The Enigmatic Muon: Investigating the Mean Lifetime of Muons and Their Fermi Constants

PHYC11 - Lab 1

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Abstract

Understanding the fundamental particles that compose our universe lies at the heart of scientific inquiry. This study investigates the mean lifetime of a muon and the Fermi coupling constant as determined by measurement. Data of muon decay time distributions were collected in three distinct data sets, each acquired during varying run times. These time spans encompassed 3 hours, 11 hours, and an extended 5-day period. It was found that with increasing allotted run time, the percent error as compared to data from the Particle Data Group in measurements of mean lifetime decreased. Similarly, the Fermi coupling constant, as calculated by the measured mean lifetimes, were found to be decreasing to a set value. This decreasing pattern shown in percent error reinforced the validity of the true known measurement of the mean lifetime of a muon as given by the Particle Data Group.

1. INTRODUCTION

The quest to understand the universe is a profound and fascinating endeavour. From the tiniest building blocks of matter to the vast expanse of space, there is so much to explore and discover. Among these building blocks is the remarkable muon, a particle that has captured the attention of scientists and researchers throughout history. To truly grasp the significance of muons, we must first look back at their discovery and the impact it has had on our understanding of the world around us.

1.1. History

It began in 1912 when a passionate physicist named Victor Hess decided to perform quite a thrilling experiment (Siegel, 2021). Hess decided to bring a particle detector up to high altitudes using a hot air balloon which is shown in figure 1. He was aware of the clutter of fast-moving particles apparent in the atmosphere and that if he could get closer to the edge, he'd be able to get an exceptional observation. The experiment turned out to be a great success with Hess discovering an abundance of particles in the upper atmosphere. This led to his extraordinary discovery of cosmic rays. Later on, Hess was awarded the Nobel Prize in 1936 for his discovery (Breisky, 2012). Within that same year, American physicists Carl D. Anderson and Seth Neddermeyer working with cosmic rays discovered a particle similar to that of an electron but had much more mass (Anderson, C. D., Neddermeyer, S. H., 1936). This particle was indeed the muon and was interestingly found in both negatively and positively charged forms. The muon's mysterious short life span was first shown by Rasetti. F later on in 1941 (T.E. Coan and J. Ye).

1.2. Muon's Importance and Applications

Given the intriguing qualities of the muons, it's reasonable to ask why it might be worth understanding their nature; in other words, it's important to understand their use in science. Muons have a major significance in modern subatomic particle physics. More recently, muons have been studied closely at the Fermi National Accelerator Laboratory, or Fermilab for short. The laboratory is shown in figure 2. The experiment, known as Muon g-2, is being used

Fig. 1: Victor F. Hess, as seen in the center sitting in the hot air balloon, departing from Vienna in 1911 (Breisky, 2012).

to study the magnetic moment of the muon (Miller, 2023). The theory has predicted that the true measurement of the muon's magnetic moment, abbreviated as g, should be approximately two. However, due to a phenomenon known as "quantum foam", where subatomic particles blip into and out of existence, cause the experimentally measured value to deviate from its estimated value. Physicists have been able to predict this deviation using the Standard Model. However, if the measured value is different from what is predicted by theory, this could uncover previously unknown particles that are at play. This is one of the many reasons why it is essential that we understand the muon well to determine uncovered ground in particle physics.

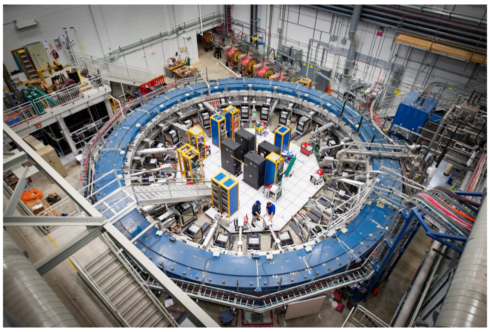

Fig. 2: Muon g-2 ring at the Fermilab particle accelerator complex in Batavia (Miller, 2023).

More directly, understanding the muon's lifetime has great importance in many applications for other fields. For instance, the muon's lifetime becomes exceptionally useful when working with Muon Tomography. Muon Scattering Tomography (MST) uses muons originating from cosmic rays to build 3D models of obstructed objects or volumes (Vlasov, 2023). By placing muon detectors at either side of a structure, the deflected muons can be detected from high-density materials. Since muons can penetrate through hundreds of metres of rock, MST is extremely useful for understanding dense structures that cannot be mapped by other standard forms of mapping. As an example, a corridor in the Great Pyramid of Giza, which was about nine metres long and two metres wide, was first discovered in 2016 with the aid of muon imaging.

