Houses Divided: a model of intergenerational transfers, differential fertility and wealth inequality*

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Abstract

Increasing income and wealth disparity in the United States has prompted a renewed focus on the mechanism of inequality. A major puzzle of this phenomenon is that wealth inequality is more pronounced than income inequality. This paper contributes to the literature by studying the impact on wealth inequality from savings, bequests, and fertility differences between the rich and the poor. We find that bequests have a significant impact on wealth inequality and the fertility difference between the rich and the poor amplifies the impact of bequests. In addition, we find that life-cycle saving and anticipated bequests interact with each other, and this interaction is important for fully understanding wealth inequality in the United States.

Keywords: Intergenerational transfers, endogenous fertility, wealth inequality, life-cycle savings.

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1. Introduction

Recent US data shows a large concentration of wealth among few of its citizens. The top 1 percent holds nearly one third of the total wealth in the economy, and that share is growing (Alvaredo, Atkinson, Piketty, Saez 2013). The top 5 percent holds over half. This trend has been accelerating in the years since the 2008 financial crisis (Saez and Zucman 2014). In addition, wealth inequality is significantly higher than labor earnings or total income inequality. In 1995 the Gini coefficient for annual labor earnings was .61 (Budria, Diaz-Gimenez, Quadrini and Rios-Rull 2002) and long term earnings was between .4 and .5 (Leonesio and Del Bene 2011; Saez, Kopzuk, Song 2010). The Gini for wealth holding was much higher, at .8 (Budria, et al. 2002). Understanding the reasons for this relatively greater level of wealth inequality is important for the economic consequences faced by highly unequal economies, such as greater societal unrest and lower intergenerational mobility.

One of the major puzzles surrounding wealth inequality is why it is so much more pronounced than income inequality. Wealthy individuals act in a different way than would be expected by traditional economic models, relatively saving more and spending less, even as they reach the end of their lifespans (Dynan, Skinner, and Zeldes 2004). In addition to this, wealthier people are much more likely to give bequests to their children at the end of their lives, even when relative wealth is accounted for. Historically the amount of wealth derived from intergenerational transfer has varied between one-tenth and one-fifth (Modigliani 1988), however, more recent estimates place it as high as one half (Gale and Scholz 1994). The top 2% of households receive nearly 70% of lifetime inheritances (Hendricks 2001).

Standard dynamic models with heterogeneous agents have a difficult time replicating this savings behavior and targeting the level of wealth inequality seen in the data. For instance, Aiyagari (1994) predicts in a calibrated simulation the top one percent will hold four percent of the wealth, while empirically the top one percent holds thirty percent. Why do rich people choose to possess such a high level of wealth instead of increasing their consumption?

Accounting for fertility decisions could be crucial to explaining wealth and income inequality. It has been made clear that there exists an inverse relationship between income and fertility (Hurd and Smith 2002). According to the U.S. Census Bureau, births per one thousand women are 98.3 for women with family income below 10,000 and 54.8 for women with family income above 75,000. This is a significant difference and has major ramifications for an overlapping generations model attempting to capture intergenerational transfers of ability and bequests. Knowles (2000) finds in a 2-period OLG model with intergenerational transfers and endogenous fertility that fertility decisions help explain the concentration of wealth in the richest families.

This paper uniquely contributes to the literature by combining multiple channels of intergenerational transfers in a model with endogenous fertility and life-cycle savings. This will provide a more accurate model of wealth inequality, income mobility and the ramifications these factors have for the members of this economy. This model incorporates a retirement period. This will add a dynamic interaction between bequests and savings, allowing a bequest motive to increase lifetime savings and expected bequest received to crowd out savings. If a child is expecting a large bequest from their parent they will choose to save less, so that the bequest and savings decisions become linked. This paper will show that bequest inequality is key for explaining the large level of wealth inequality, as well as calculate that fertility differences between rich and poor accounts for over 10 % of total wealth inequality.

In this paper, we wish to accomplish three goals. First is to build and run an overlapping generations model that includes endogenous fertility and intergenerational transfers. Second is to mirror the negative income-fertility relationship seen in the data. Third is to match the wealth-income inequality disparity seen in the data, specifically a result showing wealth inequality that is higher than income inequality.

The rest of paper is organized as follows. In section 2, we describe the existing literature. In section 3, we describe the model and its stationary equilibrium. In section 4, we calibrate a benchmark specification using moment matching. In section 5, we discuss the results. In section 6, we conduct several robustness tests. The final section concludes.

2. Literature Review

Inequality and its causes have become a political and economic touchstone in recent years. However, defining what exactly is unequal is often left unsaid by the bumper stickers. There exists unequal distributions of productivity, income, wealth, consumption, bequests, shocks, choices, etc. Some of these elements, especially income and wealth, are treated as if they are equivalent. But the data shows large differences in the distributions of income and wealth in the United States. As found by Diaz-Gimenez et al. (1997), the correlations between earnings and wealth and between income and wealth are surprisingly low, 0.230 and 0.321, respectively.

In 1992 the United State's Gini indexes for short term labor earnings, income, and wealth were, respectively, .63, .57, and .78 (Diaz-Gimenez et al. 1997), while in 1995 they were .61, .55 and .80 (Budria et al. 2002). The shares of earnings and wealth of the households in the top 1 percent of the corresponding distributions are 15 percent and 30 percent, respectively (Castaneda, Diaz-Gimenez and Rios-Rull 2003).

