Houses Divided: A Model of Intergenerational Transfers, Differential Fertility and Wealth Inequality*

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Abstract

Recent increases in income and wealth disparity in the United States has prompted a renewed focus on the mechanisms driving inequality. A major puzzle of this phenomenon is that wealth inequality is significantly larger than income inequality. This paper contributes to the existing literature by studying the impact on wealth inequality from life-cycle savings, intergenerational transfers, and fertility differences between the rich and the poor. We find that bequests have a significant impact on the level of wealth inequality and that the fertility difference between the rich and the poor amplifies the effect of bequests. In addition, we find that life-cycle saving and anticipated bequests interact with each other, and this interaction is important for fully understanding wealth inequality in the United States.

Keywords: Intergenerational transfers, endogenous fertility, wealth inequality, life-cycle savings.

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1. Introduction

It is well-known in the literature that behaviors of wealthy individuals often contradict the predictions from standard life-cycle models. The wealthy save much more and spend less than predicted by classic calibrated macroeconomic models even as they reach the end of their lifespans (Dynan et al. (2004)). In addition, they are much more likely to leave bequests to their children at the end of their lives, even when relative wealth is considered (De Nardi (2004)). Intergenerational transfers have accounted for one half of total wealth (Gale and Scholz (1994)) and the top 2% of households receive nearly 70% of lifetime inheritances (Hendricks (2001)).

In addition, recent U.S. data reveals a large concentration of wealth among a small number of its citizens. In 1995, the Gini coefficient for annual labor earnings was 0.63 (Diaz-Gimenez et al. (1997)), whereas the Gini coefficient for wealth holding was much higher, at 0.8 (Rodriguez et al. (2002)). Alvaredo et al. (2013) documents that the top 1 percent holds nearly one third and the top 5 percent holds over half of the total wealth in the economy, and that these shares have been growing for quite some time. According to Saez and Zucman (2016), this trend has been accelerating since the 2008 financial crisis.

However, standard dynamic models with heterogeneous agents not only have a difficult time replicating the savings behavior of the rich individuals, but also have trouble matching the magnitude of the wealth inequality observed in the data. For example, Aiyagari (1994) predicts in a calibrated simulation that the top one percent will hold four percent of the total wealth in the economy, an order of magnitude smaller than that observed in the data. Why do rich people choose to hold such a high level of wealth instead of increasing their consumption? Previous explanations offered in the literature include heterogeneity in patience (Krusell and Smith (1998)), heterogeneity in bequest motive (Bernheim et al. (2004)), the risk that the rich outlive their savings(De Nardi et al. (2010)), large earnings risk for the top earners (Castaneda et al. (2003)), and entrepreneurship (Cagetti and De Nardi (2006)). This paper will contribute to the literature by showing that the endogenous differences in fertility between the rich and the poor interacts with the savings and bequesting behavior in a quantitatively meaningful way. In other words, we show that the differences in fertility is an important channel that contributes to explaining the consumption behavior of the rich and the resulting concentration of wealth.

The intuitive reason why fertility differences matter is the following. The literature has established that there exists an inverse relationship between income and fertility.[1] We calculate using U.S. census data that females in the lowest percentile of the income distribution have 2.65 children, while the highest have only 1.51. We argue that this significant fertility difference between the poor and the rich amplifies the impact of bequests on wealth inequality, since the children of the rich parents have fewer siblings to share their parent's total amount of bequest. Given that rich parents have a greater amount of money to leave as bequests than their poor counterparts to begin with, this implies that the children of the rich not only get to eat from a bigger pie, but they get a greater share of the pie. Thus, fertility differences amplify the magnitude of wealth inequality.

This paper has three goals. First, to capture the interaction between fertility differences and bequests, we develop a general equilibrium over-lapping generations (OLG) model with endogenous fertility and intergenerational transfers. Our model is closest in spirit to Knowles (1999), with the key difference being that agents in our model faces retirement, thereby capturing the dynamic interaction between anticipated bequests and life-cycle savings. For example, if an agent is expecting a large bequest from her parents, she will respond by saving less for retirement than she otherwise would have. Second, we use the model to match the negative relationship between income and fertility observed in the data. Third, we match the disparity between the magnitude of inequality observed in wealth and income, and we quantify how important endogenous fertility is in explaining this disparity.

Using a calibrated version of our model, we find that the fertility differences between the rich and the poor increases the wealth Gini coefficient by about 10%, driven especially by a one quarter increase in the wealth share of the top 1%. We also quantify the importance of the bequest mechanism by showing that an alternative model in which the bequest channel has been shut down results in a much lower Gini coefficient for wealth distribution of 0.54, compared to the 0.76 we obtain in our benchmark. We also find that anticipated bequests crowd out current savings, which implies that intergenerational trans-

[1] See Hurd and Smith (2002), De la Croix and Doepke (2004), De La Croix and Doepke (2003), and Jones and Tertilt (2007), among others.

fers lead to less capital formulation in the present. In sum, this paper finds that pairing bequest motive with endogenous fertility is quantitatively important for explaining the ”excess” saving behavior of the rich and the consequent high level of wealth inequality, especially the extreme right skewed nature of the wealth distribution.

The rest of paper is organized as follows. In Section 2., we review the literature. In Section 3., we describe the model and its stationary equilibrium. In Section 4., we calibrate a benchmark specification using moment matching. In Section 5., we discuss the results. The final section concludes.

2. Literature Review

Ever since heterogenous agent macroeconomic models have been introduced to the macroeconomics literature by Bewley (1986), Huggett (1996), and Aiyagari (1994), there has been a surge in papers that use these models to explain the causes and mechanisms behind wealth inequality. It is well-known in the literature that such a model with precautionary and/or life-cycle savings as the only inequality generating mechanism fails to replicate the levels of wealth concentration observed in the data (De Nardi and Yang (2016)). Specifically, these models fail to account for the extremely long and thin top tails of the distributions and for the large number of households in the bottom tail (Castaneda et al. (2003)).

