Electron Odyssey: Navigating the Charge-To-Mass Ratio of the Electron

PHYC11 - Lab 2

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Abstract

The purpose of this study was to replicate and verify the accepted value of the charge-to-mass ratio of the electron. An electron gun (heated cathode as source) providing a cathode ray into a low-pressure helium filled glass bulb within a pair of Helmholtz coils was used to determine the charge-to-mass ratio (e/m) of the electron. The magnetic field produced by the Helmholtz coils caused the electron to under go circular motion within the helium medium. The Lorentz force was used to measure the charge-to-mass ratio. The charge-to-mass ratio was determined to be $\frac{e}{m_e} = 198337 \times 10^{11}$ C/Kg within 12.8% error of the accepted value of $\frac{e}{m_e} = 1.75882001076 \times 10^{11}$ C/kg. The error involved in the measurement was determined to be observational error due to the width of the cathode ray, the interference of Earth's magnetic field and an overestimation in the calculation of the electron's kinetic energy.

1. INTRODUCTION

The beginning of new and unexplored lands of subatomic physics were uncovered after the infamous discovery of the particle known as the electron. Its involvement in modern physics, including quantum physics as well as general particle physics, continuously grows as more interesting phenomena are discovered. It is undeniable that the electron holds immense significance in this pursuit. In particle physics, an important quantity to examine concerning charged particles is a quantity known as the charge-to-mass ratio. As the name implies, it is simply the ratio between the particle's intrinsic charge to its mass. The charge-to-mass ratio for the electron, and hence the discovery of the electron, was found in 1897 by British physicist Jeffery John Thomson (Thomson, 2024). This further led him to postulate the "Plum Pudding Model", which essentially described the atom as negatively charged particles floating in a closed positively charged space. Unfortunately, this model was later shown to be inaccurate and was replaced by the well-known Rutherford atomic model.

Although the charge-to-mass ratio may seem to be an arbitrary quantity of particles to study, it's important to note that this was the only quantity experimentally measurable during the early days of subatomic physics. Today, this ratio has significant importance in particle physics research, revealing details about the behaviour of accelerated particles. For example, particles with the same charge-to-mass ratio will move in the same path when exposed to the same electric and/or magnetic field, as predicted by classical electrodynamics. Additionally, this ratio is crucial for understanding mass spectrometry. This technique identifies chemical substances by sorting gaseous ions in electric and magnetic fields using their charge-to-mass ratios (Beynon & Brown, 2024). Its usefulness has led to many applications, such as identifying isotopes and elements and their precise masses and abundances, analysing lunar samples, dating geological samples, and more. Clearly, the charge-to-mass ratio plays a significant role in modern science.

1.1. Theory

Classically, when a charged particle is placed in a magnetic field, nothing happens. However, if a charged particle now *moves* in a magnetic field, it experiences the Lorentz force,

$$\vec{F} = q\vec{v} \times \vec{B} \tag{1}$$

Where q is the charge of the particle, $\vec{v}$ is the velocity of the particle, and $\vec{B}$ is the magnetic field the particle is experiencing. In this study, electrons are produced by a heated cathode creating an electron beam within a Helmholtz coil that produced the magnetic field. The magnetic field is constant and uniform* within the space inside the coil. The velocity of the electron will always be perpendicular to the magnetic field, causing the electron to undergo circular motion as shown in figure 1.

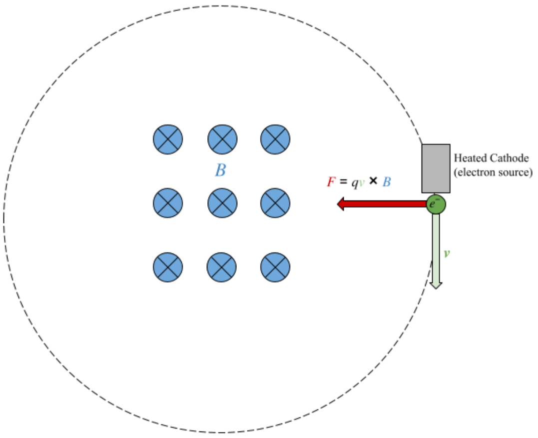

Fig. 1: Illustration of electron undergoing circular motion due to the Lorentz force. The magnetic field is pointing into the page as illustrated by the crossed circles.