Standard quantitative macroeconomic models have had difficulties in generating the observed degree of wealth concentration (De Nardi and Yang 2015). Specifically, these models fail to account for the extremely long and thin top tails of the distributions and for the large number of households in the bottom tail (Castaneda et al 2003, Quadrini and Rios-Rull 1997). However, if it is intergenerational transmission of wealth and ability that drives wealth inequality, as Kotlikoff and Summers (1981) have argued, then a focus on life-cycle saving will fail to capture the relevant causes. Overlapping generations are an improvement at mimicking the data. Huggett (1996) predicts that the top one percent will hold seven percent of the wealth. This model only accounted for accidental bequests, distributed equally to all individuals.

Allowing for transfer of ability and human capital across generations enables a yet more realistic model. Many models, including Kotlikoff and Summers (1981), Knowles (2000), De Nardi (2004) and De Nardi and Yang (2015), account for high ability (or high luck) parents being more likely to have high ability (or high luck) children. Lee, Roys and Seshadri (2015) find that parental education is positively related to their children's earnings, creating a virtuous cycle for the wealthiest, and a vicious cycle for the poorest. Introducing preference and patience heterogeneity can generate increased wealth inequality, as shown in Heer (1999), who uses a model that allows different preferences for leaving bequests depending on wealth. Papers looking at the effect of the estate tax on inter-vivos giving show a clear bequest motive (Bernheim, Lemke and Scholz. 2004), indicating that accidental bequests are not the only cause of testamentary transfer.

There has been disagreement in the literature about bequest motives, specifically whether parents are perfectly altruistic towards their children. Altonji, Hayashi and Kotlikoff (1997) strongly reject the perfect altruism hypothesis. While they find that parents increase their giving to their children by a few cents from each extra dollar they have, the giving does not show the relationship between size of transfer and the child's income that is predicted by perfect altruism. They find that a one dollar transfer from child to parent results in only a 13 cent donation from parent to child, which should be the full dollar under perfect altruism. This theory is explored by Lee and Tan (2017) who use an exogenous unforeseen increase in Social Security payments to show that there is a positive relationship between level of income and pass through rate (the amount of an additional dollar in income an individual will leave as a bequest). This varied greatly, with the average individual giving 50 % of an addition dollar while the richest gave nearly 100%.

Altonji, Hayashi and Kotlikoff (1992) found that the division of consumption and income within a family are dependent, indication that perfect altruism does not apply to operative transfers. Other studies have gone on to show that an increase in parental resources coupled with a decrease in child consumption does not lead to a corresponding increase in transfers (Altonji, et al. 1997; Cox 1987). Siblings generally receive equally divided inheritances, rather than the size of the inheritance being dependent on relative income as perfect altruism would predict (Wilhelm 1996).

There has been multiple papers that argue that bequest giving is crucial to explaining wealth differentials. Most recently, De Nardi (2004) and De Nardi and Yang (2015) incorporates bequest leaving into the utility function as a luxury good, allowing for rich parents to value bequests more. If bequests are a luxury good such that the rich gain greater utility from leaving them, then greater inequality in bequests, and subsequently wealth, is generated. This is due to the emergence of large estates, or dynasties, where wealthy parents have well educated, high productivity children who they then leave large bequests to. These persistent rich often have smaller families, leading to greater relative concentration. This is consistent with Jones and Schoonbroodt (2016) that smaller cohorts receive relatively large per child transfers from parents.

Bequests represent a large piece of intergenerational transfers. Gale and Scholz (1994) use the Survey of Consumer Finances to find the amount of inter-vivos transfers and inheritance from 1983-85. Between support given, college expenses paid and inheritance given, the amount totaled over $350 billion. Of this, inheritance was nearly 40 percent and over 60 percent of those who reported receiving inheritance were in the top decile. Their central estimate is that intended life-time transfers (which they define as inter-vivos transfers, trust accumulations, and life insurance payments to children) account for at least 20 percent of aggregate net worth, and bequests, accidental or intended, account for 31 percent more. Kopczuk and Lupton (2007) find that three-fourths of the elderly single population has a bequest motive and about four-fifths of their net wealth will be bequeathed, half of which is due to a bequest motive as opposed to accidental bequests. This ratio is consistent with Lee and Tan (2017) and Hendricks (2002). Hendricks also finds that the effects of capital income taxes are nearly invariant to assumptions about bequest motives as well as to reasonable variations in the size of bequest flows. This means that parents are likely not altruistic and care about giving amounts ante-taxation.

3. Model

This is a three period OLG model. In the first period individuals consume and incur costs to their parents. In the second period they work, pay for their children and save for retirement. In the final period they receive bequests from their parents, consume some of their wealth and leave the remainder as bequests to their children in the next period.

3.1. Consumer's Problem

An individual makes no economic decisions in the first period, but imposes a time cost on her parents.

In period two the consumer problem is:

$$V_2(\psi, n^p, x^p) = \max_{c,a,n} \left[\frac{c^{1-\sigma}}{1-\sigma} + \eta_1 n^{\eta_2} + \beta E[V_3(x)] \right]$$

subject to

$$c + a \leq \psi w(1 - n\gamma)$$
$$x = a + \frac{f(x^p)}{n^p}$$

The state variables are individual's ability ψ, the individual's number of siblings n^p and parental wealth x^p. The individual can calculate an expected bequest value as a function of parental wealth and the number of siblings in the next period, $b^p = f(x^p)/n^p$. Thus the total wealth the individual possesses going into period 3 is $x = (1+r)(a+f(x^p)/n^p)$, where a is the life-cycle saving for period 3. γ is child care time cost and n is number of children. Note that because child costs are delineated in time, higher earning parents will effectively pay more for their children, as is expected and reflected in the data. η_i are weights on the utility from children. Parents utility from children is concave, and the curvature of the utility from children is determined by the coefficient η_2.

