

# CS61A Notes

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Please find the course notes for CS61A Structure and Interpretation of Computer Programs below, closely following the Fall 2022 asynchronous lecture videos by Professors John Denero and Justin Yokota. A special thank-you to Simon Kuang (simontheftutist@berkeley.edu) for the template.

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# 1 Functions and Expressions

## 1.1 Expressions

- Call expression anatomy: `<Operator>(<Operands>)`
- Call expressions evaluate to *values*
- Order of evaluation:
  - Evaluate operator
  - Evaluate operands
  - Apply operator function onto operand values
- Primitive expressions
  - Number: 2
  - Name: add
  - String: 'hello'

## 1.2 Environment Diagrams

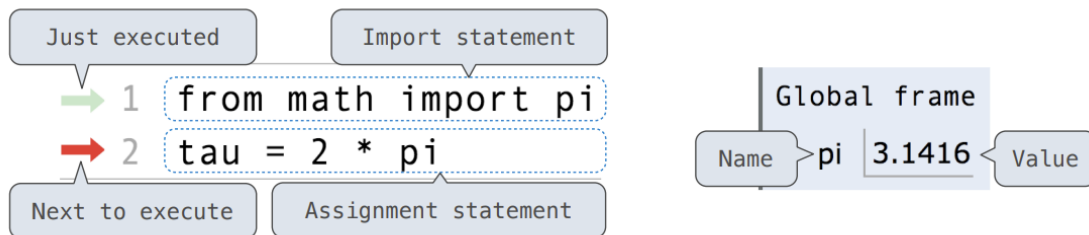


Figure 1: Example Environment Diagram

- Visualization for the interpreter's process
- Left side: statements and expressions
  - Arrows indicate evaluation order
- Right side: names bounded to values in frames
  - Names cannot be repeated in the same frame
  - An environment is a *sequence of frames*: a frame, its parent frame, and so on
  - A name evaluates to the value bound to the name in the *earliest frame* of the current environment
  - Look in the local frame, then in the parent frame, and so on until the global frame
  - Every expression is evaluated in the context of its environment
- Assignment statements: right side is evaluated, and resulting value(s) are bound to the name(s) on the left side

### 1.3 Functions

- Function signature anatomy: `<name>(<formal parameters>)`
- Execution order for applying user-defined functions:
  1. Create local frame
  2. Bind formal parameters to arguments
  3. Execute the function body in the local frame
- None represents nothing
- The return statement completes the evaluation of the call expression
- A function that does not explicitly return a value returns None
- Pure function: just returns a value
- Non-pure function: has side effects

### 1.4 Statements

- A statement is *executed by the interpreter to perform an action*

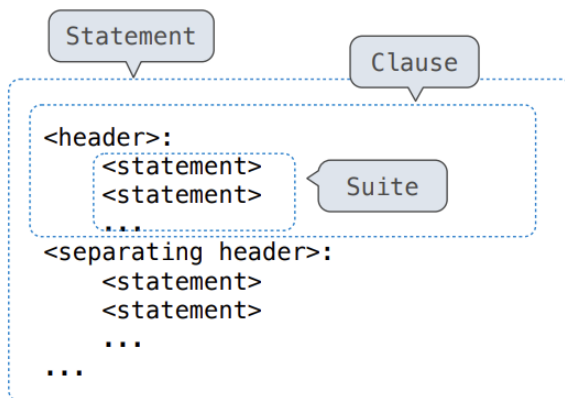


Figure 2: Compound Statement Anatomy

- Conditional statement: if clause, followed by elif/else clauses
- Execution:
  1. Evaluate header expression
  2. If true, execute the suite and skip the remaining clauses
- False values in Python: False, 0, "", None
- True values are anything else
- While statements (iteration)
- Execution:
  1. Evaluate header expression
  2. If true, execute the suite and return to step 1