Measuring the lifetime of muons also provides direct evidence of Einstein's special relativity (T.E. Coan and J. Ye). Despite their inherently short lifetimes, muons manage to persist until they reach the Earth's surface. How is this possible? The answer lies in time dilation. As muons hurtle through the atmosphere at speeds approaching that of light, their lifetimes appear extended from the perspective of an observer on Earth. However, once we bring the muon to rest relative to our reference frame, it resumes its intrinsic mean lifetime. Although this topic in specific will not be further explored in this paper, the measurement of the muon's lifetime plays a significant role in its explanation.

1.3. Objectives

The purpose of this study is to verify the known mean lifetime of muons as well as estimate their Fermi coupling constant. Using a detector manufactured by TeachSpin, data of muon decay times will be collected over different run times. The data will then be used to make a measurement on the mean lifetime of a muon. Using this measurement, an approximation for the Fermi coupling constant G_F will also be calculated. Comparisons will be made between the different run times and error will be discussed using known values from the Particle Data Group.

2. METHOD

2.1. Theory

The source for the muons that are being measured and studied in this paper are from cosmic rays. The top of the atmosphere is bombarded with high energy particles consisting mainly of protons or heavier nuclei; approximately 98% are protons and heavy nuclei mainly consisting of helium nuclei and 2% are electrons (Simpson, J.A). Primary cosmic rays collide with air molecules to produce a shower of many particles (T.E. Coan and J. Ye). Under electromagnetic and nuclear interactions, the particles produce other ones creating a cascade. This cascade can be seen in figure 3.

The particles resulting from the cosmic ray that we are primarily interested in are pions. Via the weak force, pions spontaneously decay into muons with an additional particle known as a neutrino. Of course, there are two types of pions – positively charged pions and negatively charged pions. The

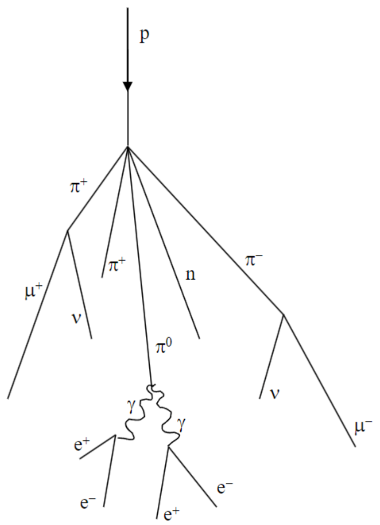

Fig. 3: Illustration of a cosmic ray cascade caused by a cosmic ray proton colliding with air molecule in the atomosphere (T.E. Coan and J. Ye).

charge of the pion determines the charge of the resulting muon as well as the additional neutrino or antineutrino. The decay is illustrated below:

$$\pi^+ \to \mu^+ v_\mu^+$$
$$\pi^- \to \mu^- \overline{v_\mu}$$

It then continues and travels a fairly large distance nearing the speed of light towards the surface of the earth. However, the muon does not stay around for long, eventually it decays into two more products; an electron or positron plus a neutrino or antineutrino via the weak force. Its second decay plays a very significant role in measuring the lifetime of the muon.

In essence, the equipment used to measure the muon's lifetime is composed of a scintillator and a detector. As the fast moving muon passes through the scintillator, it is slowed down and loses some of its kinetic energy. This motion excites the molecules in the scintillator releasing a flash of light that is picked up by the detector. Then, the muon will decay releasing an electron and a neutrino or antineutrino; the neutrinos have negligible effects on the detector and are ignored. Because the electron is much smaller than the muon, it has high kinetic energy and excites the scintillator once more, releasing a second flash of light. The interval between these successive flashes is what was measured. Although, it's important to understand that positive and negative muons interact differently in matter. Because of the negative muon's close similarity to that of an electron, it is able to bind with the scintillator's carbon and hydrogen nuclei. Once bound, it is then able to interact with the proton in the nuclei to produce a neutron and a neutrino before it decays. This means that a negative muon actually has two ways it can disappear; either it slows down and decays, or binds with nuclei in the scintillator. Consequently, the mean lifetime of a negative muon is less than that of a positive muon. Since the equipment cannot differ between negative and positive muon events, the mean lifetime measured is an average of both.