Since $\vec{v}$ and $\vec{B}$ are always orthogonal to each other, we can simplify equation (1),

$$F = qvB \tag{2}$$

Where $v = |\vec{v}|$ and $B = |\vec{B}|$ and noting that the direction of the Lorentz force is radial, supporting the circular motion. The radial acceleration that is experienced by the electron can be expressed as,

$$a = \frac{v^2}{r} \tag{3}$$

Where r is the radius of the circular trajectory. Using newton's second law in equation (2), and substituting the value for acceleration show in equation (3) gives,

$$m_e \frac{v^2}{r} = qvB$$
$$m_e \frac{v}{r} = qB$$
$$\frac{q}{m_e} = \frac{v}{rB} \tag{4}$$

Where m_e is the mass of an electron. This equation can be simplified further by finding an expression for the electrons speed. The electrons produced in the apparatus of this study are accelerated by a potential difference before exiting the electron gun. The energy given to the electron by the potential is transformed into kinetic energy. When a charged particle is placed in an electric field, the work done on the electric charge can be expressed simply as,

$$W = -q\Delta V \tag{5}$$

Where ΔV is the electric potential difference of the field. The work done on the charge is transformed into kinetic energy E_K. Thus, the energy equivalence is,

$$W = E_K$$
$$-q\Delta V = \frac{1}{2} m_e v^2$$
$$v = \sqrt{\frac{-2q\Delta V}{m_e}} \tag{6}$$

Although the negative sign may be concerning, further inspection reveals that the radicand is always positive: If the particles charge is negative then $q < 0$ and the particle will move from a lower potential to a higher potential giving $\Delta V > 0$ and vice versa. Thus, we can substitute (6) into equation (4) for the charge-to-mass ratio to give,

$$\frac{q}{m_e} = \sqrt{\frac{-2q\Delta V}{m_e}} \cdot \frac{1}{rB}$$
$$\frac{q}{m_e} = \frac{-2\Delta V}{(rB)^2} \tag{7}$$

Equation (7) gives the charge-to-mass ratio for a charged particle given a radius, potential difference and magnetic field producing the circular motion. Since this equation will be used for the electron, the charge is set to $q = -e$ which gives,

$$\frac{e}{m_e} = \frac{2\Delta V}{(rB)^2} \tag{8}$$

In essence, equation (8) is all that is needed to begin computing charge-to-mass ratios given the certain characteristics of the apparatus and electron.

*Technically, the magnetic field used in this experiment isn't quite constant as the current delivered varies slightly. As well, the magnetic field is only nearly uniform in the space where the electron will experience motion. However, these effects will be ignored for this experiment.

The expression for the magnetic field produced by the Helmholtz coil used in the experiment is,

$$B = \frac{\mu_0 NI}{a} (\frac{4}{5})^{\frac{3}{2}} \tag{9}$$

Where μ_0 is the permeability of free space, N is the number of turns in each pair of coils in the apparatus, I is the current passing through the coils and a is the coils radius.

1.2. Objectives

The objective of this study was to replicate and verify the accepted measured value of the charge-to-mass ratio of the electron given by National Institute of Standard and Technology (NIST). The accepted value is $\frac{e}{m_e} = 1.75882001076 \times 10^{11}$ C/kg ("Fundamental Physical Constants", 2018). This was executed by examining and observing the circular motion of a cathode ray under the influence of a magnetic field produced by Helmholtz coil in a low-pressure helium filled volume. Measurements were taken in varying conditions in voltage applied to the electron gun producing the cathode ray, and varying current to the Helmholtz coil, altering the magnetic field.

2. METHOD

The charge-to-mass ratio of the electron was measured using an apparatus designed by Science First which consists mainly of a electron gun (using a heated cathode), Helmholtz coils, and a low-pressure Helium filled bulb.