## 2 More Functions

### 2.1 Functions

- Function domain: the set of all inputs that are possible for arguments
- Function range: the set of all output values that are possible to return
- Function behavior: the relationship between input and output
- A function is an abstraction for its effect, behavior, or return value
- Functions are a type of value
- Higher-order function: a function that takes a function as an argument or returns a function
- Higher-order functions can be described with environment diagrams
- Self-reference: returning a function using its own name
- Parent frame of a function: the frame in which the function was *defined*

### 2.2 Lambda Expressions

- Lambda expression: an expression that evaluates to a function
- Anatomy: `lambda <arguments>: <single expression that evaluates to return value>`
- Lambda expressions cannot contain statements
- "def" vs. "lambda": only def gives the function an intrinsic name in environment diagrams (doesn't affect execution)
- Function currying: transforming a multi-argument function into a single-argument, higher-order function

### 2.3 Logical Operators

- Evaluation of `<left>` and `<right>`:
  1. Evaluate `<left>`
  2. If the result is False, then the whole expression evaluates to False
  3. Otherwise, the whole expression evaluates to `<right>`
- Evaluation of `<left>` or `<right>`:
  1. Evaluate `<left>`
  2. If the result is True, then the whole expression evaluates to True
  3. Otherwise, the whole expression evaluates to `<right>`

### 2.4 Errors

- Syntax errors: detected by Python interpreter before execution
- Runtime errors: detected by Python interpreter during execution
- Logic & behavior errors: Not detected by the Python interpreter

## 2.5 Recursion

- Recursive function: a function that calls itself, directly or indirectly
- Usage: solving problems that *have smaller instances of the same problem*
- Anatomy:
  - "def" statement header
  - Conditional statements that check for *base cases*
  - Base cases are evaluated *without recursive calls*
  - Recursive cases (all other cases) are evaluated *with recursive calls*
- The recursive leap of faith: assumption that the smaller case is solved correctly to solve the current case
- Iteration is a special case of recursion
- Tree recursion: a recursive function makes more than one recursive call

## 3 Sequences

### 3.1 Lists and Containers

- List: a compound value (sequence of values)
- A method of combining data values satisfies the *closure property* if the result of the combination can itself be combined using the same method
- i.e. Lists can contain lists as elements

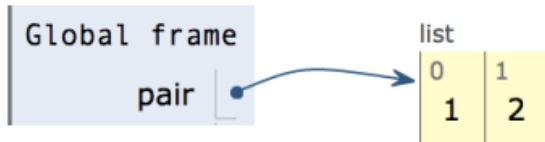


Figure 3: List Format in Environment Diagrams

### 3.2 For Statements

- Anatomy: for <name> in <expression>:
- Execution:
  1. evaluate <expression>, which must yield an iterable value (sequence)
  2. For each element in that sequence in order:
    - a) Bind <name> to that element in the current frame
    - b) Execute the suite

### 3.3 Ranges

- Range: a sequence of consecutive integers
- Starting value (inclusive) to ending value (exclusive)
- Length: ending value - starting value

### 3.4 List Comprehensions

- Combined expression that evaluates to a list
- Anatomy: [<map exp> for <name> in <iter exp> if <filter exp>]
- Anatomy without filter: [<map exp> for <name> in <iter exp>]
- For each element in <iter exp> if <filter exp> is True, then add <map exp>

### 3.5 Slicing

- Slicing a list creates new values, does not reference original list values
- Anatomy: <new list name> = <list name>[<start index>:<end index>]
- If no explicit start or end index, the beginning or end of the list is used
- Start index is inclusive, end index is exclusive

### 3.6 Aggregation

- Built-in functions:
- `sum(iterable[, start]) -> value`
- Returns the sum of an iterable + start (default start value is 0)
- `max(iterable[, key=func]) -> value`
- Returns the largest item in the iterable, based on key function (default is no function)
- `all(iterable) -> value`
- Returns True if all values in the iterable are True (including empty iterable), False otherwise

### 3.7 Strings

- String literals are surrounded with single or double quotes
- A backslash `"\"` escapes the next character
- Line feed character `"\n"` represents a new line

