Del in cylindrical and spherical coordinates

Table with the del operator in cylindrical and spherical coordinates

Operation	Cartesian coordinates (x, y, z)	Cylindrical coordinates (ρ, ϕ, z)	Spherical coordinates (r, θ, ϕ)
		$x = \rho \cos \phi$	$x = r \sin \theta \cos \phi$
Definition of coordinates		$y = \rho \sin \phi$	$y = r \sin \theta \sin \phi$
		z = z	$z = r \cos \theta$
		$\rho = \sqrt{x^2 + y^2}$	
		$\phi = \operatorname{atan2}(y, x)$	$\theta = \arccos(z/r)$
		$\lfloor z = $	
	$A_x\mathbf{\hat{x}} + A_y\mathbf{\hat{y}} + A_z\mathbf{\hat{z}}$	$A_{ ho}oldsymbol{\hat{ ho}} + A_{\phi}oldsymbol{\hat{\phi}} + A_{z}oldsymbol{\hat{z}}$	$A_r \hat{m{r}} + A_ heta \hat{m{ heta}} + A_\phi \hat{m{\phi}}$
∇f	$rac{\partial f}{\partial x}\hat{\mathbf{x}}+rac{\partial f}{\partial y}\hat{\mathbf{y}}+rac{\partial f}{\partial z}\hat{\mathbf{z}}$	$rac{\partial f}{\partial ho}\hat{oldsymbol{ ho}} + rac{1}{ ho}rac{\partial f}{\partial \phi}\hat{oldsymbol{\phi}} + rac{\partial f}{\partial z}\hat{oldsymbol{z}}$	$rac{\partial f}{\partial r}\hat{m{r}}+rac{1}{r}rac{\partial f}{\partial heta}\hat{m{ heta}}+rac{1}{r\sin heta}rac{\partial f}{\partial\phi}\hat{m{\phi}}$
$ abla \cdot \mathbf{A}$	$\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$	$rac{1}{ ho}rac{\partial(ho A_ ho)}{\partial ho}+rac{1}{ ho}rac{\partial A_\phi}{\partial\phi}+rac{\partial A_z}{\partial z}$	$\frac{1}{r^2} \frac{\partial (r^2 A_r)}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (A_{\theta} \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial A_{\phi}}{\partial \phi}$
	$(rac{\partial A_z}{\partial y} - rac{\partial A_y}{\partial z})\hat{\mathbf{x}} + (rac{\partial A_x}{\partial z} - rac{\partial A_z}{\partial x})\hat{\mathbf{y}} + (rac{\partial A_y}{\partial x} - rac{\partial A_x}{\partial y})\hat{\mathbf{z}}$	$(rac{1}{ ho}rac{\partial A_z}{\partial \phi} - rac{\partial A_\phi}{\partial z})\hat{m{ ho}}$ +	$rac{1}{r\sin heta}(rac{\partial}{\partial heta}(A_{\phi}\sin heta)-rac{\partial A_{ heta}}{\partial\phi})\hat{m{r}}$ +
$ abla imes \mathbf{A}$	$(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x})\hat{\mathbf{y}}$ +	$(rac{\partial A_{ ho}}{\partial z} - rac{\partial A_{z}}{\partial ho})\hat{m{\phi}}$ +	$rac{1}{r}(rac{1}{\sin heta}rac{\partial A_r}{\partial \phi}-rac{\partial}{\partial r}(rA_\phi))\hat{m{ heta}}$ +
	$(rac{\partial A_y}{\partial x} - rac{\partial A_x}{\partial y})\mathbf{\hat{z}}$	$(rac{1}{ ho}rac{\partial A_z}{\partial \phi}-rac{\partial A_{\phi}}{\partial z})\hat{m{ ho}} + \ (rac{\partial A_{ ho}}{\partial z}-rac{\partial A_z}{\partial ho})\hat{m{\phi}} + \ rac{1}{ ho}(rac{\partial (ho A_{\phi})}{\partial ho}-rac{\partial A_{ ho}}{\partial ho})\hat{m{z}}$	$\frac{1}{r}(\frac{\partial}{\partial r}(rA_{\theta}) - \frac{\partial A_{r}}{\partial \theta})\hat{\phi}$
