## Del in cylindrical and spherical coordinates

Table with the del operator in cylindrical and spherical coordinates

Operaion	Cartesian coordinates $(x, y, z)$	Cylindrical coordinates $(\rho, \phi, z)$	Spherical coordinates $(r, \theta, \phi)$
		$x = \rho \cos \phi$	$x = r \sin \theta \cos \phi$
Definition of coordinates		$y = \rho \sin \phi$	$y = r \sin \theta \sin \phi$
		z = z	$z = r \cos \theta$
		$\rho = \sqrt{x^2 + y^2}$	
		$\phi = \operatorname{atan2}(y, x)$	$\theta = \arccos(z/r)$
		z = z	$\phi = \operatorname{atan2}(y, x)$
	$A_x \hat{\mathbf{x}} + A_y \hat{\mathbf{y}} + A_z \hat{\mathbf{z}}$	$A_{ ho}oldsymbol{\hat{ ho}} + A_{\phi}oldsymbol{\hat{\phi}} + A_{z}oldsymbol{\hat{z}}$	$A_r \hat{m{r}} + A_ heta \hat{m{ heta}} + A_\phi \hat{m{\phi}}$
$\nabla f$	$rac{\partial f}{\partial x}\hat{\mathbf{x}}+rac{\partial f}{\partial y}\hat{\mathbf{y}}+rac{\partial f}{\partial z}\hat{\mathbf{z}}$	$rac{\partial f}{\partial  ho} \hat{m{ ho}} + rac{1}{ ho} rac{\partial f}{\partial \phi} \hat{m{\phi}} + rac{\partial f}{\partial z} \hat{m{z}}$	$rac{\partial f}{\partial r}\hat{m{r}}+rac{1}{r}rac{\partial f}{\partial heta}\hat{m{ heta}}+rac{1}{r\sin heta}rac{\partial f}{\partial\phi}\hat{m{\phi}}$
$ abla \cdot \mathbf{A}$	$\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$	$\frac{1}{\rho} \frac{\partial (\rho A_{ ho})}{\partial  ho} + \frac{1}{\rho} \frac{\partial A_{\phi}}{\partial \phi} + \frac{\partial A_{z}}{\partial z}$	$\frac{1}{r^2}\frac{\partial (r^2A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$
	$egin{array}{ll} (rac{\partial A_z}{\partial y} - rac{\partial A_y}{\partial z})\hat{\mathbf{x}} & + \ (rac{\partial A_x}{\partial z} - rac{\partial A_z}{\partial x})\hat{\mathbf{y}} & + \ (rac{\partial A_y}{\partial x} - rac{\partial A_x}{\partial y})\hat{\mathbf{z}} & + \end{array}$	$(rac{1}{ ho}rac{\partial A_z}{\partial \phi} - rac{\partial A_\phi}{\partial z})\hat{m{ ho}}$ +	$rac{1}{r\sin heta}(rac{\partial}{\partial heta}(A_{\phi}\sin heta)-rac{\partial A_{ heta}}{\partial\phi})\hat{m{r}}$ +
$ abla  imes \mathbf{A}$	$(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x})\hat{\mathbf{y}}$ +	$(rac{\partial A_{ ho}}{\partial z} - rac{\partial A_z}{\partial  ho})\hat{m{\phi}}$ +	$rac{1}{r}(rac{1}{\sin heta}rac{\partial A_r}{\partial \phi}-rac{\partial}{\partial r}(rA_\phi))\hat{m{ heta}}$ +
		$(rac{\partial A_{ ho}}{\partial z} - rac{\partial Z}{\partial  ho})\hat{m{\phi}} + rac{1}{ ho}(rac{\partial (m{ ho}A_{\phi})}{\partial m{ ho}} - rac{\partial A_{ ho}}{\partial m{\phi}})\hat{m{z}}$	$\frac{1}{r}(\frac{\partial}{\partial r}(rA_{ heta}) - \frac{\partial A_r}{\partial  heta})\hat{m{\phi}}$
$\Delta f = \nabla^2 f$	$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$	$\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho\frac{\partial f}{\partial\rho}) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial\phi^2} + \frac{\partial^2 f}{\partial z^2}$	$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial \phi^2}$
