crucial. To highlight the dependence of the rates at which these two limits are taken, we index τ by the level of disaggregation of the economy, n, and study the limiting behavior of the sequence of tail ratios,

$$ R_n(\tau_n) = \frac{\log \Pr(y_n < -\tau_n \sigma_n)}{\log \Phi(-\tau_n)}, $$

as $n \rightarrow \infty$, where y_n is the aggregate output of the economy consisting of n sectors, $\sigma_n = \text{stdev}(y_n)$ is the corresponding aggregate volatility, and $\{\tau_n\}$ is an increasing sequence of positive real numbers such that $\lim_{n\rightarrow\infty} \tau_n = \infty$.

To determine how the sequence $\{\tau_n\}$ should depend on the level of disaggregation, n, we rely on the irrelevance argument of Lucas (1977), which maintains that idiosyncratic, sectoral-level shocks in a simple economy—with no network or primitive heterogeneity—should have no aggregate impact as the number of sectors grows. Our next result uses this argument to pin down the rate of dependence of τ on n.

PROPOSITION 3: *Consider a sequence of simple economies: that is, $\mu = 0$ and $\beta_i = 1/n$ for all i.*

---

[14] In Appendix B, we provide conditions under which two economies of different sizes correspond to representations of the same economy at two different levels of disaggregation. This characterization shows that the only restriction imposed on Domar weights is an additivity constraint: the Domar weight of each aggregated sector has to be equal to the sum of the Domar weights of the subindustries that belong to that sector. This restriction has no bearing on our analysis in the text, and we do not impose it for the sake of parsimony.

(i) *If* $\lim_{n\to\infty} \tau_n/\sqrt{n} = 0$, *then* $\lim_{n\to\infty} R_n(\tau_n) = 1$ *for all light-tailed microeconomic shocks*.

(ii) *If* $\lim_{n\to\infty} \tau_n/\sqrt{n} = \infty$, *then there exist light-tailed microeconomic shocks such that* $\lim_{n\to\infty} R_n(\tau_n) = 0$.

In other words, as long as $\lim_{n\to\infty} \tau_n/\sqrt{n} = 0$, the rate at which we take the two limits is consistent with the idea that in simple economies (with no primitive or network heterogeneity across firms/sectors) light-tailed microeconomic shocks have no major macroeconomic impact—whether or not they exhibit tail risks in the sense of Definition 1. In contrast, if the rate of growth of τ_n is fast enough such that $\lim_{n\to\infty} \tau_n/\sqrt{n} = \infty$, then Lucas's (1977) argument for the irrelevance of microeconomic shocks in a simple economy would break down. Motivated by these observations, we set $\tau_n = c\sqrt{n}$ for some arbitrary constant $c > 0$ and obtain the following definition.

DEFINITION 3: *A sequence of economies exhibits macroeconomic tail risks if*

$$\lim_{n\to\infty} R_n(c\sqrt{n}) = 0,$$

for all $c > 0$.

Throughout most of the paper, we rely on this definition as the notion for the presence of micro-originated macroeconomic tail risks. In Section IIIE, we demonstrate that our main results on the role of microeconomic shocks in creating macroeconomic tail risks remain qualitatively unchanged under different choices for τ_n.

III. Micro Shocks, Macro Tail Risks

In this section, we study whether idiosyncratic microeconomic shocks can translate into sizable macroeconomic tail risks and present our main results. Taken together, our results illustrate that the severity of macroeconomic tail risks is determined by the interaction between the extent of heterogeneity in Domar weights and the distribution of microeconomic shocks.

A. *Normal Shocks*

We first focus on the case in which microeconomic shocks are normally distributed. Besides providing us with an analytically tractable example of a distribution with extremely light tails, the normal distribution serves as a natural benchmark for the rest of our results.

PROPOSITION 4: *If microeconomic shocks are normally distributed, no sequence of economies exhibits macroeconomic tail risks.*

This result is an immediate consequence of Proposition 1: aggregate output, which is a weighted sum of microeconomic shocks, is normally distributed whenever the

shocks are normal, thus implying that GDP fluctuations can be well approximated by a normal distribution, even at the tails. Crucially, this result holds irrespective of the nature of input-output linkages or sectoral size distribution.

B. *Exponential-Tailed Shocks*

Next, we focus on economies in which microeconomic shocks have exponential tails. Formally, we say microeconomic shocks have *exponential tails* if there exists a constant $\gamma > 0$ such that

$$\lim_{z\to\infty} \frac{1}{z} \log F(-z) = -\gamma.$$

For example, any microeconomic shock with a CDF satisfying $F(-z) = 1 - F(z) = Q(z)e^{-\gamma z}$ for $z \geq 0$ and some polynomial function $Q(z)$ belongs to this class. Note that even though exponential-tailed shocks belong to the class of light-tailed distributions, they exhibit tail risks according to Definition 1. Our main results in this subsection provide necessary and sufficient conditions under which these microeconomic tail risks translate into sizable macroeconomic tail risks.

To present our results, we introduce the measure of *sectoral dominance* of a given economy as

$$\delta = \frac{v_{\max}}{\|v\|/\sqrt{n}},$$

where n is the number of sectors in the economy, $v_{\max} = \max\{v_1, \ldots, v_n\}$, and $\|v\| = \left(\sum_{i=1}^{n} v_i^2\right)^{1/2}$ is the second (uncentered) moment of the economy's Domar weights. Intuitively, δ measures how important the most dominant sector in the economy is compared to the variation in the importance of all sectors as measured by $\|v\|$. The normalization factor $\sqrt{n}$ reflects the fact that δ captures the extent of this dominance *relative to a simple economy*, for which $v_{\max} = 1/n$ and $\|v\| = 1/\sqrt{n}$. This of course implies that the sectoral dominance of a simple economy is equal to 1. We further remark that even though, formally, sectoral dominance depends on the largest Domar weight, a high value of δ does not necessarily imply that a single sector is overwhelmingly important relative to the rest of the economy. Rather, the presence of a group of sectors that are large relative to the amount of dispersion in Domar weights would also translate into a high level of sectoral dominance. The next proposition contains our main results.[15]

PROPOSITION 5: *Suppose microeconomic shocks have exponential tails.*

   (*i*) *A sequence of economies exhibits macroeconomic tail risks if and only if* $\lim_{n\to\infty}\delta = \infty$.

---

[15] Whenever we work with a sequence of economies, all our key objects, including δ, $v_{\max}$, and $\|v\|$, depend on the level of disaggregation n. However, unless there is a potential risk of confusion, we simplify the notation by suppressing their dependence on n. We make the dependence on n explicit when presenting the proofs in the Appendix.

(ii) *A sequence of economies for which* $\lim_{n\to\infty} \delta/\sqrt{n} = 0$ *and* $\lim_{n\to\infty} \delta = \infty$ *exhibits macroeconomic tail risks, even though aggregate output is asymptotically normally distributed, in the sense that* $y/\sigma \to \mathcal{N}(0, 1)$ *in distribution.*

Statement (i) illustrates that in the presence of exponentially tailed shocks, sequences of economies with limited levels of sectoral dominance (i.e., $\liminf_{n\to\infty} \delta < \infty$) exhibit no macroeconomic tail risks. This is due to the fact that in the absence of a dominant sector (or a group of sectors), microeconomic tail risks wash out as a result of aggregation, with no sizable macroeconomic effects. In contrast, in economies with nontrivial sectoral dominance (where $\delta \to \infty$), the tail risks of exponential microeconomic shocks do not entirely cancel each other out, even in a very large economy, thus resulting in aggregate tail risks.[16] This result thus complements those of Gabaix (2011) and Acemoglu et al. (2012) by establishing that heterogeneity in Domar weights is essential not only in generating aggregate volatility, but also in translating microeconomic tail risk into macroeconomic tail risks. However, as we show in Section IIID, the role played by the heterogeneity in Domar weights in engendering aggregate volatility is fundamentally distinct from its role in generating tail risks.

Part (ii) of the proposition shows that significant macroeconomic tail risks can coexist with a normally distributed aggregate output, as predicted by the central limit theorem. Though it may appear contradictory at first, this coexistence is quite intuitive: the notion of asymptotic normality implied by the central limit theorem considers the likelihood of a $\tau\sigma$ deviation from the mean as the number of sectors grows, while keeping the size of the deviations τ fixed. In contrast, per our discussion in Section II, tail risks correspond to the likelihood of large deviations, formally captured by taking the limit $\tau \to \infty$. Proposition 5 thus underscores that the determinants of large deviations can be fundamentally distinct from the origins of small or moderate deviations. This result also explains how, consistent with the patterns documented for the United States in Figure 1, aggregate output can be well approximated by a normal distribution away from the tails, even though it may exhibit significantly greater likelihood of tail events.

The juxtaposition of Propositions 4 and 5 further highlights the important role that the distribution of microeconomic shocks plays in shaping aggregate tail risks. In particular, replacing normally distributed microeconomic shocks with exponential shocks—which have only slightly heavier tails—may dramatically increase the likelihood of large economic downturns. This is despite the fact that the distribution of microeconomic shocks has no impact on the standard deviation of GDP or the shape of its distribution away from the tails (as shown by part (ii) of Proposition 5).

---

[16] The fact that sectoral dominance, $\delta = v_{\max} \sqrt{n} / \|v\|$, is a sufficient statistic for the presence of macroeconomic tail risks has an intuitive interpretation as well. Recall that, by construction, our notion of tail risks does not reflect differences in the magnitude of aggregate volatility, as it compares the likelihood of large deviations relative to a normally distributed random variable of the same standard deviation. As such, $\|v\| = \text{stdev}(y)$ in the denominator of δ normalizes for the level of aggregate volatility in the economy. The $\sqrt{n}$ term ensures that the sectoral dominance of a simple economy is normalized to 1. Finally, the $v_{\max}$ term captures the fact that the likelihood of a τ-standard deviation decline in the sector with the largest Domar weight provides a lower bound for the likelihood of an equally large decline in aggregate output. The proof of Proposition 5 establishes that this lower bound is tight when microeconomic shocks have exponential tails.
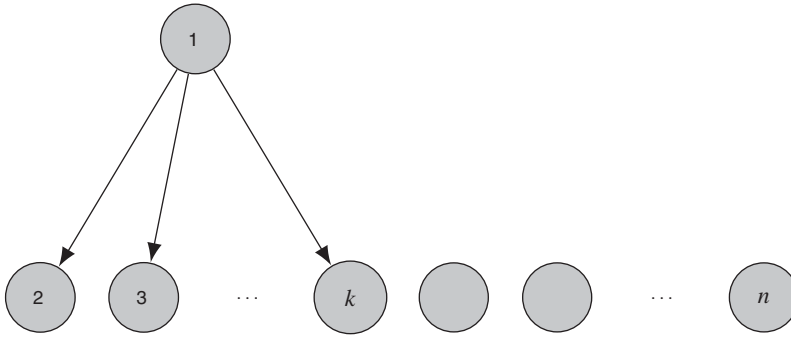
FIGURE 2. AN ECONOMY IN WHICH SECTOR 1 IS THE INPUT-SUPPLIER TO k SECTORS

Example 2: Consider the sequence of economies depicted in Figure 2 in which sector 1 is the sole supplier to k sectors, whereas the output of the rest of the sectors are not used as intermediate goods for production. Furthermore, suppose that the economies in this sequence exhibit no primitive heterogeneity, in the sense that households assign an equal weight of $\beta_i = 1/n$ to all goods i. It is easy to verify that

$$v_{\max} = v_1 = \frac{\mu k}{n(1 - \mu)} + \frac{1}{n}$$

and

$$\|v\| = \frac{1}{n(1 - \mu)} \sqrt{(\mu k + 1 - \mu)^2 + (k - 1)(1 - \mu)^2 + n - k}.$$

As a result, $\lim_{n \to \infty} \delta = \infty$ if and only if $k \to \infty$ as the level of disaggregation is increased. Thus, by Proposition 5, exponentially distributed microeconomic shocks in such a sequence of economies generate macroeconomic tail risks provided that $k \to \infty$ as $n \to \infty$. Notice that macroeconomic tail risks can be present even if sector 1 is an input-supplier to a diminishing fraction of sectors. For example, if $k = \log n$, the fraction of sectors that rely on sector 1 satisfies $\lim_{n \to \infty} k/n = 0$, which ensures that the central limit theorem applies. Indeed, in this case, Proposition 5(ii) guarantees that $y/\sigma \to \mathcal{N}(0, 1)$ in distribution.

The next example shows that macroeconomic tail risks can arise in the absence of network heterogeneity as long as the economy exhibits sufficient levels of primitive heterogeneity.

Example 3: Consider a sequence of economies with no input-output linkages (i.e., $\mu = 0$) and suppose that the weights assigned by the representative household to different goods are given by $\beta_1 = s/n$ and $\beta_i = (1 - \beta_1)/(n - 1)$ for all $i \neq 1$. Thus, the representative household values good 1 more than all other goods as long as $s > 1$. Then,

$$v_{\max} = s/n,$$

whereas

$$\|v\| = \frac{1}{n}\sqrt{s^2 + (n-s)^2/(n-1)}$$

for the economy consisting of n sectors. Therefore, provided that $s \to \infty$ (as $n \to \infty$), this sequence of economies exhibits nontrivial sectoral dominance and sizable levels of macroeconomic tail risks.

Contrasting this observation with Example 2 shows that either network or primitive heterogeneity are sufficient for the emergence of macroeconomic tail risks.

Though Proposition 5 provides a complete characterization of the conditions under which macroeconomic tail risks emerge from the aggregation of microeconomic shocks, its conditions are in terms of the limiting behavior of our measure of sectoral dominance, δ, which in turn depends on the entire distribution of Domar weights. Our next result focuses on a subclass of economies for which we can directly compute the extent of sectoral dominance.

DEFINITION 4: *An economy has Pareto Domar weights with exponent $\eta > 0$ if $v_i = ci^{-1/\eta}$ for all i and some constant $c > 0$.*

In an economy with Pareto Domar weights, the fraction of sectors with Domar weights greater than or equal to any given k is proportional to $k^{-\eta}$. Consequently, a smaller η corresponds to more heterogeneity in Domar weights and hence a (weakly) larger measure of sectoral dominance. It is easy to verify that if $\eta < 2$, the measure of sectoral dominance of such a sequence of economies grows at rate $\sqrt{n}$, whereas for $\eta > 2$, it grows at rate $n^{1/\eta}$.[17] Nevertheless, in either case, $\lim_{n\to\infty}\delta = \infty$, which leads to the following corollary to Proposition 5.

COROLLARY 1: *Consider a sequence of economies with Pareto Domar weights with common exponent η and suppose that microeconomic shocks have exponential tails.*

  (i) *The sequence exhibits macroeconomic tail risks for all $\eta > 0$.*

 (ii) *$y/\sigma \to \mathcal{N}(0, 1)$ in distribution if $\eta \geq 2$.*

Consequently, if $\eta \geq 2$, exponential-tailed microeconomic shocks lead to macroeconomic tail risks, even though aggregate output is asymptotically normally distributed.

C. *Generalization: Super-Exponential Shocks*

In the previous subsection, we focused on economies in which microeconomic shocks have exponential tails. In this subsection, we show that our main results

---

[17] In the knife edge case of $\eta = 2$, sectoral dominance grows at rate $\sqrt{n/\log n}$. See the proof of Corollary 1 for exact derivations.

presented in Proposition 5 generalize to a larger subclass of light-tailed microeconomic shocks. Specifically, here we focus on economies in which microeconomic shocks belong to the subclass of *super-exponential* distributions with shape parameter $\alpha \in (1, 2)$, in the sense that

$$(6) \qquad \lim_{z \to \infty} \frac{1}{z^\alpha} \log F(-z) = -k,$$

where F is the common CDF of microeconomic shocks and $k > 0$ is a constant.[18] For example, any shock with a CDF satisfying $F(-z) = 1 - F(z) = Q(z) \exp(-kz^\alpha)$ for some polynomial function $Q(z)$ belongs to this family. Note that we are ruling out the cases of $\alpha = 1$ and $\alpha = 2$ as, in such cases, shocks have exponential and normal tails, respectively.[19] This observation thus illustrates that microeconomic shocks belonging to this class of distributions exhibit tail risks in the sense of Definition 1, while having tails that are lighter than that of the exponential distribution.