As surveyed by De Nardi and Yang (2016), there have been many variations of the heterogeneous agent model in which various mechanisms have been introduced to better match the magnitude of wealth inequality observed in the data, such as preference heterogeneity (Krusell and Smith (1998), Heer (2001), Suen (2014)), entrepreneurship (Cagetti and De Nardi (2006)), high earnings risk for the top earners (Castaneda et al. (2003)), transmission of bequests and human capital across generations, and others. Among these, our paper is related to the literature that espouses the transmission of bequests and human capital across generations as the main mechanism behind wealth inequality. In a seminal paper, Kotlikoff and Summers (1981) have argued that if intergenerational transmission of wealth and ability is what really drives wealth inequality, then a simple model that focuses on life-cycle savings alone will fail to capture the relevant causes. Models with bequests are better at reflecting the data than these simple models. For instance, Huggett (1996) generates a higher degree of wealth concentration than a model with only accidental bequests(Aiyagari (1994)), in which the top one percent holds seven percent of the wealth.

Empirically, bequests represent a large piece of intergenerational transfers. Gale and Scholz (1994) use the Survey of Consumer Finances to find the amount of inter-vivos transfers and inheritance between 1983 and 1985. The total amount of monetary support, college expenses, and inheritance granted by the parents to their children was over $350 billion. Of this, inheritance constituted nearly 40 percent of the total amount of transfers, while over 60 percent of those who reported receiving inheritance were in the top net worth decile. Their central estimate is that intended life-time transfers (which they define as inter-vivos transfers, trust accumulations, and life insurance payments to children) account for at least 20 percent of aggregate net worth, and bequests, accidental or intended, account for 31 percent more.

The literature has been at odds as to how to model bequest, specifically, whether to model it as an accidental gift from the older generations who misjudged their expected lifespan (as in Huggett (1996)) or to model it with the so-called "bequest motive," in which the parents are making a conscious economic decision about how much to leave behind to their offspring. Based on the findings in (Bernheim et al. (2004))—which analyzes the effect of the estate tax on inter-vivos giving to support the presence of bequest motive—and Kopczuk and Lupton (2007)—which finds that three-fourths of the elderly single population has a bequest motive and about four-fifths of their net wealth will be bequeathed, half of which is due to a bequest motive as opposed to accidental bequests—we choose to use a bequest motive.

Even within the literature on bequest motive, there has been disagreements as to how to model bequest motives, specifically on whether parents are perfectly altruistic towards their children. Altonji et al. (1992) found that the division of consumption and income within a family are dependent, indication that perfect altruism does not apply to operative transfers. Other studies have gone on to show that an increase in parental resources coupled with a decrease in child consumption does not lead to a corresponding increase in transfers (Altonji et al. (1997) and Cox (1987)). Siblings generally receive equally divided inheritances, rather than the size of the inheritance being dependent on relative income as perfect altruism would predict (Wilhelm (1996)). Altonji et al. (1997) strongly reject the perfect altruism hypothesis. They find that a one dollar transfer from child to parent results in only a 13 cent donation from parent to child, which should be the full dollar under perfect altruism. Based on these empirical findings, there have been multiple papers that assume an alternative bequest motive, the ”warm-glow” motive.[2] That is, parents derive utility from giving while not caring directly about the wellbeing of the recipient. In addition, motivated by the highly skewed distribution of bequests, these papers incorporate leaving bequests into the utility function as a luxury good, allowing for rich parents to value bequests relatively more. If bequests are a luxury good such that the rich gain greater utility from leaving them, then greater inequality in bequests, and subsequently wealth, is generated. This is due to the emergence of large estates, or dynasties, where wealthy parents have well educated and highly productive children, to whom they then leave large bequests. These persistent rich often have smaller families, leading to greater relative concentration.

In addition to bequests, our paper is also related to many papers that have shown that allowing for transfer of ability and human capital across generations also goes very far toward accounting for the data. Many models, including Kotlikoff and Summers (1981), Knowles (1999), De Nardi (2004), and De Nardi and Yang (2016) account for high ability (or high luck) parents being more likely to have high ability (or high luck) children. Of special note, Lee et al. (2015) find that parental education is positively related to their children's earnings, thereby creating a virtuous cycle for the wealthiest and a vicious cycle for the poorest.

Becker (1960) is the seminal paper analyzing post-war birth decisions in an economic context. In this paper Becker differentiates between child quality and quantity, and postulates that since children do not appear to be inferior in relation to any other consumption good class, it is likely that a rise in long-run income would increase the amount spent on children. However, he compares spending on children to the spending patterns on other durable goods, where quantity elasticity is relatively smaller than quality elasticity, so he would predict that parents will spend the majority of increases in income on the quality of their children rather than the quantity. When he considered census date from the first half

[2]See De Nardi (2004) and De Nardi and Yang (2016), among others.

of the 20th century, he found most data tended to show a negative relationship between income and fertility. He ascribes this mainly to a lack of contraceptive knowledge.

Several decades later, Heckman and Walker (1990) used Swedish data to show a strong negative relationship between female education (which they support as a proxy for potential female earnings) and children ever born, despite widespread contraceptive access in that culture during the time periods considered.

A more recent review, Jones et al. (2008), finds ”overwhelming” empirical evidence that fertility is negatively related to income in most countries at most times. While there are many potential explanations for this, they find that the most common explanation is that there is a greater opportunity cost for higher income parents to having children, although they conclude that more research is needed. We do not take a strong stance on the reasons for fertility differential in this paper, but choose to use opportunity cost to generate the differential endogenously due to computational ease and because it seems to be the most convincing current argument in the literature.

The two papers that are closest to ours in spirit are De Nardi (2004) and Knowles (1999).

De Nardi (2004) uses a quantitative, general equilibrium, overlapping-generations model in which parents and children are linked by bequests and ability. The area in which our papers differ is in our treatment of fertility. In De Nardi (2004) each agent has the same number of children. In our model agents face a choice of how many children to have, and their endogenous choices affects the results in interesting ways.

Knowles (1999) uses a two period model with endogenous fertility to show the importance of fertility to inequality. In this model, individuals are children in period one, have children in period two and then die. This does not allow for retirement or life-cycle savings. In our model we show that the behavior pattern of savings and bequesting differs, interacts and is affected by the endogenous fertility choice. It is therefore useful to model bequests and saving separately in a environment with a retirement period.