		$(\Delta A_{ ho} - rac{A_{ ho}}{ ho^2} - rac{2}{ ho^2} rac{\partial A_{\phi}}{\partial \phi}) \hat{m{ ho}}$ +	$(\Delta A_r - \frac{2A_r}{r^2} - \frac{2A_\theta \cos \theta}{r^2 \sin \theta} - \frac{2}{r^2} \frac{\partial A_\theta}{\partial \theta} - \frac{2}{r^2 \sin \theta} \frac{\partial A_\phi}{\partial \phi})\hat{r} +$
$\Delta \mathbf{A} =  abla^2 \mathbf{A}$	$\Delta A_x \hat{\mathbf{x}} + \Delta A_y \hat{\mathbf{y}} + \Delta A_z \hat{\mathbf{z}}$	$(\Delta A_{\phi} - \frac{A_{\phi}}{\rho^2} + \frac{2}{\rho^2} \frac{\partial A_{\rho}}{\partial \phi})\hat{\phi}$ +	$(\Delta A_{\theta} - \frac{A_{\theta}}{r^2 \sin^2 \theta} + \frac{2}{r^2} \frac{\partial A_r}{\partial \theta} - \frac{2 \cos \theta}{r^2 \sin^2 \theta} \frac{\partial A_{\phi}}{\partial \phi}) \hat{\boldsymbol{\theta}}$ +
		$(\Delta A_z)\hat{m{z}}$	$(\Delta A_{\phi} - \frac{A_{\phi}}{r^2 \sin^2 \theta} + \frac{2}{r^2 \sin^2 \theta} \frac{\partial A_r}{\partial \phi} + \frac{2 \cos \theta}{r^2 \sin^2 \theta} \frac{\partial A_{\theta}}{\partial \phi}) \hat{\phi}$
Differential displacement	$d\mathbf{l} = dx\hat{\mathbf{x}} + dy\hat{\mathbf{y}} + dz\hat{\mathbf{z}}$	$d\mathbf{l} = d ho\hat{oldsymbol{ ho}} +  ho d\phi\hat{oldsymbol{\phi}} + dz\hat{oldsymbol{z}}$	$d\mathbf{l} = dr\hat{\mathbf{r}} + rd\theta\hat{\boldsymbol{\theta}} + r\sin\theta d\phi\hat{\boldsymbol{\phi}}$
	$d\mathbf{S} = dydz\hat{\mathbf{x}} +$	$d\mathbf{S} = \rho d\phi dz \hat{\boldsymbol{\rho}} +$	$d\mathbf{S} = r^2 \sin\theta d\theta d\phi \hat{\mathbf{r}} +$
Differential normal area	$dxdz\mathbf{\hat{y}}+$	$d ho dz \hat{m{\phi}} +$	$r\sin heta dr d\phi \hat{m{ heta}} +$
	$dxdy\mathbf{\hat{z}}$	$ ho d ho d\phi {f \hat{z}}$	$rdrd heta oldsymbol{\hat{\phi}}$
Differential volume	dv = dxdydz	$dv = \rho d\rho d\phi dz$	$dv = r^2 \sin \theta dr d\theta d\phi$

Non-trivial calculation rules:

- 1. div grad  $f = \nabla \cdot (\nabla f) = \nabla^2 f = \Delta f$  (Laplacian)
- 2. curl grad  $f = \nabla \times (\nabla f) = 0$
- 3. div curl  $\mathbf{A} = \nabla \cdot (\nabla \times \mathbf{A}) = 0$
- 4. curl curl  $\mathbf{A} = \nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) \nabla^2 \mathbf{A}$
- 5.  $\Delta fg = f\Delta g + 2\nabla f \cdot \nabla g + g\Delta f$
- 6. Lagrange's formula for the cross product:

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$$

- Note: This page uses standard physics notation; some (American mathematics) sources define  $\theta$ , as the angle with the xy-plane instead of  $\phi$ .
- Note: The function atan2(y, x) is used instead of the mathematical function arctan(y/x) due to its domain and image. The classical arctan(y/x) has an image of  $(-\pi/2, +\pi/2)$ , whereas atan2(y, x) is defined to have an image of  $(-\pi, \pi]$ .