Servo Design Report

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Prepared for:
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April 7, 2025

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1 Introduction

In the previous lab assignment, system identification was performed to understand the servo plant dynamics. Ramp tests, step responses, and random input signals were used to find a transfer function that accurately describes the plant's behaviour. The servo system under consideration is a cascaded control structure, as shown in Figure 1.1 below.

This lab assignment focuses on designing controllers $G_1(s)$ and $G_2(s)$ to meet specific performance criteria. Controller $G_2(s)$ will be designed to control the speed loop, ensuring that the system's settling time is within 1 second, achieves zero steady-state error, and maintains overshoot below 20%. Additionally, it should reject output disturbances within 1 second. Controller $G_1(s)$ will regulate the position loop, ensuring that the controller output avoids saturation and that stability is maintained across varying plant conditions without significant oscillations.

This report will document design choices, calculations of controller transfer functions, and simulation results using Simulink to verify compliance with the outlined performance specifications.

The transfer functions describing the servo plant dynamics under different test conditions were identified as follows:

Nominal Case:

$$P(s) = \frac{4.0225}{1 + 1.17s}$$

Added Weight Case:

$$P_{weight}(s) = \frac{3.5075}{1 + 6.24s}$$

Added Magnetic Field Case:

$$P_{magnet}(s) = \frac{0.9936}{1 + 0.1s}$$

A scaling factor, $a = 9.11$, was also determined during the system identification process. This is an inherent physical property of the system and is therefore included in simulations in $P_1(s)$.

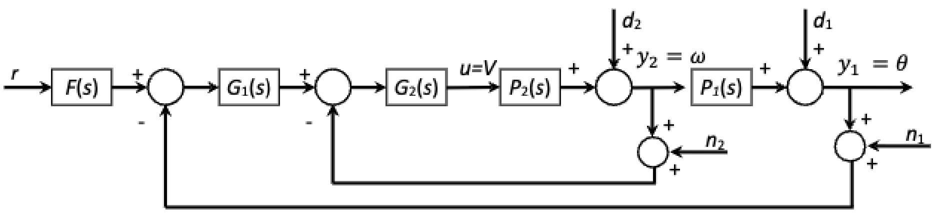

Figure 1.1: Servo system design

2 Design using proportional controller

The three plants are shown on the inverse Nichols plots in Figures 2.1, 2.2, and 2.3. According to the given specifications, the inner speed loop (the loop with controller $G_2(s)$) requires a proportional controller to meet the performance requirements. Looking at the nominal plant, a proportional gain of 3.5 is sufficient, as it lifts the low-frequency region above the -25 dB open-loop circle, which improves the system's tracking and disturbance rejection performance.

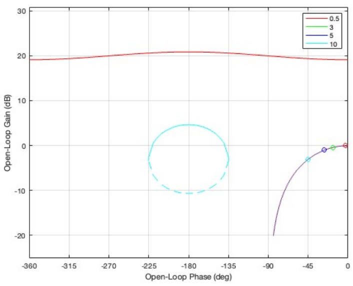

Figure 2.1: Inverse Nichols for the low plant

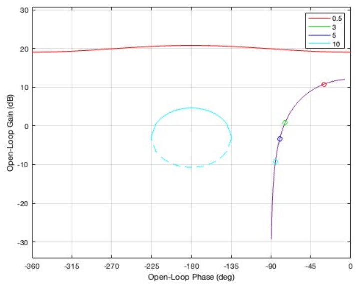

Figure 2.2: Inverse Nichols for the nominal plant

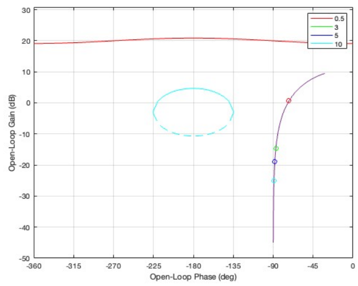

Figure 2.3: Inverse Nichols for the high plant

Applying this gain, the inverse Nichols plot of the nominal plant produces the following result.

