Houses Divided: A Model of Intergenerational Transfers, Differential Fertility and Wealth Inequality*

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Abstract

Rising income and wealth inequality across the developed world has prompted a renewed focus on the mechanisms driving inequality. This paper contributes to the existing literature by studying the impact from life-cycle savings, intergenerational transfers, and fertility differences between the rich and the poor on wealth distribution. We find that bequests increase the level of wealth inequality and that fertility differences between the rich and the poor amplify this relationship. In addition, we find expected bequests crowd out life-cycle savings and this interaction is quantitatively important for understanding wealth inequality in the United States.

Keywords: Intergenerational transfers, differential fertility, wealth inequality, life-cycle savings.

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1. Introduction

The literature shows that standard heterogeneous agents models struggle to replicate the magnitude of the wealth inequality observed in the data. For example, the Gini coefficient of the wealth distribution generated in a baseline Aiyagari (1994) model is only around 0.4, while the U.S. wealth gini coefficient is close to 0.8 (see Quadrini and Ríos-Rull (1997)). An important part of the puzzle is that the rich save more and spend less than predicted by standard models, and consequently accumulate a large amount of wealth. According to Alvaredo et al. (2013), the top 1% of households in the U.S. hold nearly one third of the total wealth and the top 5% holds over half, an order of magnitude larger than their counterparts generated in standard models.

Why is the wealth distribution so unequal? Why do rich people hold such a high amount of wealth? An important existing explanation offered in the literature is from De Nardi (2004), who emphasizes the role of bequests and intergenerational links. De Nardi (2004) finds that the rich are much more likely to leave bequests to their children compared to their poorer counterparts, even after accounting for the relative wealth between the two groups. Based on this finding, she created a model incorporating bequests into the utility function as a luxury good, and finds that this model is capable of accounting for the high concentration of wealth in the data, and that bequeathing behaviors are important in shaping the distribution of wealth. However, the De Nardi (2004) model assumes an identical fertility rate among the population, and thus abstracts from the fact that the poor tend to have more children than the rich, a dimension of heterogeneity we argue is relevant for understanding the wealth distribution. In this paper, we contribute to the literature by extending the De Nardi (2004) model to incorporate differential fertility choice among the population, and analyze the implication of differential fertility for the wealth distribution through its interaction with the bequest mechanism.

Economists have long argued that there exists an inverse relationship between income and fertility.[1] For instance, Jones and Tertilt (2008) document a strong negative relationship between income and fertility choice for all cohorts of women born between 1826 and 1960 in the U.S. census data. They estimate an overall income elasticity of fertility of about

[1] See De La Croix and Doepke (2003), De la Croix and Doepke (2004), Jones and Tertilt (2008), Zhao (2011), Zhao (2014), among others.

-0.38. We argue that this significant fertility difference between the poor and the rich can amplify the impact of bequests on wealth inequality, because not only do rich parents leave a greater amount of bequests than their poorer counterparts, but the children of rich parents have fewer siblings to share their bequests with relative to the children of poor parents.

To capture the interaction between differential fertility and bequests, and to assess its quantitative importance for understanding the wealth distribution, we develop a general equilibrium overlapping generations (OLG) model with the "warm-glow" bequest motive (similar to that used in De Nardi (2004)) and differential fertility. Using a version of our model calibrated to the U.S. economy, we find that the fertility difference between the rich and the poor increases the wealth Gini coefficient by about 5%, driven especially by about a one quarter increase in the wealth share of the top 1 %. We also quantify the importance of the bequest mechanism by showing that an alternative model in which the bequest channel has been shut down results in a much lower Gini coefficient of the wealth distribution, i.e., 0.68, compared to our benchmark value of 0.79. In addition, we find in our model that anticipated bequests crowd out life-cycle savings, which implies that intergenerational transfers can lead to less capital formulation. In sum, this paper finds that pairing bequest motive with differential fertility is quantitatively important for explaining the saving behaviors of the rich and the consequent high level of wealth inequality.