### 3.8 Dictionaries

- Dictionary: a collection of key-value pairs
- Keys must be unique within the same dictionary
- Keys cannot be a list or dictionary
- Dictionary comprehension: `{<key exp>: <value exp> for <name> in <iter exp> if <filter exp>}`



## 4 Abstraction

### 4.1 Data Abstraction

- Data abstraction: a methodology by which functions enforce an abstraction barrier between *representation* and *use*
- Identify a basic set of operations in which all manipulations of a data type can be expressed, then use only those operations for manipulating that data
- Abstraction barriers separate stages of an implementation into levels of abstraction
- Example - rational numbers:

Parts of the program that...	Treat rationals as...	Using...
Use rational numbers to perform computation	whole data values	<code>add_rational, mul_rational</code> <code>rationals_are_equal, print_rational</code>
Create rationals or implement rational operations	numerators and denominators	<code>rational, numer, denom</code>
Implement selectors and constructor for rationals	two-element lists	list literals and element selection

*Implementation of lists*

Figure 4: List Format in Environment Diagrams

## 5 Mutability

### 5.1 Objects

- Objects represent *information* and consist of attributes - *data* and *behavior*
- A type of object is called a *class*
- Object-oriented programming: a methodology for organizing larger programs by:
  - Using data abstraction
  - Bundling together information and related behavior (objects)
- Functions do one thing, objects do *many related things*
- Mutability: the ability for an object to change

### 5.2 Tuples

- Tuple: an immutable sequence
- An immutable sequence can still be changed if it *contains* a mutable value as an element

### 5.3 Identity Operators

- Identity: `<exp0> is <exp1>`
- True if both `<exp0>` and `<exp1>` evaluate to the *same object*
- Equality: `<exp0> == <exp1>`
- True if both `<exp0>` and `<exp1>` evaluate to *equal values*

### 5.4 Files, Strings, and Lists

- `.strip()` : returns a string without whitespace on the ends
- `.split()` : returns a list of strings that were separated by whitespace
- `.replace(a, b)` : returns a string with all instances of string `a` replaced by string `b`

## 6 Iterators and Generators

### 6.1 Iterators

- Any container can provide an iterator that provides elements in Order
- `iter(iterable)` : returns an iterator
- `next(iterator)` : returns the next element in an iterator
- For dictionaries, the order of items (key-value pairs) is the order in which they were added
- Iterators make few assumptions about the data, so others are more likely to be able to use your code on their data
- Iterators keep track of position within the sequence, ensuring each element is only processed once

### 6.2 Built-In Functions for Iteration

- `map(func, iterable)` : iterate over x in iterable using `func(x)`
- `filter(func, iterable)` : iterate over x in iterable if `func(x)`
- `zip(iter1, iter2)`: iterate over co-indexed (x, y) pairs,
  - Skips extras if iterables are different length
  - Can take more than two lists as arguments
- `reversed(sequence)` : iterate over x in a sequence in reverse order
- Functions to view the contents of an iterator:
- `list(iterable)` : return a list containing all x in iterable
- `tuple(iterable)` : return a tuple containing all x in iterable
- `sorted(iterable)` : return a sorted list containing all x in iterable

### 6.3 Generators

- Generator: a function that *yields* values instead of returning them
- A generator can yield multiple times
- A generator is an iterator that is created by calling a *generator function*
- "yield from" statement yields all values from an iterator/iterable

## 7 Objects

### 7.1 Classes

```
class <name>:  
    <suite>
```

- Class: a type/category of objects
- Objects are created by calling Classes
- Classes are objects as well
- Every object that is an instance of a user-defined class has a unique identity
- "is" and "is not" test if two expressions evaluate to the same object
- Binding an object to a new name using assignment does not create a new object
- Methods are functions defined in the suite of a class statement
- Defining methods:

```
class <class name>:  
    def <method name>(self, <formal parameters>):  
        <suite>
```