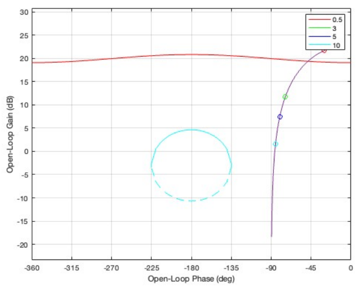

Figure 2.4: Inverse Nichols for the nominal plant with proportional gain controller

Once the speed loop was placed into closed-loop control, the inverse Nichols plot and step response of the full system were generated, with $G_1(s)$ set as a constant gain of 1.

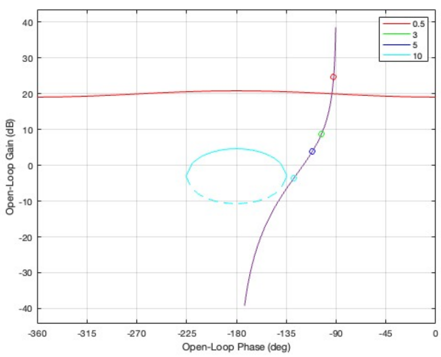

Figure 2.5: Inverse Nichols for entire system

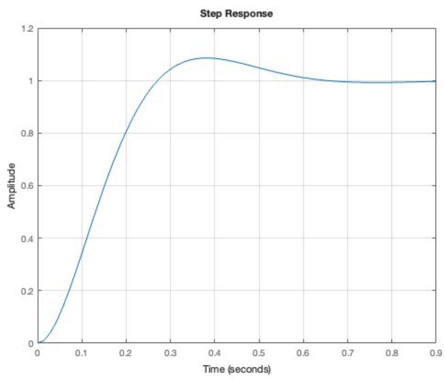

Figure 2.6: Step response for entire system

The system is able to meet the required specifications even before adding any position controller. To investigate this further, the system was simulated in Simulink, and the results are shown below.

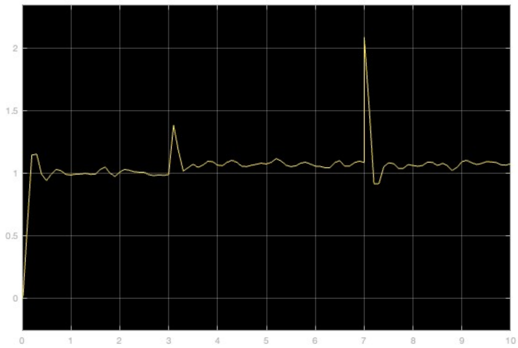

Figure 2.7: Output of the Simulink system

The system shows an overshoot of less than 20% and a settling time within 1 second. Disturbances to the speed loop and position loop are applied at 3 seconds and 7 seconds respectively, as shown in Figure 2.7. The disturbance applied to the speed loop caused a slight drop in tracking accuracy. Because of this, the controller design needs to be adjusted to handle the disturbances better.

3 Design using Proportional Integral Control

After implementing proportional control in the speed loop, it became clear that further improvements were needed—particularly in handling steady-state error and improving robustness to disturbances. To address this, a Proportional-Integral (PI) controller was designed for the outer position loop.

Just like in the speed loop, achieving good low-frequency gain is crucial for meeting the tracking accuracy specifications in the position loop. For this reason, we set a gain threshold of -20 dB. By examining the plant models on the Nichols Chart in Figure 2.2, we can determine how much additional low-frequency gain is needed to meet the requirements. This gain can be adjusted through testing to find a suitable value that meets the specifications, provided it is at least equal to or greater than the previous proportional controller gain of 3.5. After testing various values in Simulink, a gain of 18 was found to be the most effective. This value was determined purely through simulation-based tuning.

3.1 PI Controller Implementation

3.1.1 Design Objectives

The PI controller was designed to meet the following time-domain performance criteria: Settling time ≤ 1 s; Overshoot $\leq 20\%$; Zero steady-state error to step commands; and robust performance across a set of uncertain plants: P_{low}, P_{nom}, and P_{high}.

Specifications in Frequency and Time Domains

To satisfy these requirements, QFT bounds were generated based on the Nichols chart specifications. The following analysis focuses on the nominal plant.

Since the nominal plant in the speed loop is a first-order system, its pole is located at:

$$s = -\frac{1}{1.17} = -0.8547$$

To achieve a settling time of approximately 1 s, the system bandwidth should be around 4 rad/s. This implies that the open-loop response must stay outside the -3 dB circle on the Nichols Chart, which ensures sufficient bandwidth and thus a fast enough response.

