Milestone 4 - Report

'Save a City from an Asteroid'

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May 29, 2025

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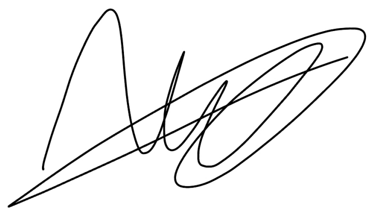

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Date

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Chapter 1

Introduction, Modelling, Control, Results and Conclusion

1.1 Introduction

This project simulates a scenario where Earth is threatened by an incoming asteroid. A rocket-based interception system must be developed to neutralise the threat before impact. All modelling and control design were conducted in MATLAB and Simulink.

To approach the problem methodically, the project was divided into three milestones. The first focused on deriving the rocket's equations of motion and estimating the asteroid's drag coefficient based on simulation data. The second milestone introduced control strategies for intercepting the asteroid under basic conditions. The final milestone presented a more complex challenge, requiring the rocket to detonate only if it approached the asteroid from within a strict strike-angle window.

This report summarises the modelling, control design and simulation results across all three milestones.

1.2 Rocket Modelling

The rocket in Figure A.1 is modelled as a planar rigid body with thrust vector control. It has mass $m = 1000\,\mathrm{kg}$, height $L_2 = 15\,\mathrm{m}$, width $L_1 = 5\,\mathrm{m}$ and a centre of mass offset $r = 3.5\,\mathrm{m}$ from the base. The thrust F is applied at an angle α relative to the rocket's body axis θ.

The dynamics were derived using Lagrangian mechanics and are presented in manipulator form. Full derivations and a diagram of the rocket are provided in Appendix A.

The resulting equations of motion are:

$$\ddot{x} = \frac{-F\sin(\theta+\alpha)}{m}, \quad \ddot{y} = \frac{F\cos(\theta+\alpha)}{m} - g, \quad \ddot{\theta} = \frac{-rF\sin(\alpha)}{I}$$

An idealised assumption was made that the rocket experiences no drag due to a "Miracle Spray" coating.

1.3 Asteroid Modelling

The asteroid is treated as a passive body subject to gravity and atmospheric drag. It has a mass of 10 000 kg and no control input.

To estimate the drag coefficient c, velocity data from the simulation was smoothed using a moving average filter. Acceleration was computed using the gradient method and the following equation was applied at each time step:

$$c = -\frac{m(\dot{x}\ddot{x} + \dot{y}\ddot{y}) + mg\dot{y}}{\dot{x}^2 + \dot{y}^2}$$

The instantaneous values were averaged to yield a final estimate used in Milestone 2 and 3 control logic.

The full derivation and filtering approach are provided in Appendix B.

1.4 Control Overview

A modular control block was implemented to handle all milestone scenarios. It received full state information for both the rocket and the asteroid, including positions, velocities, angles and angular rates. The controller also used the simulation time and scenario number to switch between control strategies.

The outputs from the controller were:

- Thrust magnitude F
- Steering angle α
- Detonation flag

Each scenario used a distinct control strategy: vertical ascent for 2a, trajectory-matching for 2b and proportional navigation with angular constraints for 3.

1.5 Scenario 2a — Vertical Ascent Interception

A vertical ascent strategy was selected due to the asteroid's distant initial position and purely horizontal velocity. This allowed a simple, open-loop approach that required minimal steering and relied on accurate timing. The rocket ascends straight up with a fixed steering angle of $\alpha = 0$ and the required thrust F is calculated to reach the asteroid's predicted vertical position at the estimated time of intercept. The asteroid was initialised at $x_0 = -3000\,\text{m}$, $y_0 = 5000\,\text{m}$, with horizontal velocity $\dot{x}_0 = 182\,\text{m/s}$ and no vertical motion. The rocket was launched from the origin at rest.