1.1. Literature Review

Ever since heterogeneous agent macroeconomic models have been introduced to the macroeconomics literature by Bewley (1986), Imrohoroglu (1989), Huggett (1993), and Aiyagari (1994), a surge in papers have used this class of models to explain the causes and mechanisms behind wealth inequality. As surveyed by De Nardi (2015), there have been many variations of the heterogeneous agent model in which introduce various mechanisms to better match the magnitude of wealth inequality observed in the data, such as preference heterogeneity (Krusell and Smith (1998), Heer (2001), Suen (2014)), entrepreneurship (Cagetti and De Nardi (2006)), high earnings risk for the top earners (Castaneda et al. (2003)), transmission of bequests across generations (Knowles (1999), De Nardi (2004), and De Nardi and Yang (2016)), and others. Among these, our paper relates to the literature espousing bequest transmission across generations as a main mechanism behind wealth inequality.

The two papers in this literature closest to ours in spirit are De Nardi (2004) and Knowles (1999). De Nardi (2004) uses a quantitative, general equilibrium, overlapping-generations model in which bequests and ability link parents and children. The element in which our papers differ is in our treatment of fertility. In De Nardi (2004), each agent has the same number of children. In our model agents have a different number of children depending on the income, impacting the results in interesting ways. Our model is also close in spirit to Knowles (1999), who uses a two period model to show the importance of fertility to inequality. In his model, there is no retirement period, which means savings that occur in his model are solely for the purpose of bequests. In contrast, the agents in our model must save for their own retirement on top of bequests. Therefore, our model captures the dynamic interaction between life-cycle savings and anticipated bequests. We show this interaction is quantitatively important for understanding the wealth distribution. In addition, our model differs from Knowles (1999) in terms of the choice of the bequest motive. While bequests are assumed to be motivated by altruism in the Knowles (1999) model, we adopt the "warm-glow" bequest motive based on the empirical literature we will discuss below.

It is well-known in the literature that intergenerational transfers account for a large fraction of wealth accumulation.[2] However, the literature has been at odds as to how to model bequest motives, specifically whether bequests are motivated by altruism. Altonji et al. (1992) found that the division of consumption and income within a family are codependent, indication that perfect altruism does not apply to operative transfers. Other studies show that an increase in parental resources coupled with a decrease in child consumption does not lead to a corresponding increase in transfers (Altonji et al. (1997) and Cox (1987)). Altonji et al. (1997) find a one dollar transfer from child to parent results in only a 13 cent donation from parent to child, which should be the full dollar under perfect altruism. Wilhelm (1996) finds siblings generally receive equally divided inheritances, rather than the size of the inheritance being dependent on relative income as perfect al-

[2]For instance, see Kotlikoff and Summers (1981), Gale and Scholz (1994), among others.

truism would predict.[3] Based on these empirical findings, multiple recent papers have assumed an alternative bequest motive: the *warm-glow* motive.[4] That is, parents derive utility from giving while not caring directly about the wellbeing of the recipient. In addition, motivated by the highly skewed distribution of bequests, these papers incorporate leaving bequests into the utility function as a luxury good, allowing for rich parents to value bequests relatively more. Following the tradition in these papers, we also adopt the "warm-glow" motive and assume bequests are a luxury good.

Our paper also relates to a growing number of papers that have shown that allowing for transfer of ability and human capital across generations is also an important element for understanding inequality. These studies include Kotlikoff and Summers (1981), Knowles (1999), De Nardi (2004), De Nardi and Yang (2016), and among others. Of special note, Lee et al. (2015) find that parental education is positively related to their children's earnings, thereby creating a virtuous cycle for the wealthiest and a vicious cycle for the poorest.

The rest of paper is organized as follows. In Section 2., we describe the model and its stationary equilibrium. In Section 3., we calibrate a benchmark specification using moment matching. In Section 4., we discuss the results. The final section concludes.

2. The Model

Consider an economy inhabited by overlapping generations of agents who live for three periods. In the first period, agents are not economically active, only incurring costs to their parents. In the second period, they make consumption and labor supply decisions, and save for retirement. In the final period, they receive bequests from their dying parents, consume some of their wealth and leave the remainder as bequests to their own children.

2.1. Consumer's Problem

[3] Note that the nature of intergenerational links can be different in developing countries that feature different institutions and less generous public insurance. For instance, Imrohoroglu and Zhao (2017) find in the Chinese data that intergenerational transfers are highly dependent on the financial and health states of parents, suggesting strong altruism between parents and children.