- Dot expressions: <exp>.<name>
  - <exp> must evaluate to an object
  - If <name> is a method, then "self" parameter is automatically supplied
  - Evaluation order:
    1. Evaluate <exp>
    2. <name> is looked up in the instance attributes of that object
    3. If not found, <name> is looked up in the class
    4. If the value is a function, a bound method is returned. If not, the value is returned
- Looking up an attribute using a string: `getattr(<object expression>, "<name>")`
- Instance attributes are attributes of a specific object instance
- Class attributes are shared across all instances of a class
- Assignment using dot expressions: `<exp1>.<name> = <exp2>`
  - The value of <exp2> is binded to the attribute <name> found in the evaluated <exp1> object

### 7.2 Inheritance

- Inheritance: a technique for relating classes together
- Often used for specialization

```
class <class name>(<base class>):  
    <suite>
```

- Subclass inherits attributes of its base class
- Certain inherited attributes may be overridden

- Base class attributes are not copied into subclasses
- Inheritance is best for representing *is-a relationships*
- Composition is best for representing *has-a relationships*
- A class may inherit from multiple base classes in Python

### 7.3 Representation

- All objects have two forms of string representations
- "str" is legible to humans (same value as what is printed with "print" function)
- "repr" is legible to the Python interpreter
- For most object types, `eval(repr(object)) == object`
- F-Strings for string interpolation
  - String interpolation: evaluating a string literal that contains expressions
  - Equivalent examples:
    - String concatenation: `'pi starts with ' + str(pi) + '...'`
    - String interpolation: `f'pi starts with {pi}...'`
    - Equivalent output: `'pi starts with 3.141592653589793...'`

### 7.4 Polymorphic Functions

- Polymorphic function: a function that applies to many different forms of data
- Examples: "str" and "repr"

### 7.5 Interfaces

- Interface: a set of shared messages, along with a specification of what they management
- Example: classes that implement `__repr__` and `__str__` methods that return string representations implement an interface for producing string representations

### 7.6 Special Method Names in Python

- `__init__` : method invoked automatically when an object is construction
- `__repr__` : method invoked to display an object as a Python expression
- `__add__` : method invoked to add one object to another
- `__bool__` : method invoked to convert an object to True or False
- `__float__` : method invoked to convert an object to a float

### 7.7 Generic Functions

- A polymorphic function may take arguments that vary in types
- Type dispatching: inspect the type of an argument to select behavior
- Type coercion: convert one value to math the type of another

## 8 Composition

### 8.1 Linked Lists

- Linked list structure: a linked list is either empty or a first value and the rest of the linked list
- Linked lists are mutable