Time to Impact

The asteroid's horizontal motion was modelled under the influence of drag, with deceleration proportional to its velocity:

$$x(t) = x_0 + \dot{x}_0 t - \frac{1}{2}\frac{c}{m}\dot{x}_0 t^2$$

Solving for $x(t) = 0$ yields the time of intercept:

$$t_{\text{impact}} = \min\left(\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right), \quad \text{with } a = \frac{1}{2}\frac{c}{m}\dot{x}_0,\ b = -\dot{x}_0,\ c = x_0$$

The remaining flight time is:

$$t_{\text{remain}} = t_{\text{impact}} - t_{\text{sim}}$$

Vertical Prediction and Required Thrust

The vertical position of the asteroid at impact is predicted by:

$$y_{\text{target}} = y_0 + \dot{y}_0 t_{\text{remain}} + \frac{1}{2}\left(-\frac{c}{m}\dot{y}_0 - g\right) t_{\text{remain}}^2$$

The rocket must reach this target height using a constant acceleration profile. The required acceleration is:

$$a = \frac{2(y_{\text{target}} - y)}{t_{\text{remain}}^2}, \quad a_{\text{total}} = a + g$$

The thrust is calculated using Newton's second law:

$$F = m \cdot a_{\text{total}}$$

This open-loop strategy assumes perfect knowledge of the asteroid's state and was found to be reliable under nominal conditions. Robustness testing under increased asteroid velocity is discussed in Appendix C.

1.6 Scenario 2b — Steering and Thrust Control

In this scenario, the asteroid began much closer to Earth, requiring dynamic control of both the thrust magnitude F and steering angle α. A purely vertical strategy was no longer viable and the rocket needed to arc toward an offset interception point.

The asteroid's initial state was:

$$x_0 = -2000\,\mathrm{m}, \quad \dot{x}_0 = 170\,\mathrm{m/s}, \quad y_0 = 4000\,\mathrm{m}, \quad \dot{y}_0 = 0$$

Intercept Time Estimation Using Drag

The controller used a closed-form solution of the drag equation for horizontal motion:

$$t_{\text{impact}} = -\frac{m}{c} \ln\left(1 - \frac{\Delta x \cdot c}{m\dot{x}_0}\right)$$

This provided a more accurate estimate than the quadratic approximation used in Scenario 2a.

Vertical Position Prediction

The vertical motion, accounting for linear drag and gravity, was computed as:

$$y_{\text{target}} = y_0 + \frac{m}{c}\left(\dot{y}_0 + \frac{mg}{c}\right)\left(1 - e^{-ct/m}\right) - \frac{mg}{c}t$$

Control Calculation

The desired steering angle was:

$$\alpha = \operatorname{atan2}(y_{\text{target}} - y_r,\, x_{\text{target}} - x_r) - \theta$$

The required vertical acceleration was computed with:

$$a_y = \frac{2(y_{\text{target}} - y_r - \dot{y}_r t)}{t^2}$$

The final thrust magnitude was:

$$F = m(a_y + g)$$

Steering PID Controller Implementation

The desired steering angle α was tracked using a PID controller implemented in Simulink. This allowed the rocket to continuously align its orientation toward the predicted interception point throughout the flight. The block diagram of the controller is shown in Figure 1.1.

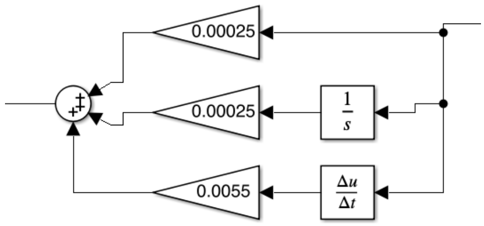

Figure 1.1: Simulink implementation of PID controller used for steering angle α

The controller receives the angular error between the desired direction and the rocket's heading and outputs a commanded steering angle. The PID gains were tuned manually to balance responsiveness and stability:

- Proportional gain K_p: scaled the immediate angular error
- Integral gain K_i: corrected for persistent offset in tracking
- Derivative gain K_d: damped rapid fluctuations in error

Gains were tuned manually by testing for minimal overshoot and rapid convergence. For example, increasing K_d improved damping during sharp steering corrections, while lowering K_i avoided windup during long tracking periods. Therefore the respective gain values were chosen as 0.00025, 0.00025 and 0.0055 respectively as seen in Figure 1.1. These had to be very small values since larger gains resulting in aggressive corrections and continuous spinning.