PROPOSITION 6: *Suppose microeconomic shocks have super-exponential tails with shape parameter* $\alpha \in (1, 2)$.

(i) *If* $\liminf_{n \to \infty} \delta < \infty$, *then the sequence of economies exhibits no macroeconomic tail risks.*

(ii) *If* $\lim_{n \to \infty} \delta / n^{(\alpha-1)/\alpha} = \infty$, *then the sequence of economies exhibits macroeconomic tail risks.*

Therefore, the insights from Proposition 5 generalize to economies that are subject to super-exponential shocks. As with exponential-tailed shocks, Proposition 6 shows that sectoral dominance δ plays a central role in translating microeconomic tail risks into macroeconomic tail risks.

D. *Macroeconomic Tail Risks and Aggregate Volatility*

Our results thus far establish that sectoral heterogeneity plays a key role in translating microeconomic shocks into macroeconomic tail risks. At the same time, as argued by Gabaix (2011) and Acemoglu et al. (2012), such a heterogeneity also underpins micro-originated aggregate volatility. Despite this similarity, it is important to note that the role played by sectoral heterogeneity in generating macroeconomic tail risks is quite distinct from its role in bringing forth aggregate volatility.

---

[18] We provide a characterization for a broader class of super-exponential distributions in the proof of Proposition 6.

[19] We estimated α in equation (6) for five-factor TFP growth rate of 459 four-digit manufacturing industries in the NBER productivity database via maximum likelihood (Fagiolo, Napoletano, and Roventini 2008). The mean and median of this parameter are, respectively, 1.42 and 1.25 (with the 25 and 75 percentiles equal to 0.97 and 1.60, respectively). These estimates therefore indicate that the exponential distribution provides a better approximation to the tails of the industry TFP shock distributions than the normal.

To see this distinction, recall from Proposition 5 that in the presence of exponential-tailed microeconomic shocks, a sequence of economies with sufficiently high levels of sectoral dominance exhibits macroeconomic tail risks: microeconomic shocks translate into sizable aggregate tail risks if and only if

$$(7) \qquad \lim_{n \to \infty} \delta = \lim_{n \to \infty} \frac{v_{\max}}{\|v\|/\sqrt{n}} = \infty.$$

As already mentioned in Section IIIB, this condition holds whenever a sector or a group of sectors play a significant role in determining macroeconomic outcomes relative to the variation in the Domar weights of all sectors.

On the other hand, the characterization in equation (2) implies that aggregate volatility is equal to $\sigma = \text{stdev}(y) = \|v\|$. Therefore, as argued by Gabaix (2011) and Acemoglu et al. (2012), micro shocks generate aggregate volatility if $\|v\|$ decays to zero at a rate slower than $1/\sqrt{n}$, that is, if

$$(8) \qquad \lim_{n \to \infty} \frac{\|v\|}{1/\sqrt{n}} = \infty.$$

Contrasting (8) with (7) highlights that even though the microeconomic origins of both aggregate volatility and tail risks are shaped by the extent of heterogeneity in Domar weights, different aspects of this heterogeneity matter in each case: while the level of macroeconomic tail risks depends on the largest Domar weight in the economy, aggregate volatility is determined by the second moment of the distribution of Domar weights. Furthermore, recall from the discussion in Sections IIIA and IIIB that the nature of microeconomic shocks plays a critical role in shaping the extent of macroeconomic tail risks. In contrast, as far as aggregate volatility is concerned, the shape and distributions of micro shocks (beyond their variance) are immaterial.

Taken together, these observations imply that a sequence of economies may exhibit macroeconomic tail risks even if it does not display nontrivial levels of aggregate volatility, and vice versa. The following examples illustrate these possibilities.

Example 4: Consider a sequence of economies with Pareto Domar weights with common exponent η, that is, $v_i = c i^{-1/\eta}$ for all sectors i. It is easy to verify that as long as $\eta > 2$, aggregate volatility in this sequence of economies decays at the rate $1/\sqrt{n}$ (or more precisely, $\limsup_{n \to \infty} \frac{\|v\|}{1/\sqrt{n}} < \infty$), regardless of the distribution of microeconomic shocks. Hence, microeconomic shocks in such a sequence of economies have no meaningful impact on aggregate volatility. This is despite the fact that, as established in Corollary 1, exponentially tailed microeconomic shocks lead to macroeconomic tail risks for all positive values of η.

Example 5: Consider any sequence of economies for which (8) is satisfied. As already argued, in this case, microeconomic shocks lead to nontrivial aggregate volatility, regardless of how these shocks are distributed. Yet, by Proposition 4, when microeconomic shocks are normally distributed, the sequence of economies exhibits no macroeconomic tail risks.

We end this discussion by showing that the distinct natures of aggregate volatility and macroeconomic tail risks mean that structural changes in an economy can lead to a reduction in the former while simultaneously increasing the latter.

Example 6: Suppose that a structural change in the economy results in a reduction in $\|v\|$ while v_{max} remains constant. This will reduce the economy's aggregate volatility, but increase its sectoral dominance, leading to aggregate fluctuations that are generally less volatile but are more likely to experience large deviations. Clearly, the same can be true even if v_{max} declines, provided that this is less than the reduction in $\|v\|$. The possibility of simultaneous declines in aggregate volatility and increases in the likelihood of large economic downturns suggests a different perspective on the experience of the Great Moderation and the Great Recession in the United States, where a persistent decline in the standard deviation of GDP since the 1970s was followed by the most severe recession that the US economy had experienced since the Great Depression.

E. *Alternative Measures and Quantifying Tail Risks*

In Section II, we relied on Proposition 2 and the irrelevance argument of Lucas (1977) to define the presence of micro-originated macroeconomic tail risks in terms of the asymptotic behavior of $R_n(\tau_n)$ when $\tau_n = c\sqrt{n}$. We conclude this section by establishing that even though a sequence τ_n that grows at rate $\sqrt{n}$ is arguably the most natural choice, our main results on the microeconomic origins of macroeconomic tail risks remain qualitatively unchanged if one opts for an alternative choice of τ_n. More importantly, we also show that the behavior of $R_n(\tau_n)$ under different choices for τ_n provides a natural quantification of the extent of tail risks.

To capture these ideas formally, we say a sequence of economies exhibits *macroeconomic tail risks with respect to* $\{\tau_n\}$ if $\lim_{n\to\infty} R_n(\tau_n) = 0$, where $\{\tau_n\}$ is a sequence of positive real numbers such that $\lim_{n\to\infty} \tau_n = \infty$. It is immediate that this notion reduces to Definition 3 if $\tau_n = c\sqrt{n}$ for some constant $c > 0$. On the other hand, different choices of $\{\tau_n\}$ translate to changing the burden for generating tail risks: a lower rate of growth of τ_n corresponds to a greater burden, as the definition would now require the deviation of aggregate output's distribution from normality to start closer to the mean of the distribution (that is, at fewer standard deviations away from the mean).

We have the following generalization to Propositions 4–6.

PROPOSITION 7: *Let $\{\tau_n\}$ be a sequence of positive real numbers such that $\lim_{n\to\infty} \tau_n = \infty$.*

(i) *Suppose microeconomic shocks are normally distributed. Then no sequence of economies exhibits macroeconomic tail risks with respect to $\{\tau_n\}$.*

(ii) *Suppose microeconomic shocks have exponential tails. Then a sequence of economies exhibits macroeconomic tail risks with respect to $\{\tau_n\}$ if and only if*

$$\lim_{n\to\infty} \frac{\delta_n}{\sqrt{n}\,/\,\tau_n} = \infty.$$

(iii) *Suppose microeconomic shocks have super-exponential tails with shape parameter $\alpha \in (1,2)$. If*

$$\lim_{n \to \infty} \frac{\delta_n}{\sqrt{n} \, / \, \tau_n^{(2/\alpha - 1)}} = \infty,$$

then the sequence of economies exhibits macroeconomic tail risks with respect to $\{\tau_n\}$.

This result thus reemphasizes that, regardless of the choice of $\{\tau_n\}$, the emergence of macroeconomic tail risks is tightly related to two factors: the distribution of microeconomic shocks and the extent of sectoral heterogeneity. Furthermore, part (ii) of the proposition shows that the measure of sectoral dominance, δ, continues to serve as a sufficient static for the presence of tail risks when microeconomic shocks have exponential tails. This result also confirms that, as hinted by Definition 3, the presence of macroeconomic tail risks with respect to some threshold sequence $\{\tau_n\}$ is only related to that sequence's rate of growth (and not to the actual values of τ_n).

Parts (ii) and (iii) of Proposition 7 further show that if a sequence of economies exhibits tail risks with respect to some $\{\tau_n\}$, it will also do so with respect to any sequence $\{\tau_n'\}$ that grows at a faster rate (that is, $\lim_{n \to \infty} \tau_n'/\tau_n = \infty$). As such, the presence or absence of macroeconomic tail risks with respect to different sequences provides a natural quantification of these risks, as we formalize next.

DEFINITION 5: *Consider two sequences of economies with tail risk ratios R_n and $\overline{R}_n$, respectively.*

(i) *R_n exhibits weakly more macroeconomic tail risks than $\overline{R}_n$ if $\lim_{n \to \infty} R_n(\tau_n)$ $= 0$ whenever $\lim_{n \to \infty} \overline{R}_n(\tau_n) = 0$.*

(ii) *R_n exhibits strictly more macroeconomic tail risks than $\overline{R}_n$ if it exhibits weakly more macroeconomic tail risks, and in addition, there exists a sequence $\tau_n^* \to \infty$ such that $\lim_{n \to \infty} R_n(\tau_n^*) = 0$ and $\lim_{n \to \infty} \overline{R}_n(\tau_n^*) > 0$.*

Put differently, if a sequence of economies exhibits strictly more macroeconomic tail risks than another, there exists a sequence τ_n^* such that the likelihood of a $\tau_n^* \sigma_n$ deviation in the former can be arbitrarily greater than the latter for large enough values of n.

COROLLARY 2: *Suppose microeconomic shocks have exponential tails. Then a sequence of economies with sectoral dominance δ_n exhibits strictly more macroeconomic tail risks than a sequence of economies with sectoral dominance $\overline{\delta}_n$ if and only if $\lim_{n \to \infty} \delta_n / \overline{\delta}_n = \infty$.*

This result thus provides a refinement of Proposition 5: the limiting behavior of the measure of sectoral dominance not only captures whether the economy exhibits macroeconomic tail risks, but also provides a sufficient statistic for the extent of such risks. This corollary also provides a (partial) ordering of different sequences

of economies in terms of the extent of macroeconomic tail risks irrespective of the choice of τ_n.

IV. Tail Comovements

Our results in the previous section show that sufficient levels of sectoral dominance can translate microeconomic tail risk into macroeconomic tail risks. Deep recessions such as the Great Depression, however, involve not only large drops in aggregate output, but also significant simultaneous contractions across a range of sectors within the economy. In this section, we investigate this issue and argue that intersectoral input-output linkages play a key role in translating idiosyncratic microeconomic shocks into such simultaneous sectoral contractions.

We start our analysis by formally defining *tail comovements* as the likelihood that all sectors experience a simultaneous τ standard deviation decline in their respective outputs conditional on a $\tau\sigma$ drop in aggregate output. Specifically, for an economy consisting of *n* sectors, we define

$$(9) \qquad C(\tau) \;=\; \Pr\left(\hat{x}_i \;<\; E\hat{x}_i - \tau\hat{\sigma}_i \quad \text{for all} \quad i \,\middle|\, y \;<\; -\tau\sigma\right),$$

where $\hat{x}_i = \log(x_i)$ is the log output of sector *i*, $\hat{\sigma}_i = \text{stdev}(\hat{x}_i)$ is output volatility of sector *i*, and $\sigma = \text{stdev}(y)$ is the economy's aggregate volatility. This statistic measures whether a large contraction in aggregate output would necessarily imply that all sectoral outputs also experience a large decline with high probability.[20] Therefore, in an economy with high levels of tail comovements, micro-originated recessions are very similar to recessions that result from economy-wide aggregate shocks.

In the remainder of this section, we show that the extent of tail comovements, as measured by (9), is determined by the nature of input-output linkages across different sectors. We establish that, keeping the distribution of microeconomic shocks and the heterogeneity in Domar weights constant, increasing the extent of sectoral interconnections leads to higher levels of tail comovements.

A. *Sectoral Interconnections*

Before presenting our main results, we introduce a formal comparison of the extent of sectoral interconnections across two economies.

Recall from equation (2) that Domar weights serve as sufficient statistics for the role of microeconomic shocks in shaping the behavior of aggregate output. Furthermore, equation (3) illustrates that the Domar weights are determined by the preference parameters, $(\beta_1,\ldots,\beta_n)$, and the economy's input-output linkages as summarized by its Leontief inverse **L**. Consequently, two economies may exhibit different levels of primitive and network heterogeneity, even though their Domar

---

[20] As discussed in footnote 6, two standard deviation contractions in the US economy are associated with about 10 percent of four-digit manufacturing industries experiencing similarly large declines. Though it is possible to define tail comovements as the conditional likelihood that 10 percent (or any other fraction) of sectors experience large declines, it is conceptually simpler and mathematically more convenient to focus on the likelihood that all sectors experience such a decline.

weights are identical. To provide a comparison across such economies, we define the following concept.

DEFINITION 6: *Consider two economies with identical Domar weights; i.e.,* $v_i = v_i'$ *for all i. The latter economy is more interconnected than the former if there exists a stochastic matrix* $\mathbf{B}$ *such that*

$$(10) \qquad\qquad\qquad \mathbf{L}' = \mathbf{BL},$$

where $\mathbf{L}$ *and* $\mathbf{L}'$ *are the corresponding Leontief inverse matrices of the two economies, respectively.*[21]

Intuitively, premultiplication of the Leontief inverse matrix $\mathbf{L}$ by the stochastic matrix $\mathbf{B}$ ensures that the entries of the resulting Leontief matrix $\mathbf{L}'$ are more evenly distributed while at the same time its diagonal elements are smaller than the corresponding elements of $\mathbf{L}$. Therefore, the resulting economy not only exhibits more intersectoral linkages, but also the intensity of such linkages are more equally distributed across pairs of sectors. The following examples clarify these properties.

Example 7: Consider two economies with identical Domar weights and input-output matrices $\mathbf{A} = [a_{ij}]$ and $\mathbf{A}' = [a_{ij}']$. Furthermore, suppose that input-output linkages in the two economies are related via

$$a_{ij}' = \frac{1}{(1-\rho)}\left(a_{ij} + \frac{\rho(1-\mu)}{n\mu} - \frac{\rho}{\mu}\,\mathbf{1}_{\{i=j\}}\right)$$

for some constant $0 \leq \rho \leq \mu\min_i a_{ii}$, with $\mathbf{1}_{\{\cdot\}}$ denoting the indicator function (where the restriction on ρ ensures that $a_{ij}' \geq 0$ for all i and j). This transformation reduces the value of a_{ii} for all sectors i (i.e., $a_{ii}' < a_{ii}$), and redistributes it evenly across pairs of sectors $j \neq i$, making input-output linkages more uniformly distributed.

Indeed, it is easy to show that this intuitive argument is consistent with our formal notion of sectoral interconnections in Definition 6: the Leontief inverse matrices of the two economies are related to one another via equation (10) for the stochastic matrix $\mathbf{B} = [b_{ij}]$ whose elements are given by $b_{ij} = \rho/n + (1-\rho)\mathbf{1}_{\{i=j\}}$. This implies that the latter economy is more interconnected.