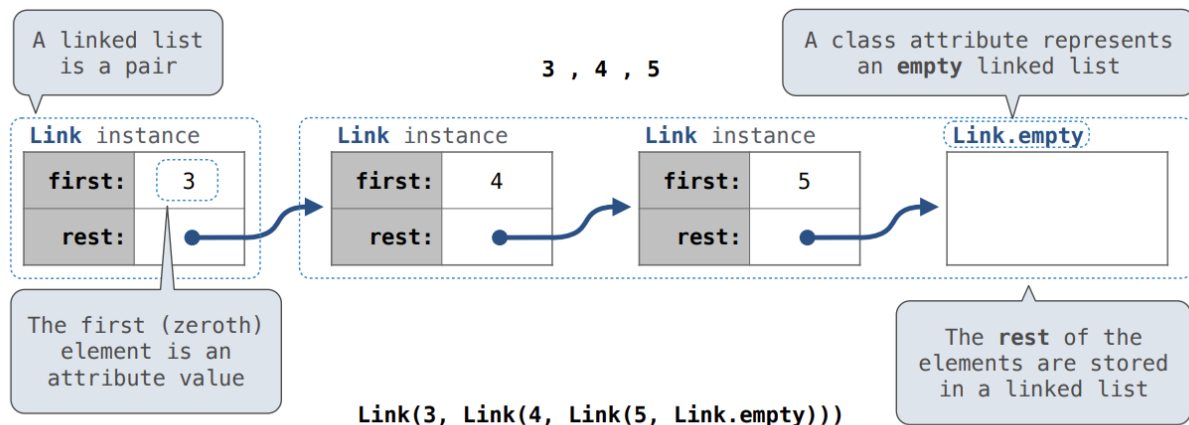


Figure 5: Linked List Example

```
class Link:
    empty = () # some zero-length sequence
    def __init__(self, first, rest=empty):
        assert rest is Link.empty or isinstance(rest, Link) # verify rest is a linked list
        self.first = first
        self.rest = rest
```

### 8.2 Trees

- Recursive description of trees
  - A tree has a root label and a list of branches
  - Each branch is a tree
  - A tree with zero branches is called a leaf
  - A tree starts at the root
- Relative description of trees
  - Each location in a tree is called a node
  - Each node has a label that can be any value
  - One node can be the parent/child of another
  - The top node is the root node
- Tree processing often uses recursion, with the leaf as a base case
- Example tree implementation:

```

class Tree:
    def __init__(self, label, branches=[]):
        self.label = label
        for branch in branches:
            assert isinstance(branch, Tree)
        self.branches = list(branches)
        for branch in branches:
            def tree(label, branches=[]):
                for branch in branches:
                    assert is_tree(branch)
                return [label] + list(branches)
            def label(tree):
                return tree[0]
            def branches(tree):
                return tree[1:]

```

- Pruning: removing subtrees from a tree
- Pruning can be used before recursive processing to speed up computation

### 8.3 Modular Design

- A design principle: isolate different parts of a program that address different concerns
- Each modular component can be developed and tested independently
- Example:

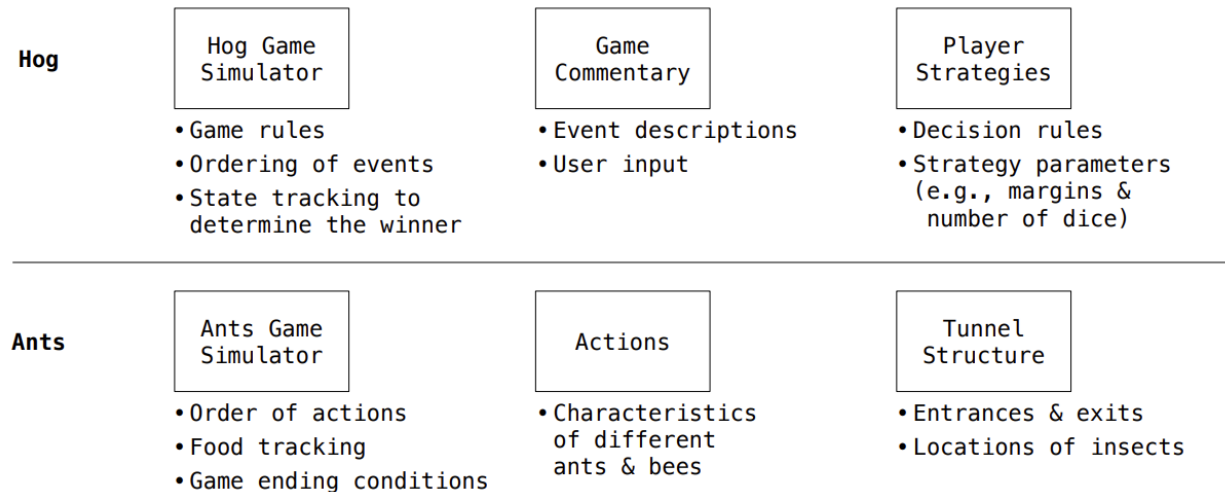


Figure 6: Modular Design Example

## 9 Efficiency

### 9.1 Memoization

- Memoization: "remembering" the results that have been computed before
- Example - fibonacci numbers:
- Store the  $n^{th}$  fibonacci number in a cache so that its value can be used to compute larger fibonacci numbers later