An individual's ψ (effective units of labor representing human capital, luck or inherent ability) is one of several states. An individuals ability state will depend on their parental ability state. This will be an AR(1) process represented by a Markov matrix. The probability a parent with ability i will have a child with ability j is represented by ψ_j^i.

The transfer of ability across generations is captured by:

$$\psi = \rho\psi^p + \epsilon$$

where

$$\epsilon \sim N(0, \sigma^2), \text{i.i.d.}$$

In period three the consumer problem is:

$$V_3(x) = \max_{c,b} \left[\frac{c^{1-\sigma}}{1-\sigma} + \phi_1 (b + \phi_2)^{1-\sigma} \right]$$

subject to

$$c + b \leq (1+r)x$$

Where b is total bequests and transfers allocated to your children, b/n is the amount left to each of their n children and where r is the interest rate. Parents have "warm glow" altruism, where they enjoy giving to their children but do not have perfect altruism.

The term ϕ_1 reflects the parent's concern about leaving bequests and transfers to her children, while ϕ_2 measures the extent to which bequests are a luxury good. This is in congruence with De Nardi (2004).

3.2. Firm's Problem

Firms are identical and profit maximizing. Their production technology is Cobb-Douglas:

$$Y = K^{\theta} L^{1-\theta}$$

The marginal product of capital is:

$$r = \theta K^{\theta-1} L^{1-\theta}$$

And the marginal product of one effective time-unit of labor is:

$$w = (1-\theta) K^{\theta} L^{-\theta}$$

3.3. Stationary Equilibrium

A steady state in this economy consists of a sequence of allocations $[c_t, a_t, b_t]_{t=0}^{\infty}$, aggregate inputs $[K_t, L_t]_{t=0}^{\infty}$ and prices $[w_t, r_t]_{t=0}^{\infty}$ such that the allocations solve each individual maximization problem subject to prices and their budget constraint, the inputs solve each firm's problem given prices and the capital and labor markets clear.

The level of capital in an economy is the sum of each individual in the second period's savings in assets and each individual in the third period's bequests (which are given in assets) when the capital market clears.

This means that the optimal choice of second period savings and third period bequests have to be calculated off the interest rates, which is determined by the sum of assets in the economy. The Capital Market Clearing condition is:

$$K_t = \int_\psi \int_{n^p} \int_{x^p} (a) \partial \kappa_2(\psi, n^p, x^p) + \int_x (b) \partial \kappa_3(x)$$

where κ_i is the population density function for generation i, a is what an individual has at the end of period 2 after consumption, and b is what they leave behind at the end of period 3 after consumption.

Labor is inelastic, there is no leisure in the utility function and every individual possesses a normalized time endowment. However, since children cost time, the amount of fertility chosen will impact the labor force. The effective labor force is dependent on the relative population density:

$$L_t = \int_\psi \int_{n^p} \int_{x^p} (1 - n\gamma) \partial \kappa_2(\psi, n^p, x^p)$$

4. Calibration

There are seven ability groups in this model. Our ability transfer follows an AR(1) process, which can be estimated by a Markov transition matrix. This matrix and income dispersal vector is created using Tauchen (1986), which was calculated using the intergenerational income persistence found in the data independently by Zimmerman (1992) and Solon (1992) of .4. We calibrate the variance to match the long-run income gini coefficient of

.44. The transition matrix and ability levels are shown in the appendix.

Our ability levels are normalized using $384,839 as median lifetime income between 1981-1992 (Leonesio and Del Bene 2011). Other parameters for our benchmark model are:

$Parameter$	$Value$	$Source$
σ	1.5	Hansen and Singleton (1982)
ϕ_1	-40575.63	Gale and Scholz (1994)
ϕ_2	1051777472	Hurd and Smith (2002)
γ	0.1	Haveman and Wolfe (1995)
β	0.37	Auerbach and Kotlikoff (1995)
θ	0.33	Auerbach and Kotlikoff (1995)
η_1	$.99$	Black, Kolesnikova, Sanders and Taylor (2013)
η_2	-1.6	Agee and Crocker (1996)

Child time cost was estimated to be 1/10 of the households time allocation per child. This is taken from the estimates of time costs of children in Haveman and Wolfe (1995) and is consistent with Knowles (2000).

The subjective discount factor β is chosen to match the capital-output ratio in the US, which is 3.0 according to Auerbach and Kotlikoff (1995). The β that is used in the model is 0.37 (i.e. $0.9755^{40} = 0.37$), while the capital share θ is .33.

Bequest parameter ϕ_1 calibrated to match the bequest/wealth ratio of 31% , calculated by Gale and Scholz (1994). ϕ_2 is calibrated to match the 90 percentile of the bequest distribution seen in the data. That value is $187,600 (Hurd and Smith 2002). We do not include inter-vivos transfers or college expenses to be conservative in our estimates of the importance of intergenerational transfers.

η_i is set to match the first moment of the fertility distribution.

5. Results

I run the simulation twice, once with the endogenous differential fertility indicated in our model section and once with enforced identical fertility across ability groups. We calculate the income and wealth distributions for each simulation.

5.1. Fertility

The comparison Fertility levels are taken from Black, Kolesnikova, Sanders and Taylor (2013) calculated from Children Ever Born, Women Aged 40-50 from the 1990 U.S. census (the last time this question was asked). The values of γ and η_2 taken from the data generate a fertility distribution that, while not exact, does a acceptable job of representing the distribution.

Fertility by Ability Group

ψ	1	2	3	4	5	6	7
$Endogenous$	2.50	2.50	2.48	2.20	1.60	1.70	1.60
$Identical$	2.20	2.20	2.20	2.20	2.20	2.20	2.20
$BKST$	–	2.51	2.32	2.1	2.16	–	–

Our fertility matching is reasonably close for as simple as our fertility modeling is. Most importantly we achieve the negative income-fertility relationship shown in the data.

