Del in cylindrical and spherical coordinates

Table with the del operator in cylindrical and spherical coordinates

Operaion	Cartesian coordinates (x, y, z)	Cylindrical coordinates (ρ, ϕ, z)	Spherical coordinates (r, θ, ϕ)
Definition of coordinates		$\left[\begin{array}{lcc} x & = & \rho\cos\phi \\ y & = & \rho\sin\phi \\ z & = & z \end{array}\right.$ $\left[\begin{array}{lcc} \rho & = & \sqrt{x^2+y^2} \\ \phi & = & \operatorname{atan2}(y, x) \\ z & = & z \end{array}\right.$	$\left[\begin{array}{lcc} x & = & r\sin\theta\cos\phi \\ y & = & r\sin\theta\sin\phi \\ z & = & r\cos\theta \end{array}\right.$ $\left[\begin{array}{lcc} r & = & \sqrt{x^2+y^2+z^2} \\ \theta & = & \arccos(z/r) \\ \phi & = & \operatorname{atan2}(y, x) \end{array}\right.$
	$A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}}$	$A_\rho\hat{\boldsymbol{\rho}} + A_\phi\hat{\boldsymbol{\phi}} + A_z\hat{\boldsymbol{z}}$	$A_r\hat{\boldsymbol{r}} + A_\theta\hat{\boldsymbol{\theta}} + A_\phi\hat{\boldsymbol{\phi}}$
∇f	$\frac{\partial f}{\partial x}\hat{\mathbf{x}} + \frac{\partial f}{\partial y}\hat{\mathbf{y}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$	$\frac{\partial f}{\partial \rho}\hat{\boldsymbol{\rho}} + \frac{1}{\rho}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\phi}} + \frac{\partial f}{\partial z}\hat{\boldsymbol{z}}$	$\frac{\partial f}{\partial r}\hat{\boldsymbol{r}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\phi}}$
$\nabla \cdot \mathbf{A}$	$\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$	$\frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial \rho} + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$	$\frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$
$\nabla \times \mathbf{A}$	$(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z})\hat{\mathbf{x}} \quad +$ $(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x})\hat{\mathbf{y}} \quad +$ $(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y})\hat{\mathbf{z}}$	$(\frac{1}{\rho}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z})\hat{\boldsymbol{\rho}} \quad +$ $(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho})\hat{\boldsymbol{\phi}} \quad +$ $\frac{1}{\rho}(\frac{\partial(\rho A_\phi)}{\partial \rho} - \frac{\partial A_\rho}{\partial \phi})\hat{\boldsymbol{z}}$	$\frac{1}{r\sin\theta}(\frac{\partial}{\partial\theta}(A_\phi\sin\theta) - \frac{\partial A_\theta}{\partial\phi})\hat{\boldsymbol{r}} \quad +$ $\frac{1}{r}(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial}{\partial r}(rA_\phi))\hat{\boldsymbol{\theta}} \quad +$ $\frac{1}{r}(\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta})\hat{\boldsymbol{\phi}}$
$\Delta f = \nabla^2 f$	$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$	$\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho\frac{\partial f}{\partial\rho}) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial\phi^2} + \frac{\partial^2 f}{\partial z^2}$	$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial f}{\partial r}) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial f}{\partial\theta}) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}$
$\Delta\mathbf{A} = \nabla^2\mathbf{A}$	$\Delta A_x\hat{\mathbf{x}} + \Delta A_y\hat{\mathbf{y}} + \Delta A_z\hat{\mathbf{z}}$	$(\Delta A_\rho - \frac{A_\rho}{\rho^2} - \frac{2}{\rho^2}\frac{\partial A_\phi}{\partial\phi})\hat{\boldsymbol{\rho}} \quad +$ $(\Delta A_\phi - \frac{A_\phi}{\rho^2} + \frac{2}{\rho^2}\frac{\partial A_\rho}{\partial\phi})\hat{\boldsymbol{\phi}} \quad +$ $(\Delta A_z)\hat{\boldsymbol{z}}$	$(\Delta A_r - \frac{2A_r}{r^2} - \frac{2A_\theta\cos\theta}{r^2\sin\theta} - \frac{2}{r^2}\frac{\partial A_\theta}{\partial\theta} - \frac{2}{r^2\sin\theta}\frac{\partial A_\phi}{\partial\phi})\hat{\boldsymbol{r}} \quad +$ $(\Delta A_\theta - \frac{A_\theta}{r^2\sin^2\theta} + \frac{2}{r^2}\frac{\partial A_r}{\partial\theta} - \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial A_\phi}{\partial\phi})\hat{\boldsymbol{\theta}} \quad +$ $(\Delta A_\phi - \frac{A_\phi}{r^2\sin^2\theta} + \frac{2}{r^2\sin^2\theta}\frac{\partial A_r}{\partial\phi} + \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial A_\theta}{\partial\phi})\hat{\boldsymbol{\phi}}$
Differential displacement	$d\mathbf{l} = dx\hat{\mathbf{x}} + dy\hat{\mathbf{y}} + dz\hat{\mathbf{z}}$	$d\mathbf{l} = d\rho\hat{\boldsymbol{\rho}} + \rho d\phi\hat{\boldsymbol{\phi}} + dz\hat{\boldsymbol{z}}$	$d\mathbf{l} = dr\hat{\mathbf{r}} + rd\theta\hat{\boldsymbol{\theta}} + r\sin\theta d\phi\hat{\boldsymbol{\phi}}$
Differential normal area	$d\mathbf{S} = \quad dydz\hat{\mathbf{x}} +$ $dxdz\hat{\mathbf{y}} +$ $dxdy\hat{\mathbf{z}}$	$d\mathbf{S} = \quad \rho d\phi dz\hat{\boldsymbol{\rho}} +$ $d\rho dz\hat{\boldsymbol{\phi}} +$ $\rho d\rho d\phi\hat{\boldsymbol{z}}$	$d\mathbf{S} = \quad r^2\sin\theta d\theta d\phi\hat{\mathbf{r}} +$ $r\sin\theta drd\phi\hat{\boldsymbol{\theta}} +$ $rdrd\theta\hat{\boldsymbol{\phi}}$
Differential volume	$dv = dxdydz$	$dv = \rho d\rho d\phi dz$	$dv = r^2\sin\theta drd\theta d\phi$

Non-trivial calculation rules:

1. div grad $f = \nabla \cdot (\nabla f) = \nabla^2 f = \Delta f$ (Laplacian)
2. curl grad $f = \nabla \times (\nabla f) = 0$
3. div curl $\mathbf{A} = \nabla \cdot (\nabla \times \mathbf{A}) = 0$
4. curl curl $\mathbf{A} = \nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2\mathbf{A}$
5. $\Delta fg = f\Delta g + 2\nabla f \cdot \nabla g + g\Delta f$
6. Lagrange's formula for the cross product:
   $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$

- Note: This page uses standard physics notation; some (American mathematics) sources define θ, as the angle with the xy-plane instead of ϕ.
- Note: The function $\operatorname{atan2}(y, x)$ is used instead of the mathematical function $\arctan(y/x)$ due to its domain and image. The classical $\arctan(y/x)$ has an image of $(-\pi/2, +\pi/2)$, whereas $\operatorname{atan2}(y, x)$ is defined to have an image of $(-\pi, \pi]$.