To limit overshoot to $\leq 20\%$, the system must have a damping ratio of at least 0.456, which corresponds to a phase margin of roughly 55°.

To reject output disturbances and still settle within 1 s, the response should also stay outside the 3 dB sensitivity circle. Additionally, the system should have low gain in the mid-frequency range (from 1 to 10 rad/s) to avoid amplifying noise and disturbance effects.

3.1.2 Controller Structure

The PI controller takes the form:

$$G_{\mathrm{PI}}(s) = \frac{K_p s + K_i}{s}$$

where K_p provides a fast proportional response, and K_i introduces an integrator to eliminate steady-state error.

After tuning, the selected gains were:

$$K_p = 1, \quad K_i = 0.12$$

This places the PI zero at:

$$s = -\frac{K_i}{K_p} = -0.12$$

which improves phase margin near the crossover frequency and provides sufficient damping across all plant variations.

The PI controller zero was placed near the pole of P_{high} at -0.1603 to improve damping and reduce its long response time of around 25 s. This compensated for the high speed-loop gain, which increased bandwidth but reduced phase margin. Although tuned specifically for P_{high}, the controller improved the performance of all three plants, as they remained within the QFT bounds. No additional gain was needed due to the system's built-in scaling factor of 9.11, and the integrator ensured perfect steady-state tracking.

Implementing the PI controller resulted in the following Nichols charts:

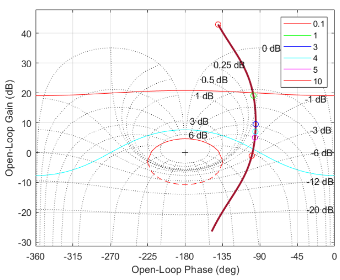

Figure 3.1: Nominal Plant with PI Control

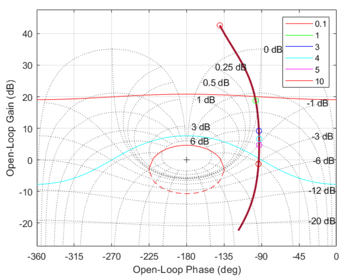

Figure 3.2: Low Plant with PI Control

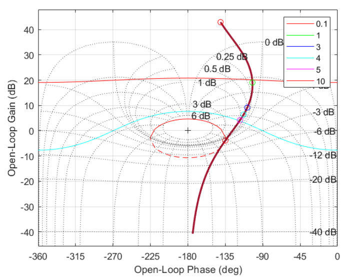

Figure 3.3: High Plant with PI Control

As seen in Figure 3.1, the nominal plant controlled by the PI controller has a good phase margin and meets all the specifications. In Figure 3.2, the low plant just touches the sensitivity circle but still maintains the required phase margin. This shows that the added phase margin from the PI controller is crucial.

To verify that the system meets the specifications, the closed-loop step response was plotted.

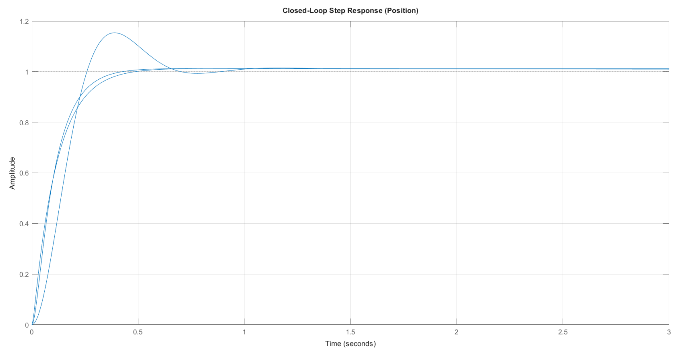

Figure 3.4: Closed-Loop Step Response

All responses show overshoot $\leq 20\%$, settle within 1 s, and track a step input without steady-state error.

3.1.3 Discrete-Time Implementation

To implement the controller in Simulink and prepare it for digital control, the continuous-time PI controller was discretised using MATLAB's built-in `pid()` function:

```
Ts = 0.01;  % Sample time
Gz_position = pid(Kp, Ki, 0, Ts); // G1 PI Control
```

This method was chosen over a manual `c2d()` conversion because `pid()` constructs a numerically stable difference equation representation, which is better suited for digital simulation and implementation. This approach successfully preserved the desired closed-loop performance in the z-domain, avoiding the instability and oscillations that were observed with earlier Tustin-based conversions.