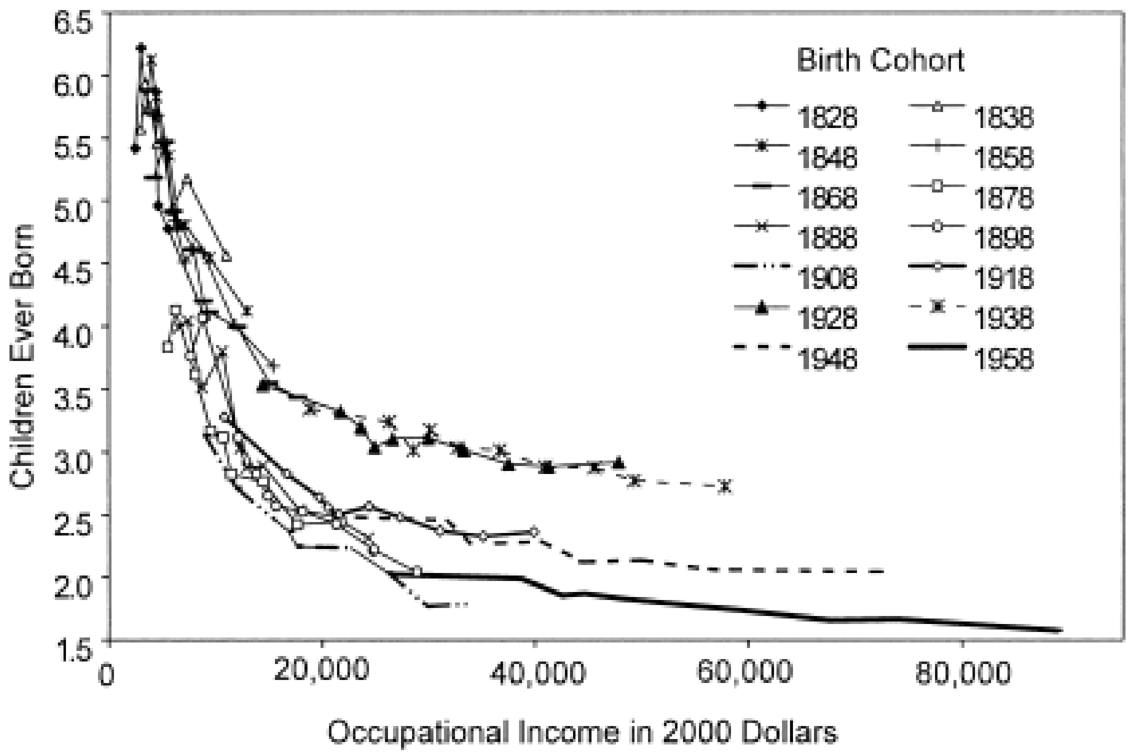

Figure 1: Taken from *Demography and the Economy*, a NBER publication. Source: Jones and Tertilt (2008)

5.2. Inequality

Share of Total Wealth- Benchmark and Identical Fertility

$Percentile$	60-80%	80-100%	90-95%	96-99%	$> 99\%$	$GiniCoef.$
$Benchmark$	0.18	0.65	0.11	0.23	0.18	0.62
$Identical$	0.16	0.61	0.14	0.22	0.11	0.54
$*$	0.13	0.79	0.13	0.24	0.30	0.78
$**$	0.00	1.00	0.28	0.36	0.22	0.90

* Empirical Study by Diaz-Gimenez, et al. (1997), **Knowles (2000)

The Wealth Gini calculated by our simulation was 0.62, compared with a Gini of 0.54 in the identical fertility scenario and 0.78 in the actual data. A distribution can be seen below.

Our lifetime labor income Gini was 0.437 in the case of differential fertility, and 0.438 in the case of identical fertility. This is compared to 0.44 in the data.

We match the data fairly well until the top 5 percent of the distribution. Our richest hold less wealth than they should, but overall our model does a good job of matching the actual distribution of American wealth, and does a better job than the comparison model with identical fertility. Differential fertility is important to explain the wealth-income distribution disparity.

The intuition behind this result is as follows. When children are receiving their bequests, they have more siblings when they are poor and fewer siblings when they are rich relative to identical fertility being imposed. This leads to less division of estates than would otherwise be the case for the richest groups, causing increased concentration of wealth at the highest income levels and greater diffusion at the lower income levels.

5.3. Fertility Matters

As found by Knowles (2000), we find that fertility can have important ramifications for the economy, increasing the Gini coefficient of wealth by over 10%. The addition of life-cycle saving and retirement consumption in our model leads to some key differences between our findings. Specifically his economy had no life-cycle savings, leading to the vast majority of wealth inequality being driven by bequests. Our bequest Gini is higher than our wealth Gini at about .90, indicating that the higher the bequest-capital ratio in the economy, the higher the overall wealth inequality.

In addition our differential fertility economy had a much higher bequest Gini than the identical fertility economy, .90 versus .77. Our savings Gini was both more equitable and less impacted by the differences between fertility rates, at .48 and .50 respectively, indicating a slight drop in savings among the rich when higher bequests were expected. The intuition behind this result is that the richest group are expecting higher bequests because they have fewer siblings to split their parents estate with. Moreover We find that expected bequests have a crowding out effect on savings. When expected bequests increase, individuals choose to consume more now and save less for retirement. The effect of bequests on wealth inequality is thus understated, as this reduction in saving makes the calculated wealth inequality understated in its welfare effects.