3. Model

Consider an economy inhabited by overlapping generations of agents who live for three periods. In the first period, agents are not economically active, only incurring costs to their parents. In the second period, they make fertility and labor supply decisions, and save for retirement. In the final period, they receive bequests from their parents, consume some of their wealth and leave the remainder as bequests to their own children, who in turn will receive them in the following period.

3.1. Consumer's Problem

3.1.1. Period One

An individual makes no economic decisions in the first period, but imposes a time cost on her parents. She inherits an ability level from her parents. An individual's ability ψ (effective units of labor representing human capital, luck or inherent ability) depends on their parental ability ψ^p, and the log of ability is assumed to follow the AR(1) process,

$$\log(\psi) = \rho \log(\psi^p) + \epsilon_\psi$$

where

$$\epsilon_\psi \sim N\left(0, \sigma_\psi^2\right), \quad \text{i.i.d.}$$

3.1.2. Period Two

Individuals in the second period differ along three dimensions: earning ability ψ, number of siblings n^p, and current wealth of their elderly parents x^p. They face the following utility maximization problem:

$$V_2(\psi, n^p, x^p) = \max_{c,a,n\geq 0} \left[\frac{c^{1-\sigma}}{1-\sigma} + \lambda_1 n^{\lambda_2} + \beta \mathbb{E}[V_3(x)] \right]$$

subject to

$$c + a \leq \psi w(1 - \gamma n)$$
$$x = a(1+r) + \frac{B(x^p)}{n^p}$$

The agent derives utility from current consumption c, in which the utility function is

CRRA with elasticity σ. Second, the agent receives utility from having a number of children n. λ_1 is the relative weight on the utility derived from children, and λ_2 controls the curvature of the utility from children.

Agents are given a time allocation set to unity. In the budget constraint, γ is the time cost of child care. Hence, $(1 - n\gamma)$ is the amount of time available to be allocated to the labor force. Since there is no labor-leisure choice in this model, $\psi(1 - n\gamma)$ is the total amount of effective labor supplied. w is the real wage per effective unit of labor. Note that because child costs are delineated in time, higher earning parents will effectively be paying more for their children, as is expected and reflected in the data. We also restrict a, the amount saved, to be strictly non-negative, thereby imposing a imperfect capital market. In other words, agents cannot borrow to finance their retirement.

The second constraint of the maximization problem describes how total amount of wealth in the third period x is determined. It is the sum of life-cycle savings a, the interest accrued on those savings, and the bequest received from dying parents $b^p = B(x^p)/n^p$. Here, $B(x^p)$ denotes the bequest policy function that will be obtained from solving the Period 3 utility maximization problem.

From the second period maximization problem, we obtain three policy functions. $C_2(\psi, n^p, x^p)$ is optimal consumption, $A(\psi, n^p, x^p)$ is optimal asset accumulation, and $N(\psi, n^p, x^p)$ is optimal fertility.

3.1.3. Period Three

Agents retire in the third period, and their state in this period can be captured by a single variable, x, the amount of wealth held. They face the following utility-maximization problem:

$$V_3(x) = \max_{c,b} \left[\frac{c^{1-\sigma}}{1-\sigma} + \phi_1 (b + \phi_2)^{1-\sigma} \right]$$

subject to

$$c + b \leq x$$

Where b is total bequests and transfers allocated to children during period three. Parents have ”warm glow” motive, where they enjoy giving to their children but do not directly care about the children’s wellbeing, meaning they will not give more to poorer children and less to richer children, but rather give an equal amount to each child, which is consistent with the behavior observed in the data.

The term ϕ_1 measures the weight on the bequest motive, while ϕ_2 measures the extent to which bequests are a luxury good.

From this maximization problem we obtain two policy functions. Optimal consumption $C_3(x)$ and optimal bequests $B(x)$.

3.2. Firm's Problem

Firms are identical and act competitively. Their production technology is Cobb-Douglas, with capital share (θ), inputs of technology (A), capital (K) and labor (L) generating output (Y):

$$Y = AK^{\theta}L^{1-\theta}$$

.

The profit-maximizing behaviors of firms imply that the marginal product of capital is:

$$r = A\theta K^{\theta-1}L^{1-\theta}$$

,

and the marginal product of one effective time-unit of labor is:

$$w = A(1-\theta)K^{\theta}L^{-\theta}$$

.

3.3. Stationary Equilibrium

Let Φ_2 and Φ_3 represent the population distributions of individuals in period 2 and 3. A steady state in this economy consists of a sequence of allocations $[c_2, c_3, a, b, n]$, aggregate inputs $[K, L]$ and prices $[w, r]$ such that

1. Given prices, the allocations $[c_2, c_3, a, b, n]$ solve each individual's maximization problem.

2. Given prices, aggregate capital and labor $[K, L]$ solve the firm's problem.

3. Markets clear:

$$K_t = \int_{\psi}\int_{n^p}\int_{x^p} A(\psi, n^p, x^p)(1+r)d\Phi_2(\psi, n^p, x^p) + \int_{x} B(x)d\Phi_3(x)$$

$$L_t = \int_{\psi}\int_{n^p}\int_{x^p} (1-\gamma N(\psi, n^p, x^p))d\Phi_2(\psi, n^p, x^p)$$

Where $N(\cdot)$ is the policy function for optimal fertility, $A(\cdot)$ is the policy function for optimal savings and $B(\cdot)$ is the policy function for optimal bequests, γ is time cost of children and r is the interest rate.