4 Simulink Simulation

4.1 Integration in Simulink

The controller was connected to the cascaded loop structure shown in Figure 4.1.

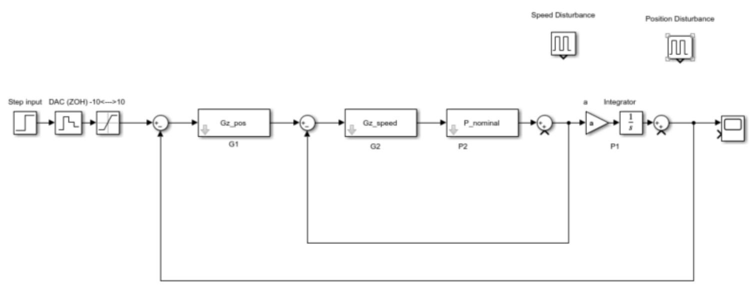

Figure 4.1: Simulink Implementation

To simulate switching plants, a subsystem was created:

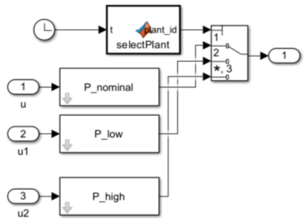

Figure 4.2: Simulating the switching of the plant models

Figure 4.2 works by receiving inputs $u1 - u2$ from the controller $G2$. The Matlab Function block then controls the switching at different time intervals ($t = 5, t = 10, t = 15$ and outputs the signal (1) back into the system. This allows us to simulate the response when the plant model changes.

4.2 Simulations

The following responses were recorded of our system, simulating the closed loop step response with speed and position disturbances.

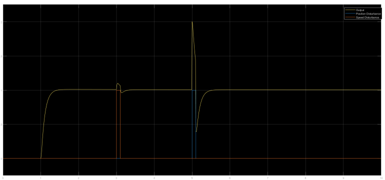

Figure 4.3: Step Response of nominal plant with speed and position disturbances

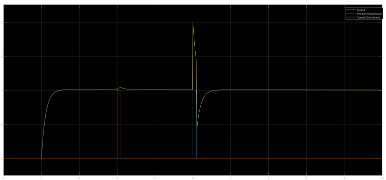

Figure 4.4: Step Response of low plant with speed and position disturbances

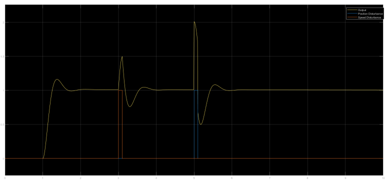

Figure 4.5: Step Response of high plant with speed and position disturbances

As we can see in figures 4.3-4.5, all the responses settle within $1s$ and have $\leq 20\%$ overshoot. Additionally, they reject disturbances and settle to the original value within $1s$.

We can use the subsystem created in 4.2 to simulate the switching between plant models.

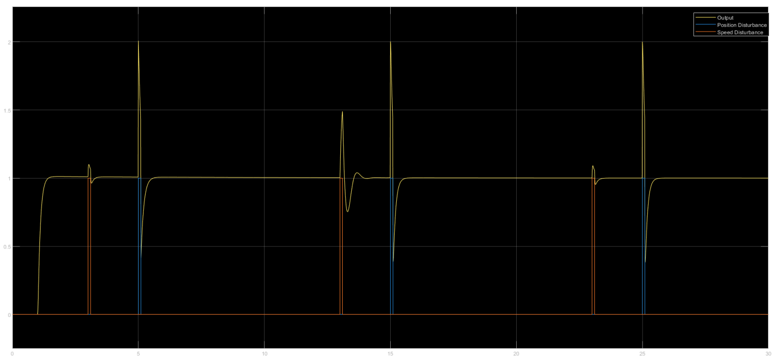

Figure 4.6: Step Response of system while switching plant models, with speed and position disturbances

As seen in Figure 4.6, the system handles the switching perfectly, while also handling the disturbances. Therefore, the designed and implemented controller is very robust and meets all the requirements.

The controller successfully met all time-domain specifications across different plant conditions, including during plant switching. Simulation results confirm the system's robustness, fast settling, low overshoot, and excellent disturbance rejection.