2.1. Apparatus

The general configuration of the electron gun can be seen in figure 2. A cathode is heated to excite the electrons on the plate as shown in the top of the figure. The cathode to anode configuration are connected to a high voltage power supply to sustain an electric field. However, only some electrons make it through since most of the area is shielded with only a small aperture open in the middle. Thus, many electrons actually impact the anode and the ones that pass through culminate the electron beam. As explained previously, the electron is given energy as it falls through the potential. The total energy given is simply $e\Delta V$. In this study, it is assumed that all of this energy is converted into the kinetic energy of the electron.

The electron beam was directed downwards in the low-pressure helium bulb. As the electrons passed through the medium, they interact with the helium particles exciting them into higher energy levels. As they undergo de-excitation, energy is released in the form of light that is observed as in the region of spectra pertaining to a green wavelength. Thus, the helium fluoresced when struck with electrons to aid in demonstrating its trajectory. This can be seen in figure 3.

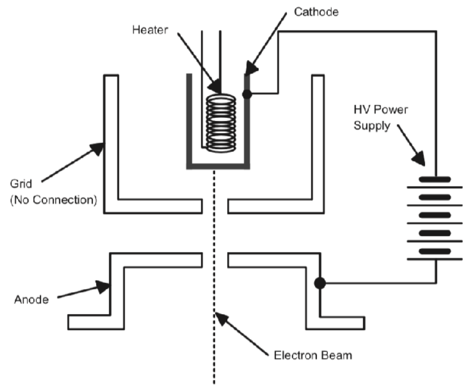

Fig. 2: Electron gun configuration in the apparatus.

The helium filled bulb had a diameter of 13cm. Inside the bulb, there was a scale to measure the diameter that illuminates on impact of electrons to aid in observations. The scale had half centimeter increments that had ranged from 5cm to 11cm.

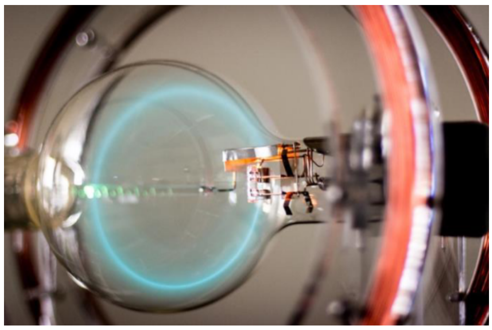

Fig. 3: Electron circular trajectory illuminated by the interactions of the electrons with the low-pressure helium inside the bulb.

The entire bulb was placed inside a set of Helmholtz coils made from copper wire to produce the magnetic field. Both of them had, in total, 132 turns per coil. Each coil had a radius of 14.75cm and were separated a distance of 14.75cm apart. The strength of the magnetic field was given by equation (9). This can also be expressed as approximately 0.79mT/A. The accelerating voltage was controlled by the *Voltage Adjust* knob on the bottom-left of the on-board microprocessor underneath the apparatus as shown in figure 4. The accelerating voltage ranged from 0-500 volts. The current applied to the Helmholtz coiled was controlled using the *Current Adjust* knob, which can be seen in the bottom right of the microprocessor. The current ranged from 0-3 amperes.

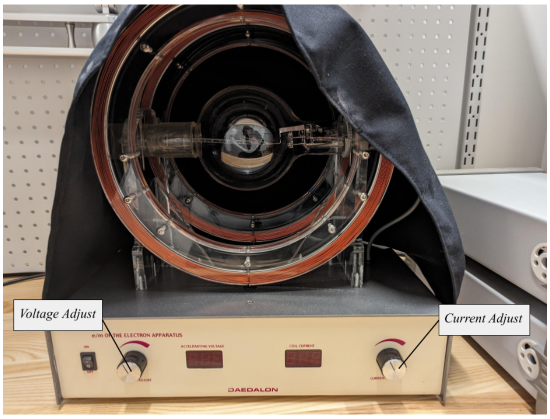

Fig. 4: The apparatus including the microprocessor on the bottom. The *Voltage Adjust* and *Current Adjust* knobs are seen in the bottom left and right, respectively.