Furthermore, this crowding out effect depends on the individual's discount parameter β. This could have a major impact on models that use calibrated levels of β differential to generate inequality. These models could be understating the value of β if they look only at savings levels and ignore expected bequests. There is an explicit solution of this in the appendix.

6. Robustness Checks

6.1. Educational Transfers

In order to see the effect of an inter-vivos transfer on the results of this model I added a simplified educational transfer. This can be conceptualized as sending their child to private school and paying for their college vs public school and taking on debt. For the sake of ease it is modeled as a policy function that is a one-to-one mapping to the bequest transfer, calibrated to match the actual educational investment flows reported by Gale and Scholz (1994).

$$V_2(\psi, n^p, x^p) = \max_{c,a,n} \left[\frac{c^{1-\sigma}}{1-\sigma} + \eta_1 n^{\eta_2} + \beta E[V_3(x)] \right]$$

subject to

$$c + a + e \leq \psi w(1 - n\gamma) + e^p(1 + r)$$
$$x = a + \frac{f(x^p)}{n^p}$$

Adding the educational transfer increases inequality, and especially increases the wealth of the far right of the distribution. The top 1 % see nearly a 25% increase in their wealth share as a result. However, the savings among this group fell slightly, as the increase in intergenerational transfer led to higher bequest levels and therefore lower savings. Modeling an educational transfer more richly will therefore be a top priority in my future research, as it seems crucial to explaining the large levels of wealth inequality.

Share of Total Wealth- Educational Transfer

Percentile	60-80%	80-100%	90-95%	96-99%	> 99%	*GiniCoef.*
Benchmark	0.18	0.65	0.11	0.23	0.18	0.62
Identical	0.16	0.61	0.14	0.22	0.11	0.54
EduTransfer	0.16	0.67	0.11	0.23	0.22	0.64

6.2. Accidental Bequests

In this section we change the nature of our bequest motivation to discover how the choice of warm glow giving impacted our results.

We ran our model again with accidental bequests and no bequest motive. The probability of dying and leaving an accidental bequest was independent of income and set to match the bequest capital ratio discussed above. The individuals in the economy made savings decisions intending to consume all their savings during retirement, but would die leaving a percentage of their savings unconsumed, leading to bequests. The percentage of their saving left unconsumed passes equally to their children. Accidental bequests struggle to match the distribution at the top tail. This is due to a precipitous drop in bequest inequality, from .90 to .36, leading to a corresponding drop in wealth inequality. See the appendix for a graphical representation.

Share of Total Wealth- Accidental Bequests

Percentile	60-80%	80-100%	90-95%	96-99%	> 99%	*GiniCoef.*
Benchmark	0.18	0.65	0.11	0.23	0.18	0.62
Identical	0.16	0.61	0.14	0.22	0.11	0.54
Accidental	0.19	0.43	0.11	0.17	0.05	0.37

6.3. Direct Altruism

Another element we desired to analyze was the motivation for bequests. We had used ”warm glow” utility, where bequests grant utility directly to givers, regardless of the circumstances of the receivers. In order to check the effects of this, we rewrote our model with direct altruism, with the child’s value function being a piece of the parental value function:

$$V_3(x) = MAX_{c,b}[\frac{c^{1-\sigma}}{1-\sigma} + \phi_3 E[V_3^c(x')]$$

subject to

$$c + b \leq (1+r)a + b/n^p$$

I recalibrated the model, setting ϕ_3 to target the bequest to capital ratio, and η_i to match the first moment and elasticity of the fertility distribution.

$Parameter$	$Value$	$Source$
ϕ_3	3.08	Gale and Scholz (1994)
η_1	.98	Black, Kolesnikova, Sanders and Taylor (2013)
η_2	.2	Black, Kolesnikova, Sanders and Taylor (2013)

Using an altruistic bequest motive decreases inequality. The economic intuition behind this result is that rich people have a much lower incentive to give to their often high ability and already wealthy children, while the poor attempt to give much more. The measurement of the bequest Gini coefficient falls from .9 to .47 when compared with the benchmark model, while savings remains about the same at .48, indicating that the rich elderly are choosing to consume their savings rather than give it to their often high earning children.

Share of Total Wealth- Altruistic Giving Motive

$Percentile$	60-80%	80-100%	90-95%	96-99%	$> 99\%$	$GiniCoef.$
$Benchmark$	0.18	0.65	0.11	0.23	0.18	0.62
$Identical$	0.16	0.61	0.14	0.22	0.11	0.54
$Altruism$	0.66	0.39	0.10	0.08	0.03	0.35

7. Conclusion

This paper had three goals. First was to build and run a simplified overlapping generations model that includes differential fertility and intergenerational transfers. This was done using a three period model with childhood, adulthood and retirement, where individuals chose fertility endogenously and gave bequests to their children. Second was to represent the negative income-fertility relationship seen in the data. This was done using a combination of child time costs and concave utility from having children. Third was to match the wealth-income inequality disparity seen in the data, with wealth inequality being higher than income inequality. This was accomplished to a degree. While this model cannot replicate the large degree of wealth inequality exactly, it does much better than most macroeconomic models considered in Diaz-Gimenez, Quadrini, and Rios-Rull (1997) at explaining this puzzle.

These results show that adding differential fertility to a model focusing on the explanations for income and wealth inequality is quite important. Differential fertility leads to greater wealth inequality despite similar income inequality and can lead to models that more accurately reflect the data. This is crucial when making policy recommendations. Ignoring the fertility differences between rich and poor can only result in an incomplete picture of inequality.