The output of the controller was saturated to lie within the physical bounds:

$$\alpha \in [-45^\circ,\ +45^\circ]$$

This approach successfully guided the rocket through curved trajectories toward the target, with gains selected to minimise overshoot and instability during flight.

Detonation Trigger

The system triggered detonation if the Euclidean distance to the asteroid was within 150 m:

$$\sqrt{(x_{\text{ast}} - x_r)^2 + (y_{\text{ast}} - y_r)^2} \leq 150$$

This control approach enabled successful interceptions across all 10 tests. However, the strategy was more sensitive to prediction error and drag estimation compared to the fixed-angle solution in Scenario 2a.

1.7 Scenario 3 — Proportional Navigation and Line-of-Sight Control

Scenario 3 introduced a directional constraint: the rocket must not only intercept the asteroid, but do so from within a $\pm 30^\circ$ window relative to the asteroid's heading. This required a more complex guidance system that accounted for both positional and angular accuracy. Proportional Navigation (PN) was implemented alongside Line-of-Sight (LOS) logic and a PID controller to guide the steering angle [1].

Relative Geometry and LOS Rate

The rocket receives full state vectors of both itself and the asteroid. Using this, the relative position and velocity vectors are calculated as:

$$\vec{r}_{\text{rel}} = \vec{r}_{\text{ast}} - \vec{r}_{\text{rocket}}, \quad \vec{v}_{\text{rel}} = \vec{v}_{\text{ast}} - \vec{v}_{\text{rocket}}$$

The LOS angle and its angular rate are then:

$$\lambda = \operatorname{atan2}(y_{\text{rel}}, x_{\text{rel}}), \quad \dot{\lambda} = \frac{x_{\text{rel}}\dot{y}_{\text{rel}} - y_{\text{rel}}\dot{x}_{\text{rel}}}{\|\vec{r}_{\text{rel}}\|^2}$$

The closing velocity, which quantifies how quickly the rocket is converging on the asteroid, is computed as:

$$v_c = \max\left(-\frac{\vec{v}_{\text{rel}} \cdot \vec{r}_{\text{rel}}}{\|\vec{r}_{\text{rel}}\|}, 0.1\right)$$

A lower bound was used to avoid division-by-zero or stagnation near intercept.

Acceleration Calculation

The PN strategy outputs lateral and tangential accelerations designed to steer the rocket smoothly onto an intercept course:

$$a_\perp = N v_c \dot{\lambda}, \quad a_t = K_t v_c$$

The gain N is increased when the rocket is near the asteroid (e.g., range < 1500 m) to allow more aggressive course corrections during the final approach. The unit vectors used to orient these accelerations are:

$$\hat{r}_{\text{rel}} = \frac{\vec{r}_{\text{rel}}}{\|\vec{r}_{\text{rel}}\|}, \quad \hat{r}_\perp = \begin{bmatrix} -\hat{r}_y \\ \hat{r}_x \end{bmatrix}$$

The overall desired acceleration vector includes gravity compensation:

$$\vec{a}_{\text{des}} = a_t \hat{r}_{\text{rel}} + a_\perp \hat{r}_\perp + \begin{bmatrix} 0 \\ g \end{bmatrix}$$

Steering and Thrust Control

The required force to achieve the desired acceleration is:

$$\vec{F}_{\text{inertial}} = m \cdot \vec{a}_{\text{des}}$$

To convert this to commands for the rocket actuators, the force vector is rotated into the rocket's body frame:

$$\vec{F}_{\text{body}} = R(\theta)^\top \vec{F}_{\text{inertial}}$$

The required thrust magnitude and steering angle are then:

$$F = \|\vec{F}_{\text{body}}\|, \quad \alpha_{\text{des}} = \operatorname{atan2}(F_y, F_x)$$

A discrete PID controller is used to track α_{des}, accounting for angular error, integral windup and damping:

$$\alpha_{\text{cmd}} = K_p e_\alpha + K_i \int e_\alpha \, dt + K_d \frac{de_\alpha}{dt}, \quad e_\alpha = \alpha_{\text{des}} - \theta$$

The error is wrapped to $[-\pi, \pi]$ using a modulo function to avoid discontinuities and the output is saturated to:

$$\alpha \in [-45°, +45°]$$

PID gains were tuned manually with $K_p = 0.05$, $K_i = 1 \times 10^{-6}$ and $K_d = 0.05$. This achieved a balance between responsiveness and stability, though occasional overshoot was observed during sharp direction changes.