[4] See De Nardi (2004), De Nardi and Yang (2016), among others.

2.1.1. Period One

An individual makes no economic decisions in the first period, but imposes a time cost on her parents. She inherits an ability level from her parents. An individual's ability ψ (effective units of labor representing human capital, luck or inherent ability) depends on their parental ability ψ^p, and the log of ability is assumed to follow the AR(1) process,

$$\log(\psi) = \rho \log(\psi^p) + \epsilon_\psi$$

where

$$\epsilon_\psi \sim N\left(0, \sigma_\psi^2\right), \quad \text{i.i.d.}$$

in which ρ is the intergenerational persistence of productivity. We discretize the AR(1) into 11-state Markov chain using the method introduced in Tauchen (1986), and the corresponding transition matrix we obtain is denoted by $M[\psi, \psi']$.

2.1.2. Period Two

Individuals in the second period differ along three dimensions: earning ability ψ, number of siblings n^p (or the parent's fertility), and current wealth of their elderly parents x^p. In this period, they jointly choose current consumption and save for period three. In addition, they raise n number of children, which is assumed to be an exogenous function of their earning ability ψ, that is, $n = n(\psi)$.[5] Therefore, the value function of an individual in period two can be specified as follows:

$$V_2(\psi, n^p, x^p) = \max_{c,a} \left[\frac{c^{1-\sigma}}{1-\sigma} + \beta V_3(x)\right]$$

subject to

$$c + a \leq \psi w(1 - \gamma n(\psi))$$
$$x = a + \frac{b^p(x^p)}{n^p}.$$

[5]We also analyze an extended model with endogenous fertility later to explore the sensitivity of our main results to the assumption of exogenous fertility.

Here, the current utility flow is derived from consumption c according the CRRA form, and β stands for the time discount factor. Agents are given a time allocation set to unity. In the budget constraint, γ is the time cost per child per parent, and thus $(1-\gamma n(\psi))$ simply represents the amount of time available to be allocated to the labor force. This implies that $\psi(1-\gamma n(\psi))$ is the total amount of effective labor supplied, with w measuring the real wage per effective unit of labor. Note that because child costs are delineated in time, higher earning parents will effectively be paying more for their children, as is expected and reflected in the data. We also restrict a, the amount saved, to be strictly non-negative, thereby imposing imperfect capital market. In other words, agents cannot borrow to finance their retirement.

The second constraint of the maximization problem describes how total amount of wealth in the third period x is determined. It is the sum of life-cycle savings a, and the share of bequests received from dying parents b^p/n^p. Here, b^p denotes the bequest left by the parent, which is a function of the parent's total wealth x^p at the beginning of the third period. It is obtained from solving the utility maximization problem for the third period. It is important to note that the bequest is shared by all children of the parent, and thus what each child receives is negatively affected by the number of siblings she has. From the utility maximization problem in Period 2, we obtain two policy functions: optimal consumption $C_2\left(\psi, n^p, x^p\right)$ and optimal asset accumulation $A(\psi, n^p, x^p)$.

2.1.3. Period Three

Individuals retire in the third period and jointly choose current consumption and the amount of bequests for her children. Their state in this period can be captured by a single variable, x, the amount of wealth held, which is simply the sum of life-cycle savings and the share of bequests received from their dying parents at the beginning of Period 3. Individuals in Period 3 face the following utility-maximization problem:

$$V_3(x) = \max_{c,b}\left[\frac{c^{1-\sigma}}{1-\sigma} + \phi_1\left(b+\phi_2\right)^{1-\sigma}\right]$$