2.2. Research Procedure

Before collecting data, the apparatus was allowed to begin preheating the cathode during its 30-second self-test. Once the self-test was completed, the accelerating voltage was raised while the current was kept at 0A until the electron beam inside the tube was visible. The voltage that this had occurred at was at 160V. Then, the current was adjusted in the Helmholtz coils until the trajectory of the electron curved into a circular path hitting the 11cm (the largest diameter that can be properly measured in the apparatus) tick on the scale. The first set of data was collected by maintaining a constant voltage while adjusting the current to meet varying diameters of circular motion demonstrated by the electrons. The second set data was collected by holding the current constant, then adjusting the accelerating voltage to different values to reach each tick on the diameter scale. The measurements taken of the radii of the electron's circular trajectory were taken once the tick on the scale was at a maximum brightness (since each tick fluoresces upon impact of electrons).

Within the first data set, the constant voltage was set to 200V, then for subsequent data, the voltage increased by 30V increments. Due to limitations on the range that the current was able to be adjusted to, certain small diameters for High voltages (limiting towards 500V) were not able to be measured and were left out of the data set. In the second data set, the current was first held constant at 1.60 Amperes, and following data are 0.20 Ampere increments. However, due to limitations on the maximum range of the accelerating voltage, measurements of larger diameter could not be attained as current increased (approached 3.00 Ampere).

After measurements were taken in each data set, the charge-to-mass ratio was calculated using equation (8), utilizing equation (9) for the strength of the magnetic field. The data was plotted using the measured diameter in centimeters on the x-axis, and the corresponding measured charge-to-mass ratio (e/m) on the y-axis.

3. RESULTS

The first data set where the voltage is held constant at different values is shown in figure 5. In the graphs, the measurement of the charge-to-mass ratio of the electron lies on the y-axis, and the diameter of the electrons circular trajectory is presented on the x-axis in their respective units. The graphs include the measured e/m data, as well as the accepted value provided by NIST as indicated as the black horizontal line. The dashed line represents the mean of the data. The mean was calculated by simply taking the average of the measured e/m ratios for the given voltage. The error in the measured diameter is also shown in error bars. Since the scale within the low-pressure helium filled bulb had increments of 0.5cm, the uncertainty in measurement is given as ± 0.25cm. It was found at *higher* voltages that the mean measured e/m ratio approached closer to the accepted value.

The second set of data where the current is held constant and the accelerating voltage was varied is shown in figure 6. In these graphs, use of axes is the same as in the first data set. Similarly the error in the measured diameter of the electron's circular trajectory is also the same as in the first data set (± 0.25cm). Interestingly, a similar relationship to what was found in the first data set was seen in the second: It was found that at *higher* current, the mean measured e/m ratio approached closer to the accepted value.

Further inspection into the results from both data sets can be seen in figure 7. In this figure, the average from each graph in data set 1 and 2 are shown with respect to increasing voltage and current, respectively. The overall mean (dashed line) is the average of the results within each of these graphs. As before, the accepted value is also displayed in the graphs as the black line. The data in this figure clearly showed the general relationship present in both data sets: Increasing the voltage and current in the respective data set 1 and 2, further approached the measured charge-to-mass ratio to the accepted value. The error in the graphs shown in figure 7 was calculated using the following expression,

$$Error = \frac{|(\frac{e}{m_e})_{true} - (\frac{e}{m_e})_{measured}|}{(\frac{e}{m_e})_{true}} \times 100\% \quad (10)$$

Where $(\frac{e}{m_e})_{true}$ is the accepted value given by NIST. The overall mean values in data set 1 and 2 were found to be $\frac{e}{m_e} = 1.97696 \times 10^{11}$ C/Kg and