Example 8: Consider an economy with no input-output linkages, that is, $a_{ij} = 0$ for all pairs of sectors $i \neq j$, so that the economy's Leontief inverse matrix is given by $\mathbf{L} = (1-\mu)^{-1}\mathbf{I}$, where μ is the share of material goods in the firms' production technology and $\mathbf{I}$ denotes the identity matrix. Domar weights in this economy are proportional to the corresponding preference parameters, i.e., $v_i = \beta_i/(1-\mu)$ for all i.

---

[21] A square matrix is *stochastic* if it is element-wise nonnegative and each of its rows add up to 1. The assumption that $\mathbf{B}$ is stochastic guarantees that the share of material inputs, μ, is equal in the two economies.

This economy is less interconnected—in the sense of Definition 6—than all other economies with identical Domar weights. To see this, consider an economy with Leontief inverse matrix $\mathbf{L}'$ with a pair of sectors $i \neq j$ such that $a'_{ij} > 0$ and $v'_k = v_k$ for all k. Since the Leontief inverse matrices of the two economies satisfy equation (10) with $\mathbf{B} = (1 - \mu)\mathbf{L}'$, it is then immediate that the latter economy is more interconnected.

We end this discussion with a remark on the relationship between primitive and network heterogeneity. Recall that Definition 6 provides a comparison for the extent of sectoral interconnections between two economies with identical Domar weights. Consequently, in order for all Domar weights to remain unchanged, the transformation in (10) not only impacts the nature of input-output linkages (as summarized by the Leontief inverse matrices), but also alters the economy's primitive heterogeneity. More specifically, if an economy is more interconnected than another, the preference parameters of the two economies have to be related via $\beta_i = \sum_{j=1}^{n} b_{ji}\beta'_j$, so that $v_i = v'_i$ for all i, implying that preference shares in the more interconnected economy are less evenly distributed across the n goods.

B. *Input-Output Linkages and Tail Comovements*

We now present the main result of this section.

PROPOSITION 8: *If an economy is more interconnected than another economy with identical Domar weights, then it exhibits more tail comovements.*

This result thus highlights the importance of input-output linkages in creating tail comovements across different sectors: given two economies with identical Domar weights, the one with greater levels of sectoral interconnections exhibits more tail comovements, in spite of the fact that the two economies are indistinguishable at the aggregate level.

Proposition 8 also clarifies a key distinction between the nature of economic fluctuations in (i) economies with no input-output linkages but a significant level of primitive heterogeneity (such as the baseline, island model of Gabaix 2011) on the one hand; and (ii) economies with a high level of network heterogeneity (such as the ones studied by Acemoglu et al. 2012) on the other. While large economic downturns of the first type mostly arise as a consequence of negative shocks to sectors with high β_i, fluctuations in the latter category are due to the propagation of shocks over the economy's input-output linkages. Even though the two mechanisms may not be distinguishable in the aggregate, they lead to significantly different levels of tail comovements.

To further clarify this point, consider the economy with no input-output linkages studied in Example 8, which is reminiscent of the island model of Gabaix (2011). As our arguments in Section III highlight, so long as preference parameters are heterogeneous enough, the economy exhibits nontrivial levels of macroeconomic tail risks. Nevertheless, Example 8 and Proposition 8 together imply that such an economy exhibits the least amount of tail comovements relative to all other economies with the same Domar weights. In other words, although an economy with significant network

heterogeneity experiences large economic downturns at the same frequency as an economy with identical Domar weights yet no network heterogeneity, its downturns are associated with severe contractions across a larger collection of sectors.

We end this section by remarking that even though we presented Proposition 8 for a pair of economies with a given number of sectors n, an identical result holds for two sequences of economies as n grows.

COROLLARY 3: *Consider two sequences of economies with identical Domar weights. If all economies in the first sequence are more interconnected than their corresponding economies in the second sequence, then* $\liminf_{n \to \infty} C_n(\tau_n)/C'_n(\tau_n) \geq 1$ *for all sequences* $\{\tau_n\}$.

V. Heavy-Tailed Shocks: An Equivalence Result

In this section, we extend our previous results by showing that the presence of primitive or network heterogeneity can translate light-tailed (e.g., exponential-tailed) idiosyncratic shocks into aggregate effects that can only arise with heavy-tailed shocks in the absence of such heterogeneity. In other words, we show that sufficient levels of heterogeneity in the economy's Domar weights have the same effect on the size of macroeconomic tail risks as subjecting firms to shocks with significantly heavier tails.

To present this result, we focus on an important subclass of heavy-tailed microeconomic shocks, namely shocks with Pareto tails. Formally, we say microeconomic shocks have *Pareto tails* if

$$\lim_{z \to \infty} \frac{1}{\log z} \, \log \, F(-z) \, = \, -\lambda,$$

where $\lambda > 2$ is the corresponding Pareto index. The smaller is λ, the heavier is the tail of the distribution. The condition that $\lambda > 2$ is meant to guarantee that the standard deviation of microeconomic shocks is well defined and finite. We start with a simple observation regarding Pareto-tailed shocks.

PROPOSITION 9: *If microeconomic shocks are Pareto tailed, any sequence of economies exhibits macroeconomic tail risks.*

The intuition for this result is simple: when microeconomic shocks have Pareto tails, the likelihood that at least one sector is hit with a large shock is high. As a result, regardless of the extent of heterogeneity in Domar weights, aggregate output experiences large declines with a relatively high probability, thus generating macroeconomic tail risks. This is in contrast to the case of exponentially tailed shocks, in which macroeconomic tail risks can emerge only if the economy exhibits sizable sectoral dominance. Nonetheless, we have the following result.

PROPOSITION 10: *For a sequence of simple economies subject to Pareto-tailed microeconomic shocks, there exists a sequence of economies subject to exponential-tailed shocks that exhibits (strictly) more macroeconomic tail risks in the sense of Definition 5.*

This result reiterates that macroeconomic tail risks can emerge not just due to (aggregate or idiosyncratic) shocks that are drawn from heavy-tailed distributions, but rather as a consequence of the interplay between relatively light-tailed distributions and heterogeneity in Domar weights. It thus underscores that sufficient levels of sectoral dominance can fundamentally reshape the distribution of aggregate output by concentrating risk at the tails and increasing the likelihood of large economic downturns from infinitesimal to substantial. This observational equivalence result thus provides a novel solution to what Bernanke, Gertler, and Gilchrist (1996) refer to as the "small shocks, large cycles puzzle" by showing that substantial levels of primitive or network heterogeneity can mimic large aggregate shocks.

Proposition 10 further highlights the distinction between our main results and those of Fama (1963) and Ibragimov and Walden (2007), who observe that the presence of Pareto-tailed shocks with extremely heavy tails and infinite variances (that is, when the Pareto index satisfies $\lambda < 2$) leads to departures from normality. In contrast to these papers, our main results show that sufficient heterogeneity in Domar weights translates light-tailed microeconomic shocks into aggregate effects that are observationally equivalent to those that arise due to heavy-tailed shocks.

We end this discussion with the following corollary to Proposition 10.

COROLLARY 4: *For a sequence of simple economies subject to Pareto-tailed shocks, there exists a sequence of economies with Pareto Domar weights and subject to exponential-tailed shocks which exhibits an identical level of macroeconomic tail risks.*

In other words, Pareto Domar weights have the same impact on the level of macroeconomic tail risks as Pareto-tailed shocks in a simple economy.

VI. A Simple Quantitative Illustration

We now provide a simple quantitative exercise to highlight whether and how microeconomic shocks can lead to macroeconomic tail risks. We show that the extent of heterogeneity in Domar weights in the US data is capable of generating departures from the normal distribution similar to the patterns documented in Figure 1. We also provide an illustration of the extent of tail comovements implied by the input-output linkages in the US data.

Throughout this section, we use the 2007 commodity-by-commodity direct requirements table and the corresponding sectoral sales data compiled by the Bureau of Economic Analysis. Using the sales data, we compute each sector's Domar weight as the ratio of its sales over GDP. The direct requirements table gives us the equivalent of our input-output matrix $\mathbf{A}$, with the typical (i,j) entry corresponding—under the Cobb-Douglas technology assumption—to the value of spending on commodity j per dollar of production of commodity i.[22] Though, for the sake of simplicity, we have thus far assumed that the row sums of $\mathbf{A}$ are equal to 1 (i.e., $\sum_{j=1}^{n} a_{ij} = 1$), we

---

[22] To better approximate the private sector of the economy, in this analysis we exclude 13 sectors corresponding to housing, residential structures, and federal and local government activities. See Acemoglu et al. (2012) for some basic descriptive statistics about the US economy's input-output matrices.
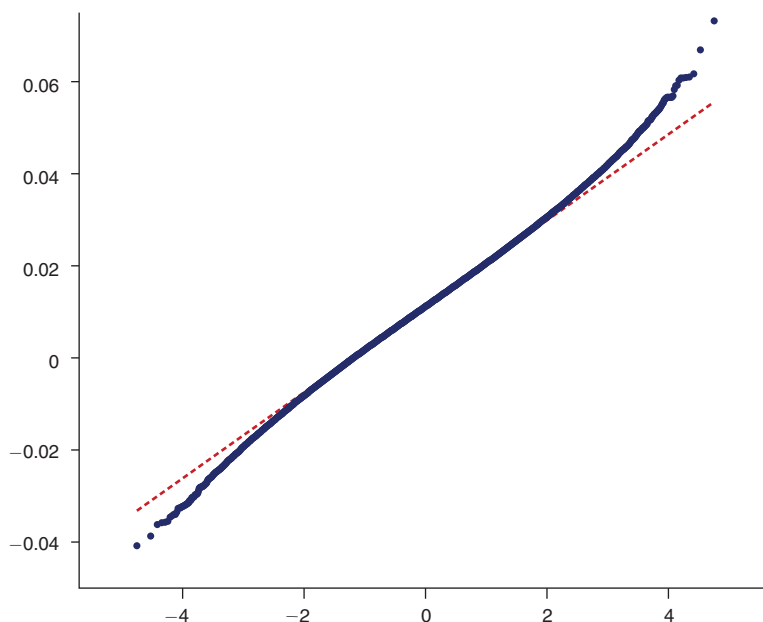
FIGURE 3. Q-Q PLOT OF AGGREGATE OUTPUT ON THE PRESENCE OF EXPONENTIAL SHOCKS

Note: The figure depicts the Q-Q plot of aggregate output against the normal distribution in the presence of exponential shocks, with Domar weights matched to the corresponding values in the US data.

drop this restriction in this section and instead work with the matrix implied by the direct requirements table.[23]

We first study the distribution of aggregate output in our model economy when microeconomic (sectoral) shocks are drawn from a symmetric exponential distribution, with the mean and variance chosen such that the first two moments of the economy's aggregate output match the first two moments of the US postwar GDP growth rate. The resulting Q-Q plot is depicted in Figure 3.[24] Confirming our theoretical results, the distribution of aggregate output exhibits systematic departures from the normal line at the tails, starting from around two standard deviations away from the mean. The Anderson-Darling, Kolmogorov-Smirnov, and Carmér-von Mises tests all reject normality at the 1 percent level.

We next investigate the contribution of network heterogeneity in the US data to the extent of macroeconomic tail risks. We focus on the distribution of aggregate output in a counterfactual economy with no primitive heterogeneity, where Domar weights are given by the column sums of the economy's Leontief inverse (divided by n). The resulting Q-Q plot is depicted in panel A of Figure 4. As the figure indicates, the distribution of aggregate output exhibits nontrivial departures from normality at both ends, highlighting the role of network heterogeneity in the emergence of macroeconomic tail risks. Indeed, all three goodness-of-fit tests reject normality at the 1 percent

---

[23] This choice is inconsequential for any of the main points we emphasize in this section, which remain essentially unchanged if we transform **A** by normalizing its row sums to 1 and then impose $\mu = 0.4$.

[24] We take 50,000 draws from the implied distributions to construct these figures and test statistics.

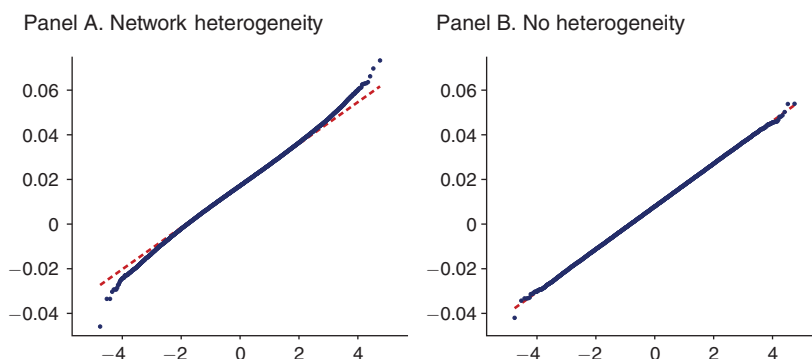Panel A. Network heterogeneity   Panel B. No heterogeneity

FIGURE 4. Q-Q PLOTS OF AGGREGATE OUTPUT IN THE PRESENCE OF EXPONENTIAL SHOCKS

Notes: Panel A depicts the Q-Q plot of the counterfactual economy with no primitive heterogeneity, with Domar weights set equal to the corresponding column sums of the Leontief inverse matrix divided by n. Panel B depicts the same plot for the counterfactual economy with all Domar weights set equal to $1/n$.

level. Finally, panel B of Figure 4 depicts the Q-Q plot of aggregate output when both sources of heterogeneity are shut down, with all Domar weights set equal to $1/n$. Consistent with our theoretical results, aggregate output in this case does not exhibit any meaningful departures from normality (with the three goodness-of-fit tests failing to reject normality at the 10 percent level) despite the fact that microeconomic shocks have an exponential distribution. Taken together, this exercise confirms that the extent of network heterogeneity in the US data is consistent with the proposition that modest levels of tail risk at the sectoral level can lead to macroeconomic tail risks.