Detonation Constraint

Detonation was conditioned not only on proximity, but also on angular alignment with the asteroid's velocity vector. The relative bearing is defined as:

$$\phi = \text{atan2}(y_{\text{rel}}, x_{\text{rel}}) - \theta_{\text{ast}}$$

Detonation occurred only if:

$$|\phi| \leq 30° \quad \text{or} \quad |\phi - 180°| \leq 30°$$

Additionally, a safety fallback was implemented: if the rocket altitude dropped below 500 m, it would apply full thrust straight upward ($\alpha = 0$) to avoid collision with Earth.

1.8 Milestone 1 Results

In Milestone 1, the rocket dynamics were derived using Lagrangian mechanics and expressed in manipulator form:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = Q(F, \alpha)$$

where $q = [x, y, \theta]^\top$. The resulting scalar equations of motion were:

$$m\ddot{x} = -F \sin(\theta + \alpha) \tag{1.1}$$

$$m(\ddot{y} + g) = F \cos(\theta + \alpha) \tag{1.2}$$

$$I\ddot{\theta} = -Fr \sin(\alpha) \tag{1.3}$$

These equations governed the rocket's motion throughout all subsequent milestone controllers. Full matrix definitions and derivations are provided in Appendix A.

Drag Coefficient Estimation

To model the asteroid's motion, the drag coefficient c was estimated using velocity and acceleration data from the simulation. The data was smoothed using a moving average filter and acceleration was computed using MATLAB's `gradient` function.

The estimated value of c was obtained by averaging instantaneous values computed over time (full expression in Appendix B). The final result is shown in Figure A.2, included in the appendix.

The drag coefficient was estimated as $c \approx 102$, consistent with expected values and used in later trajectory prediction and control calculations.

1.9 Milestone 2a Results

The vertical interception controller for Scenario 2a performed reliably and consistently across all simulations. The rocket followed a straight vertical trajectory with a fixed steering angle $\alpha = 0$ and triggered detonation within 150 m of the asteroid, meeting the milestone requirements as seen in Figure 1.2.

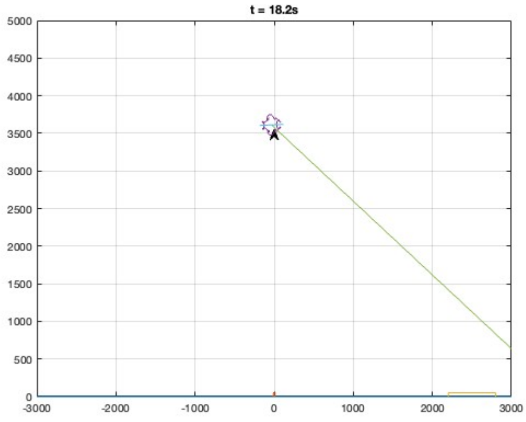

Figure 1.2: Successful interception and detonation in Milestone 2a

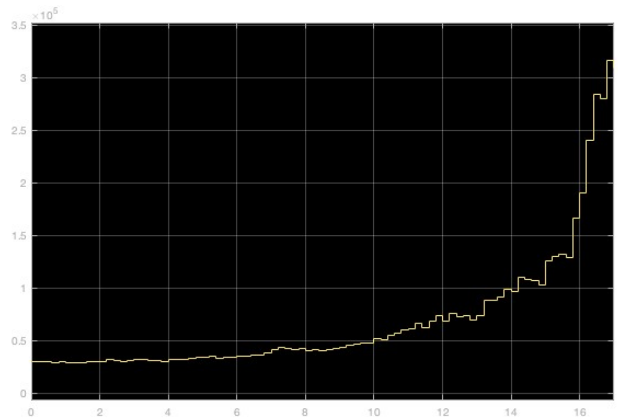

Figure 1.3: Thrust force applied by the rocket over time (nominal asteroid speed)

The thrust increased as the rocket approached the intercept point as seen in Figure 1.3, reflecting the required acceleration to reach the asteroid's projected position in time.