subject to

$$c + b \leq (1+r)x,$$

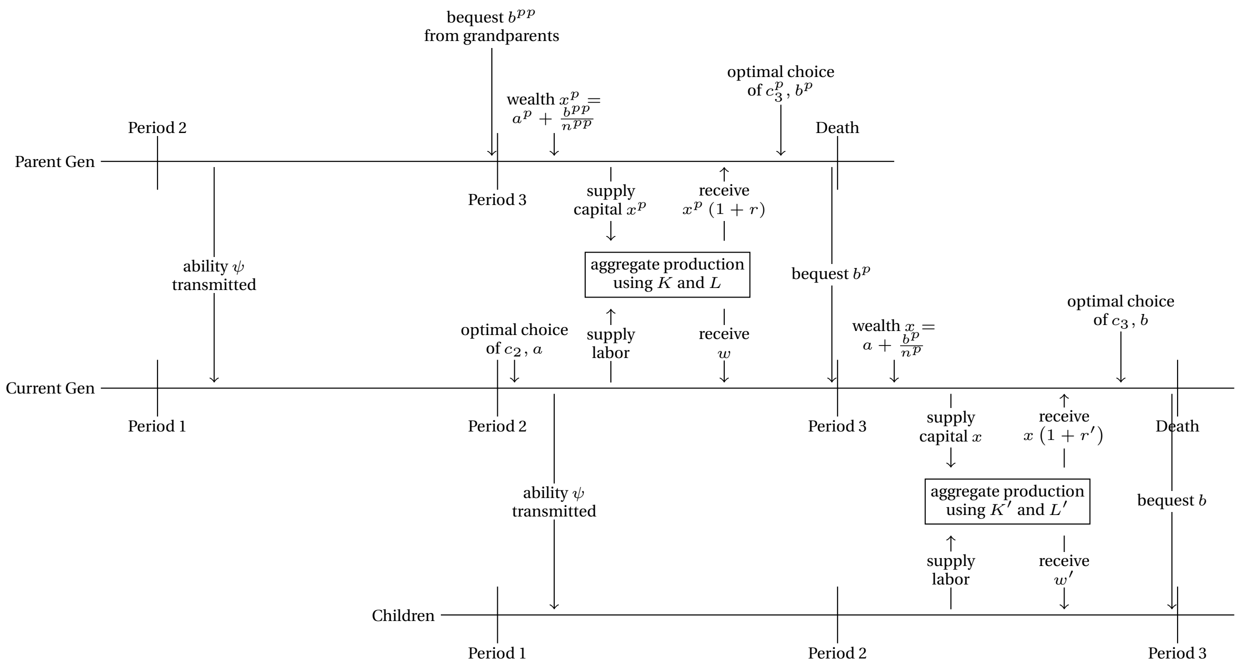

Figure 1: Sequence of Events for Current Generation

where b is the total amount of bequests left for children in the next period. Here we follow De Nardi (2004) and assume that parents have "warm glow" motive, where they enjoy giving to their children but do not directly care about the children's wellbeing, and in addition bequest is assumed to be a luxury good. As we reviewed in the introduction, this assumption is consistent with sizable empirical evidence. The term ϕ_1 measures the relative weight placed on the bequest motive, while ϕ_2 measures the extent to which bequests are a luxury good. From this maximization problem, we obtain two policy functions: optimal consumption $C_3(x)$ and optimal bequests $B(x)$.

Figure 1 contains the timeline summing up the sequence of events that happen throughout the lifecycle.

2.2. Firm's Problem

Firms are identical and act competitively. Their production technology is Cobb-Douglas, which combines aggregate capital K and aggregate labor L to produce output Y as follows

$$Y = zK^{\theta}L^{1-\theta}$$

in which θ is the capital share and z is the total factor productivity (TFP).

The profit-maximizing behaviors of firms imply that

$$r = z\theta K^{\theta-1}L^{1-\theta} - \delta$$

and

$$w = z(1-\theta)K^{\theta}L^{-\theta},$$

where δ represents the capital depreciation rate.

2.3. Stationary Equilibrium

Let Φ_2 and Φ_3 represent the population distributions of individuals in period 2 and 3. A steady state in this economy consists of a sequence of allocations $[c_2, c_3, a, b]$, aggregate inputs $[K, L]$ and prices $[w, r]$ such that

1. Given prices, the allocations $[c_2, c_3, a, b]$ solve each individual's utility maximization problem
2. Given prices, aggregate capital and labor $[K, L]$ solve the firm's problem.
3. Markets clear:

$$K' = \int_{\psi}\int_{n^p}\int_{x^p}\left[A\left(\psi, n^p, x^p\right) + \frac{B^p\left(x^p\right)}{n^p}\right] d\Phi_2\left(\psi, n^p, x^p\right)$$

$$L' = \widehat{n}\int_{\psi}\int_{n^p}\int_{x^p}(1-\gamma N'(\psi, n^p, x^p))\psi d\Phi_2(\psi, n^p, x^p)$$

where $\widehat{n}$ is the average number of children the current period two individuals have.