The characterization in Corollary 1 provides an alternative way to asses the role of microeconomic interactions in the emergence of large economic downturns. Recall that, according to this result, Pareto distributed Domar weights can translate exponential-tailed microeconomic shocks into macroeconomic tail risks. Motivated by this observation, Figure 5 plots the empirical countercumulative distribution (defined as 1 minus the empirical cumulative distribution function) of the Domar weights in US data on the log-log scale. It also includes the nonparametric estimates for the empirical countercumulative distribution using the Nadaraya-Watson kernel regression (Nadaraya 1964; Watson 1964) with a bandwidth selected using least squares cross-validation. The tail of the distribution of Domar weights appears to be approximately linear, corresponding to a Pareto distribution. Taking the tail to correspond to 20 percent of the sample, we estimate the Pareto index, η, using an ordinary least squares regression with the Gabaix and Ibragimov (2011) correction. This yields an estimate of $\hat{\eta}_{OLS} = 1.45$ (standard error $= 0.24$), which is very close to the average slope implied by the nonparametric Nadaraya-Watson regression for the same part of the sample, $\hat{\eta}_{NW} = 1.36$. This exercise thus suggests that US Domar weights have a distribution that is consistent with exponentially tailed sectoral shocks generating macroeconomic tail risks.[25]

---

[25] Based on the results of Acemoglu et al. (2012), our estimates for the Pareto index of the distribution of Domar weights also imply the presence of significant levels of micro-originated aggregate volatility (measured as standard deviation of aggregate output).
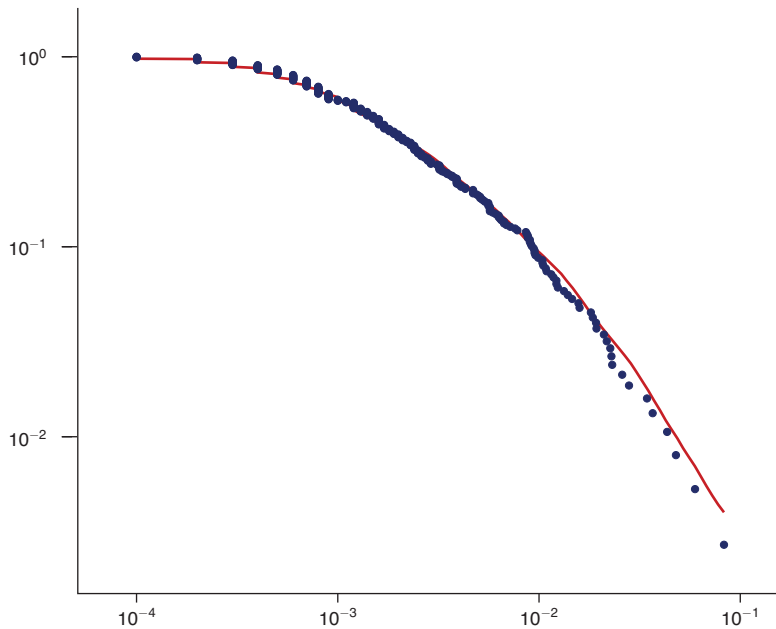
FIGURE 5. THE EMPIRICAL COUNTERCUMULATIVE DISTRIBUTION OF SECTORAL DOMAR WEIGHTS

As a final exercise, we assess the implications of input-output linkages observed in the US data for the extent of tail comovements. Assuming exponentially distributed sectoral shocks, we compute the probability that 10 percent or more of sectors experience a two standard deviation decline when aggregate output itself declines by two standard deviations or more (recall from footnote 6 that 10 percent is approximately the fraction of manufacturing sectors experiencing such a decline in the two sharpest US recessions). Given the Domar weights and input-output matrix of the US economy, we find this number to be equal to 0.17 percent. We then compute the same number for the counterfactual economy in which Domar weights are identical to that of the US economy but are entirely driven by primitive heterogeneity. Given that, by construction, this counterfactual economy is less interconnected than the US economy, our theoretical results imply that it should also display less tail comovements. Indeed, in this case the conditional probability that 10 percent or more of sectors experience a two standard deviation decline is effectively equal to 0, up to four digits after the decimal point.

VII. Dynamic Model

We conclude our analysis by showing that the main results of the static model studied in the earlier sections extend to a dynamic economy in the spirit of Long and Plosser (1983) and Horvath (2000), where it takes each firm one period to transform inputs to output.

Consider a discrete-time dynamic economy consisting of n sectors, each with a Cobb-Douglas production technology. In contrast to the static model in (1),

production does not take place instantaneously, but with one period delay following the purchase of inputs. Specifically, the production technology of sector i at time t is given by

$$(11) \qquad x_{it+1} = \Xi_{it} \zeta_i l_{it}^{1-\mu} \prod_{j=1}^{n} (x_{ijt})^{\mu a_{ij}},$$

where Ξ_{it} denotes the productivity shock to sector i at time t, and ζ_i is a normalization constant.[26] We assume that Ξ_{it} is known when period t decisions are made and that $\log(\Xi_{it})$ follows an AR(1) process,

$$(12) \qquad \log(\Xi_{it}) = \varphi \log(\Xi_{it-1}) + \tilde{\epsilon}_{it},$$

where $\varphi \in [0, 1]$ and $\tilde{\epsilon}_{it}$s are drawn independently across time and sectors from a common distribution F with zero mean and unit variance.

In addition to the firms, the economy comprises a representative household with logarithmic preferences over the n goods, who maximizes her expected lifetime utility,

$$(13) \qquad U = E_0 \sum_{t=0}^{\infty} \rho^t \left[\sum_{i=1}^{n} \beta_i \log(c_{it}) \right],$$

where $\rho \in (0, 1)$ denotes her discount factor. To ensure that markets are complete, we assume that there are n risk-free bonds in zero net supply, each denominated in the units of one of the sectors. The representative household owns shares in all sectors. The household's intertemporal budget constraint is thus given by

$$(14) \quad \sum_{i=1}^{n} p_{it} c_{it} + \sum_{i=1}^{n} q_{it} s_{it+1} + \sum_{i=1}^{n} \pi_{it} b_{it+1} \leq w_t + \sum_{i=1}^{n} s_{it}(q_{it} + d_{it}) + \sum_{i=1}^{n} p_{it} b_{it},$$

where p_{it} is the price of good i at time t; s_{it} denotes the household's time t shareholdings in sector i; d_{it} is the dividend paid on one share in that sector; q_{it} is the corresponding share price; and finally b_{it} and π_{it} denote the household's holding and the price of the ith bond at time t, respectively.

The equilibrium of this economy is defined as a sequence of prices and quantities such that all markets clear at all times. We have the following dynamic counterpart to Proposition 1.

PROPOSITION 11: *For any $\rho \in (0, 1)$ and $\varphi \in [0, 1]$, the time-discounted sum of aggregate output in the dynamic economy satisfies*

$$(1 - \rho\varphi) \sum_{t=0}^{\infty} \rho^t \log(\text{GDP}_{t+1}) = \sum_{t=0}^{\infty} \rho^t \sum_{i=1}^{n} v_{it} \tilde{\epsilon}_{it} + \text{constant},$$

---

[26] As in the static economy presented in Section I, the value of ζ_i is immaterial for our results. To simplify our expressions, we set $\zeta_i = (1 - \rho\mu)^{-(1-\mu)} \prod_{j=1}^{n} (\rho\mu a_{ij})^{-\mu a_{ij}}$, where ρ denotes the representative household's discount factor.

where

$$(15) \qquad v_{it} = \frac{p_{it} x_{it}}{\mathrm{GDP}_t} = \sum_{j=1}^{n} \beta_j \tilde{\ell}_{ji}$$

is sector i's Domar weight at time t and $\tilde{\mathbf{L}} = (\mathbf{I} - \rho \mu \mathbf{A})^{-1}$. *In particular, if microeconomic shocks follow a unit root process (i.e.,* $\varphi = 1$), *then*

$$(1 - \rho) \sum_{t=0}^{\infty} \rho^t \log(\mathrm{GDP}_{t+1}) = \sum_{t=0}^{\infty} \rho^t \sum_{i=1}^{n} v_{it} \tilde{\epsilon}_{it} + \text{constant.}$$

This result illustrates that, in line with our results for the static economy, the time-discounted sum of aggregate output of the dynamic economy is a linear combination of innovations to sectoral productivities, with the weights given by the Domar weights. Furthermore, as in Proposition 1, the Domar weight of sector i at any given time t only depends on the corresponding column of the economy's Leontief inverse matrix and the representative household's preference parameters ($\beta_1, \ldots, \beta_n$). The key difference is that the relevant Leontief inverse includes an adjustment for the delay in production using the discount factor of the representative household: the Leontief inverse of the dynamic economy takes the form of $\tilde{\mathbf{L}} = (\mathbf{I} - \rho \mu \mathbf{A})^{-1}$ instead of $\mathbf{L} = (\mathbf{I} - \mu \mathbf{A})^{-1}$ for the static economy. Nonetheless, the parallel between equations (15) and (3) underscores the central roles played by primitive and network heterogeneity in determining the distribution of Domar weights in the dynamic and static economies.

A comparison of Propositions 1 and 11 further illustrates an even closer relationship between the two models: the distribution of time-discounted sum of aggregate output in the dynamic economy can be approximated by that of the aggregate output in the static economy as the discount factor ρ tends to one. More specifically, it is easy to see that $(1 - \rho \varphi)\sqrt{1 - \rho^2} \sum_{t=0}^{\infty} \rho^t \log(\mathrm{GDP}_{t+1})$ can be arbitrarily closely approximated by $\sum_{i=1}^{n} v_i \epsilon_i$ for large enough values of ρ, where $v_i = \sum_{j=1}^{n} \beta_j \ell_{ji}$ is the Domar weight of sector i in the static economy and

$$\epsilon_i = \sqrt{1 - \rho^2} \sum_{t=0}^{\infty} \rho^t \tilde{\epsilon}_{it}$$

is independent and identically distributed across sectors.[27] Crucially for our analysis, ϵ_i inherits the tail properties of the innovations to sectoral productivities $\tilde{\epsilon}_{it}$; for example, normal- or exponential-tailed innovations translate into normal or exponential-tailed ϵ_is, respectively.

VIII. Conclusions

In this paper we have argued that the interplay of microeconomic shocks and sectoral heterogeneity can lead to systematic departures in the likelihood of large

---

[27] The normalization constant $\sqrt{1 - \rho^2}$ is meant to ensure that stdev(ϵ_i) $= 1$ regardless of the value of ρ.

economic downturns from what is implied by the normal distribution. Our results show that such macroeconomic tail risks emerge under two intuitive conditions. First, microeconomic shocks themselves need to exhibit tail risks, as combinations of normal shocks always result in normally distributed aggregates. Second, there needs to be sufficient heterogeneity in sectoral Domar weights, ensuring that these microeconomic tail risks do not wash out in the aggregate. Crucially, our results illustrate that departures from normality at the tails can emerge even if GDP fluctuations are approximately normally distributed away from the tails, highlighting the qualitatively different nature of large economic downturns from small or moderate fluctuations.

We also show that even though the distribution of sectoral Domar weights is a sufficient statistic for the extent of macroeconomic tail risks, the likelihood that many sectors experience large, simultaneous contractions is determined by the nature of intersectoral input-output linkages. More specifically, we demonstrate that, keeping the empirical distribution of Domar weights constant, an increase in the extent of sectoral interconnections leads to greater tail comovements. This result thus illustrates that when sectoral heterogeneity, at least partly, reflects network heterogeneity, large recessions involve not only significant GDP contractions, but also large simultaneous declines across a wide range of industries within the economy.

Finally, our quantitative results show that when calibrated to the values of primitive and network heterogeneity observed in the US data, our model—despite its stylized nature—generates significant macroeconomic tail risks and tail comovements.

We see our paper as a first step in a systematic investigation of the origins of macroeconomic tail risks. Though many commentators view large economic downturns as more consequential than a series of small or moderate recessions, there is relatively little work on understanding whether and how these different types of downturns are distinct from one another. Our results suggest that despite their similar origins rooted in the interplay of microeconomic shocks and sectoral heterogeneity, large economic downturns can indeed be rather different from regular business-cycle fluctuations: unlike small or moderate recessions, the extent of macroeconomic tail risks is highly sensitive to the distribution of microeconomic shocks. This perspective naturally lends itself to an investigation of whether certain structural changes can stabilize the economy during regular times while simultaneously increasing tail risks.

Several important issues remain open to future research. First, the tractability of our model permits the introduction of various market imperfections into this general framework. This would not only enable an investigation of whether, in the presence of realistic market structures, network and primitive heterogeneities play richer (and more distinct) roles, but also whether large economic downturns necessitate different microeconomic and macroeconomic policy responses. Second, our analysis was simplified by the log-linear nature of our model economy. An interesting question is whether reasonable nonlinear interactions could exacerbate macroeconomic tail risks. One possibility is to generalize the Cobb-Douglas production technologies, in which case even though versions of equations (2) and (3) would continue to apply, the Domar weights would change endogenously in response to microeconomic shocks. Indeed, depending on elasticities, a sector's Domar weight

may grow in response to a negative shock to that sector, thus increasing the likelihood of large economic downturns. Third, while our main focus has been on the role of input-output linkages, one would expect that other types of interactions between microeconomic units may also have major implications for aggregate tail risks and tail comovements. Two natural candidates are the linkages between financial institutions and between the financial sector and the rest of the economy. For example, the nonlinear financial contagion mechanisms proposed in recent papers such as Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015) and Elliott, Golub, and Jackson (2014) or the nonlinearities emphasized by Brunnermeier, Eisenbach, and Sannikov (2013), may lead to macroeconomic tail risks that are distinct in nature from those studied in the current framework. Finally, given that our work was motivated by a stylized look at US economic fluctuations, a more systematic empirical investigation to measure and describe the nature of macroeconomic tail risks would be a natural next step. A critical challenge would be to empirically distinguish the economic mechanisms proposed here from alternative approaches to the origins of large aggregate shocks (as in the rare disasters literature), time-varying model parameters or volatility (Engle 1982, Cogley and Sargent 2005), and nonlinear financial interactions.

Appendix A: Proofs

LEMMA A.1: *Let Φ denote the CDF of the standard normal distribution. Then,*

$$\lim_{z \to \infty} \frac{1}{z^2} \log \Phi(-z) = -1/2.$$

PROOF:
It is well known that $\lim_{z \to \infty} z\Phi(-z)/\phi(z) = 1$, where ϕ denotes the standard normal density function (e.g., Grimmett and Stirzaker 2001, p. 98). Consequently,

$$\lim_{z \to \infty} \frac{\log z + \log \Phi(-z)}{\log \phi(z)} = 1,$$

which in turn implies that

$$\lim_{z \to \infty} \frac{\log z + \log \Phi(-z)}{\log \sqrt{2\pi} + z^2/2} = -1.$$

The statement of the lemma follows immediately. ∎

LEMMA A.2: $\sum_{j=1}^{n} \ell_{ij} = 1/(1-\mu)$ *for all i, where* $\mathbf{L} = [\ell_{ij}]$ *is the economy's Leontief inverse.*

PROOF:
The fact that $\mu < 1$ implies that the Leontief inverse can be written as

$$\mathbf{L} = (\mathbf{I} - \mu\mathbf{A})^{-1} = \sum_{k=0}^{\infty} (\mu\mathbf{A})^k.$$

As a result,

$$\sum_{j=1}^{n} \ell_{ij} = \sum_{k=0}^{\infty} \mu^k \sum_{j=1}^{n} (\mathbf{A}^k)_{ij} = \sum_{k=1}^{\infty} \mu^k = \frac{1}{1-\mu},$$

where the second equality is a consequence of the assumption that all firms employ production technologies that exhibit constant returns to scale. ∎

A. *Proof of Proposition 1*

The first-order conditions of firms in sector i imply that

(A1) $$x_{ij} = \mu a_{ij} p_i x_i / p_j;$$

$$l_i = (1-\mu) p_i x_i / w,$$

where w denotes the market wage. Plugging the above into firm i's production function and taking logarithms yields

$$\log p_i + \epsilon_i = (1-\mu) \log w + \mu \sum_{j=1}^{n} a_{ij} \log p_j.$$

Solving for the equilibrium prices in the system of equations above, we obtain

$$\log p_i = (1-\mu) \log w \sum_{j=1}^{n} \ell_{ij} - \sum_{j=1}^{n} \ell_{ij} \epsilon_j,$$

where ℓ_{ij} is the (i,j) element of the economy's Leontief inverse. Consequently, by Lemma A.2,

(A2) $$\log p_i = \log w - \sum_{j=1}^{n} \ell_{ij} \epsilon_j.$$

Multiplying both sides of the equation above by β_i and summing over all sectors i leads to

$$\sum_{i=1}^{n} \beta_i \log p_i = \log w - \sum_{i=1}^{n} \sum_{j=1}^{n} \beta_i \ell_{ij} \epsilon_j.$$

Given that (i) labor is the only primary factor of production; (ii) all firms make zero profits; and (iii) total labor supply is normalized to 1, GDP in this economy is equal to the market wage, w. Therefore, by normalizing the price index $\prod_{i=1}^{n} p_i^{\beta_i} = 1$, we obtain $\log(\text{GDP}) = \sum_{j=1}^{n} v_j \epsilon_j$, where $v_j = \sum_{i=1}^{n} \beta_i \ell_{ij}$.