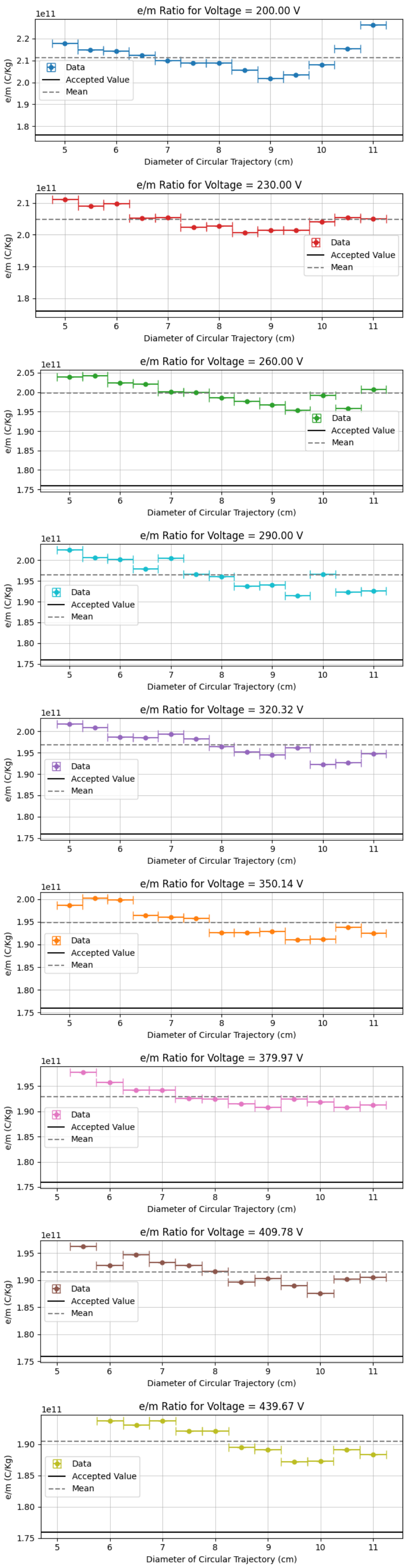

Fig. 5: Data set 1: Charge-to-mass ratio data collected at constant voltages and varying currents.

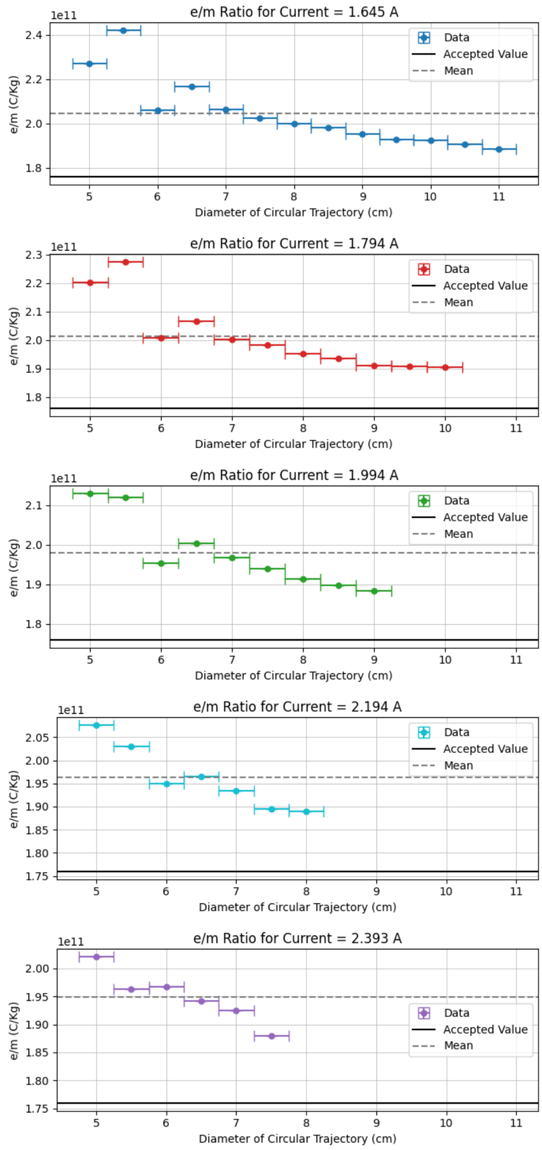

Fig. 6: Data set 2: Charge-to-mass ratio data collected at constant current and varying voltage.

$\frac{e}{m_e} = 1.98979 \times 10^{11}$ C/Kg, respectively. Thus, equation (10) gave the resulting error in each data sets to be approximately $\approx 12.4\%$ for data set 1, and $\approx 13.1\%$ for data set 2. In full consideration of the total mean including all data from both sets, it was found that the total mean was $\frac{e}{m_e} = 198337 \times 10^{11}$ C/Kg. Equation (10) gave the final resulting error in the total mean to be $\approx 12.8\%$.