4. The distributions Φ_2 and Φ_3 are stationary in the steady state and evolve according to the following laws of motions:

$$\Phi_2\left(\psi', n^{p'}, x^{p'}\right) = \frac{1}{\widehat{n}} \int_{\psi} \int_{n^p} \int_{x^p} I_{x^{p'}=A(\psi,n^p,x^p)+\frac{b^p(x^p)}{n^p}} I_{n^{p'}=n(\psi)} M\left[\psi, \psi'\right] n(\psi) d\Phi_2\left(\psi, n^p, x^p\right)$$
$$\Phi_3(x^{p'}) = \frac{1}{\widehat{n}} \int_{\psi} \int_{n^p} \int_{x^p} I_{x=A(\psi,n^p,x^p)+\frac{b^p(x^p)}{n^p}} d\Phi_2(\psi, n^p, x^p)$$

where $M[\cdot]$ is the Markov transition matrix, I's are the indicator functions. The ability distribution in the next period depends on the current period young's fertility. In the third period, an individual's wealth is what he has saved in the previous period, as well as what he has received in bequests from his parents. Note that the distribution of the elderly's wealth holdings is identical to the distribution of the young's parental wealth holdings (i.e., $x = x^{p'}$).

The rest of the paper focuses on stationary equilibrium analysis. Since analytical results are not obtainable, numerical methods are used to solve the model.

3. Calibration

We calibrate the model to match the current U.S. economy, and the calibration strategy we adopt here is the following. The values of some standard parameters are predetermined based on previous studies, and the values of the rest of the parameters are then simultaneously chosen to match some key empirical moments in the U.S. economy.

3.1. Demographics and Preferences

One period in our model is equivalent to 30 years. Individuals enter the economy when they are 30 years old (Period 2). They retire at 60 years old (Period 3) and die at the end of the third period (at 90 years old).

The parameter in CRRA utility, σ, is set to 1.5 based on the existing macro literature. The subjective discount factor β is calibrated to match an annual interest rate of 0.04, which gives us an annual discount factor of 0.90. We calibrate our bequest parameters to ensure

Table 1: The Benchmark Calibration

Parameter	Value	Source
z	1.0	Normalization
σ	1.5	Macro Literature
θ	0.36	Macro Literature
δ	0.04	Macro Literature
γ	0.2	Haveman and Wolfe (1995)
ρ	0.4	Solon (1992)
Parameter	**Value**	**Moment to match**
β	0.90	annual interest rate: 0.04
ϕ_1	-0.33	bequest/wealth ratio: 0.31
ϕ_2	0.086	90 percentile of the bequest distribution: 0.08
σ_ψ^2	1.15	Income Gini: 0.63

that the level and distribution of bequests generated from our benchmark model matches their respective data counterparts. Specifically, ϕ_1 is calibrated to match the aggregate bequest to wealth ratio: 0.31 according to the estimation by Gale and Scholz (1994). ϕ_2 is calibrated to match the 90th percentile of the bequest distribution seen in the data, estimated by Hurd and Smith (2002). We do not include inter-vivos transfers or college expenses to be conservative in our estimates of the importance of intergenerational transfers.

We use the 1990 U.S. census data to calibrate the fertility choices for each group in our benchmark model[6]. We follow the approach in Jones and Tertilt (2008) and use the Children Ever Born to a woman as the fertility measure. Specifically, we use the sample of currently married women ages 40-50 (birth cohort 1940-50), and then organize the respondents into 11 ability groups corresponding to our model distribution by Occupational Income, corrected for a 2% growth rate.[7] We believe the propensity of death on childbirth

[6]Courtesy of Steven Ruggles, Katie Genadek, Ronald Goeken, Josiah Grover, and Matthew Sobek. Integrated Public Use Micro data Series: Version 6.0 [dataset]. Minneapolis: University of Minnesota, 2015. http://doi.org/10.18128/D010.V6.0.