2.2. Equipment

The detector used to measure the muons mean lifetime consists mainly of a scintillator and a photomultiplier tube (PMT)(T.E. Coan and J. Ye). As explained previously, when the muon enters the detector and passes through the scintillator, its kinetic energy is reduced via ionisation and atomic excitation of the scintillator molecules. During deexcitation, the light emitted from the scintillator is within the blue and near-UV portion of the electromagnetic spectrum. The first flash of light detected by the PMT is sent to the discriminator box where it is amplified, then compared to a reference voltage. After passing the discriminator, it triggers the FPGA timer clock. The stopped muon then decays producing an electron which excites the scintillator once more. This second flash of light is detected by the PMT and goes through the same process of amplification and voltage comparison before triggering the FPGA timer clock once more. The time interval between the two triggers is sent out through a usb port to the PC to determine the muon lifetime. The block diagram of the electronics involved is shown in figure 4.

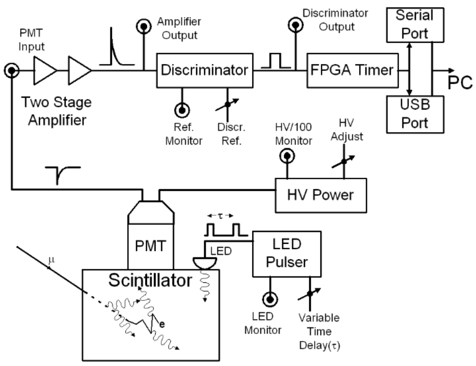

Fig. 4: Block diagram of electronic components in the detector.

2.3. Research Procedure

In order to find the mean time of the muon, the decay time distribution must be recorded (T.E. Coan and J. Ye). The reason for the necessity of the decay time distribution can be explained mathematically.

Given at some time t the amount of muons is $N(t)$, and λ is our constant decay rate, then it must be true that the change in the amount of muons dN is simply:

$$dN = -N(t)\lambda dt \tag{1}$$

$$\frac{dN}{N(t)} = -\lambda dt \tag{2}$$

We can integrate both sides to obtain an expression for $N(t)$

$$N(t) = N_0 e^{-\lambda t} \tag{3}$$

Where N_0 is the initial number of muons at $t = 0$. In this case, the lifetime of a muon, known as τ, is just the reciprocal of the decay rate λ

$$\tau = \frac{1}{\lambda}$$

However, in this experiment a set amount of muons decaying over time is not being observed. Instead, different lifetimes of muons are recorded as they enter the detector at random times. Conveniently, the decay time distribution of the muons entering has the same exponential form as $N(t)$. If we let $D(t)$ represent the probability that a muon decays at a time t, then the probability that a muon will decay in a time interval t and $t + dt$ is simply $D(t)dt$. If we use our expression for $N(t)$ in (3) and sub it into equation (1) we obtain

$$-dN = N_0 \lambda e^{-\lambda t} dt \tag{4}$$

$$\frac{-dN}{N_0} = \lambda e^{-\lambda t} dt \tag{5}$$

The left hand side of equation 5 is simply the decay probability in the time t and $t + dt$, thus we obtain that

$$D(t) = \lambda e^{-\lambda t} dt \tag{6}$$

Notice that this expression holds true regardless of what our initial amount N_0 of muons is. Therefore, in this experiment it is of great importance to obtain the distribution of decay times and to attempt to find the exponential that best fits the data of the form

$$f(t) = ae^{-bt} + c$$

Where a, b and c are just some real constants. The constant b plays the same role as the decay rate so in order to measure the mean lifetime of the muon its reciprocal is measured.

A python program was coded to take the recorded data of decay times and plot them. Because the decay times are distributed on the positive integer line, data was organised into bins to find the exponential curve. For all data plots, a bin size of 20 was applied to the horizontal axis which range from 0 to 40000 nanoseconds. As illustrated in the manual, decay times of less than 80 nanoseconds were removed from the data due to the lack of accuracy in measurement from the equipment in such time intervals. Error in the decay events per bin were determined by taking the root of the total events within that bin. The error along the time scale was set to the interval given by the bin size. A python package known as SciPy was used to fit the general exponential form to the data plots. The constant parameters of the exponential function are shown in the graphs listed in the results sections.