Additional robustness testing was also performed to evaluate the controller's performance under more demanding conditions. These results are included in Appendix C.

1.10 Milestone 2b Results

Milestone 2b introduced a tighter timeframe and required dynamic control of both thrust magnitude F and steering angle α. The previously used fixed-angle vertical strategy was no longer sufficient and a fully adaptive controller was applied.

As shown in Figure 1.4, the rocket successfully intercepted the asteroid before it could reach Earth, satisfying all scenario requirements.

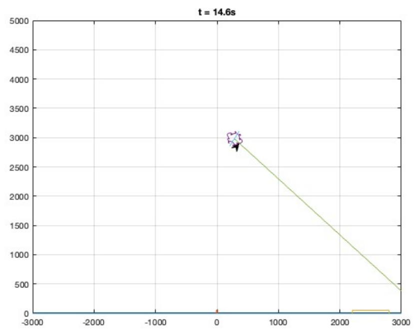

Figure 1.4: Successful interception and detonation in Milestone 2b

Both the thrust and steering angle evolved in real time during the flight. The thrust increased closer to intercept, while the steering angle adapted continuously to guide the rocket laterally. These plots and further analysis are included in Appendix D.

The system remained stable throughout, with all 10 simulation runs resulting in successful interception within the required distance and angle limits.

1.11 Milestone 3 Results

Milestone 3 introduced a strike-angle constraint: the rocket had to approach the asteroid from within a 30° arc relative to its heading.

Out of 10 simulations, 5 runs met both positional and directional requirements. One successful example is shown in Figure 1.5.

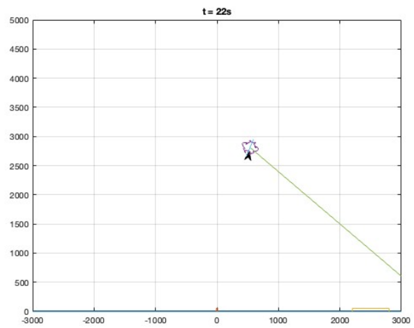

Figure 1.5: Successful interception and detonation in Milestone 3

The proportional navigation controller was able to adjust thrust and steering in real-time to reach the target under angular constraints. However, tuning proved difficult due to variations between runs. Additional analysis of the steering and thrust behaviour is provided in Appendix E.

Table 1.1: Summary of Scenario Results

Scenario	Strategy	Success Rate	Intercept Time (s)	Thrust Peak (N)
2a	Vertical Ascent Interception	10/10	18.2	3.2×10^5
2b	Steering and Thrust Control (PID)	10/10	14.6	0.8×10^6
3	PN + PID + LOS	5/10	22	2.3×10^6

A summary of scenario outcomes is provided in Table 1.1, including success rates and key metrics.

1.12 Conclusion

This project successfully demonstrated the interception of an asteroid using a simulated rocket controller across progressively challenging scenarios. The rocket and asteroid were accurately modelled and the asteroid's drag coefficient was effectively estimated from simulation data.

Controllers were designed and implemented to meet the specific requirements of each milestone. Milestones 2a and 2b performed reliably, showcasing the effectiveness of vertical and angle-controlled interception strategies. Milestone 3 introduced directional constraints and parameter uncertainty, which led to more varied outcomes. While consistent performance proved more difficult, the control strategy still achieved multiple successful interceptions.

Key challenges included gain tuning under time pressure and ensuring stability when strict angular constraints were applied. Future improvements could involve adaptive control strategies, more robust angle enforcement, or closed-loop trajectory replanning to enhance reliability under uncertainty.

Bibliography

[1] P. Kennedy and R. Kennedy, “Direct versus indirect line of sight (los) stabilization,” *IEEE Transactions on Control Systems Technology*, vol. 11, no. 1, pp. 3–15, 2003.

[2] E. M. I. Staff, “Save a city from an asteroid – milestone 1 brief,” 2025, accessed from project handout.