The next step of this research will be welfare analysis. How does the addition of endogenous fertility affect the overall well being of individuals in this economy, in an absolute sense and relatively between rich and poor? Further extensions of this model could look at tax policy. If rich people have fewer children, this could have significant impact on how the estate tax is viewed, and whether an inheritance tax would be more equitable and efficient. Another addition could be to add discrete education investment by parents into their children, which could have an impact on the debate between private and public education provision, and lead to different levels of investment depending on both wealth and family size. Finally, incorporating accidental fertility, where the chances of an unplanned pregnancy are correlated with education and ability, could help replicate the fertility differences between rich and poor more comprehensively and accurately.

8. Appendices

8.1. Appendix A: Computational Algorithm

I solved for the Steady State Equilibrium as follows:

1. I create grids for savings a, ability ψ, total parental wealth x^p (the grid for x is twice savings and bequests due to it being the addition of the two) and parental ability ψ^p. There is 2000 grid points for wealth and 7 for ability, leading to a lattice of 98,000,000 points. Wealth ranges from \$0 to \$40m.

2. I choose a initial per-capita capital (K/L) to generate my initial wage and interest rate.

3. I define my ability transition Markov matrix using Tauchen (1986) and data from Zimmerman (1992) and Solon (1992), calibrated for distribution of earnings.

4. I allocate values generated from the Tauchen process to my 7 ability groups.

5. I allocate an initial guess of population density across my grid.

6. I calculate my 3rd period value function and use nearest neighbor grid search on this constrained optimization to determine the optimal choice variable of bequests to children. I use this to calculate the value of wealth in the 3rd period.

7a. I then calculate the 2nd period value function, using the values generated from step 6.

7b. Using a grid search I determine optimal consumption, savings and fertility for each distinct combination of state variables.

8. I then run a loop of population distribution updating, using the knowledge of optimal consumption, fertility, bequests and savings, to allow total wealth to become new parental wealth x^p, ability to become new parental ability ψ^p and using the Markov transition matrix to calculate the quantity of new children with the state variables $[\psi, \psi^p, x^p]$.

9a. In each loop I calculate the change in population distribution, and sum the total changes to see if the distribution is converging.

9b. In each loop I update the per capita capital and calculate new values of r and w.

$$L_t = \int_{\phi} \int_{n^p} \int_{x^p} \int_{a} \phi(\psi, n^p, x^p, a)(1 - n\gamma_{\psi})$$

$$K_t = \int_{\phi}\int_{n^p}\int_{x^p} \phi(\psi, n^p, x^p)(a) + \int_{x^p} \phi(x^p) b^p$$

10. I then sum across the different dimensions of my grid to see the steady state distribution across wealth and ability, and use parental wealth, savings, bequests and ability to calculate the distribution of the variables in steady state.

8.2. Appendix B: Solving Explicitly with 2 Ability Groups

My two policy functions are:

$$b_c = \frac{a(1+r) + b_p/n_p^* - \frac{\alpha_c \phi_2}{\phi_1(1-\sigma)}^{1/\sigma}}{1 + \frac{\alpha_c \phi_2^{1-\sigma}}{\phi_1(1-\sigma)}^{1/\sigma}}$$

and

$$c^3 = \frac{a(1+r) + b_p/n_p^* + \phi_2}{1 + \frac{\phi_1(1-\sigma)}{\alpha_c \phi_2^{1-\sigma}}^{1/\sigma}}$$

Which allows me to calculate discounted utility for each possible value of savings in period 2. Using this, I can find the optimal level of savings in period 2.

$$\frac{\partial U_2(c,n)}{\partial c} = \alpha_c c^{-\sigma}$$

$$\beta \frac{\partial U_3(a)}{\partial a} = \beta[\alpha_c[\frac{1+r}{1 + \frac{\phi_1(1-\sigma)}{\alpha_c \phi_2^{1-\sigma}}^{1/\sigma}}]^{-\sigma} + \frac{\phi_1(1-\sigma)}{\phi_2}[1 + \frac{a(1+r) + b_p/n_p^* - \frac{\alpha_c \phi_2}{\phi_1(1-\sigma)}^{1/\sigma}}{\phi_2[1 + \frac{\alpha_c \phi_2^{1-\sigma}}{\phi_1(1-\sigma)}^{1/\sigma}]}]^{-\sigma} \frac{1+r}{1 + \frac{\alpha_c \phi_2^{1-\sigma}}{\phi_1(1-\sigma)}^{1/\sigma}}]$$

The marginal utility of the consumption in the second period and the shadow utility of saving for the third period must be equal.

$$\frac{\partial U_2(c,n)}{\partial c} = \beta \frac{\partial U_3(a)}{\partial a}$$

subject to

$$c + nc_i + a = w(1 - b_1 n)$$

This gives me two equations and two unknowns (c, a) in period two, which is able to be solved computationally.

For 2 ability groups, since in a steady state the quantity and quality of labor will stay constant over time:

$$Q^{*h} = \frac{1}{1 - H_h F_h} L_h F_l Q^{*l}$$

and

$$Q^{*l} = \frac{1}{1 - L_l F_l} H_l F_h Q^{*h}$$

So,

$$L_h F_l H_l F_h = (1 - H_h F_h)(1 - L_l F_l)$$

and

$$L_l F_h H_h F_l = (1 - H_l F_l)(1 - L_h F_h)$$

Since $L_l + L_h = 1$ and $H_l + H_h = 1$

in order for a steady state to exist, so the transition probabilities of each group can be determined, given fertility.