Once the mean lifetime is obtained, we can then continue to compute its Fermi coupling constant G_F. For a good approximation, the relation used in this experiment to compute the Fermi coupling constant was (T.E. Coan and J. Ye)

$$\tau = \frac{192\pi^3\hbar^7}{G_F^2 m_\mu^5 c^4} \tag{7}$$

Here, m_μ is the mass of the muon while the other constants are their standard values. The equation is simply rearranged to measure the Fermi coupling constant G_F

$$G_F = \sqrt{\frac{192\pi^3\hbar^7}{\tau m_\mu^5 c^4}} \tag{8}$$

However, the Fermi coupling constant G_F is more commonly presented in GeV^{-2} ; In other words, it is presented as $G_F/(\hbar c)^3$. Thus, our equation is changed to the form of

$$\frac{G_F}{(\hbar c)^3} = \sqrt{\frac{192\pi^3\hbar}{\tau(m_\mu c^2)^5}} \tag{9}$$

The constants used for the calculation were obtained from the Particle Data Group (R. L. Workman et al., 2022):

$m_\mu = 0.1057\ GeV/c^2$
$c = 299792458\ ms^{-1}$
$\hbar = 6.582 \times 10^{-25}\ GeV \cdot s$

3. RESULTS

For each data set presented, there is additional information presented in the graphs. When analysing the decay time distribution, as explained in the method section, the data is fitted to an exponential curve of the form:

$$f(t) = ae^{-bt} + c$$

In the graphs, the constant values a, b and c determined by the SciPy optimization package are listed in the top right corner. As the constant b plays the same role as λ in the derivation listen in the research procedure subsection, the estimated mean time life for the muon is the reciprocal of b. This is shown as τ, which is also presented in the top right corner of the graphs. The total decay events are also listed above the constants which shows the total amount of muons that had slowed down and decayed in the detector. The true mean lifetime of the muon that we will be using to calculate error as mentioned before is from the Particle Data Group. The muon mean lifetime is $\tau_{\mu} = 2.1969\ \mu s$ (R. L. Workman et al., 2022).

The first set of data collected was a run time of 3 hours on January 23, 2024 at approximately 1:46pm. The total amount of decay events detected was 229. The data is shown in figure 5.

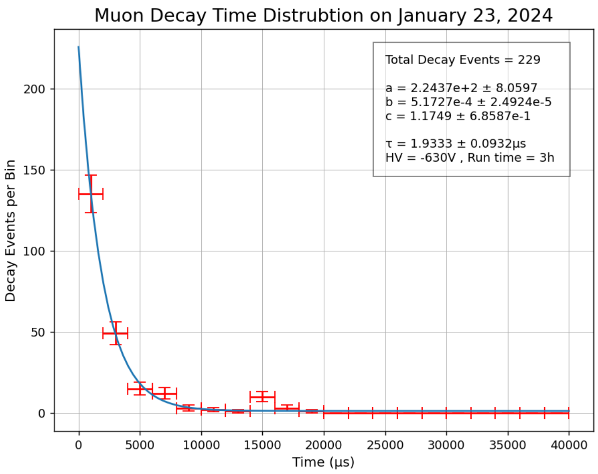

Fig. 5: Data collection on January 23, 2024. Run time was for 3 hours with high voltage set to -630V. Data was fit to to an exponential via SciPy optimization package. Uncertainties given by SciPy in parameters are listed in the top right.

As illustrated, the lifetime for this dataset was found to be τ =1.9333 $\pm$ 0.0932μs. Comparing with the true value τ_{μ}, the error was found by the simple expression

$$Error = \frac{\tau_{\mu} - \tau}{\tau_{\mu}} \times 100\% \tag{10}$$

Therefore, the error in the mean life time found using data collected on January 23 was found to be approximately 12%.

The corresponding Fermi constant G_F was found using equation 9. The value found was

$$\frac{G_F}{(\hbar c)^3} = \sqrt{\frac{192\pi^3(6.582 \times 10^{-25})}{(1.9333 \times 10^{-6})(0.1057)^5}}$$

$$G_F \approx 1.2394 \times 10^{-5} GeV^{-2}$$

Moving to the next data set, data was collected for a run time of 11 hours. It was collected on January 30, 2024 and was intially started at approximately 2:02pm. The total amount of decay events detected during this run was 957. The data is illustrated in figure 6.