Next, we show that v_i also coincides with the Domar weight of sector i. The market-clearing condition for good i is given by $x_i = c_i + \sum_{j=1}^{n} x_{ji}$. Consequently,

$$p_i x_i = \beta_i w + \mu \sum_{j=1}^{n} a_{ji} p_j x_j,$$

where we are using the fact that $x_{ji} = \mu a_{ji} p_j x_j / p_i$ and $c_i = \beta_i w / p_i$. Solving the system of equations above for sectoral sales implies

$$p_i x_i = w \sum_{j=1}^{n} \beta_j \ell_{ji},$$

thus establishing that $v_i = p_i x_i / w = p_i x_i / \text{GDP}$. ∎

B. *Proof of Proposition 2*

The fact that microeconomic shocks are independent across sectors implies that

$$\Pr(y < -\tau\sigma) \geq \Pr(v_i \epsilon_i < -\tau\sigma) \cdot \Pr\left(\sum_{j \neq i} v_j \epsilon_j \leq 0\right)$$

for any arbitrarily chosen sector i. As a result, $\Pr(y < -\tau\sigma) \geq (1/2)\Pr(v_i \epsilon_i < -\tau\sigma)$, where we are using the assumption that the distribution of microeconomic shocks is symmetric around the origin. Consequently,

$$R(\tau) \leq \frac{\log(1/2)}{\log\Phi(-\tau)} + \frac{\log F(-\tau\sigma/v_i)}{\log \Phi(-\tau)},$$

and in particular,

(A3) $$R(\tau) \leq \frac{\log(1/2)}{\log \Phi(-\tau)} + \frac{\log F(-\tau\sigma/v_{\max})}{\log \Phi(-\tau)}.$$

Taking limits from both sides of the inequality above and using Lemma A.1, we have

$$\limsup_{\tau\to\infty} R(\tau) \leq \limsup_{\tau\to\infty} \frac{\log F(-\tau\sigma/v_{\max})}{-\tau^2/2}.$$

Invoking Lemma A.1 one more time implies

$$\limsup_{\tau\to\infty} R(\tau) \leq \left(\frac{\sigma}{v_{\max}}\right)^2 \limsup_{\tau\to\infty} \frac{\log F(-\tau\sigma/v_{\max})}{\log \Phi(-\tau\sigma/v_{\max})}.$$

On the other hand, the assumption that microeconomic shocks exhibit tail risks means that $\log F(-z)/\log \Phi(-z) \to 0$ as $z \to \infty$. Therefore, the right-hand side of the inequality above is equal to 0. ∎

C. *Proof of Proposition 3*

We first state what is known as Cramér's Theorem (Petrov 1975, p. 218) and then proceed to the proof of Proposition 3.

THEOREM A.1 (Cramér's Theorem): *Let* $\{\epsilon_1, \epsilon_2, \ldots\}$ *be a sequence of i.i.d. light-tailed random variables with zero mean and unit variance and let* $G_n(z) = \Pr\left(\sum_{i=1}^{n} \epsilon_i < z\sqrt{n}\right)$. *Then, for any sequence* $z_n \geq 0$ *such that* $z_n/\sqrt{n} \to 0$,

$$\frac{G_n(-z_n)}{\Phi(-z_n)} = \exp\left(-\frac{z_n^3}{\sqrt{n}}\Lambda\left(-\frac{z_n}{\sqrt{n}}\right)\right)\left[1 + O\left(\frac{z_n + 1}{\sqrt{n}}\right)\right]$$

as $n \to \infty$, *where* $\Lambda(z)$ *is a power series with coefficients depending on the cumulants of* ϵ_i *that converges for sufficiently small values of* $|z|$.

PROOF OF PROPOSITION 3:

Part (i).—Recall that in a simple economy, all Domar weights are equal to $v_i = 1/n$. Therefore, by Proposition 1, aggregate output is given by $y_n = (1/n)\sum_{i=1}^{n}\epsilon_i$, which in turn implies that $\sigma_n = 1/\sqrt{n}$. Thus, as long as $\lim_{n\to\infty}\tau_n/\sqrt{n} = 0$, Theorem A.1 implies that

$$\Pr(y_n < -\tau_n\sigma_n) = \Phi(-\tau_n)\exp\left(-\frac{\tau_n^3}{\sqrt{n}}\Lambda\left(-\frac{\tau_n}{\sqrt{n}}\right)\right)\left[1 + O\left(\frac{\tau_n + 1}{\sqrt{n}}\right)\right]$$

as $n \to \infty$. Consequently,

$$\log\Pr(y_n < -\tau_n\sigma_n) = \log\Phi(-\tau_n) - \frac{\tau_n^3}{\sqrt{n}}\Lambda(-\tau_n/\sqrt{n}) + O\left(\frac{\tau_n}{\sqrt{n}}\right).$$

Dividing both sides by $\log\Phi(-\tau_n)$ and using Lemma A.1, we have

$$R_n(\tau_n) = 1 + \frac{2\tau_n}{\sqrt{n}}\Lambda(-\tau_n/\sqrt{n}) + o(1).$$

Now the fact that $\Lambda(z)$ is convergent for small enough values of z guarantees that the second term on the right-hand side above converges to zero as long as $\lim_{n\to\infty}\tau_n/\sqrt{n} = 0$, thus implying that $\lim_{n\to\infty}R_n(\tau_n) = 1$. ∎

Part (ii).—Recall from the proof of Proposition 2 that inequality (A3) is satisfied for any economy and all τ. Therefore, for any sequence $\{\tau_n\}$ such that $\lim_{n\to\infty}\tau_n = \infty$,

$$\limsup_{n\to\infty}R_n(\tau_n) \leq \limsup_{n\to\infty}\frac{2\log 2}{\tau_n^2} + \limsup_{n\to\infty}\frac{\log F(-\tau_n\sqrt{n})}{\log\Phi(-\tau_n)}$$

$$= \limsup_{n\to\infty}\frac{\log F(-\tau_n\sqrt{n})}{-\tau_n^2/2},$$

where we are using the fact that $v_{\max} = 1/n$ and $\sigma_n = 1/\sqrt{n}$ and invoking Lemma A.1. Now suppose microeconomic shocks have exponential tails; that is, $\lim_{z \to \infty}(1/z) \log F(-z) = -\gamma$ for some $\gamma > 0$. As a result,

$$\limsup_{n \to \infty} R_n(\tau_n) \leq \limsup_{n \to \infty} \frac{2\gamma \sqrt{n}}{\tau_n}.$$

Consequently, $\lim_{n \to \infty} R_n(\tau_n) = 0$ for any arbitrary sequence $\{\tau_n\}$ such that $\lim_{n \to \infty} \tau_n/\sqrt{n} = \infty$. ∎

D. *Proof of Proposition 4*

Recall from Proposition 1 that $y = \sum_{i=1}^{n} v_i \epsilon_i$, where v_i is the Domar weight of sector i. This linear relationship implies that y is normally distributed whenever all microeconomic shocks have a normal distribution. Consequently, $R(\tau) = 1$ regardless of the value of τ and the number of sectors n, which implies that for any given sequence of economies and all constants $c > 0$, it must be the case that $\lim_{n \to \infty} R_n(c\sqrt{n}) = 1$. ∎

E. *Proof of Proposition 5*

Part (i).—We start with proving sufficiency. First, note that in any given economy, $\sigma = \|v\|$. Therefore, inequality (A3) can be rewritten as

$$(A4) \qquad R(\tau) \leq \frac{\log(1/2)}{\log \Phi(-\tau)} + \frac{\log F(-\tau \sqrt{n}/\delta)}{\log \Phi(-\tau)},$$

where $\delta = v_{\max}\sqrt{n}/\|v\|$ is the economy's measure of sectoral dominance. Now, consider a sequence of economies as $n \to \infty$ and suppose that $\tau_n = c\sqrt{n}$ for some constant $c > 0$. The assumption that microeconomic shocks have exponential tails, alongside inequality (A4), implies that

$$\limsup_{n \to \infty} R_n(c\sqrt{n}) \leq \limsup_{n \to \infty} \frac{2\gamma}{c\delta_n},$$

where δ_n is the measure of sectoral dominance of the n-sector economy in the sequence. Thus, if $\lim_{n \to \infty} \delta_n = \infty$, the right-hand side of the inequality above is equal to 0, establishing that the sequence of economies exhibits macroeconomic tail risks.

To prove the reverse implication, we follow steps similar to those of Teicher (1984) to obtain exponential bounds for large deviations of sums of independent random variables. Consider an arbitrary economy consisting of n sectors. The assumption that microeconomic shocks have, symmetric exponential tails with exponent $\gamma > 0$ guarantees that there exists a constant $\hat{\gamma} \leq \gamma$ such that

$$(A5) \qquad F(-z) = 1 - F(z) < e^{-\hat{\gamma}z}$$

for all $z > 0$. On the other end,

$$\frac{1}{2}E\,|\epsilon_i|^k \;=\; \int_0^\infty z^k\,dF(z) \;=\; \int_0^\infty kz^{k-1}(1-F(z))\,dz$$

for $k \geq 2$, where are using the fact that $\lim_{z\to\infty} z^k(1-F(z)) = 0$. Thus, by (A5), there exists a positive constant $h = 1/\hat{\gamma}$ such that

(A6) $$\frac{1}{2}E|\epsilon_i|^k \;\leq\; \int_0^\infty kz^{k-1}e^{-z/h}\,dz \;=\; h^k k!$$

for all $k \geq 2$. Consequently, for any positive constant d,

$$E\big(e^{dv_i\epsilon_i}\big) \;=\; \sum_{k=0}^\infty \frac{(dv_i)^k}{k!}E\epsilon_i^k \;\leq\; 1 + \sum_{k=2}^\infty \frac{(dv_i)^k}{k!}E|\epsilon_i|^k \;\leq\; 1 + 2\sum_{k=2}^\infty (dhv_i)^k,$$

where the last inequality is a consequence of (A6). Thus, as long as $dhv_{\max} < 1$,

(A7) $$E\big(e^{dv_i\epsilon_i}\big) \;\leq\; 1 + \frac{2(dhv_i)^2}{1-dhv_i} \;\leq\; \exp\!\left(\frac{2(dhv_i)^2}{1-dhv_{\max}}\right)$$

for all i, where we are using the fact that $1 + z \leq e^z$ for all z. On the other hand, Chernoff's inequality, alongside Proposition 1 and the assumption that shocks are symmetrically distributed around the origin, implies that

$$\Pr(y < -\tau\sigma) \;\leq\; e^{-d\tau\sigma}E\big(e^{-dy}\big) \;=\; e^{-d\tau\sigma}E\big(e^{dy}\big) \;=\; e^{-d\tau\sigma}\prod_{i=1}^n E\big(e^{dv_i\epsilon_i}\big).$$

Combining this inequality with (A7) yields

$$\log\Pr(y < -\tau\sigma) \;\leq\; -d\tau\sigma + \sum_{i=1}^n \frac{2(dhv_i)^2}{1-dhv_{\max}} \;=\; -d\tau\sigma + \frac{2(dh\|v\|)^2}{1-dhv_{\max}}.$$

Letting $d = \tau\sigma(4h^2\|v\|^2 + h\tau\sigma v_{\max})^{-1}$ (which satisfies the condition required for deriving (A7), $dhv_{\max} < 1$) leads to

$$\log\Pr(y < -\tau\sigma) \;\leq\; \frac{-\tau^2\|v\|}{8h^2\|v\| + 2h\tau v_{\max}},$$

where we are using the fact that $\|v\| = \sigma$. Therefore, the τ-tail ratio of any economy consisting of n sectors satisfies

(A8) $$R(\tau) \;\geq\; \frac{\tau^2(4h^2 + h\tau\delta/\sqrt{n})^{-1}}{-2\log\Phi(-\tau)}$$

for all $\tau > 0$, where $\delta = v_{\max}\sqrt{n}/\|v\|$ is the economy's measure of sectoral dominance.

Now, consider an arbitrary sequence of economies as $n \to \infty$ and let $\tau_n = c\sqrt{n}$ for some constant $c > 0$. Inequality (A8) then implies

$$\limsup_{n\to\infty} R_n(c\sqrt{n}) \geq \limsup_{n\to\infty} \frac{1}{4h^2 + hc\delta_n},$$

where we have once again used Lemma A.1. Consequently, if $\liminf_{n\to\infty}\delta_n < \infty$, the right-hand side of the inequality above would be strictly positive, establishing that $\limsup_{n\to\infty} R_n(c\sqrt{n}) > 0$; that is, the sequence of economies does not exhibit macroeconomic tail risks. ∎

Part (ii).—Since $\lim_{n\to\infty}\delta_n = \infty$, part (i) of the proposition implies that the sequence of economies exhibits macroeconomic tail risks. On the other hand, recall that for any given economy consisting of n sectors, $\delta/\sqrt{n} = v_{\max}/\|v\|$. Therefore, by Theorem 1 of Acemoglu et al. (2012), in any sequence of economies that satisfies $\lim_{n\to\infty}\delta_n/\sqrt{n} = 0$, the random variable y_n/σ_n converges in distribution to the standard normal distribution as $n \to \infty$. ∎

F. *Proof of Corollary 1*

Consider an economy consisting of n sectors. By definition, the Domar weight of sector i is given by $v_i = ci^{-1/\eta}$, where c is a properly chosen normalization constant, so that $\sum_{i=1}^{n} v_i = 1/(1-\mu)$.[28] It is then immediate that $v_{\max} = c$ and $\|v\| = c\sqrt{\sum_{i=1}^{n} i^{-2/\eta}}$. Consequently, the economy's measure of sectoral dominance is given by

$$\delta = \left(\frac{1}{n}\sum_{i=1}^{n} i^{-2/\eta}\right)^{-1/2}.$$

Now consider a sequence of economies with Pareto Domar weights with common Pareto index η. We analyze the behavior of the measure of sectoral dominance in such a sequence of economies under three separate cases.

First, suppose that $\eta \in (0, 2)$. In this case, the summation $\sum_{i=1}^{n} i^{-2/\eta}$ is convergent and hence bounded from above. It is thus immediate that $\lim_{n\to\infty}\delta_n = \infty$, where δ_n is the measure of sectoral dominance of the n-sector economy in the sequence.

Next, suppose that $\eta = 2$. In this case, $\sum_{i=1}^{n} i^{-2/\eta}$ is nothing but the harmonic series and is therefore upper bounded by $1 + \log n$ for all n. Consequently,

$$\delta_n \geq \left(\frac{1}{n}(1 + \log n)\right)^{-1/2}$$

for all n, which implies that $\lim_{n\to\infty}\delta_n = \infty$.

---

[28] Recall that, by Lemma A.2, $\sum_{i=1}^{n}\ell_{ji} = 1/(1-\mu)$ for all sectors j, where μ is the share of intermediate inputs in the firms' production technologies. Therefore, in any economy, $\sum_{i=1}^{n} v_i = \sum_{i=1}^{n}\sum_{j=1}^{n}\beta_j\ell_{ji} = 1/(1-\mu)$.

Finally, if $\eta > 2$, then there exists a constant $\bar{c} > 0$, independent of n, such that

$$\sum_{i=1}^{n} i^{-2/\eta} \leq \bar{c} n^{1-2/\eta}$$

for all n. Hence, $\delta_n \geq n^{1/\eta}/\sqrt{\bar{c}}$, thus once again implying that $\lim_{n\to\infty} \delta_n = \infty$. ∎

G. *Proof of Proposition 6*

We prove this result for a more general class of super-exponential shocks than the ones considered in Section IIIC. In particular, we assume that the CDF of microeconomic shocks satisfies

$$\lim_{z\to\infty} \frac{1}{\rho(z)} \log F(-z) = -1$$

for some nonnegative, increasing function $\rho(z)$ such that

$$\lim_{z\to\infty} \rho(z)/z = \infty;$$

$$\lim_{z\to\infty} \rho(z)/z^2 = 0.$$

These conditions guarantee that the tail of the distribution is lighter than that of the exponential distribution, but heavier than that of the normal distribution. It is immediate that the class of distributions that satisfy (6) corresponds to the special case in which $\rho(z) = kz^{\alpha}$ for $\alpha \in (1, 2)$ and $k > 0$.