### 9.2 Exponentiation

- Taking advantage of repeated multiplication
- One more multiplication allows the problem size to be doubled
- Example:

```
# doubling input doubles the time
def exp(b, n):
    if n == 0:
        return 1
    else:
        return b * exp(b, n-1)

# doubling the input increases the time by one step
def exp_fast(b, n):
    if n == 0:
        return 1
    elif n % 2 == 0:
        return square(exp_fast(b, n//2))
    else:
        return b * exp_fast(b, n-1)

def square(x):
    return x * x
```

### 9.3 Orders of Growth

**Constant growth** -  $\Theta(1), O(1)$ : increasing  $n$  doesn't increase time

**Logarithmic growth** -  $\Theta(\log n), O(\log n)$ : doubling  $n$  increases time by a constant

**Linear growth** -  $\Theta(n), O(n)$ : incrementing  $n$  increases time by a constant

**Quadratic growth** -  $\Theta(n^2), O(n^2)$ : incrementing  $n$  increases time by  $n * \text{constant}$

- Example: functions that process all *pairs* of values in a sequence

**Exponential growth** -  $\Theta(b^n), O(b^n)$ : incrementing  $n$  multiplies time by a constant

- Example: tree-recursive functions (fibonacci without memoization)



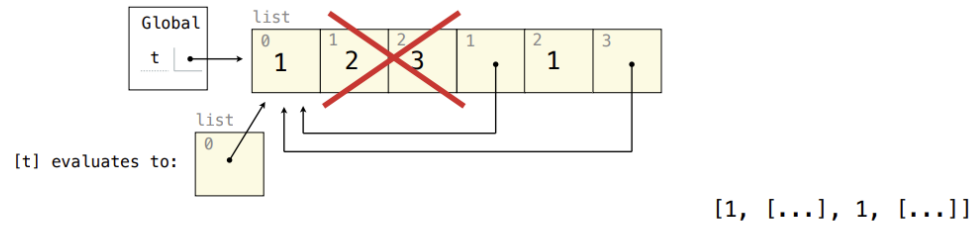
## 9.4 Space

- Active environments:
- Environments for any function calls currently being evaluated
- Parent environments of functions named in active environments

## 10 Data Examples

### 10.1 Lists in Lists in Environment Diagrams

```
t = [1, 2, 3]
t[1:3] = [t]
t.extend(t)
```



```
t = [[1, 2], [3, 4]]
t[0].append(t[1:2])
```

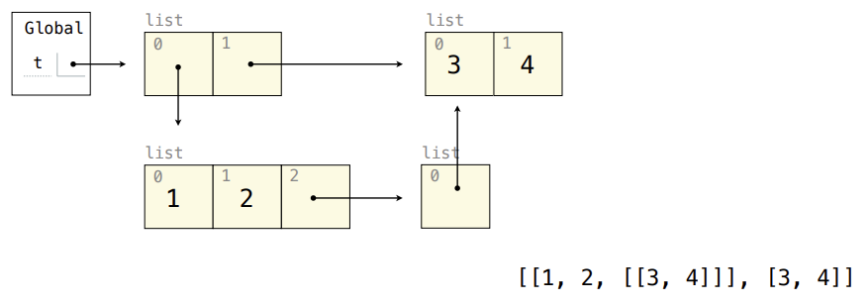


Figure 7: Example of Lists within Lists in an Environment Diagram

### 10.2 Objects

## 11 Scheme

### 11.1 Scheme

- Scheme is a dialect of Lisp
- Scheme programs consist of expressions:
  - Primitive expressions: 2, 3.3, true, +, quotient
  - Combinations: (quotient 10 2), (not true)
- Call expressions include an operator and 0 or more operands in parentheses
- Special forms: combinations that are not call expressions
- Examples: **if**, **and**, **or**, binding symbols, defining procedures, **cond**, **begin**

```
(if <predicate> <consequent> <alternative>)
```

```
(and <exp1> ... <expn>)
```

```
(or <exp1> ... <expn>)
```

```
(define <symbol> <expression>)
```

```
(define (<symbol> <formal parameters>) <body>)
```

```
(cond (<predicate 1> <consequent 1>)  
      (<predicate 2> (begin <consequent 2a> ... <consequent 2m>))  
      ...  
      (else <consequent n>))
```