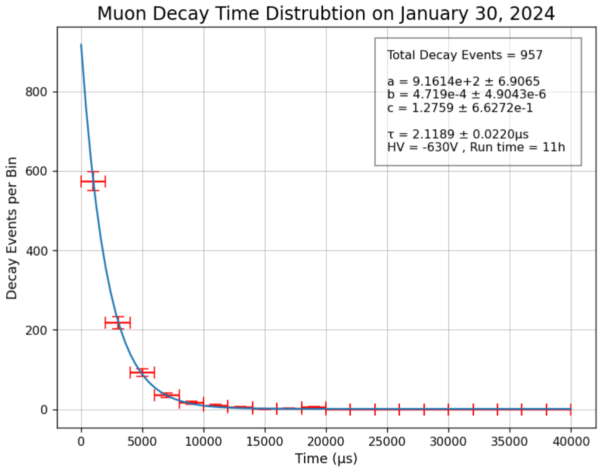

Fig. 6: Data collection on January 30, 2024. Run time was for 11 hours with high voltage set to -630V. Data was fit to to an exponential via SciPy optimization package. Uncertainties given by SciPy in parameters are listed in the top right.

As shown, the mean lifetime for the muon in the 11 hour run was found to be $\tau = 2.1189 \pm 0.0220\mu s$. As before, we can calculate the error in the measurement found by referencing equation 10. The error was found to be approximately 3.6%, a considerable difference when comparing the results collected in the 3 hour run time.

The Fermi coupling constant was found using equation 9

$$\frac{G_F}{(\hbar c)^3} = \sqrt{\frac{192\pi^3(6.582 \times 10^{-25})}{(2.1189 \times 10^{-6})(0.1057)^5}}$$

$$G_F \approx 1.1839 \times 10^{-5} GeV^{-2}$$

As shown, there was a noticeable reduction in the Fermi coupling constant compared to the 3 hour run time data set.

Finally, the last data set was collected for a total run time of approximately 5 days. The data collection began on February 1, 2024 at approximately

3:14 pm and continued until February 6, 2024 at around 1:19pm. The total amount of decay events detected during the allotted time was 8754. The data is shown in figure 7

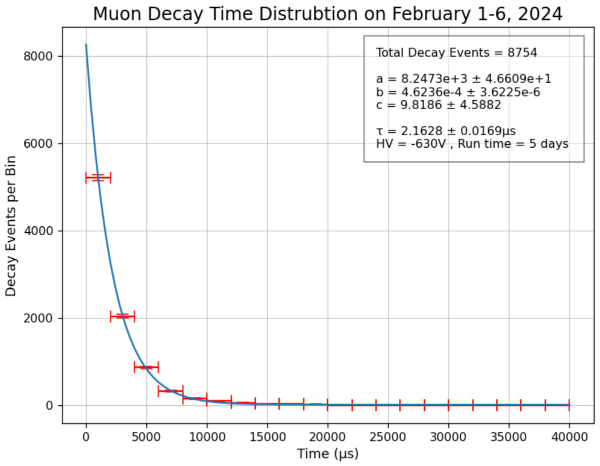

Fig. 7: Data collection on February 1 - 6, 2024. Run time was for 5 days with high voltage set to -630V. Data was fit to to an exponential via SciPy optimization package. Uncertainties given by SciPy in parameters are listed in the top right.

The data set from this elapsed time attained the muon mean lifetime value of $\tau = 2.1628 \pm 0.0169\mu s$. To analyse the error in the measurement compared to the value given by the Particle Data Group, we will use equation 10. The error was found to be approximately 1.5%. The measurement made by the 5 day long data set appeared to have the lowest error. The corresponding Fermi coupling constant for the mean life measurement of this data set was found using equation 9. The value for G_F was found to be

$$\frac{G_F}{(\hbar c)^3} = \sqrt{\frac{192\pi^3(6.582 \times 10^{-25})}{(2.1628 \times 10^{-6})(0.1057)^5}}$$

$$G_F \approx 1.1718 \times 10^{-5} GeV^{-2}$$

4. DISCUSSION

4.1.