$$L_h F_l (1 - H_h) F_h = (1 - H_h F_h)(1 - (1 - L_h) F_l)$$

and

$$(1 - L_h) F_h H_h F_l = (1 - (1 - H_h) F_l)(1 - L_h F_h)$$

$$L_h = \frac{(F_h F_l - F_l)(1 - F_l)}{(F_h F_l - F_h)^2} + \frac{1 - F_l}{F_h F_l - F_h}$$

$$L_l = 1 - \frac{(F_h F_l - F_l)(1 - F_l)}{(F_h F_l - F_h)^2} - \frac{1 - F_l}{F_h F_l - F_h}$$

$$H_h = \frac{(F_h F_l - F_l)(1 - F_l)}{(F_h F_l - F_h)^2} + \frac{1 - F_l}{F_h F_l - F_h}$$

$$H_l = 1 - \frac{(F_h F_l - F_l)(1 - F_l)}{(F_h F_l - F_h)^2} - \frac{1 - F_l}{F_h F_l - F_h}$$

Since

$$L_t = \psi_t^h Q^{*h} + \psi_t^l Q^{*l}$$

We can now solve for the amount of labor in the economy while in a steady state. This will allow us to solve for w*

$$w^* = \psi^i (1 - \theta)(\frac{K_t}{L_t})^\theta$$

which, given ability level and along with

$$r^* = \theta(\frac{K_t}{L_t})^{\theta - 1}$$

allows us to computationally calculate the optimal choice variables c, c', a, and b_c.

8.3. Appendix C: Graphical Representation of Wealth

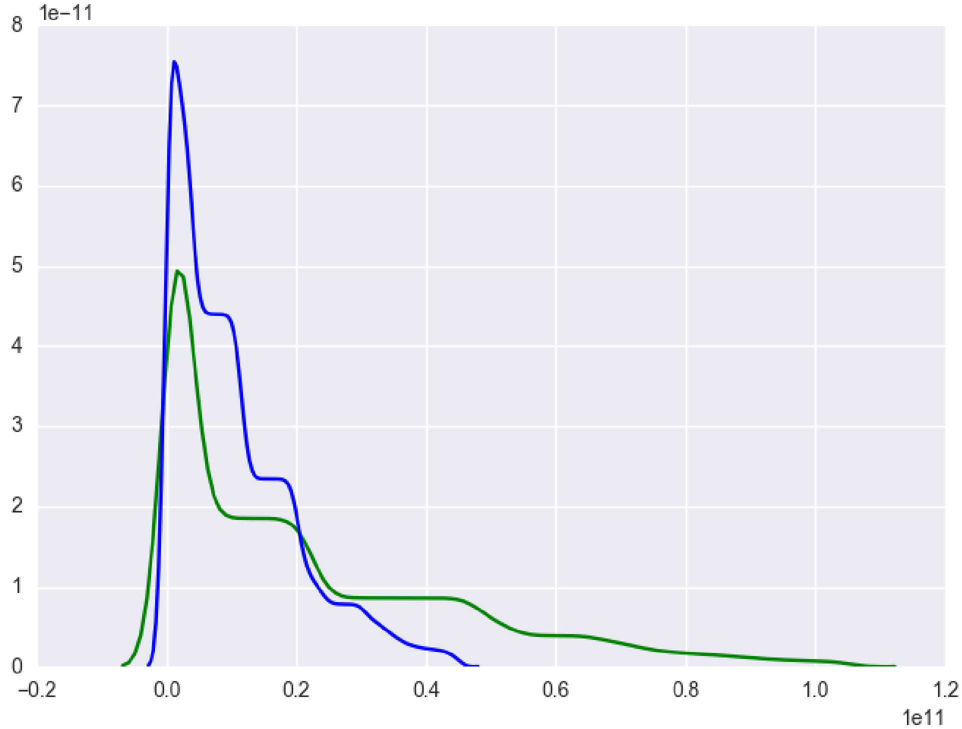

Figure 2: Wealth under the differential fertility assumption (green) and the identical fertility assumption (blue)

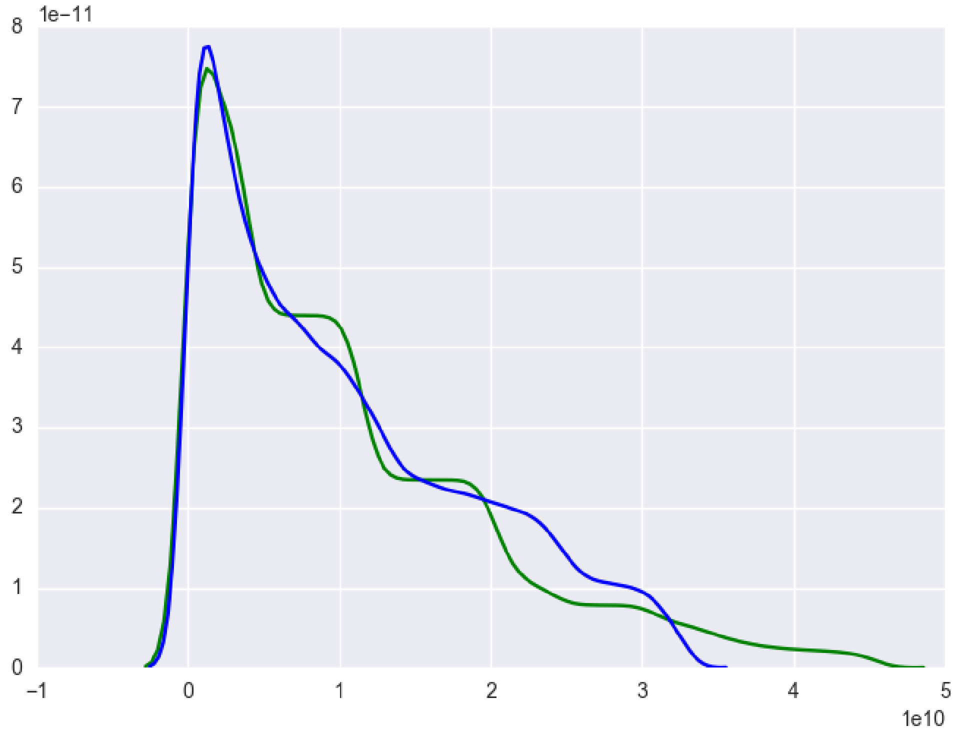

Figure 3: Wealth under the identical fertility assumption (green) and the accidental bequest assumption (blue)