$\Delta f = \nabla^2 f$	$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$	$\frac{1}{ ho} \frac{\partial}{\partial ho} (ho \frac{\partial f}{\partial ho}) + \frac{1}{ ho^2} \frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$	$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial f}{\partial r}) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\frac{\partial f}{\partial \theta}) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial \phi^2}$
		$(\Delta A_ ho - rac{A_ ho}{ ho^2} - rac{2}{ ho^2} rac{\partial A_\phi}{\partial \phi}) \hat{m{ ho}}$ +	$ (\Delta A_r - \frac{2A_r}{r^2} - \frac{2A_\theta \cos \theta}{r^2 \sin \theta} - \frac{2}{r^2} \frac{\partial A_\theta}{\partial \theta} - \frac{2}{r^2 \sin \theta} \frac{\partial A_\phi}{\partial \phi}) \hat{r} + $
$\Delta \mathbf{A} = abla^2 \mathbf{A}$	$\Delta A_x \hat{\mathbf{x}} + \Delta A_y \hat{\mathbf{y}} + \Delta A_z \hat{\mathbf{z}}$	$(\Delta A_{\phi} - \frac{A_{\phi}}{\rho^2} + \frac{2}{\rho^2} \frac{\partial A_{\rho}}{\partial \phi})\hat{\phi} +$	$(\Delta A_{ heta} - rac{A_{ heta}}{r^2 \sin^2 heta} + rac{2}{r^2} rac{\partial A_r}{\partial heta} - rac{2 \cos heta}{r^2 \sin^2 heta} rac{\partial A_{\phi}}{\partial \phi}) \hat{m{ heta}}$
		$(\Delta A_z)\hat{m{z}}$	$(\Delta A_{\phi} - \frac{A_{\phi}}{r^2 \sin^2 \theta} + \frac{2}{r^2 \sin^2 \theta} \frac{\partial A_r}{\partial \phi} + \frac{2 \cos \theta}{r^2 \sin^2 \theta} \frac{\partial A_{\theta}}{\partial \phi}) \hat{\phi}$
Differential displacement	$d\mathbf{l} = dx\hat{\mathbf{x}} + dy\hat{\mathbf{y}} + dz\hat{\mathbf{z}}$	$d\mathbf{l} = d ho\hat{oldsymbol{ ho}} + ho d\phi\hat{oldsymbol{\phi}} + dz\hat{oldsymbol{z}}$	$d\mathbf{l} = dr\hat{\mathbf{r}} + rd\theta\hat{\boldsymbol{\theta}} + r\sin\theta d\phi\hat{\boldsymbol{\phi}}$
	$d\mathbf{S} = dydz\hat{\mathbf{x}} +$	$d\mathbf{S} = ho d\phi dz \hat{m{ ho}} +$	$d\mathbf{S} = r^2 \sin\theta d\theta d\phi \hat{\mathbf{r}} +$
Differential normal area	$dxdz\mathbf{\hat{y}}+$	$d ho dz \hat{m{\phi}} +$	$r\sin heta dr d\phi oldsymbol{\hat{ heta}} +$
	$dxdy\mathbf{\hat{z}}$	$ ho d ho d\phi {f \hat{z}}$	$rdrd heta\hat{oldsymbol{\phi}}$
Differential volume	dv = dx dy dz	$dv = \rho d\rho d\phi dz$	$dv = r^2 \sin \theta dr d\theta d\phi$

Non-trivial calculation rules:

- 1. div grad $f = \nabla \cdot (\nabla f) = \nabla^2 f = \Delta f$ (Laplacian)
- 2. curl grad $f = \nabla \times (\nabla f) = 0$
- 3. div curl $\mathbf{A} = \nabla \cdot (\nabla \times \mathbf{A}) = 0$
- 4. curl curl $\mathbf{A} = \nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) \nabla^2 \mathbf{A}$
- 5. $\Delta fg = f\Delta g + 2\nabla f \cdot \nabla g + g\Delta f$
- 6. Lagrange's formula for the cross product:

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$$

- Note: This page uses standard physics notation; some (American mathematics) sources define θ , as the angle with the xy-plane instead of ϕ .
- Note: The function atan2(y, x) is used instead of the mathematical function arctan(y/x) due to its domain and image. The classical arctan(y/x) has an image of $(-\pi/2, +\pi/2)$, whereas atan2(y, x) is defined to have an image of $(-\pi, \pi]$.