[7]Here we follow Jones and Tertilt (2008) closely and use the husband's occupational income to avoid the selection bias in women's employment status.

during this time period is low enough that the child mortality risk is not a significant issue. We take the mean fertility rate for each group and assign it to the corresponding group of agents in our benchmark model to generate the appropriate level of differential fertility by income.[8] The resulting fertility-income relationship from our calibration exercise is reported in Table 3, which is consistent with the estimation results in Jones and Tertilt (2008). For instance, the income elasticity of fertility is estimated to be -0.20 to -0.21 for the cohorts of women born between 1940 and 1950 in Jones and Tertilt (2008), while the implied income elasticity of fertility from our calibrated fertility distribution is -0.22.

3.2. Technology and Earning Ability

The capital share θ is set to 0.36, and the capital depreciation rate is set to 0.04. Both are commonly used values in the macro literature. The value of TFP parameter, z, is normalized to one.

We approximate the AR(1) process for earning ability ψ by an 11-state Markov chain using the method introduced in Tauchen (1986). The coefficient of intergenerational persistence, ρ, is set to 0.4 according to the estimates in Solon (1992). We calibrate the income variance σ_ψ^2 so that the income Gini coefficient generated from the model matches the value of 0.63 that Castaneda et al. (2003) estimated using the 1992 Survey of Consumer Finances data. We report the resulting ability levels in Table 3 and the corresponding transition matrix can be seen in Section A of the Appendix. In addition, we set the time cost of children γ to be 0.2 of parental time per child based on the empirical estimates of Haveman and Wolfe (1995).

The key parameter values and their sources are summarized in Table 1.

4. Quantitative Results

We start this section by reviewing the main properties of the benchmark model at the steady state, with special attention given to its implications for wealth inequality. We then run counter-factual computational experiments to highlight the impact of differential fer-

[8] Note that the fertility choice in our model is the per parent fertility so we follow the tradition in the fertility literature and halve these fertility rates calculated from the data when using them in the model.

Table 2: Benchmark Model Statistics

Name	Model	Data
Annual Interest Rate	0.04	0.04
US Aggregate Bequest/Wealth Ratio	0.31	0.31
90th Percentile of the Bequest Distribution	0.91	0.90
Average fertility rate per household	2.3	2.3
Gini Coefficient of the US Income Distribution	0.64	0.63
Income Elasticity of Fertility	-0.22	-0.20/-0.21

tility and bequests on wealth inequality.

4.1. Some Key Properties of the Benchmark Economy

A key element of our theory is the negative income-fertility relationship, which is best measured by the income elasticity of fertility. As we mentioned previously, the income elasticity of fertility implied by our benchmark model is very close to its empirical counterpart estimated by Jones and Tertilt (2008). Another important part of our theory is the skewed distribution of bequests with a long right tail. We ensure the model matches the bequest distribution we observe in the data by modelling bequests as luxury goods in the fashion of De Nardi (2004) and De Nardi and Yang (2016). In addition, our calibration strategy implies that our benchmark model matches the bequest-capital ratio and the 90th percentile of bequest amount.

Table 2 contains some key statistics of the benchmark economy together with their data counterparts. As can be seen, our calibrated benchmark model matches the key empirical moments from the US economy fairly well. Table 3 summarizes the ability distribution generated by our benchmark model, along with how the average fertility calculated by ability groups match up against the data. The first row represents the relative value of the ability ψ_i for Group i, in which the value for Group 6 is normalized to unity. The second row is the share of the population whose ability is equal to or less than that group. Hence, Group 11—the highest ability group in our model—corresponds to the top 0.4% and the

Table 3: Fertility-Income Relationship from the Benchmark Model

Ability Group i	1	2	3	4	5	6	7	8	9	10	11
ψ_i	0.02	0.04	0.09	0.21	0.46	1.0	2.19	4.81	10.56	23.16	50.80
Cumulative Mass	0.004	0.015	0.064	0.185	0.383	0.617	0.815	0.937	0.985	0.996	1.0
Fertility per Parent	1.6	1.6	1.4	1.4	1.25	1.15	1.08	1.07	0.96	0.86	0.88

top two groups together correspond to the top 2% of the population.