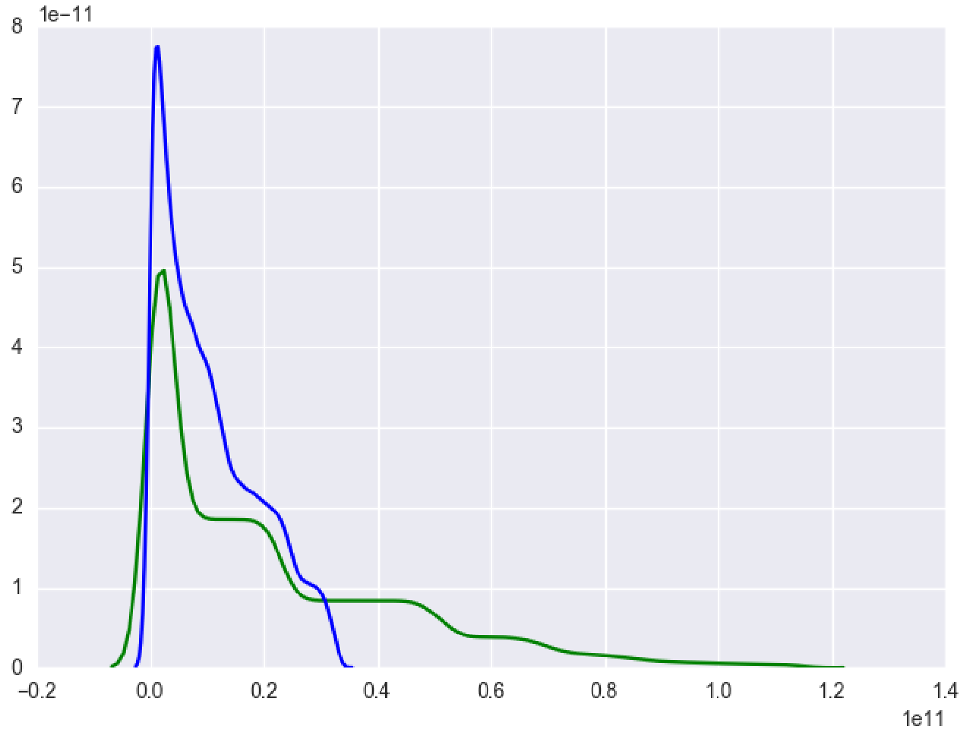

Figure 4: Wealth under the warm glow bequest motive assumption (green) and the direct altruism assumption (blue)

8.4. Appendix D: Markov Matrix and Ability Distribution

	ψ_7	ψ_6	ψ_5	ψ_4	ψ_3	ψ_2	ψ_1
ψ_7^p	0.0720	0.2937	0.4058	0.1907	0.0302	0.0016	0.0000
ψ_6^p	0.0302	0.1907	0.4058	0.2937	0.0720	0.0059	0.0002
ψ_5^p	0.0106	0.1040	0.3415	0.3804	0.1440	0.0183	0.0008
ψ_4^p	0.0031	0.0477	0.2418	0.4146	0.2418	0.0477	0.0032
ψ_3^p	0.0008	0.0183	0.1440	0.3804	0.3415	0.1041	0.0110
ψ_2^p	0.0002	0.0059	0.0720	0.2937	0.4058	0.1907	0.0318
ψ_1^p	0.0000	0.0016	0.0302	0.1907	0.4057	0.2937	0.0780

Our ability values are as follows.

ψ	1	2	3	4	5	6	7
Log	-1.2111	-0.8074	-0.4037	0.0000	0.4037	0.8074	1.2111
Actual	0.2979	0.4460	0.6678	1.0000	1.4974	2.2421	3.3572

8.5. Appendix E: Solution for expected bequests crowding out savings

$$V_2(\psi, n^p, x^p) = \max_{c,a,n} \left[\frac{c^{1-\sigma}}{1-\sigma} + \eta_1 n^{\eta_2} + \beta E[V_3(x)] \right]$$

Using non-satiation and substituting:

$$V_2(\psi, n^p, x^p) = \max_{c,a,n} \left[\frac{(\psi w(1-n\gamma) - a)^{1-\sigma}}{1-\sigma} + \eta_1 n^{\eta_2} + \beta E[V_3(x)] \right]$$

Solving for optimal savings using concavity to ensure uniqueness:

$$\frac{\partial V_2(\psi, n^p, x^p)}{\partial a} = -(\psi w(1-n\gamma) - a)^{-\sigma} + \beta(a + b^p - b)^{-\sigma}] = 0$$

Solving for a*

$$a* = \frac{\psi w(1-n\gamma) - \beta^{-\frac{1}{\sigma}}(b^p - b)}{1 + \beta^{-\frac{1}{\sigma}}}$$

Comparative statics:

$$\frac{\partial a*}{\partial b^p} = -\frac{\beta^{-\frac{1}{\sigma}}}{1 + \beta^{-\frac{1}{\sigma}}} \leq 0$$

The lower β is the stronger the crowding out effect. Impatient children of rich parents will save very little, anticipating large bequests.

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