4. The distributions Φ_2 and Φ_3 are stationary in the steady state and evolve according to the followings law of motions:

$$\Delta \int_{\psi}\int_{n^p}\int_{x^p} \Phi_2(\psi', n^{p'}, x^{p'}) = \varphi_{a,b,n,m} \int_{\psi}\int_{n^p}\int_{x^p} \Phi_2(\psi, n^p, x^p)$$

$\varphi_{a,b,n,m}$ is a transformation that govern the inter-temporal movement of individuals across the state space.

$$\varphi_{a,b,n,m} = \begin{cases} \int_{\psi} \Phi_2(\psi') = \int_{\psi} M[\psi, \psi]\Phi_2(\psi, n^p, x^p) \\ \int_{n^p} \Phi_2(n^{p'}) = \int_{\psi}\int_{n^p}\int_{x^p} N(\psi, n^p, x^p)\Phi_2(\psi, n^p, x^p) \\ \int_{x^p} \Phi_2(x^{p'}) = \int_{\psi}\int_{n^p}\int_{x^p} (A(\psi, n^p, x^p)(1+r) + \mathbb{E}[B(x^p)])\Phi_2(\psi, n^p, x^p) \end{cases}$$

Where $M[\cdot]$ is the Markov transition matrix, $N(\cdot)$ is the policy function for optimal fertility, $A(\cdot)$ is the policy function for optimal savings and $B(\cdot)$ is the policy function for optimal bequests and r is the interest rate. Δ is the reciprocal of population growth, to ensure there is no population growth in the steady state.

The next period distribution of young depends on the current period of young's fertility decisions, specifically using the Markov transition matrix to generate the new distribution of ability, the saving policy function and expected bequests to generate expected parental wealth and generating optimal fertility from their ability and optimal savings levels.

In the third period, an individuals wealth is what he has saved in the previous period, as well as what he has received in bequests from his parents.

$$\int_x \Phi_3(x^{'}) = \int_\psi \int_{n^p} \int_{x^p} (A(\psi, n^p, x^p)(1+r) + \mathbb{E}[B(x^p)])\Phi_2(\psi, n^p, x^p)$$

Note that the distribution of the elderly's wealth holdings is identical to the distribution of the young's parental wealth holdings.

4. Calibration

The parameter values and their sources are summarized in Table 1.

We obtain measurements for the US level of intergenerational income persistence, income gini, fertility median, fertility elasticity, the effect of anticipated bequests on optimal savings, bequest/capital ratio, bequest distribution, and the time cost of child care.

4.1. Demographics

The model period is 30 years. Individuals enter the economy when they are 30 years old (period 2). They retire at 60 years old (period 3) and die at the end of the period (at 90 years old).

There are 7 ability groups. This number was chosen for computational efficiency while still generating a useful distribution of groups. For example, the highest earning group is representative of the top .6% of the population, and the top two groups together account for the top 6.4%. Our ability levels are normalized so the median ability is unity. The values of each ability level are listed below.

Table 1: The Benchmark Calibration

Parameter	Value	Source
β	0.9454	De Nardi and Fang (2016)
σ	1.7	SCF
θ	0.33	Macro Literature
γ	0.1	Haveman and Wolfe (1995)
ρ	0.4	Zimmerman (1992) and Solon (1992)
Parameter	Value	Moment to match
ϕ_1	-17.6	bequest/wealth ratio: 0.31
ϕ_2	5.15	90 percentile of the bequest distribution: 0.3
η_1	-0.0057	First Moment of Fertility Distribution: 2.04
η_2	-9.07	Income-Fertility Elasticity: -0.24
σ_ψ^2	1.15	Income Gini: 0.63

At the steady state the model will match two fertility targets: median number of children and fertility elasticity with regards to income.

Using the 1990 U.S. Census, we calculated the "children ever born" (CEB) from the 1945-1955 birth cohort. Since most of our income calibration is done using data from 90s, this is appropriate. Though using CEB has the possibility of misstating fertility due to mortality, the large decrease in childbirth deaths post WWII has made this measurement error acceptable. We segregated using occupational income (to correct for them being out of work at the current time) and found the first two moments for each of our ability groupings.

Table 2: CEB mean and variance by Ability Group

Ability	1	2	3	4	5	6	7
Mean	2.65	2.26	2.41	2.04	1.76	1.80	1.51
Variance	2.32	1.94	2.06	1.76	1.58	1.63	1.47

Not only does fertility decline with income, the variance does as well. η_1 is set to match the first moment of the fertility distribution, 2.04 children per household and η_2 is set to match the income-fertility elasticity of -0.24.

Child time cost γ is set to 1/10 of the household's time, based on the empirical estimates of time costs of children in Haveman and Wolfe (1995) Haveman and Wolfe (1995).

We approximate the AR(1) process for earning ability ψ by a 7-state Markov chain using the method introduced in Tauchen (1986) Tauchen (1986). The coefficient of intergenerational persistence, ρ, is set to 0.4 according to the estimates in Zimmerman (1992) Zimmerman (1992) and Solon (1992) Solon (1992). We calibrate the income variance to match the income gini coefficient in the 1992 SCF data, 0.63, taken from Castaneda et al. (2003). The transition matrix and normalized ability levels are shown in the appendix.

4.2. Preferences and Technology

The subjective discount factor β is chosen to match the amount used in De Nardi and Yang (2016) De Nardi and Yang (2016). The resulting value of β is 0.1856, which is equivalent to an annual discount factor of 0.9454 (i.e. $0.9454^{30} = 0.1856$). The capital share θ is set to the widely accepted 0.33 according to the quantitative macro literature.

σ is calibrated using the 2013 Survey of Consumer Finance. The variable of interest is a survey question asking respondents about how much they feel they need to have saved, which is represented by A_i^*. The explanatory variable asks them how much they expect to receive from a substantial inheritance or transfer of assets in the future, represented by $E[B_i]$. We control for age, partners age, mother's age, partner's mother's age (second order polynomials), liquid assets, retirement accounts (IRAs, Pensions, etc), saving accounts, bonds, equity, total income (adjusted if an "abnormal year"), received inheritance in the past, race and education. These are represented by χ_i.

$$A_i^* = \beta_1 E[B_i] + \beta_i \chi_i + \epsilon_i$$

***-Significant at the 1% level

Regression 1 is a basic reduced form OLS. Regression 2-4 uses a bootstrapping stan-

Table 3: SCF Regression Results

	(1)	(2)	(3)	(4)
β_1	$-.0227^{***}$	-.0226	-.0276	-.0374
S.E.	(.0075)	(.0604)	(.0766)	(.1104)
Bootstrapped S.E.	No	Yes	Yes	Yes
Groups	1-7	1-7	5-7	6-7

dard error technique with a correction for multiple imputation. Regression 3 and 4 are subsamples corresponding to our top 3 (top 30%) and top 2 (top 6.4 %) income groups respectively (which give almost all bequests in both the data and the model). When the standard error is calculated correctly, we do not find a significant result. However, we believe the coefficients are indicative of the effect that we expect, and have future plans for a stronger identification strategy.