Part (i).—Since super-exponential microeconomic shocks exhibit tails that are lighter than that of the exponential distribution, any given deviation from the mean is less likely compared to an identical deviation under the assumption that microeconomic shocks have exponential tails. Hence, in the presence of shocks with super-exponential tails, there exists a constant $h > 0$ such that inequality (A8) is satisfied for large enough values of $\tau > 0$. Consequently, for any given sequence of economies, we have

$$\limsup_{n\to\infty} R_n(c\sqrt{n}) \geq \limsup_{n\to\infty} \frac{1}{4h^2 + hc\delta_n}.$$

As a result, if $\liminf_{n\to\infty} \delta_n < \infty$, then $\limsup_{n\to\infty} R_n(c\sqrt{n}) > 0$ for all $c > 0$, guaranteeing that the sequence of economies does not exhibit macroeconomic tail risks. ∎

Part (ii).—Recall that any economy satisfies inequality (A4). Therefore, for any sequence of economies and any constant $c > 0$,

$$\limsup_{n\to\infty} R_n(c\sqrt{n}) \leq -2 \limsup_{n\to\infty} \frac{1}{c^2 n} \log F(-cn\delta_n^{-1}),$$

where δ_n is the measure of sectoral dominance of the n-sector economy in the sequence. Given that shocks have super-exponential tails, we have

$$\limsup_{n\to\infty} R_n(c\sqrt{n}) \leq 2 \limsup_{n\to\infty} \frac{\rho(cn\delta_n^{-1})}{c^2 n}.$$

Therefore, if $\lim_{n\to\infty} \rho(cn\delta_n^{-1})/n = 0$, the right-hand side of the expression above is equal to 0, implying that the sequence of economies exhibits macroeconomic tail risks. Setting $\rho(z) = kz^\alpha$ proves the result for the subclass of super-exponential shocks that satisfy (6). ∎

H. *Proof of Proposition 7*

Part (i).—This follows immediately from an argument identical to that of the proof of Proposition 4. ∎

Part (ii).—Recall that any given economy satisfies inequality (A4). As a result,

$$\text{(A9)} \qquad \limsup_{n\to\infty} R_n(\tau_n) \leq 2\gamma \limsup_{n\to\infty} \frac{\sqrt{n}}{\tau_n \delta_n},$$

where we are using Lemma A.1 and the assumption that microeconomic shocks have exponential tails. Therefore, if $\lim_{n\to\infty} \tau_n \delta_n/\sqrt{n} = \infty$, the right-hand side of the inequality above is equal to 0, thus implying that the sequence of economies exhibits macroeconomic trial risks with respect to $\{\tau_n\}$.

To prove the reverse implication, recall from (A8) that in the presence of microeconomic shocks with exponential tails,

$$R_n(\tau_n) \geq \frac{\tau_n^2 (4h^2 + h\tau_n \delta_n/\sqrt{n})^{-1}}{-2\log \Phi(-\tau_n)}.$$

Therefore, by Lemma A.1,

$$\limsup_{n\to\infty} R_n(\tau_n) \geq \limsup_{n\to\infty} \frac{1}{4h^2 + h\tau_n \delta_n/\sqrt{n}}.$$

Now, the fact that $\liminf_{n\to\infty} \tau_n \delta_n/\sqrt{n} < \infty$ implies that the right-hand side of the inequality above is strictly positive, and as a result, the sequence of economies does not exhibit macroeconomic tail risks with respect to $\{\tau_n\}$. ∎

Part (iii).—The proof is similar to that of part (ii) of Proposition 6. By inequality (A4) and Lemma A.1,

$$\limsup_{n\to\infty} R_n(\tau_n) \leq -2 \limsup_{n\to\infty} \frac{1}{\tau_n^2} \log F(-\tau_n \sqrt{n}/\delta_n) = 2 \limsup_{n\to\infty} \frac{\rho(\tau_n \sqrt{n}/\delta_n)}{\tau_n^2},$$

where the equality is a consequence of the fact that shocks have super-exponential tails. Setting $\rho(z) = kz^\alpha$, we have

$$\limsup_{n\to\infty} R_n(\tau_n) \le 2k \limsup_{n\to\infty} n^{\alpha/2}\, \delta_n^{-\alpha}\, \tau_n^{\alpha-2}.$$

Now the assumption that $\lim_{n\to\infty} \delta_n \tau_n^{2/\alpha-1}/\sqrt{n} = \infty$ implies that the right-hand side of the inequality above is equal to 0. ∎

I. *Proof of Corollary 2*

Consider two sequences of economies with τ-tail ratios R_n and $\overline{R}_n$ that are subject to exponential shocks and suppose that $\lim_{n\to\infty} \delta_n/\overline{\delta}_n = \infty$, where δ_n and $\overline{\delta}_n$ denote the sectoral dominance of the economy consisting of n sector in the two sequences. Pick an arbitrary sequence $\tau_n \to \infty$ such that $\lim_{n\to\infty} \overline{R}_n(\tau_n) = 0$. Inequality (A8) implies that for any such sequence,

$$\limsup_{n\to\infty} \frac{1}{4h^2 + h\tau_n \overline{\delta}_n/\sqrt{n}} \le \limsup_{n\to\infty} \overline{R}_n(\tau_n) = 0,$$

which implies that $\lim_{n\to\infty} \tau_n \overline{\delta}_n/\sqrt{n} = \infty$. As a result, it is immediate that $\lim_{n\to\infty} \tau_n \delta_n/\sqrt{n} = \infty$. On the other hand, inequality (A9) guarantees that

$$\limsup_{n\to\infty} R_n(\tau_n) \le 2\gamma \limsup_{n\to\infty} \frac{\sqrt{n}}{\tau_n \delta_n} = 0,$$

proving that the first sequence exhibits (weakly) more macroeconomic tail risks than the latter.

To prove that the inequality holds strictly, set $\tau_n^* = \sqrt{n}/\overline{\delta}_n$. Inequalities (A4) and (A8) immediately imply that

$$\limsup_{n\to\infty} \overline{R}_n(\tau_n^*) \ge \limsup_{n\to\infty} \frac{1}{4h^2 + h\tau_n^*\overline{\delta}_n/\sqrt{n}} = \frac{1}{4h^2 + h};$$

$$\limsup_{n\to\infty} R_n(\tau_n^*) \le 2\gamma \limsup_{n\to\infty} \overline{\delta}_n/\delta_n = 0,$$

thus, guaranteeing that the first sequence of economies exhibits strictly more macroeconomic tail risks in the sense of Definition 5.

To prove the converse implications, suppose that the first sequence of economies exhibits strictly more macroeconomic tail risks. By Definition 5, there exists a sequence $\tau_n^* \to \infty$ such that $\lim_{n\to\infty} R_n(\tau_n^*) = 0$, while at the same time $\lim_{n\to\infty} \overline{R}_n(\tau_n^*) > 0$. These assertions alongside inequalities (A4) and (A8) respectively imply that

$$2\gamma \lim_{n\to\infty} \frac{\sqrt{n}}{\tau_n^* \overline{\delta}_n} \ge \lim_{n\to\infty} \overline{R}_n(\tau_n^*) > 0;$$

$$\lim_{n\to\infty} \frac{1}{4h^2 + h\tau_n^*\delta_n/\sqrt{n}} \le \lim_{n\to\infty} R_n(\tau_n^*) = 0.$$

As a result, $\lim_{n\to\infty} \tau_n^* \bar{\delta}_n / \sqrt{n} < \infty$, even though $\lim_{n\to\infty} \tau_n^* \delta_n / \sqrt{n} = \infty$. These two equations guarantee that $\lim_{n\to\infty} \delta_n / \bar{\delta}_n = \infty$. ∎

J. *Proof of Proposition 8*

We start by stating and proving three simple lemmas.

LEMMA A.3: *The log output of sector i is given by $\hat{x}_i = \log x_i = \log v_i + \sum_{j=1}^n \ell_{ij} \epsilon_j$.*

PROOF:
Recall from Proposition 1 that the equilibrium sales of sector i satisfy $p_i x_i = v_i w$, where w is the market wage and v_i is sector i's Domar weight. On the other hand, by equation (A2), the equilibrium price of good i satisfies $\log p_i = \log w - \sum_{j=1}^n \ell_{ij} \epsilon_j$, thus implying that $\log x_i = \log v_i + \sum_{j=1}^n \ell_{ij} \epsilon_j$. ∎

LEMMA A.4: *Let z_i and q_{ij} be nonnegative numbers for all i and j. Then,*

$$\sqrt{\sum_{j=1}^n \left(\sum_{i=1}^n z_i q_{ij} \right)^2} \le \sum_{i=1}^n z_i \left(\sum_{j=1}^n q_{ij}^2 \right)^{1/2}.$$

PROOF:
A simple application of the Cauchy-Schwarz inequality guarantees that

$$\sum_{j=1}^n q_{ij} q_{kj} \le \left(\sum_{j=1}^n q_{ij}^2 \right)^{1/2} \left(\sum_{j=1}^n q_{kj}^2 \right)^{1/2}$$

for all i and k. Multiplying both sides of the inequality above by $z_i z_k$ and summing over all i and k implies

$$\sum_{j=1}^n \left(\sum_{i=1}^n z_i q_{ij} \right) \left(\sum_{k=1}^n z_k q_{kj} \right) \le \left(\sum_{i=1}^n z_i \left(\sum_{j=1}^n q_{ij}^2 \right)^{1/2} \right) \left(\sum_{k=1}^n z_k \left(\sum_{j=1}^n q_{kj}^2 \right)^{1/2} \right).$$

Taking square roots from both sides of the inequality above proves the result. ∎

LEMMA A.5: *In any given economy,*

$$C(\tau) = \frac{\Pr(\hat{x}_i < E\hat{x}_i - \tau \hat{\sigma}_i \text{ for all } i)}{\Pr(y < -\tau \sigma)},$$

where $C(\tau)$ is the measure of tail comovement defined in (9), $\hat{\sigma}_i = \text{stdev}(\hat{x}_i)$ is output volatility of sector i, and $\sigma = \text{stdev}(y)$ is the economy's aggregate volatility.

PROOF:

From the definition of conditional probability, it is immediate that the statement of the lemma follows once we show that whenever $\hat{x}_i < E\hat{x}_i - \tau\hat{\sigma}_i$ for all sectors i, then $y < -\tau\sigma$.

To this end, suppose that $\hat{x}_i < E\hat{x}_i - \tau\hat{\sigma}_i$ for all i. Recall from Lemma A.3 that $\hat{x}_i = \log v_i + \sum_{j=1}^n \ell_{ij}\epsilon_j$, which implies that $\hat{\sigma}_i = \sqrt{\sum_{j=1}^n \ell_{ij}^2}$. Therefore,

$$\sum_{j=1}^n \ell_{ij}\epsilon_j < -\tau\left(\sum_{j=1}^n \ell_{ij}^2\right)^{1/2}.$$

Multiplying both sides of the inequality above by β_i and summing over all sectors implies

$$y < -\tau\sum_{i=1}^n \beta_i\left(\sum_{j=1}^n \ell_{ij}^2\right)^{1/2},$$

where we are using the fact that $y = \sum_{i,j}\beta_i\ell_{ij}\epsilon_j$, established in Proposition 1. Therefore, by Lemma A.4,

$$y < -\tau\sqrt{\sum_{j=1}^n\left(\sum_{i=1}^n \beta_i\ell_{ij}\right)^2}.$$

Finally, the fact that $v_j = \sum_{i=1}^n \beta_i\ell_{ij}$ implies that the right-hand side of the inequality above is simply equal to $\|v\|$, which is the volatility of aggregate output, $\sigma = \text{stdev}(y)$. ∎

PROOF OF PROPOSITION 8:

Consider two economies with identical Domar weights (i.e., $v_i = v_i'$ for all i) and denote their corresponding measures of tail comovement with C and C', respectively. Furthermore, assume that the latter economy exhibits more sectoral interconnectivity relative to the former in the sense of Definition 6. In particular, there exists a stochastic matrix $\mathbf{B} = [b_{ij}]$ such that

$$\ell_{ij}' = \sum_{k=1}^n b_{ik}\ell_{kj}$$

for all pairs of sectors i and j, where $\mathbf{L} = [\ell_{ij}]$ and $\mathbf{L}' = [\ell_{ij}']$ are the two economies' corresponding Leontief inverse matrices.

To compare the extent of tail comovements in the two economies, recall from Lemma A.5 that

$$C(\tau) = \frac{\Pr\left(\hat{x}_i < E\hat{x}_i - \tau\hat{\sigma}_i \text{ for all } i\right)}{\Pr(y < -\tau\sigma)},$$

where $\hat{\sigma}_i = \text{stdev}(\hat{x}_i)$ is the output volatility of sector i. On the other hand, by Lemma A.3, the log output of sector i is given by $\hat{x}_i = \log v_i + \sum_{j=1}^n \ell_{ij} \epsilon_j$. As a result,

$$\Pr(\hat{x}_i < E\hat{x}_i - \tau\hat{\sigma}_i \text{ for all } i) = \Pr\left(\sum_{j=1}^n \ell_{ij} \epsilon_j < -\tau\hat{\sigma}_i \text{ for all } i\right).$$

Consider the event that

$$\sum_{j=1}^n \ell_{ij} \epsilon_j < -\tau\hat{\sigma}_i \quad \text{for all } i,$$

and pick some arbitrary sector k. Multiplying both sides of the inequality above by b_{ki} and summing over all sectors i implies

$$\sum_{j=1}^n \ell'_{kj} \epsilon_j < -\tau \sum_{i=1}^n b_{ki} \left(\sum_{j=1}^n \ell_{ij}^2\right)^{1/2},$$

where we are using the fact that $\hat{\sigma}_i = (\sum_j \ell_{ij}^2)^{1/2}$. Hence, by Lemma A.4,

$$\sum_{j=1}^n \ell'_{kj} \epsilon_j < -\tau\sqrt{\sum_{j=1}^n \left(\sum_{i=1}^n b_{ki}\ell_{ij}\right)^2} = -\tau\left(\sum_{j=1}^n \ell_{kj}'^2\right)^{1/2}.$$

Since the right-hand side of the inequality above is simply equal to $-\tau\hat{\sigma}'_k$, we have

$$\Pr(\hat{x}_i < E\hat{x}_i - \tau\hat{\sigma}_i \text{ for all } i) \leq \Pr(\hat{x}'_i < E\hat{x}'_i - \tau\hat{\sigma}'_i \text{ for all } i),$$

and hence,

$$C(\tau) \leq \frac{\Pr(\hat{x}'_i < E\hat{x}'_i - \tau\hat{\sigma}'_i \text{ for all } i)}{\Pr(y < -\tau\sigma)}.$$

Finally, since the two economies have identical Domar weights, it is immediate that the distribution of aggregate output is also identical in the two economies, and as a result, $\Pr(y < -\tau\sigma) = \Pr(y' < -\tau\sigma')$. Using Lemma A.5 one more time then implies that $C(\tau) \leq C'(\tau)$. ∎

K. *Proof of Corollary 3*

This result is an immediate consequence of Proposition 8. That every economy in the first sequence exhibits more sectoral interconnectivity relative to the corresponding economy in the second sequence implies that $C_n(\tau_n) \geq C'_n(\tau_n)$. As a result, it is immediate that $\liminf_{n\to\infty} C_n(\tau_n)/C'_n(\tau_n) \geq 1$ for all sequences $\{\tau_n\}$. ∎

L. *Proof of Proposition 9*

Inequality (A4) implies that, when microeconomic shocks have Pareto tails with Pareto index λ, we have

$$\limsup_{n\to\infty} R_n(c\sqrt{n}) \le 2\lambda \limsup_{n\to\infty} \frac{1}{c^2 n} \log(cn/\delta_n).$$

Given that $\delta_n \ge 1$,

$$\limsup_{n\to\infty} R_n(c\sqrt{n}) \le 2\lambda \limsup_{n\to\infty} \frac{1}{c^2 n} \log(cn).$$

It is then immediate that $\lim_{n\to\infty} R_n(c\sqrt{n}) = 0$ for all $c > 0$. ∎

M. *Proof of Proposition 10*

We first state a theorem, the proof of which can be found in Nagaev (1979, Theorem 1.9), followed by a simple lemma.