4. DISCUSSION

This study was able to determine the charge-to-mass ratio of the electron within reasonable error to the accepted value given the apparatus. The mea-

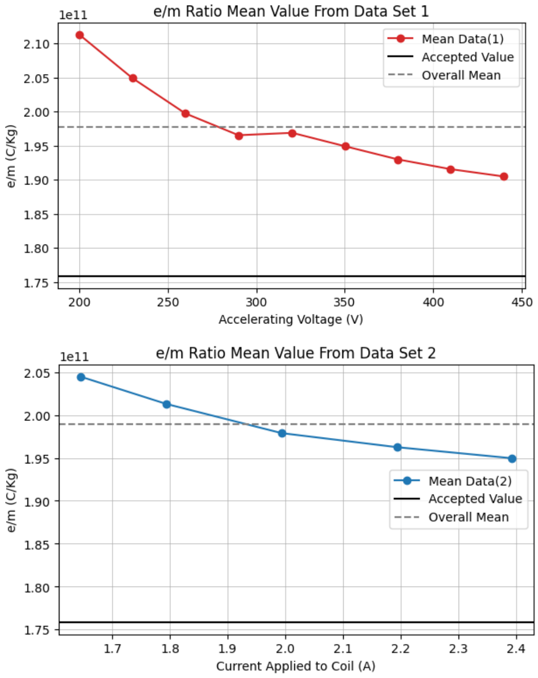

Fig. 7: The mean charge-to-mass ratio data collected from both data sets. Data set 1 pertaining to constant voltage and varying current. Data set 2 pertaining to constant current and varying voltage.

sured charge-to-mass ratio was found to be slightly larger than the accepted value with approximately 12% error. Most values shown in the graphs in figures 5, 6 and 7 had measured charge-to-mass ratios above the accepted value. However, a general trend was observed in all graphs: the measured charge-to-mass ratio approached from above towards the accepted value when voltage or current was increased. Meaning, the increase in high voltage in the electron gun or increasing the current in the Helmholtz coils strengthening the effects of the magnetic field both reduce error in the measurement. Predictions based of the data presented in this study suggest that using a similar apparatus at higher accelerating voltages and or higher currents, the data will approach the horizontal line shown by the accepted value.

4.1. Analysis of Error

The reason for error can be explained from many factors. Firstly, the Earth's own magnetic field was not taken into consideration when computing the charge-to-mass ratio. The Earth's magnetic field does have a considerable effect on the electrons trajectory by adding another component to the net magnetic field as shown,

$$\vec{B}_{net} = \vec{B}_{coil} + \vec{B}_{earth}$$
$$\implies F_{net} = q\vec{v} \times \vec{B}_{net} = q\vec{v} \times (\vec{B}_{coil} + \vec{B}_{earth})$$

This implies that the true circular trajectory is perturbed from the original trajectory that was assessed in this study. This perturbation causes a slight curve in the circular motion of the electron creating a slight spiral motion. Thus, the observed measurement of the diameter of the motion would be off from the true measurement that the Helmholtz coil itself would produce.

Another factor worth mentioning is the width of the cathode ray and kinetic energy supplied to the electron. In this study, it was assumed that the velocity was given to the electron during its acceleration in the electron gun given by equation (6). However, there are some flaws in this assumption. The assumption further implied that the electric field between the cathode and anode was uniform. This is only the case with two non-deformed and large enough plates with a small separation. Though, the aperture found in the electron gun slightly alters the electric field from being perfectly uniform and vertical with respect to the plates as shown in figure 8.

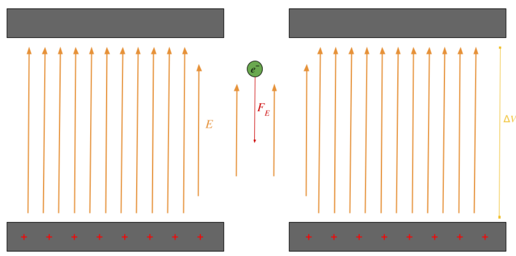

Fig. 8: Illustration showing the non-uniform electric field present within the electron gun. The electric field E becomes weaker in magnitude towards the center as shown by shorter arrows and larger spacing. The force from the electric field F_E the electron experiences is shown.