Using a .03 coefficient, and the comparative statics discussed in the appendix, we calibrate σ to be 1.7.

Bequest parameter ϕ_1 calibrated to match the bequest/wealth ratio of .31 , calculated by Gale and Scholz (1994) Gale and Scholz (1994). ϕ_2 is calibrated to match the 90 percentile of the bequest distribution seen in the data. That value is $187,600 Hurd and Smith (2002). We do not include inter-vivos transfers or college expenses to be conservative in our estimates of the importance of intergenerational transfers.

5. Quantitative Results

We start this section by reviewing the main properties of the benchmark model in steady state, with special attention given to its implications for wealth inequality. We then run counter-factual experiments to highlight the role of fertility difference and bequests in shaping wealth inequality. In counter-factual I, we impose identical fertility to show the effects of endogenous fertility on the distribution on wealth, savings and bequests.

In counter-factual II we eliminate the bequest motive to show the impact of bequests on wealth and savings. From these results two things become clear. Fertility has an significant impact on inequality, and bequests and savings interact in important ways.

5.1. The Benchmark Model

Table 3 summarizes the benchmark model's income distribution. The first row is the value of the ability weight for that group, when group 4 is normalized to unity. The second row is the share of the population that is equal to or larger than that group. So the highest ability level corresponds to the top .6% and the top two corresponds to the top 6.4% of the population. These ability levels are generated from the Tauchen (1986) process, using the parameter .4 for income persistence ρ, and a calibrated value of 1.15 for income variance σ_ψ^2 to match the income gini seen in the data.

Table 4: Income Distribution

Ability Group	1	2	3	4	5	6	7
Benchmark	0.02	0.07	0.27	1.0	3.7	13.7	50.8
CumPopShare	0.006	0.064	.300	0.680	0.929	0.993	1.0

Table 4 summarizes the benchmark model's wealth distribution.

Table 5: Wealth Distribution

Percentile	60−80 %	80−100 %	90−95 %	96−99 %	>99%	Gini Coef.
Data[3]	0.13	0.79	0.13	0.24	0.30	0.78
Benchmark	0.13	0.80	0.19	0.27	0.24	0.76

A graphical distribution of wealth can be seen in the appendix.

We match the data fairly well. Our richest 1% hold less wealth than the data, but overall our model does a good job of matching the actual distribution of American wealth, and does a better job than the comparison models with identical fertility or no bequest motive, especially at the extreme right of the distribution. Our richest 1 percent have a 10% greater share of wealth under endogenous fertility. Differential fertility is important to explain the wealth-income distribution disparity.

Our bequest Gini is higher than our wealth Gini at about .94 versus .76, indicating that the higher the bequest-capital ratio in the economy, the higher the overall wealth inequality. This is intuitive, since it is the richest who give the vast majority of bequests, so the higher the overall level of bequests, the more the richest benefit from them.

This result is achieved with a very conservative estimate of intergenerational transfer (not including college costs or inter-vivo transfers) and only taking into account time cost of children. If one were to include all intergenerational transfers and the costs of child care having an impact on child earnings, the differential will increase.

The intuition behind this result is as follows. When children are receiving their bequests, they have more siblings when they are poor and fewer siblings when they are rich relative to identical fertility being imposed. This leads to less division of estates than would otherwise be the case for the richest groups, causing increased concentration of wealth at the highest income levels and greater diffusion at the lower income levels.

The reason the rich have fewer children than the poor is a question that remains without a definitive answer in the literature. Explanations range from preference heterogeneity, desiring quality over quantity, to old age care, to education about contraceptives. The driving force behind the differential in this model is the opportunity cost of children. Since children impose a time cost, they are more expensive for the rich to have than the poor. This is not a judgment into the actual reasons for fertility differences, just a simple way to generate the desired fertility distribution.

Table 5 presents the fertility-income relationship in the benchmark model.

Table 6: Mean Fertility by Ability Group

ψ	1	2	3	4	5	6	7
$Data$[4]	2.65	2.26	2.41	2.04	1.76	1.80	1.51
Endogenous	2.7	2.4	2.5	2.0	1.8	1.7	1.5
Identical	2.04	2.04	2.04	2.04	2.04	2.04	2.04