Appendix A

Appendix

A Rocket Modelling Details

Rocket Parameters

Table A.1: Rocket physical parameters

Parameter	Symbol	Value
Mass	m	1000 kg
Width	L_1	5 m
Height	L_2	15 m
CoM offset	r	3.5 m
Gravity	g	9.81 m/s^2

Equations of Motion Derivation

The rocket is modelled as a planar rigid body with mass m, height L_2, width L_1, and a centre of mass offset r. Its state is described by generalised coordinates $q = [x, y, \theta]^\top$, with corresponding velocities $\dot{q} = [\dot{x}, \dot{y}, \dot{\theta}]^\top$.

The position vector in the inertial frame is:

$$\vec{r} = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}, \quad \vec{v} = \frac{\partial \vec{r}}{\partial q}\dot{q}$$

The angular velocity vector is:

$$\vec{\omega} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta} \end{bmatrix}$$

The moment of inertia about the centre of mass is:

$$J_{zz} = \frac{1}{12} m(L_1^2 + L_2^2) + m\left(\frac{L_2}{2} - r\right)^2$$

The inertia tensor becomes:

$$I_r = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & J_{zz} \end{bmatrix}$$

The total kinetic energy is the sum of translational and rotational parts:

$$T_{\text{Linear}} = \frac{1}{2} m \vec{v}^{\top} \vec{v}$$
$$T_{\text{Angular}} = \frac{1}{2} \vec{\omega}^{\top} I_r \vec{\omega}$$
$$T = T_{\text{Linear}} + T_{\text{Angular}} = \frac{1}{2} m \vec{v}^{\top} \vec{v} + \frac{1}{2} \vec{\omega}^{\top} I_r \vec{\omega}$$

The potential energy is:

$$V = mgy$$

The Lagrangian is:

$$L = T - V$$

Using Lagrangian mechanics, the equations of motion are expressed in manipulator form:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = Q(F,\alpha)$$

The system matrices are:

$$M = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & J_{zz} \end{bmatrix}, \quad C = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad G = \begin{bmatrix} 0 \\ mg \\ 0 \end{bmatrix}$$

The generalised force vector is:

$$Q = \begin{bmatrix} -F\sin(\theta+\alpha) \\ F\cos(\theta+\alpha) \\ -rF\sin(\alpha) \end{bmatrix}$$

Rocket Diagram

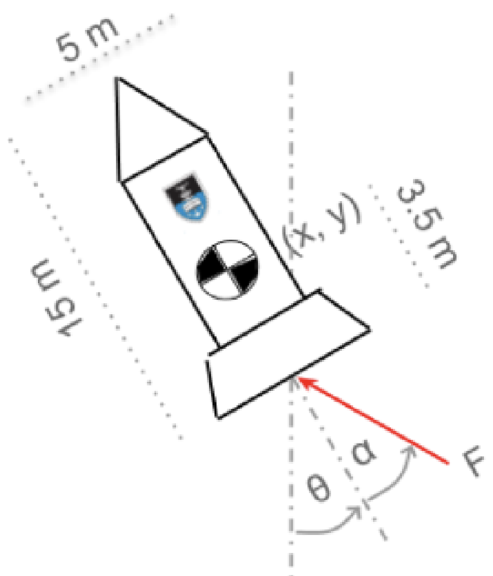

Figure A.1: Rocket schematic showing thrust vector and CoM [2]

B Drag Coefficient Derivation

The asteroid is acted upon by two forces: gravitational force and aerodynamic drag. The drag force is proportional to the asteroid's speed and acts opposite to its direction of motion:

$$F_{\text{drag}} \propto c\sqrt{\dot{x}^2 + \dot{y}^2}$$

To estimate the drag coefficient c, Newton's second law was applied in both the horizontal and vertical directions:

$$m\ddot{x} = -c\|\vec{v}\|\dot{x}, \quad m\ddot{y} = -c\|\vec{v}\|\dot{y} - mg$$

Multiplying each equation by its respective velocity and summing yields:

$$m(\dot{x}\ddot{x} + \dot{y}\ddot{y}) = -c\|\vec{v}\|(\dot{x}^2 + \dot{y}^2) - mg\dot{y}$$

Rearranging to solve for c:

$$c = -\frac{m(\dot{x}\ddot{x} + \dot{y}\ddot{y}) + mg\dot{y}}{\dot{x}^2 + \dot{y}^2}$$

The velocity data was smoothed using a moving average filter (window size 20), and accelerations were computed using MATLAB's `gradient` function to accommodate uneven time steps. The final value of c was obtained by averaging the instantaneous values over time.