4.2. Wealth Inequality in the Benchmark Economy

In this section, we examine the wealth distribution generated in our benchmark model. We compute the proportion of overall wealth held by each percentile group in our benchmark model and compare it against the data. Some key statistics of the wealth distribution are reported in Table 4.[9] The richest 1% from our benchmark model hold less wealth than the data, but overall our model does moderately accurate job of matching the actual distribution of wealth in the U.S., especially among the top 20%. As can be seen in the last column, our benchmark model also matches the Gini coefficient of the wealth distribution closely. It is important to note that these statistics of the wealth distribution are not used as our targeted moments in the calibration.

To understand the role of differential fertility and bequests in shaping the U.S. wealth inequality, in the rest of this section we conduct two counter-factual computational experiments in which each of the two factors is assumed away respectively. In the first counterfactual experiment, we impose identical fertility to show the effects of differential fertility on the distribution of wealth and bequests. In the second counter-factual experiment, we eliminate the bequest motive to highlight the impact of bequests on wealth inequality. From these two counter-factual experiments, two things become clear. We find that bequests significantly increase the level of wealth inequality, and fertility differences between the rich and the poor amplify this effect, especially for the far right of the wealth distribution. In addition, we find that life-cycle saving and anticipated bequests interact with each other, with expected bequests crowding out life-cycle saving for retirement.

[9] A graphical distribution of wealth generated from our benchmark model can be seen in Figure 2.

Table 4: Wealth Distribution: Model vs Data

Percentile	$< 60\%$	$60-80$ %	$> 80\%$	$90-95$ %	$95-99$ %	>99%	Gini Coef.
Data	0.08	0.13	0.79	0.13	0.24	0.30	0.78
Benchmark Model	0.05	0.13	0.82	0.18	0.30	0.19	0.79
Identical Fertility	0.07	0.16	0.77	0.17	0.26	0.15	0.75
No Bequest	0.11	0.18	0.71	0.15	0.23	0.14	0.68

Data source: Diaz-Gimenez et al. (1997)

This interaction is quantitatively important for fully understanding wealth inequality in the United States.

4.3. Counter-factual Experiment I: Identical Fertility

To highlight the important role of fertility differences across the income groups in amplifying the impact of bequests on wealth inequality, we consider a counter-factual experiment in which fertility is assumed to be identical across the income distribution. That is, we force everyone in the model to have the same fertility choice, 1.15 per parent, and recalibrate the model using exactly the same strategy and the same empirical moments as in the benchmark model.

The main results from this counter-factual experiment are also reported in Table 4. We find that allowing for differential fertility can have important ramifications for the wealth distribution, as evidenced by the Gini coefficient of wealth distribution, and the share of wealth held by the top 1% respectively increasing by around 4% and by about a quarter.

The reason why the counter-factual model with identical fertility performs worse than the benchmark model with differential fertility can be best understood when we analyze the distribution of bequests generated from the two models. Table 5 highlights the differences. In the benchmark model, we obtain a extremely skewed distribution of bequests in which the top 1 % are responsible for 43% of total bequests at the steady state. In fact, the top 10 % are responsible for almost all the bequests. As a result, the Gini coefficient is very high at 0.96. In contrast, the counter-factual model with identical fertility obtains a

Table 5: Bequest Distribution: Benchmark vs. Identical Fertility

Percentile	$< 90\%$	$90-95$ %	$95-99$ %	>99%	Gini Coef.
Benchmark	<0.01	0.09	0.48	0.43	0.96
Identical Fertility	0.12	0.23	0.39	0.26	0.90

lower value of Gini coefficient of 0.90. This decrease mainly results from the fact that the share of total bequests from the top 1% and the top 5% drop significantly. We argue that this change in the distribution of bequests is an important reason why the counter-factual model generates a lower wealth inequality. The intuition behind this result is the following. When children are receiving their bequests, poor children have more siblings and rich children have fewer siblings relative to the identical fertility case. This leads to less division of estates than would otherwise be the case for the richest groups, causing increased concentration of wealth at the highest income levels and greater diffusion at the lower income levels.[10]