THEOREM A.2: *Let $\{\epsilon_1, \epsilon_2, \ldots\}$ be a sequence of zero-mean, unit variance i.i.d. Pareto-tailed random variables with Pareto index $\lambda > 2$. Furthermore, suppose that $E|\epsilon_i|^{2+\xi} < \infty$ for some $\xi > 0$. Then,*

$$\Pr(S_n \ge z_n) = \left(1 - \Phi(z_n/\sqrt{n})\right)(1 + o(1)) + n(1 - F(z_n))(1 + o(1)),$$

as $n \to \infty$ for $z_n \ge \sqrt{n}$, where $S_n = \epsilon_1 + \cdots + \epsilon_n$.

LEMMA A.6: *For a sequence of simple economies with Pareto-tailed microeconomic shocks,*

(*i*) *if $\lim_{n\to\infty} \tau_n/\sqrt{\log n} < \infty$, then $\lim_{n\to\infty} R_n(\tau_n) > 0$;*

(*ii*) *if $\lim_{n\to\infty} \tau_n/\sqrt{\log n} = \infty$, then $\lim_{n\to\infty} R_n(\tau_n) = 0$.*

PROOF:

By Proposition 1, all sectoral Domar weights in a simple economy (that is, an economy with symmetric preferences and no input-output linkages) are identical and are given by $v_i = 1/n$. Consequently, $y_n = (1/n)\sum_{i=1}^{n} \epsilon_i$ and $\sigma_n = 1/\sqrt{n}$, which means that $\Pr(y_n < -\tau_n\sigma_n) = \Pr(S_n > \tau_n\sqrt{n})$. Therefore, for any sequence $\{\tau_n\}$ such that $\tau_n \ge 1$ and $\lim_{n\to\infty} \tau_n = \infty$, Theorem A.2 implies that

(A10) $\Pr(y_n < -\tau_n\sigma_n) = \Phi(-\tau_n)(1 + o(1)) + nF(-\tau_n\sqrt{n})(1 + o(1)).$

Now, to prove part (i), suppose that $\lim_{n\to\infty} \tau_n/\sqrt{\log n} < \infty$. Under this assumption,

$$\lim_{n\to\infty} \frac{\log \Phi(-\tau_n)}{\log[nF(-\tau_n\sqrt{n})]} = \lim_{n\to\infty} \frac{\tau_n^2/2}{(\lambda/2 - 1)\log n + \lambda \log \tau_n} < \infty,$$

implying that the second term on the right-hand side of (A10) never dominates the first term as $n \to \infty$. As a result,

$$\lim_{n\to\infty} \frac{\Pr(y_n < -\tau_n\sigma_n)}{\Phi(-\tau_n)} < \infty,$$

implying that $\lim_{n\to\infty} R_n(\tau_n) > 0$.

Next, to prove part (ii), suppose that $\lim_{n\to\infty} \tau_n/\sqrt{\log n} = \infty$. For such a sequence, we have

$$\lim_{n\to\infty} \frac{\log \Phi(-\tau_n)}{\log[nF(-\tau_n\sqrt{n})]} = \lim_{n\to\infty} \frac{\tau_n^2/2}{(\lambda/2 - 1)\log n + \lambda \log \tau_n} = \infty,$$

implying that the second term on the right-hand side of (A10) dominates the first term as $n \to \infty$, that is,

$$\Pr(y_n < -\tau_n\sigma_n) = nF(-\tau_n\sqrt{n})(1 + o(1)).$$

Consequently,

$$\lim_{n\to\infty} R_n(\tau_n) = \lim_{n\to\infty} \frac{\log[nF(-\tau_n\sqrt{n})]}{\log \Phi(-\tau_n)} = 0. \ \blacksquare$$

PROOF OF PROPOSITION 10:

Consider the following two sequences of economies: (i) a sequence of simple economies that are subject to Pareto-tailed microeconomic shocks, with τ-tail ratios denoted by $\overline{R}_n$; and (ii) a sequence of economies subject to exponential-tailed shocks, with τ-tail ratios denoted by R_n and measures of sectoral dominance satisfying $\lim_{n\to\infty} \delta_n\sqrt{(\log n)/n} = \infty$.

Pick a sequence $\{\tau_n\}$ such that $\lim_{n\to\infty} \overline{R}_n(\tau_n) = 0$. Part (i) of Lemma A.6 implies that $\lim_{n\to\infty} \tau_n/\sqrt{\log n} = \infty$. On the other hand, recall from inequality (A4) that

$$\limsup_{n\to\infty} R_n(\tau_n) \leq 2\gamma \limsup_{n\to\infty} \frac{\sqrt{n}}{\tau_n\delta_n}.$$

As a result, it is immediate that $\lim_{n\to\infty} R_n(\tau_n) = 0$, satisfying requirement (i) of Definition 5.

To establish that requirement (ii) of Definition 5 is also satisfied, let $\tau_n^* = \sqrt{\log n}$. From part (i) of Lemma A.6, we have that $\lim_{n\to\infty} \overline{R}_n(\tau_n^*) > 0$. On the other hand, inequality (A9) implies that $\limsup_{n\to\infty} R_n(\tau_n^*) = 0$, thus establishing that the economy subject to exponential-tailed shocks exhibits strictly more macroeconomic tail risks than the one subject to shocks with Pareto tails. $\blacksquare$

N. *Proof of Corollary 4*

As in the proof of Proposition 10, consider the following two sequences of economies: (i) a sequence of simple economies that are subject to Pareto-tailed microeconomic shocks, with τ-tail ratios denoted by $\bar{R}_n$; and (ii) a sequence of economies with Pareto Domar weights of common exponent $\eta = 2$ and subject to exponential-tailed shocks, with τ-tail ratios denoted by R_n. To show that the two economies exhibit identical levels of macroeconomic tail risks, it is sufficient to show that $\lim_{n\to\infty} R_n(\tau_n) = 0$ if and only if $\lim_{n\to\infty} \bar{R}_n(\tau_n) = 0$ for any given sequence $\{\tau_n\}$ such that $\tau_n \to \infty$.

First, suppose $\{\tau_n\}$ is such that $\lim_{n\to\infty} \bar{R}_n(\tau_n) = 0$. From part (i) of Lemma A.6 it is immediate that $\lim_{n\to\infty} \tau_n / \sqrt{\log n} = \infty$. On the other hand, it is easy to verify that for a sequence of economies with Pareto Domar weights of exponent $\eta = 2$, we have

$$(A11) \qquad 0 < \liminf_{n\to\infty} \delta_n \sqrt{(\log n)/n} \leq \limsup_{n\to\infty} \delta_n \sqrt{(\log n)/n} < \infty.$$

As a result, inequality (A9) implies that $\lim_{n\to\infty} R_n(\tau_n) = 0$.

To prove the converse, suppose that $\{\tau_n\}$ is such that $\lim_{n\to\infty} R_n(\tau_n) = 0$. From (A8), we have

$$\lim_{n\to\infty} \frac{1}{4h^2 + h\tau_n \delta_n/\sqrt{n}} \leq \lim_{n\to\infty} R_n(\tau_n) = 0.$$

Therefore, $\lim_{n\to\infty} \tau_n \delta_n/\sqrt{n} = \infty$. Combining this with (A11) thus implies that $\lim_{n\to\infty} \tau_n / \sqrt{\log n} = \infty$. As a result, part (ii) of Lemma A.6 implies that $\lim_{n\to\infty} \bar{R}_n(\tau_n) = 0$, completing the proof. ∎

O. *Proof of Proposition 11*

The representative household maximizes her expected discounted utility (13) subject to her intertemporal budget constraint (14). The first-order conditions of the household's problem thus imply that

$$p_{it} c_{it} = \beta_i \rho^t / \lambda_t;$$

$$\lambda_t \pi_{it} = E_t[\lambda_{t+1} p_{it+1}],$$

where λ_t denotes the Lagrange multiplier corresponding to the household's time t budget constraint. On the other hand, the fact that value added at time t is given by $\text{GDP}_t = \sum_{i=1}^{n} p_{it} c_{it}$ implies that

$$(A12) \qquad c_{it} = \frac{\beta_i}{p_{it}} \text{GDP}_t,$$

with the gross interest rate of the i-th bond between periods t and $t+1$ satisfying

$$(A13) \qquad \frac{1}{R_{it}} = \rho E_t\left[\frac{p_{it+1}}{\text{GDP}_{t+1}}\right] \text{GDP}_t.$$

On the supply side, the representative firm in sector i maximizes

$$\Pi_{it} \;=\; \frac{1}{R_{it}}\, x_{it+1} - w_t l_{it} - \sum_{j=1}^{n} p_{jt} x_{ijt}$$

subject to its production technology (11), where note that firm i's revenue has to be discounted by R_{it} as output is produced with one-period delay. The first-order conditions of the representative firm in sector i alongside (A13) imply that

$$(A14) \qquad\qquad x_{ijt} = \rho \mu a_{ij} \left(\frac{\mathrm{GDP}_t}{p_{jt}} \right) E_t \left[\frac{p_{it+1} x_{it+1}}{\mathrm{GDP}_{t+1}} \right];$$

$$(A15) \qquad\qquad l_{it} = \rho (1-\mu) \left(\frac{\mathrm{GDP}_t}{w_t} \right) E_t \left[\frac{p_{it+1} x_{it+1}}{\mathrm{GDP}_{t+1}} \right],$$

where note that the assumption that Ξ_{it} is known at time t implies that x_{it+1} is also known at time t.

Since the market-clearing condition for good i at time t is given by $x_{it} = c_{it} + \sum_{j=1}^{n} x_{jit}$, equations (A12) and (A14) imply that

$$(A16) \qquad\qquad \frac{p_{it} x_{it}}{\mathrm{GDP}_t} = \beta_i + \rho \mu \sum_{j=1}^{n} a_{ji} E_t \left[\frac{p_{jt+1} x_{jt+1}}{\mathrm{GDP}_{t+1}} \right].$$

On the other hand, the transversality condition corresponding to firm i's problem requires that

$$\lim_{t \to \infty} \rho^t E_0 \left[\frac{p_{it} x_{it}}{\mathrm{GDP}_t} \right] = 0.$$

As a result, equation (A16) has a unique solution with the Domar weight of sector i at time t given by

$$v_{it} = \frac{p_{it} x_{it}}{\mathrm{GDP}_t} = \sum_{j=1}^{n} \beta_j \tilde{\ell}_{ji},$$

where $\tilde{\mathbf{L}} = (\mathbf{I} - \rho \mu \mathbf{A})^{-1}$ is the Leontief inverse matrix of the dynamic economy, thus proving (15).

To characterize firms' equilibrium output, note that equations (A14) and (A15) imply that sector i's input and labor demands are given by

$$x_{ijt} \;=\; \rho \mu \, a_{ij} (v_i / v_j) \, x_{jt};$$

$$l_{it} \;=\; (1 - \rho \mu) \, v_i,$$

respectively, where we are using the fact that sectoral Domar weights remain unchanged over time. Plugging the expressions above into sector i's production function, we obtain

$$\log(x_{it+1}) = \log(\Xi_{it}) + \mu \sum_{j=1}^{n} a_{ij} \log(x_{jt}) + \log(v_i) - \mu \sum_{j=1}^{n} a_{ij} \log(v_j).$$

As a result, the vector of log sectoral outputs at time $t + 1$ is given by

$$(A17) \quad \log(x_{t+1}) = \sum_{\tau=0}^{t} (\mu\mathbf{A})^{\tau} \log(\Xi_{t-\tau}) + (\mu\mathbf{A})^{t+1} \log x_0 + \left[\mathbf{I} - (\mu\mathbf{A})^{t+1}\right] \log v.$$

On the other hand, the fact that $v_i = p_{it}x_{it}/\text{GDP}_t$ implies that

$$\log(\text{GDP}_t) = \log p_{it} + \log x_{it} - \log v_i.$$

Multiplying both sides of the equation above by β_i and normalizing the price index $\prod_{i=1}^{n} p_{it}^{\beta_i} = 1$ leads to

$$\log(\text{GDP}_t) = \sum_{i=1}^{n} \beta_i \log x_{it} - \sum_{i=1}^{n} \beta_i \log v_i.$$

Combining the above with $(A17)$ thus implies that for $t \geq 0$,

$$\log(\text{GDP}_{t+1}) = \beta' \sum_{\tau=0}^{t} (\mu\mathbf{A})^{\tau} \log(\Xi_{t-\tau}) + \beta'(\mu\mathbf{A})^{t+1}(\log x_0 - \log v).$$

As a result, equation (12) implies that for $t \geq 1$,

$$\log(\text{GDP}_{t+1}) - \varphi \log(\text{GDP}_t) = \beta' \sum_{\tau=0}^{t} (\mu\mathbf{A})^{t-\tau} \tilde{\epsilon}_{\tau}$$

$$+ \beta'(\mu\mathbf{A})^{t}(\mu\mathbf{A} - \varphi\mathbf{I})(\log x_0 - \log v),$$

with the convention that $\tilde{\epsilon}_{i0} = \log(\Xi_{i0})$. Multiplying both sides of the equation above by ρ^t and summing over t results in

$$(1 - \rho\varphi) \sum_{t=0}^{\infty} \rho^t \log(\text{GDP}_{t+1}) = \beta'(\mathbf{I} - \rho\mu\mathbf{A})^{-1} \sum_{t=0}^{\infty} \rho^t \tilde{\epsilon}_t$$

$$+ (1 - \rho\varphi)\beta'(\mathbf{I} - \rho\mu\mathbf{A})^{-1}(\mu\mathbf{A})(\log x_0 - \log v).$$

The juxtaposition of the equation above with the fact that $v' = \beta' \tilde{\mathbf{L}}$ completes the proof. ∎

Appendix B: Aggregation and Disaggregation

In this part of the Appendix, we illustrate how representations corresponding to two different levels of disaggregation of the same economy are related to one another.

Suppose we observe an economy at two levels of disaggregations n and $N < n$. This means that each sector/industry in the latter representation is a disjoint collection of subindustries in the former. We use upper- and lowercase letters to denote the aggregated industries and their comprising subindustries, respectively. In particular, we write $i \in I$ to indicate that disaggregated sector i is one of the subindustries that is assigned to the more aggregated sector indexed I. These two representations correspond to different levels of disaggregation of the same economy only if a certain set of intuitive consistency restrictions are satisfied.

A first set of restrictions require that (i) the dollar value of intersectoral trade between any two aggregated sectors is equal to the dollar value of trade between the subindustries that comprise them; (ii) households' expenditure on the output of a given aggregated sector is equal to their expenditure on disaggregated goods that are assigned to that sector; and (iii) each aggregated industry's expenditure on labor is equal to the total wage bill of its comprising subindustries. In other words,

$$(\text{B1}) \qquad\qquad p_J x_{IJ} = \sum_{i \in I} \sum_{j \in J} p_j x_{ij};$$

$$(\text{B2}) \qquad\qquad p_I c_I = \sum_{i \in I} p_i c_i;$$

$$(\text{B3}) \qquad\qquad w l_I = \sum_{i \in I} w l_i$$

for all aggregated sectors I and J. Taken together, these requirements—which we refer to as *flow conditions*—guarantee that the value of trade between different sectors and between households and sectors is independent of the level of disaggregation.[29]

The second restriction requires that any aggregated sector's contribution to value added to be equal to sum of the contributions of that sector's subindustries, that is,

$$\text{value added}_I = \sum_{i \in I} \text{value added}_i$$

for all I. We refer to this requirement as the *value added condition*. Similar to the flow conditions, this restriction simply requires that the value added measurement to be independent of economy's level of aggregation.