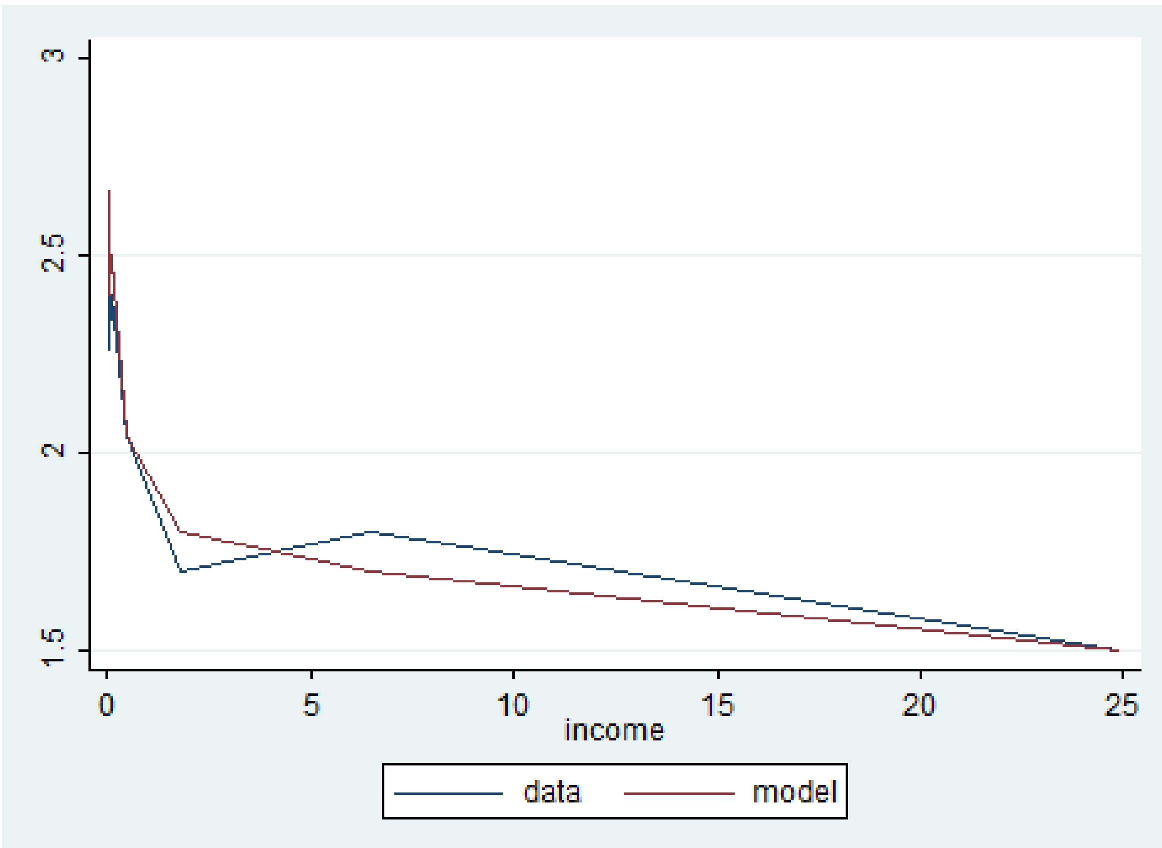

Figure 1: Fertility under the Benchmark Model

We achieve a negative income-fertility elasticity of -.11. This compares to -.24 in the data.

5.2. Counter-factual Experiment I: Identical Fertility

To highlight the role of fertility difference in amplifying the impact of bequests on wealth inequality, now we consider a counter-factual experiment in which fertility is assumed to be identical across the income distribution.

The counter-factual economy follows the same set-up as the benchmark economy. Preferences, technology and demographics are all the same, except that instead of endogenous fertility, in this economy everyone has the same number of children, which is equivalent to the median number of children in the benchmark economy. The benefit to this is it shows what a traditional OLG model looks like that does not take into account fertility, and assumes that every agent has the same number of children.

The counter-factual economy is recalibrated to match the same targets as the benchmark economy, but does not allow endogenous fertility. Instead all households choose the same fertility level, set to be the median of the fertility distribution.

Table 7: Wealth Distribution

Percentile	60−80 %	80−100 %	90−95 %	96−99 %	>99%	Gini Coef.
Data[5]	0.13	0.79	0.13	0.24	0.30	0.78
Benchmark	0.13	0.80	0.19	0.27	0.24	0.76
Identical	0.15	0.74	0.19	0.25	0.19	0.69

We find that fertility can have important ramifications for the economy, increasing the Gini coefficient of wealth by around 10%, and increasing the share of wealth held by the top 1% by a similar amount.

Table 5 highlights the distribution of bequests in this economy. As expected, bequests are extremely skewed, with the top 1 percent giving about 44%, and the top 20 percent giving almost all the bequests in this economy.

Our benchmark economy had a higher bequest Gini than the identical fertility econ-

Table 8: Share of Total Bequests- Benchmark and Identical Fertility

Percentile	80−100 %	90−95 %	96−99 %	>99%	Gini Coef.
Benchmark	0.99	0.21	0.34	0.44	0.94
Identical	0.99	0.23	0.34	0.27	0.87

omy, .94 versus .87. Note that in order to maintain the bequest/wealth level in our calibration, all groups except the two richest give higher bequests under the identical assumption. Therefore in the identical scenario bequests are shifted left in the distribution.

In contrast, savings behavior is much more equitable between the endogenous fertility benchmark and the identical fertility comparison. The gini for savings for the benchmark model is .736, only .003 higher than in the identical model. This is despite the fact the rich are losing more to child care in the identical model.

The intuition behind this result is that the richest group are expecting higher bequests in the Benchmark model because they have fewer siblings to split their parents estate with. We find that expected bequests have a crowding out effect on savings. When expected bequests increase, individuals choose to consume more now and save less for retirement. The effect of bequests on wealth inequality is thus understated, as this reduction in saving makes the calculated wealth inequality underestimated in its welfare effects. Effectively bequests have a twofold effect. The recipient gains the amount of the bequest but also decreases their savings. So bequests have a greater effect on welfare inequity than they do on wealth inequality.

Furthermore, this crowding out effect depends on the individual's discount parameter β. This could have a major impact on models that use calibrated levels of β differential to generate inequality. These models could be understating the value of β if they look only at savings levels and ignore expected bequests. There is an explicit solution of this in the appendix.

5.3. Counter-factual Experiment II: No Bequests

To highlight the role of bequests on wealth inequality, now we consider a counter-factual experiment in which there is no bequest motive.

Table 9: Wealth Distribution of Benchmark and No Bequest comparison

Percentile	60–80 %	80–100 %	90–95 %	96–99 %	>99%	Gini Coef.
Benchmark	0.13	0.80	0.19	0.27	0.24	0.76
NoBequest	0.18	0.61	0.14	0.21	0.14	0.54

The economy is identical to the identical fertility model, except that ϕ_1 is set to null. As this results in no bequest motive, there will be no bequests given.

Removing bequests leads to a major drop in wealth inequality. The drop in inequality is led by a large drop in wealth holding of the very rich. Specifically the top 1% drop their wealth share from 24% to 14%, and the top 20 drop from 80% to 61%. This is consistent with what was found above, that bequests are a major driver of inequality, and when bequests are removed, the wealth share of the richest drops.

6. Conclusion

This paper pursued three goals. First, to build and run a simplified overlapping generations model that includes differential fertility and intergenerational transfers. We did this using a three period model with childhood, adulthood and retirement, where individuals chose fertility endogenously and gave bequests to their children.

Second, to represent the negative income-fertility relationship seen in the data. We did this using a combination of child time costs and concave utility from having children.

Third, to match the wealth-income inequality disparity seen in the data, with wealth inequality being higher than income inequality. We accomplished this to a degree. While this model cannot replicate the large degree of wealth held by the top 1% exactly, it does much better than most macroeconomic models considered in Diaz-Gimenez, Quadrini, and Rios-Rull (1997) at explaining this puzzle.