This alteration accelerates the electron in a slightly different direction and magnitude. Additionally, after the electron does pass through the aperture, it is now moving *away* from the positive plate. This means it will de-accelerate upon leaving the electron gun and hence decrease the expected kinetic energy of the electron. In equation (4), we can see that an overestimation of its kinetic energy, and thus its velocity, increases the charge-to-mass ratio accordingly since they are linearly proportional,

$$\frac{q}{m_e} \propto v_e$$

This expression explains the reason why the results found in this study overestimated the charge-to-mass ratio of the electron. These factors not only effect the assumptions of the kinetic energy of the electron, but gives a dispersion of different electron circular motions as seen in the width of the electron beam. Thus, measurements made on the scale within the low-pressure helium filled bulb are more uncertain due to the dispersion of electrons increasing the cathode ray's width. Understandably, these effects would diminish as the accelerating voltage is increased which aligns with the results found in this study.

4.2. Further Research

Interesting further research into this topic may include replicating this study in way to minimize the suggested factors of error mentioned above. For instance, in order to minimize the effects of Earth's gravitational field, it may be worth considering to position the apparatus such that the magnetic field produced by the Helmholtz coil is in line with the magnetic field of the Earth. This would no longer deform the electrons circular path but only work against or with the magnetic field produced in the apparatus.

Additionally, the results of this study had predicted that working with higher voltages, as well as higher currents, give closer results to the accepted value. A future study could build on this idea working with a new apparatus that works with much larger limits on the accelerating voltages and currents involved. That way, a much more accurate charge-to-mass ratio for the electron may be measured. Armed with the value of e/m, the next step may be to find one of the variables that create this ratio in order to find the other. A study such as the Millikan Oil Drop experiment could be use to accurately measure the charge of an electron. Using the measured value of the charge-to-mass ratio, one could make an accurate prediction of the electron's mass.

5. CONCLUSION

After consideration of all data presented in this study, the charge-to-mass ratio was measured to be $\frac{e}{m_e} = 198337 \times 10^{11}$ C/Kg. The measured value from this study was within $\approx 12.8\%$ error to the accepted value, $\frac{e}{m_e} = 1.75882001076 \times 10^{11}$ C/kg provided by NIST ("Fundamental Physical Constants", 2018). Of course since the charge is negative, the value of $\frac{e}{m_e}$ is negative as well.

The error in measurement was a result of many factors. The effects from the magnetic field produce from the Earth does produce error using this apparatus. The Earth's magnetic field adds another component to the net magnetic field changing the direction and magnitude of the Lorentz force experienced by the electron. Additionally, flawed assumptions were made in the electron's kinetic energy and trajectory. The configuration of the physical system under the assumptions made overestimated the kinetic energy of the exiting electron. There was also a noticeable difference in trajectories of many electrons as seen in the cathode ray's width, decreasing the accuracy of diameter measurements.

Nonetheless, this study has reinforced the validity of the accepted value of the charge-to-mass ratio of the electron. The pattern of all data shown in this study was observed to approach a limiting value that coincides well to the accepted value. The charge-to-mass quantity of a electron, or any subatomic particle for that matter, is of extreme importance in physics. Understanding this quantity allows physicists to predict characteristics of the particle without knowing specific intrinsic ones. Harnessing this quantity, physicists have a looking glass into the mysterious subatomic world to seek and discover previously unknown particles.

REFERENCES

Beynon, J. H., & Brown, L. (2024, March 1). *Mass spectrometry*. Encyclopædia Britannica. https://www.britannica.com/science/mass-spectrometry

National Institute of Standards and Technology. (2018). *Fundamental Physical Constants*. The NIST Reference on Constants, Units, and Uncertainty. https://physics.nist.gov/cgi-bin/cuu/Value?esme

Thomson, G. P. (2024, February 3). *J.J. Thomson*. Encyclopædia Britannica. https://www.britannica.com/biography/J-J-Thomson