Using the Worstcase Solver - Demo 2

Timothy J. Wheeler
Dept. of Mechanical Engineering
University of California, Berkeley

Contents

Introduction

Consider a dynamic system of the form

$$\dot{x} = f(x,u)$$

$$y = g(x,u),$$

where x(0)=0. Given positive scalars B and T and a positive definite matrix C, the goal is to maximize

$$x(T)'Cx(T),$$

subject to the constraints

$$\|u\|_{2,T} := \int_0^T \|u(t)\|_2 \, dt \leq B.$$

Of course, since we are only interested in the value of `x` at time `T`, we only need to consider inputs defined on the interval `[0,T]`.

Create a model of the system.

First, `polynomial` variables are created using the `pvar` command. Then, these variables are used to define the functions `f` and `g`, which are also `polynomial` variables.

```
pvar x1 x2 u
states = [x1;x2];
inputs = u;
f = [ -x1 + x2 - x1*x2^2 ; -x2*x1^2 - x2 + u ];
g = states;
```

Then, a `polysys` object is created from the polynomials `f` and `g`.

```
sys = polysys(f,g,states,inputs);
```

The `polynomial` objects `states` and `inputs` specify the ordering of the variables. That is, by setting `states(1) = x1`, we specify that `f(1)` is the time derivative of `x1`.

Optimization parameters.

Use the following values for the optimization parameters (defined above):

```
T = 10;
B = 1;
C = eye(2);
```

The time vector `t` specifies the time window (`T=t(end)`) and the points at which the system trajectory is computed.

```
t = linspace(0,T,1000)';
```

Set options for worstcase solver.

Create a `@wcoptions` object that contains the default options.

```
opt = wcoptions();
```

Specify the maximum number of iterations and tell the solver to not display any information while solving.

```
opt.MaxIter = 500;
opt.PlotProgress = 'none';
```

Specify the desired cost function.

```
opt.Objective = 'Final';
opt.FinalCostMatrix = C;
```

Specify the bound on the input.

```
opt.InputL2Norm = B;
```

Find worst input.

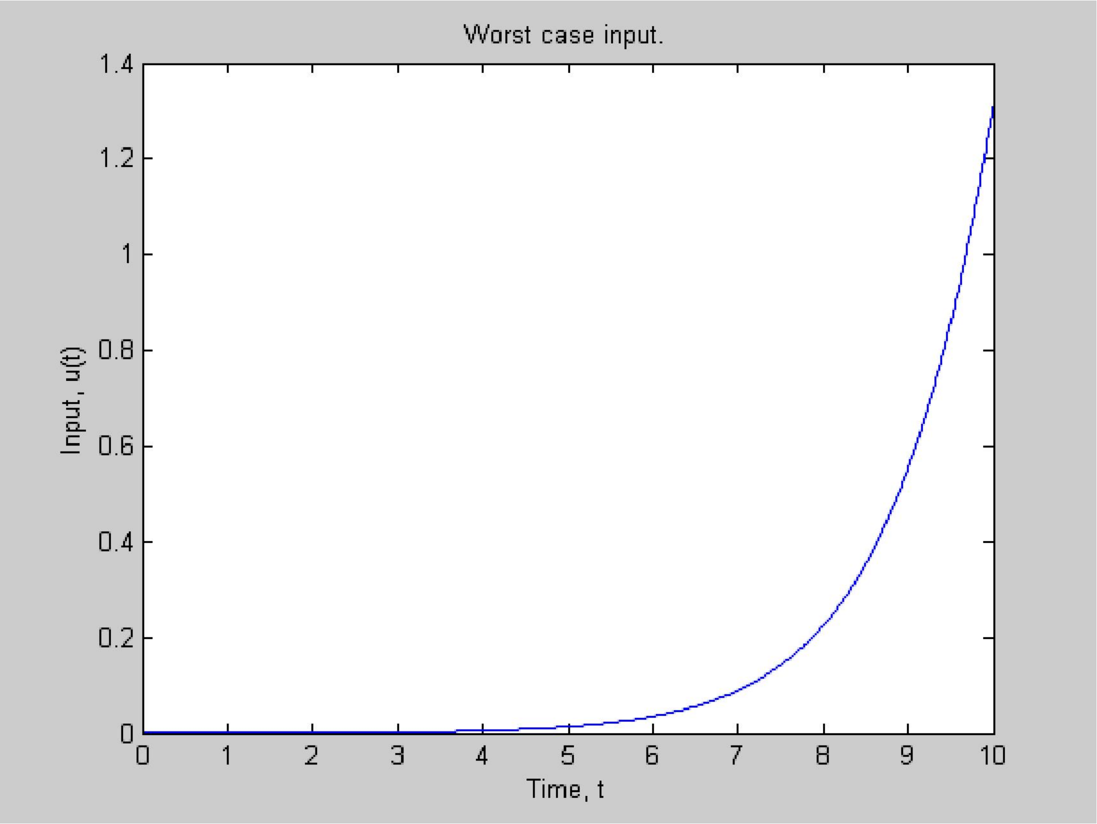
```
[tOut,x,y,u] = worstcase(sys,t,opt);
```

Display results.

```
cost = x(end,:)*C*x(end,:)';
fprintf( '||u|| = %0.4f, cost = %0.4f\n', B, cost );
```

```
figure;
plot(tOut,u)
xlabel('Time, t')
ylabel('Input, u(t)')
title('Worst case input.')
```

```
||u|| = 1.0000, cost = 0.5727
```



Published with MATLAB® 7.6