---

[29] One may also want to impose the restriction that the dollar value of output of any aggregated industry is equal to the total dollar value of output of all its subindustries, that is, $p_I x_I = \sum_{i \in I} p_i x_i$. Market clearing, alongside conditions (B1) and (B2), guarantees that this restriction is automatically satisfied.

The final restriction, which we refer to as *orthogonality condition*, requires the measured productivity of any aggregated industry to be orthogonal to productivities of subindustries that lie outside of it, that is, for any given I,

$$\epsilon_I \perp \epsilon_j \quad \forall j \notin I.$$

Thus, the orthogonality condition ensures that changing the productivity of some subindustry j only impacts the measured TFP of the aggregated industry to which it belongs.

DEFINITION B.1: *The N-sector economy is an admissible aggregate representation of the n-sector economy if the flow, value added, and orthogonality conditions are satisfied.*

PROPOSITION B.1: *Consider two economies consisting of n and $N < n$ sectors, with each sector in the latter representation corresponding to a disjoint collection of sectors in the former. The N-sector economy is an admissible aggregate representation of the n-sector economy if and only if*

(B4) $$v_I = \sum_{i \in I} v_i;$$

(B5) $$a_{IJ} = \frac{1}{v_I} \sum_{i \in I} \sum_{j \in J} v_i a_{ij};$$

(B6) $$\epsilon_I = \frac{1}{v_I} \sum_{i \in I} v_i \epsilon_i;$$

(B7) $$\beta_I = \sum_{i \in I} \beta_i$$

for all pairs of aggregated industries I and J.

This result illustrates that the consistency requirements imposed by flow, value added, and orthogonality conditions uniquely identify the structural parameters of the aggregated representation of the economy in terms of their disaggregated counterparts. Furthermore, it provides necessary and sufficient conditions under which two economies consisting of different numbers of sectors correspond to two levels of disaggregation of the same economy. In particular, equations (B5) and (B4) illustrate how input-output linkages and Domar weights of the more aggregated representation relate to those of the more disaggregated one.

A. *Proof of Proposition B.1*

It is straightforward to verify that conditions (B4)–(B7) jointly imply that the flow, value added, and orthogonality conditions are satisfied. In what follows, we prove the converse implication.

Recall that the market-clearing condition for good i is given by $x_i = c_i + \sum_{j=1}^{n} x_{ji}$. Multiplying both sides by p_i and summing over all $i \in I$ implies that

$$\sum_{i \in I} p_i x_i = p_I c_I + \sum_{J=1}^{N} p_I x_{JI},$$

where we are using (B1) and (B2) to obtain the right-hand side. On the other hand, the market-clearing condition for aggregated good I implies that

$$p_I x_I = p_I c_I + \sum_{J=1}^{n} p_I x_{JI}.$$

Combining the two equations above, we obtain

$$p_I x_I = \sum_{i \in I} p_i x_i$$

for all aggregated industries I. Note that the value added condition requires that the value added of the entire economy has to be independent of its representation. Therefore, dividing both sides of the equation by GDP and using the fact that each sector's Domar weight is defined as its sales over GDP establishes (B4).

To establish (B5), recall from (A1) that

$$p_j x_{ij} = \mu a_{ij} p_i x_i;$$

$$p_J x_{IJ} = \mu a_{IJ} p_I x_I.$$

As a result, (B1) implies that

$$a_{IJ} p_I x_I = \sum_{i \in I} \sum_{j \in J} a_{ij} p_j x_i.$$

Dividing both sides of the equation above by GDP and using the definition of sectoral Domar weights establishes (B5).

To establish (B6), once again note that the value added of the economy has to be independent of its representation. Furthermore, recall from Proposition 1 that the economy's log value added is equal to the linear combination of microeconomic shocks, with the coefficients given by the sectoral Domar weights. As a result,

$$\sum_{I=1}^{N} v_I \epsilon_I = \sum_{i=1}^{n} v_i \epsilon_i.$$

The orthogonality condition guarantees that (B6) is satisfied.

Finally, to establish (B7), recall from flow condition (B2) that $p_I c_I = \sum_{i \in I} p_i c_i$. On the other hand, the representative household's first-order conditions in the two representations imply that $c_i = \beta_i w / p_i$ and $c_I = \beta_I w / p_I$, which guarantees that $\beta_I = \sum_{i \in I} \beta_i.$ ∎

REFERENCES

Acemoglu, Daron, Ufuk Akcigit, and William Kerr. 2016. "Networks and the Macroeconomy: An Empirical Exploration." In *National Bureau of Economic Research Macroeconomics Annual*, Vol. 30, edited by Martin Eichenbaum and Jonathan Parker, 276–335. Chicago: University of Chicago Press.

Acemoglu, Daron, Vasco M. Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. 2012. "The Network Origins of Aggregate Fluctuations." *Econometrica* 80 (5): 1977–2016.

Acemoglu, Daron, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. 2010. "Cascades in Networks and Aggregate Volatility." National Bureau of Economic Research Working Paper 16516.

Acemoglu, Daron, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. 2015. "Systemic Risk and Stability in Financial Networks." *American Economic Review* 105 (2): 564–608.

Acemoglu, Daron, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. 2016. "Networks, Shocks, and Systemic Risk." In *The Oxford Handbook of the Economics of Networks*, edited by Yann Bramoullé, Andrea Galeotti, and Brian Rogers, 569–607. New York: Oxford University Press.

Acemoglu, Daron, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. 2017. "Microeconomic Origins of Macroeconomic Tail Risks: Dataset." *American Economic Review.* https://doi.org/10.1257/aer.20151086.

Ascari, Guido, Giorgio Fagiolo, and Andrea Roventini. 2015. "Fat-Tail Distributions and Business-Cycle Models." *Macroeconomic Dynamics* 19 (2): 465–76.

Atalay, Enghin. 2015. "How Important are Sectoral Shocks?" US Census Bureau Center for Economic Studies Paper CES-WP-14–31.

Atalay, Enghin, and Thorsten Drautzberg. 2015. "Accounting for the Sources of Macroeconomic Tail Risks." http://laef.ucsb.edu/pages/conferences/mbcc15/papers/drautzburg.pdf.

Barro, Robert J. 2006. "Rare Disasters and Asset Markets in the Twentieth Century." *Quarterly Journal of Economics* 121 (3): 823–66.

Bernanke, Ben, Mark Gertler, and Simon Gilchrist. 1996. "The Financial Accelerator and the Flight to Quality." *Review of Economics and Statistics* 78 (1): 1–15.

Brunnermeier, Markus K., Thomas M. Eisenbach, and Yuliy Sannikov. 2013. "Macroeconomics with Financial Frictions: A Survey." In *Advances in Economics and Econometrics: Tenth World Congress, Volume II: Applied Economics*, edited by Daron Acemoglu, Manuel Arellano, and Eddie Dekel, 3–96. New York: Cambridge University Press.

Burlon, Lorenzo. 2012. "How Do Aggregate Fluctuations Depend on the Network Structure of the Economy?" University of Barcelona Working Paper 278.

Canning, David, L. A. N. Amaral, Y. Lee, and H. E. Stanley. 1998. "Scaling the Volatility of GDP Growth Rates." *Economics Letters* 60 (3): 335–41.

Carvalho, Vasco M. 2010. "Aggregate Fluctuations and the Network Structure of Intersectoral Trade." Universitat Pompeau Fabra Working Paper 1206.

Carvalho, Vasco M. 2014. "From Micro to Macro via Production Networks." *Journal of Economic Perspectives* 28 (4): 23–48.

Carvalho, Vasco, and Xavier Gabaix. 2013. "The Great Diversification and Its Undoing." *American Economic Review* 103 (5): 1697–727.

Carvalho, Vasco M., Makoto Nirei, Yukiko Saito, and Alireza Tahbaz-Salehi. 2015. "Supply Chain Disruptions: Evidence from the Great East Japan Earthquake." http://www.columbia.edu/~at2761/JapanEQ.pdf (accessed November 7, 2016).

Christiano, Lawrence J. 2007. "On the Fit of New Keynesian Models: Comment." *Journal of Business and Economic Statistics* 25 (2): 143–51.

Christiano, Lawrence, Martin Eichenbaum, and Sergio Rebelo. 2011. "When Is the Government Spending Multiplier Large?" *Journal of Political Economy* 119 (1): 78–121.

Christiano, Lawrence J., Martin S. Eichenbaum, and Mathias Trabandt. 2015. "Understanding the Great Recession." *American Economic Journal: Macroeconomics* 7 (1): 110–67.

Cogley, Timothy, and Thomas J. Sargent. 2005. "Drifts and Volatilities: Monetary Policies and Outcomes in the Post WWII US." *Review of Economic Dynamics* 8 (2): 262–302.

Cole, Harold L., and Lee E. Ohanian. 1999. "The Great Depression in the United States from a Neoclassical Perspective." *Federal Reserve Bank of Minneapolis Quarterly Review* 23 (1): 2–24.

Cole, Harold L., and Lee E. Ohanian. 2002. "The Great U.K. Depression: A Puzzle and Possible Resolution." *Review of Economic Dynamics* 5 (1): 19–44.

Cúrdia, Vasco, Marco Del Negro, and Daniel L. Greenwald. 2014. "Rare Shocks, Great Recessions." *Journal of Applied Econometrics* 29 (7): 1031–52.

di Giovanni, Julian, Andrei A. Levchenko, and Isabelle Mejean. 2014. "Firms, Destinations, and Aggregate Fluctuations." *Econometrica* 82 (4): 1303–40.

Domar, E. D. 1961. "On the Measurement of Technological Change." *Economic Journal* 71: 709–29.

Dupor, Bill. 1999. "Aggregation and Irrelevance in Multi-sector Models." *Journal of Monetary Economics* 43 (2): 391–409.

Durlauf, Steven N. 1993. "Nonergodic Economic Growth." *Review of Economic Studies* 60 (2): 349–66.

Eggertsson, Gauti B., and Paul Krugman. 2012. "Debt, Deleveraging, and the Liquidity Trap: A Fisher-Minsky-Koo Approach." *Quarterly Journal of Economics* 127 (3): 1469–513.

Eggertsson, Gauti B., and Neil R. Mehrotra. 2015. "A Model of Secular Stagnation." National Bureau of Economic Research Working Paper 20574.

Elliott, Matthew, Benjamin Golub, and Matthew O. Jackson. 2014. "Financial Networks and Contagion." *American Economic Review* 104 (10): 3115–53.

Engle, Robert F. 1982. "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation." *Econometrica* 50 (4): 987–1007.

Fagiolo, Giorgio, Mauro Napoletano, and Andrea Roventini. 2008. "Are Output Growth-Rate Distributions Fat-Tailed? Some Evidence from OECD Countries." *Journal of Applied Econometrics* 23 (5): 639–69.

Fama, E. F. 1963. "Mandelbrot and the Stable Paretian Hypothesis." *Journal of Business* 36 (0): 420–29.

Farhi, Emmanuel, and Xavier Gabaix. 2016. "Rare Disasters and Exchange Rates." *Quarterly Journal of Economics* 131 (1): 1–52.

Foerster, Andrew T., Pierre-Daniel G. Sarte, and Mark W. Watson. 2011. "Sectoral versus Aggregate Shocks: A Structural Factor Analysis of Industrial Production." *Journal of Political Economy* 119 (1): 1–38.

Foss, Sergey, Dmitry Korshunov, and Stan Zachary. 2011. *An Introduction to Heavy-Tailed and Subexponential Distributions.* New York: Springer.

Gabaix, Xavier. 2011. "The Granular Origins of Aggregate Fluctuations." *Econometrica* 79 (3): 733–72.

Gabaix, Xavier. 2012. "Variable Rare Disasters: An Exactly Solved Framework for Ten Puzzles in Macro-Finance." *Quarterly Journal of Economics* 127 (2): 645–700.

Gabaix, Xavier, Parameswaran Gopikrishnan, Vasiliki Plerou, and H. Eugene Stanley. 2003. "A Theory of Power-Law Distributions in Financial Market Fluctuations." *Nature* 423: 267–30.

Gabaix, Xavier, Parameswaran Gopikrishnan, Vasiliki Plerou, and H. Eugene Stanley. 2006. "Institutional Investors and Stock Market Volatility." *Quarterly Journal of Economics* 121 (2): 461–504.

Gabaix, Xavier, and Rustam Ibragimov. 2011. "Rank 1/2: A Simple Way to Improve the OLS Estimation of Tail Exponents." *Journal of Business and Economic Statistics* 29 (1): 24–39.

Gourio, Francois. 2012. "Disaster Risk and Business Cycles." *American Economic Review* 102 (6): 2734–66.

Grimmett, Geoffrey, and David Stirzaker. 2001. *Probability and Random Process.* 3rd edition. New York: Oxford University Press.

Horvath, Michael. 1998. "Cyclicality and Sectoral Linkages: Aggregate Fluctuations from Independent Sectoral Shocks." *Review of Economic Dynamics* 1 (4): 781–808.

Horvath, Michael. 2000. "Sectoral Shocks and Aggregate Fluctuations." *Journal of Monetary Economics* 45 (1): 69–106.

Hulten, Charles R. 1978. "Growth Accounting with Intermediate Inputs." *Review of Economic Studies* 45 (3): 511–18.

Ibragimov, Rustam, and Johan Walden. 2007. "The Limits of Diversification When Losses May Be Large." *Journal of Banking and Finance* 31 (8): 2551–69.

Jovanovic, Boyan. 1987. "Micro Shocks and Aggregate Risk." *Quarterly Journal of Economics* 102 (2): 395–409.

Kehoe, Timothy J., and Edward C. Prescott. 2002. "Great Depressions of the 20th Century." *Review of Economic Dynamics* 5 (1): 1–18.

Kelly, Bryan, and Hao Jiang. 2014. "Tail Risk and Asset Prices." *Review of Financial Studies* 27 (10): 2841–71.

Lee, Youngki, Luís A. Nunes Amaral, David Canning, Martin Meyer, and H. Eugene Stanley. 1998. "Universal Features in the Growth Dynamics of Complex Organizations." *Physical Review of Letters* 81 (15): 3275–78.

Long, John B., Jr., and Charles I. Plosser. 1983. "Real Business Cycles." *Journal of Political Economy* 91 (1): 39–69.

Lucas, Robert E., Jr. 1977. "Understanding Business Cycles." *Carnegie-Rochester Conference Series on Public Policy* 5 (0): 7–29.

Nadaraya, E. A. 1964. "On Estimating Regression." *Theory of Probability and Its Applications* 9 (1): 141–42.

Nagaev, Sergey V. 1979. "Large Deviations of Sums of Independent Random Variables." *Annals of Probability* 7 (5): 745–89.

Nakamura, Emi, Jón Steinsson, Robert Barro, and Jose Ursua. 2013. "Crises and Recoveries in an Empirical Model of Consumption Disasters." *American Economic Journal: Macroeconomics* 5 (3): 35–74.

Petrov, Valentin V. 1975. *Sums of Independent Random Variables*. New York: Springer Berlin Heidelberg.

Rietz, Thomas A. 1988. "The Equity Risk Premium: A Solution." *Journal of Monetary Economics* 22 (1): 117–31.

Stute, Winfried, Wenceslao Gonzáles Manteiga, and Manuel Presedo Quindimil. 1992. "Bootstrap Based Goodness-of-Fit Tests." *Metrika* 40 (1): 243–56.

Teicher, Henry. 1984. "Exponential Bounds for Large Deviations of Sums of Unbounded Random Variables." *Sankhyā: The Indian Journal of Statistics, Series A* 46 (1): 41–53.

Watson, Geoffrey S. 1964. "Smooth Regression Analysis." *Sankhyā: The Indian Journal of Statistics, Series A* 26 (4): 359–72.