Drag Estimation

This demonstrates the controller's ability to generalise beyond nominal parameters, indicating a degree of built-in robustness in the vertical ascent strategy.

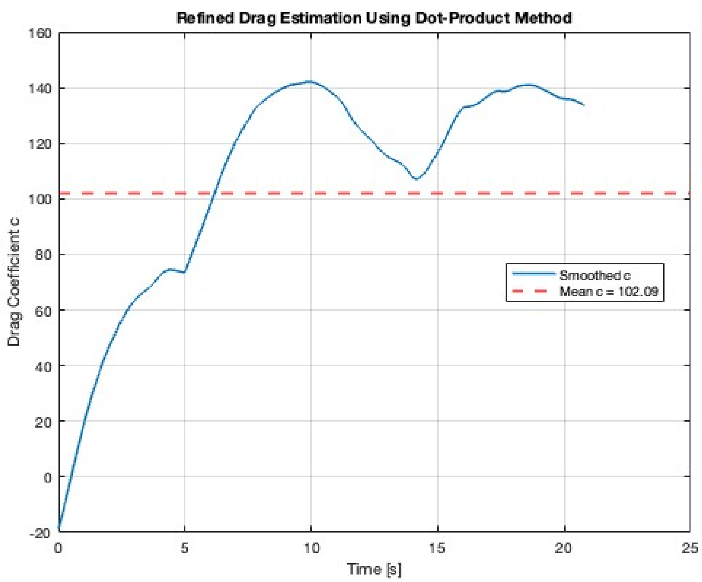

Figure A.2: Estimated drag coefficient c over time. The red line represents the smoothed average value.

C Milestone 2a — Robustness Test

To assess the robustness of the Scenario 2a controller, a test was conducted where the asteroid's initial horizontal speed was increased from 182 m/s to 200 m/s.

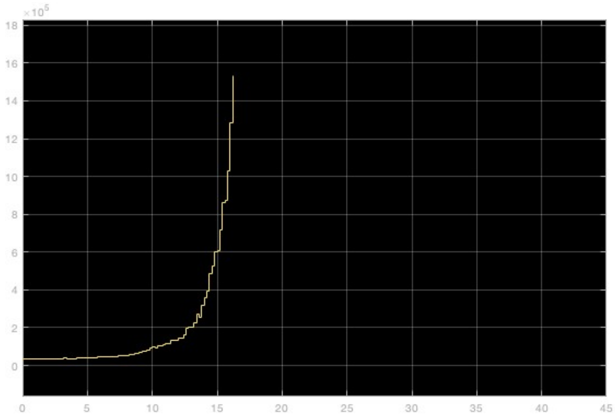

Figure A.3: Thrust force applied with increased asteroid speed (200 m/s)

As seen in Figure A.3, the required thrust reached nearly $F \approx 1.6 \times 10^6$ N, which is much greater than the nominal value of $F \approx 3.2 \times 10^5$ N. Despite the increased demand, the rocket was able to intercept successfully without any controller modifications.

D Milestone 2b — Thrust and Steering Behaviour

The following plots illustrate the control behaviour during Milestone 2b:

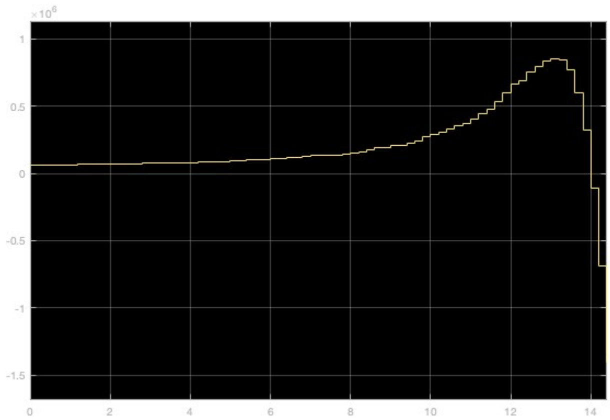

Figure A.4: Thrust force profile during Milestone 2b

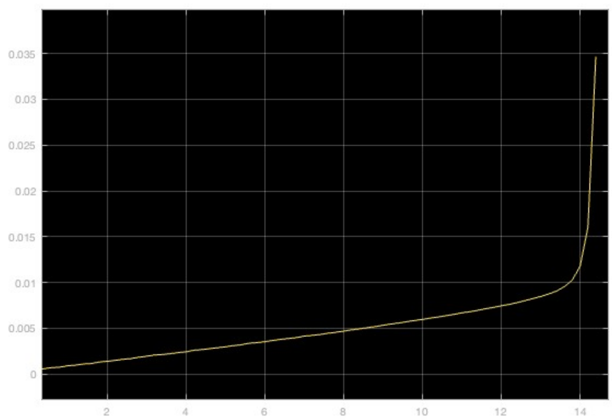

Figure A.5: Steering angle α throughout the flight

Figure A.5 shows that the steering angle α varied continuously, staying within the limit of $\pm 45°$. This confirms the controller's ability to track the target trajectory while maintaining actuation limits.

Figure A.4 shows how the thrust increased as the rocket closed in on the asteroid, with some fluctuation due to the dual-axis control. The maximum force required was slightly higher than in Milestone 2a.

The average intercept time was approximately 14.4 seconds, and the detonation distance was consistently within the 150 m radius. All 10 test runs were successful.

E Milestone 3 — Steering and Thrust Behaviour

The following plots correspond to the successful run shown in Figure 1.5 of the main report:

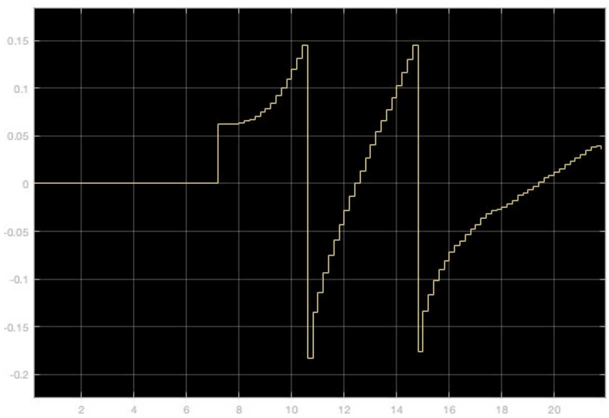

Figure A.6: Steering angle α during Milestone 3

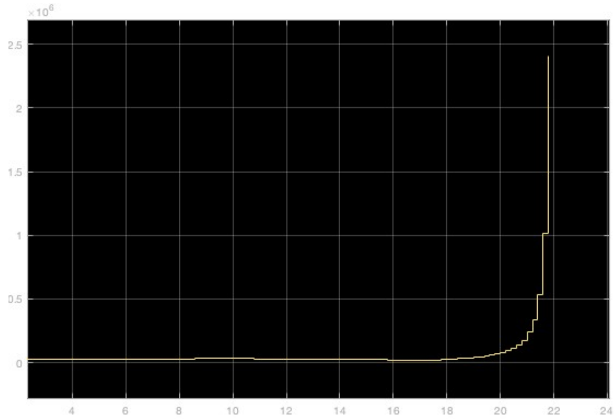

Figure A.7: Thrust force profile during Milestone 3

The steering angle plot in Figure A.6 shows frequent sharp changes. These correspond to aggressive flips as the controller attempted to align with the valid strike angle zone. The PID control loop occasionally overshot, especially when the initial heading was misaligned or when LOS rate changed rapidly.

Figure A.7 shows that the thrust varied more significantly than in previous milestones due to the combined tangential and normal acceleration components in the proportional navigation logic. Peak thrust during this run reached approximately $F \approx 2.3 \times 10^5$ N.

The average intercept time across successful runs was 22 seconds. Unsuccessful attempts often failed due to angular misalignment or unstable controller gains under high-speed scenarios.