These results show that adding differential fertility to a model focusing on the explanations for income and wealth inequality is quite important. Differential fertility leads to greater wealth inequality despite similar income inequality and can lead to models that more accurately reflect the data. This is crucial when making policy recommendations. Ignoring the fertility differences between rich and poor can only result in an incomplete picture of inequality.

The next step of this research will be welfare analysis. How does the addition of endogenous fertility affect the overall well-being of individuals in this economy, in an absolute sense, and relatively between rich and poor? Further extensions of this model could look at tax policy. If rich people have fewer children, this could have significant impact on how the estate tax is viewed, and whether an inheritance tax would be more equitable and efficient.

Another addition could be to add discrete education investment by parents into their children, which could have an impact on the debate between private and public education provision, and lead to different levels of investment depending on both wealth and family size. Finally, incorporating accidental fertility, where the chances of an unplanned pregnancy are correlated with education and ability, could help replicate the fertility differences between rich and poor more comprehensively and accurately.

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8. Appendices

8.1. Appendix A: Computational Algorithm

We solved for the Steady State Equilibrium as follows:

1. We create grids for ability ψ, parental wealth x^p, and parental ability ψ^p. There is 400 grid points for wealth, 15 for fertility and 7 for ability, leading to a lattice of 42000 points.

2. We choose a initial per-capita capital (K/L) to generate our initial wage and interest rate.

3. We define our ability transition Markov matrix using Tauchen (1986) and data from Zimmerman (1992) and Solon (1992), calibrated for distribution of earnings.

4. We allocate values generated from the Tauchen process to our 7 ability groups.

5. We allocate an initial guess of population density across our grid.

6. We calculate our 3rd period value function and use nearest neighbor grid search on this constrained optimization to determine the optimal choice variable of bequests to children. We use this to calculate the value of wealth in the 3rd period.

7a. We then calculate the 2nd period value function, using the values generated from step 6.

7b. Using a grid search We determine optimal consumption, savings and fertility for each distinct combination of state variables.

8. We then run a loop of population distribution updating, using the knowledge of optimal consumption, fertility, bequests and savings, to allow total wealth to become new parental wealth x^p, ability to become new parental ability ψ^p and using the Markov transition matrix to calculate the quantity of new children with the state variables $[\psi, \psi^p, x^p]$.

9a. In each loop We calculate the change in population distribution, and sum the total changes to see if the distribution is converging.

9b. In each loop We update the per capita capital and calculate new values of r and w.

$$L_t = \int_\phi \int_{n^p} \int_{x^p} \int_a \phi(\psi, n^p, x^p, a)(1 - n\gamma_\psi)$$

$$K_t = \int_\phi \int_{n^p} \int_{x^p} \phi(\psi, n^p, x^p)(a) + \int_{x^p} \phi(x^p) b^p$$

10. We then sum across the different dimensions of our grid to see the steady state distribution across wealth and ability, and use parental wealth, savings, bequests and ability to calculate the distribution of the variables in steady state.

8.2. Appendix B: Solving Explicitly with 2 Ability Groups

Our two policy functions are:

$$b_c = \frac{a(1+r) + b_p/n_p^* - \frac{\alpha_c \phi_2}{\phi_1(1-\sigma)}^{1/\sigma}}{1 + \frac{\alpha_c \phi_2^{1-\sigma}}{\phi_1(1-\sigma)}^{1/\sigma}}$$

and

$$c^3 = \frac{a(1+r) + b_p/n_p^* + \phi_2}{1 + \frac{\phi_1(1-\sigma)}{\alpha_c \phi_2^{1-\sigma}}^{1/\sigma}}$$

Which allows me to calculate discounted utility for each possible value of savings in period 2. Using this, We can find the optimal level of savings in period 2.

$$\frac{\partial U_2(c,n)}{\partial c} = \alpha_c c^{-\sigma}$$

$$\beta\frac{\partial U_3(a)}{\partial a} = \beta[\alpha_c[\frac{1+r}{1+\frac{\phi_1(1-\sigma)}{\alpha_c\phi_2^{1-\sigma}}^{1/\sigma}}]^{-\sigma}+\frac{\phi_1(1-\sigma)}{\phi_2}[1+\frac{a(1+r)+b_p/n_p^*-\frac{\alpha_c\phi_2}{\phi_1(1-\sigma)}^{1/\sigma}}{\phi_2[1+\frac{\alpha_c\phi_2^{1-\sigma}}{\phi_1(1-\sigma)}^{1/\sigma}]}]^{-\sigma}\frac{1+r}{1+\frac{\alpha_c\phi_2^{1-\sigma}}{\phi_1(1-\sigma)}^{1/\sigma}}]$$

The marginal utility of the consumption in the second period and the shadow utility of saving for the third period must be equal.

$$\frac{\partial U_2(c,n)}{\partial c} = \beta\frac{\partial U_3(a)}{\partial a}$$

subject to

$$c + nc_i + a = w(1 - b_1 n)$$

This gives me two equations and two unknowns (c, a) in period two, which is able to be solved computationally.

For 2 ability groups, since in a steady state the quantity and quality of labor will stay constant over time:

$$Q^{*h} = \frac{1}{1-H_hF_h}L_hF_lQ^{*l}$$

and

$$Q^{*l} = \frac{1}{1-L_lF_l}H_lF_hQ^{*h}$$

So,

$$L_hF_lH_lF_h = (1-H_hF_h)(1-L_lF_l)$$

and

$$L_lF_hH_hF_l = (1-H_lF_l)(1-L_hF_h)$$

Since $L_l + L_h = 1$ and $H_l + H_h = 1$

in order for a steady state to exist, so the transition probabilities of each group can be determined, given fertility.