- Lambda expressions:

```
(lambda (<formal parameters>) <body>)
```
- Let expression: binds symbols to values just for one expression

```
(let <symbol> <expression>)
```

### 11.2 Scheme Lists

- Scheme lists are similar to Python linked-lists
  - cons**: two-argument procedure that creates a linked list
  - car**: procedure that returns the first element of a list
  - cdr**: procedure that returns the rest of a list
  - nil**: the empty list
- Example list:

```
> (cons 1 (cons 2 (cons 3 (cons 4 nil))))  
(1 2 3 4)  
> (define x (cons 1 (cons 2 nil)))  
> x  
(1 2)  
> (car x)  
1  
> (cdr x)  
(2)
```

### 11.3 Symbolic Programming

- Symbols normally refer to values, single quote is used to refer to symbols directly

```
> (define a 1)
> (define b 2)
> (list 'a b)
(a 2)

> '(a b c)
(a b c)
> (car '(a b c))
a
> (cdr '(a b c))
(b c)
```

### 11.4 Built-in List Processing Procedures

**(append s t):** list the elements of s and t (can be called on more than two lists)

**(map f s):** call a procedure f on each element of a list s and list results

**(filter f s):** call a procedure f on each element of a list s and list the elements for which true is the result

**(apply f s):** call a procedure f using elements of list s as its arguments

## 12 Exceptions

### 12.1 Raise

- Raise statements raise an exception in Python

```
raise <expression>
```

- <expression> must evaluate to a subclass or instance of BaseException
  - **TypeError**: incorrect type of argument was passed into a function
  - **NameError**: a name wasn't found
  - **KeyError**: a key wasn't found in a Dictionary
  - **RecursionError**: too many recursive calls

### 12.2 Try

- Try statements handle Exceptions

```
try:  
    <try suite>  
except <exception class> as <name>:  
    <except suite>  
...
```

- Execution order:
  1. <try suite> is executed first
  2. If, in the try suite, an exception is raised that is not handled otherwise and if the class exception inherits from <exception class>, then
  3. <except suite> is executed, with <name> bound to the exception

## 13 Programming Languages

### 13.1 Programming Languages

**Machine languages:** interpreted by the hardware itself

**High-level languages:** interpreted by another program or compiled into another language

- Abstracts away system details
- Creates independence from hardware and operating system

**Syntax:** legal statements and expressions in the language

**Semantics:** execution/evaluation rules for those statements and expressions

### 13.2 Parsing

- Parsing: turning text into an expression
- Syntactic analysis: identifying the hierarchical structure of an expression
- Evaluation: the computation of the value of an expression

### 13.3 Interpreters

- Programs specify the logic of a computational device
- An interpreter is a *general computing machine*