After a close evaluation of the data presented, it was found to support known true values of mean muon lifetime and Fermi coupling constants. The pattern suggested by the data found implies that with increasing run times, the percent error found in comparison to the mean muon lifetime given by the Particle Data Group is reduced. Understandably, it was found that the lowest error in decay lifetime was found in the 5-day-long data set shown in figure 7. The measured mean lifetime for this data set was approximately $\tau = 2.1628$. In comparison with Particle Data Group's result, $\tau_\mu = 2.1969$, the percent error was significantly low being only 1.5%. Similarly, the Fermi coupling constant determined from the measured mean decay time from each data set decreased as the run times of the data collection increased. With the measured muon decay time decreasing in error, it is suggested that the Fermi coupling constants were also approaching the true value. In consideration of these findings, the results given by the datasets are aligned with their known measured values.

4.2. Analysis of Error

In the 3-hour data collection on January 23, it was found that the obtained value for mean lifetime had approximately 12% error. This data is shown in the figure 5. The error in the following data sets for 11-hour and 5-day run times had shown lower errors. The collected data for these data sets can be seen in figures 6 and 7 respectively. The measurement error arises because the measured mean lifetime is consistently lower than the true value provided by the Particle Data Group. However, this error was expected due to a few reasons.

As previously explained, the nature of positive and negative muons has a significant effect on the measured lifetime. Negative muons, when interacting with the scintillator, tend to have a shorter average lifetime due to their ability to bind with scintillator molecules instead of decaying. Thus, when taking the average of the muon decay time distribution, it is expected to be slightly lower than the true muon lifetime. This is clearly seen in the data sets collected. Interestingly, as the runtime for data collection increases, this error diminishes. The mean lifetime approaches the true value from below. Nonetheless, the effect of negative muons is still a significant factor to consider as the result of the measured decay time is less than the true measured value. Additionally, the detector records signals from any light emitted by the scintillator, regardless of the particle responsible. This means that the detector cannot distinguish whether a muon slows down and then decays, if two muons follow each other, or if other particles produce similar effects. This effect is worth considering as it can lead to "decay times" that do not align with the expected muon decay time distribution.

Furthermore, due to results that are still unknown, interference was observed in the detector's initial results that are not shown in this paper. It was hypothesised that the source of interference was caused by radioactive materials that were being worked on in a nearby lab room. Interestingly, when the radioactive materials were removed and placed a considerable distance away from the detector, the interference was still present. Consequently, the detector was set at a lower voltage to lessen the undetermined interference; instead of setting the high voltage to -800V on the detector, it was reduced to -630V. Although this may have caused minor alterations to the datasets, it is worth noting its effect as a source of error.

4.3. Further Research

Regarding future endeavours into this topic, there are many other interesting details that should be explored. For instance, the reduction in error with increasing run times of data collection brings forth the idea to do much longer run times on scales of weeks or months. Doing so would increase the accuracy in measurement of the measured mean lifetime of the muon as well as the Fermi coupling constant. Additionally, further research towards data collection during different times of the day is also of interest. This could be done by collecting muon decay times during day and night separately and analysing for any significant differences. Finally, other research could include demonstrations of the relativistic effects of the muon in its venture from the upper atmosphere to the surface of the earth. Although the detector used for this paper is not well equipped for such a study, it would be of great interest to be able to measure the time dilation effects by other means.

5. CONCLUSION

In this paper, data sets of muon decay times were collected over varying run times to validate the previously known mean lifetime for a muon and to determine the Fermi coupling constants implied by the measurement. The true value of the mean lifetime for a muon was taken from the Particle Data Group. In the first data set of an allotted run time of 3 hours, the measured mean lifetime was found to have approximately 12% error. The second data set, given a longer allotted run time of 11 hours, was measured to have a 3.5% error which showed a significant decrease when compared to the first data set. The final data set, having the longest run time of 5 days, presented the lowest percent error of 1.5% and a measured mean lifetime value of $\tau = 2.1628$. In the evaluation of all data sets, the Fermi coupling constant calculated from each measured muon lifetime approached a set value as run time increased. The suggested pattern of the data presented in the study significantly aligns with the mean lifetime of a muon as given by the Particle Data Group. Hence, the study aids in the validation of this measured value. With validation of intrinsic properties of fundamental particles such as the muon, the broader scientific community is aided in its reliability. By bridging theory and empirical evidence, this study contributes to the ongoing quest for deeper insights into the subatomic realm.

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