$$L_h F_l(1 - H_h)F_h = (1 - H_h F_h)(1 - (1 - L_h)F_l)$$

and

$$(1 - L_h)F_h H_h F_l = (1 - (1 - H_h)F_l)(1 - L_h F_h)$$

$$L_h = \frac{(F_h F_l - F_l)(1 - F_l)}{(F_h F_l - F_h)^2} + \frac{1 - F_l}{F_h F_l - F_h}$$

$$L_l = 1 - \frac{(F_h F_l - F_l)(1 - F_l)}{(F_h F_l - F_h)^2} - \frac{1 - F_l}{F_h F_l - F_h}$$

$$H_h = \frac{(F_h F_l - F_l)(1 - F_l)}{(F_h F_l - F_h)^2} + \frac{1 - F_l}{F_h F_l - F_h}$$

$$H_l = 1 - \frac{(F_h F_l - F_l)(1 - F_l)}{(F_h F_l - F_h)^2} - \frac{1 - F_l}{F_h F_l - F_h}$$

Since

$$L_t = \psi_t^h Q^{*h} + \psi_t^l Q^{*l}$$

We can now solve for the amount of labor in the economy while in a steady state. This will allow us to solve for w*

$$w^* = \psi^i (1 - \theta)(\frac{K_t}{L_t})^\theta$$

which, given ability level and along with

$$r^* = \theta(\frac{K_t}{L_t})^{\theta - 1}$$

allows us to computationally calculate the optimal choice variables $c, c', a,$ and b_c.

8.3. Appendix C: Graphical Representation of Wealth

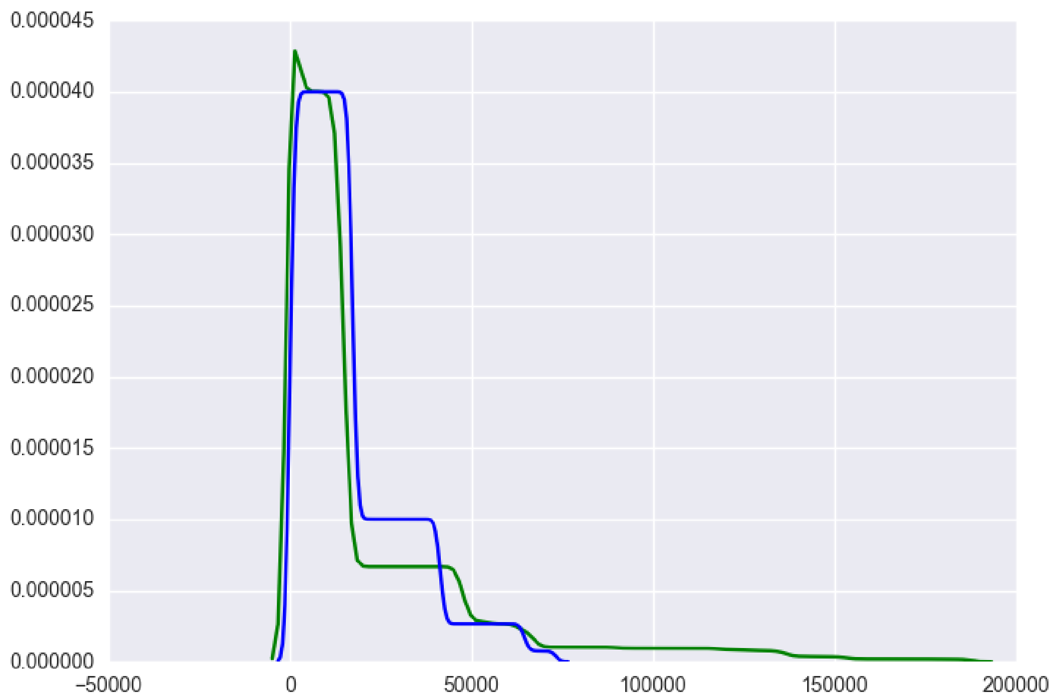

Figure 2: Wealth under the Benchmark Model (green) and the No Bequest Model (blue)

8.4. Appendix D: Markov Matrix and Ability Distribution

	ψ_7	ψ_6	ψ_5	ψ_4	ψ_3	ψ_2	ψ_1
ψ_7^p	0.0720	0.2937	0.4058	0.1907	0.0302	0.0016	0.0000
ψ_6^p	0.0302	0.1907	0.4058	0.2937	0.0720	0.0059	0.0002
ψ_5^p	0.0106	0.1040	0.3415	0.3804	0.1440	0.0183	0.0008
ψ_4^p	0.0031	0.0477	0.2418	0.4146	0.2418	0.0477	0.0032
ψ_3^p	0.0008	0.0183	0.1440	0.3804	0.3415	0.1041	0.0110
ψ_2^p	0.0002	0.0059	0.0720	0.2937	0.4058	0.1907	0.0318
ψ_1^p	0.0000	0.0016	0.0302	0.1907	0.4057	0.2937	0.0780

Our ability values are as follows.

ψ	1	2	3	4	5	6	7
Log	-1.2111	-0.8074	-0.4037	0.0000	0.4037	0.8074	1.2111
$Actual$	0.2979	0.4460	0.6678	1.0000	1.4974	2.2421	3.3572

8.5. Appendix E: Solution for expected bequests crowding out savings

$$V_2(\psi, n^p, x^p) = \max_{c,a,n} \left[\frac{c^{1-\sigma}}{1-\sigma} + \eta_1 n^{\eta_2} + \beta E[V_3(x)] \right]$$

Using non-satiation and substituting:

$$V_2(\psi, n^p, x^p) = \max_{c,a,n} \left[\frac{(\psi w(1-n\gamma) - a)^{1-\sigma}}{1-\sigma} + \eta_1 n^{\eta_2} + \beta E[V_3(x)] \right]$$

Solving for optimal savings using concavity to ensure uniqueness:

$$\frac{\partial V_2(\psi, n^p, x^p)}{\partial a} = -(\psi w(1-n\gamma) - a)^{-\sigma} + \beta((1+r)^T a + b^p - b)^{-\sigma}] = 0$$

Solving for a*

$$a* = \frac{\psi w(1-n\gamma) - \beta^\sigma (b^p - b)}{(1+r)^T(1+\beta^\sigma)}$$

Comparative statics:

$$\frac{\partial a*}{\partial b^p} = -\frac{\beta^\sigma}{(1+r)^T(1+\beta^\sigma)} \leq 0$$

The lower β is the stronger the crowding out effect. Impatient children of rich parents will save very little, anticipating large bequests.