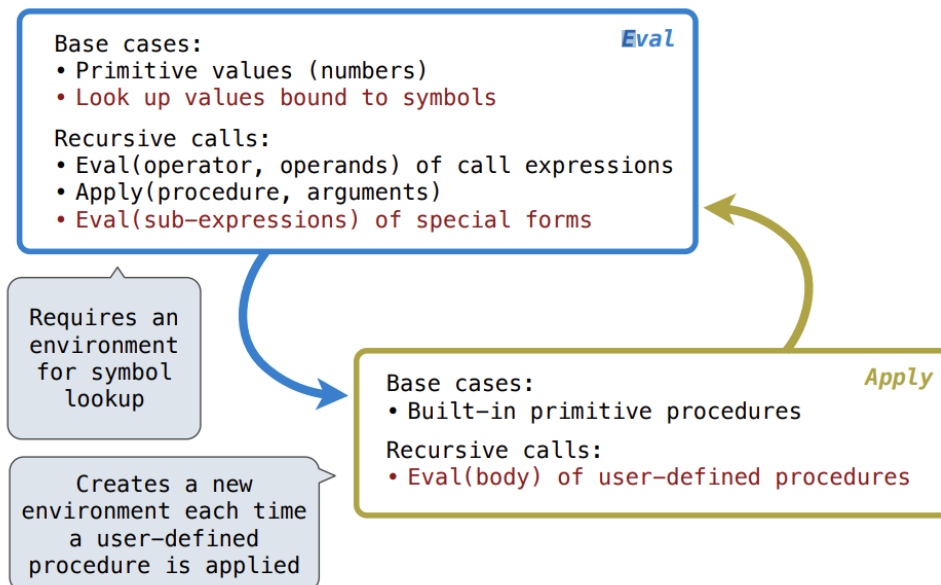


Figure 8: Interpreter Structure

## 14 Tail Recursion

### 14.1 Dynamic Scope

**Lexical scope:** the parent of a frame is the environment in which a procedure was *defined*

```
(define <symbol> (lambda <formal parameters> <body>))
```

**Dynamic scope:** the parent of a frame is the environment in which a procedure was *called*

```
(define <symbol> (mu <formal parameters> <body>))
```

### 14.2 Tail Recursion

- Referential transparency: the value of an expression does not change when we substitute one of its subexpressions with the value of that subexpression
- Tail recursion eliminates the "middleman" frames to save space
- Tail call: a call expression in a tail context
  - The last sub-expression in a **lambda** expression's body
  - Sub-expressions <consequent> and <alternative> in an **if** expression that is in a tail context
  - All non-predicate sub-expressions in a tail context **cond**
  - The last sub-expression in a tail context **and**, **or**, **begin**, or **let**

### 14.3 Macros

- Macro: an operation performed on the source code of a program before evaluation
- Scheme example:

```
> (define-macro (twice expr) (list 'begin expr expr))
> (twice (print 2))
2
2
```

## 15 SQL

### 15.1 Databases

- Table: a collection of rows that have a value for each column
- SQL is a *declarative* programming languages
  - **Declarative language (SQL, Prolog):** a program is a description of the desired result, and the program figures out how to generate the result
  - **Imperative language (Python, Scheme):** a program is a description of computational processes that the interpreter carries out

### 15.2 SQL

- A **select** statement creates a new table, either from scratch or by projecting a table
  - Always includes a comma-separated list of column descriptions

```
select [expression] as [name], [expression] as [name]; ...  
select [columns] from [table] where [condition] order by [order];
```

- A **create table** statement gives a global name to a table

```
create table [name] as [select statement];
```

- Two or more tables can be joined together

```
select parent from parents, dogs where child = name and fur = "curly"
```

### 15.3 Aliases and Dot Expressions

- Aliases and dot expressions clear up ambiguity with column names
- Example of using aliases and dot expressions when joining a table with itself:

```
select a.child as first, b.child as second  
from parents as a, parents as b  
where a.parent = b.parent and a.child < b.child;
```

- Other statements: **analyze, delete, explain, insert, replace, update**, etc.
- Expressions can contain function calls and arithmetic Operators
- String values can be combined to form longer strings

```
> select "hello," || " world";  
hello, world
```

### 15.4 Aggregate Functions

- An aggregate function in the [columns] clause computes a value from a group of rows

```
select [columns] from [table] where [condition] order by [order];
```

- Aggregate functions: **max, min, avg**



## 15.5 Grouping Rows

- Rows in a table can be grouped, then aggregation can be performed on each group
- Number of groups is the number of unique values of an expression
- A **having** clause filters the set of groups that are aggregated

```
select [columns] from [table] group by [expression] having [filter expression]
```