It is also interesting to examine the life-cycle saving behaviors in the two versions of the model. We find that life-cycle saving and anticipated bequests interact with each other, which is important for understanding the wealth distribution. That is, anticipated bequests have a crowding out effect on life-cycle saving. As shown in Table 6, the Gini coefficient for the distribution of life-cycle saving from the benchmark model is 0.72 and the top 20% account for 0.74 of the savings, which is lower than that from the counter-factual model with identical fertility. On the surface, this result is puzzling because you would expect the rich from the counter-factual model to be saving less than the rich from the benchmark model. That is, the rich in this counter-factual economy are forced to have more children than otherwise they would have, therefore they spend more time raising children and receive less labor income than in the benchmark model. Assuming the same saving rates in the two models, the life-cycle saving distribution should be less unequal in the model with identical fertility.

The reason why the distribution of life-cycle saving becomes more unequal after shut-

[10]The reason the rich have fewer children than the poor is a question that remains without a definitive answer in the literature. Please see Jones et al. (2008) for a complete literature review on this topic.

Table 6: Distribution of Life-Cycle Saving

Percentile	$< 60\%$	60−80 %	$> 80\%$	90−95 %	95−99 %	>99%	Gini Coef.
Benchmark Model	0.10	0.16	0.74	0.14	0.25	0.18	0.72
Identical Fertility	0.09	0.15	0.76	0.15	0.26	0.17	0.73
No Bequest	0.11	0.18	0.71	0.15	0.23	0.14	0.68

ting down differential fertility is because of the interaction between anticipated bequests and life-cycle saving. In other words, the more unequal life-cycle saving distribution seen in the identical fertility model is simply the endogenous response to the less unequal distribution of bequests in this counter-factual model. Given that the wealth distribution is jointly determined by the distributions of life-cycle saving and bequests, the crowding out effect from anticipated bequests on life-cycle saving weakens the impact of bequests on wealth inequality. See B for a more complete explanation of this effect.

4.4. Counter-factual Experiment II: No Bequests

To highlight the role of bequests on wealth inequality, we now consider a counter-factual experiment in which there is no bequest motive and identical fertility. This allows us to parse out the effects of fertility and bequests. Computationally, we set ϕ_1 equal to null, which means no bequests ever take place at the steady state, and recalibrate the rest of the parameters in the same way as in the benchmark model. Results from this experiment are also shown in Table 4. As can be seen, removing bequests leads to a major drop in wealth inequality. The drop in inequality is led by a large drop in wealth holding of the very rich. Specifically the top 1% drop their wealth share from 19% to 14%, and the top 20 drop from 82% to 71%. This result is a further testament to what we conclude from our benchmark model, which is that bequests are a major driver of inequality.

Table 7: Income-Fertility Relationship in the Endogenous Fertility Model

Ability Group i	1	2	3	4	5	6	7	8	9	10	11
ψ_i	0.02	0.04	0.09	0.21	0.46	1.0	2.19	4.81	10.56	23.16	50.80
Cumulative Mass	0.004	0.015	0.064	0.185	0.383	0.617	0.815	0.937	0.985	0.996	1.0
Fertility per Parent	1.62	1.40	1.35	1.35	1.24	1.15	1.15	1.07	0.95	0.85	0.85

5. An Extended Model with Endogenous Fertility

We assume that fertility choices are exogenous in our benchmark model. This assumption significantly simplifies our analysis, and helps us avoid the complicated theoretical issues that arose in the literature on the negative income-fertility relationship (see Jones et al. (2008) for a complete review of this literature). In this section, we consider an extended version of the model to assess the sensitivity of our main results with regard to this assumption.

In this extended model, we endogenize the fertility choices by simply assuming that the number of children directly enters into agents' utility function. Specifically, the second period problem facing agents becomes:

$$V_2\left(\psi, n^p, x^p\right) = \max_{c,a,n\geq 0}\left[\frac{c^{1-\sigma}}{1-\sigma} + \lambda_1 n^{\lambda_2} + \beta\left[V_3(x)\right]\right]$$

subject to

$$c + a \leq \psi w(1-\gamma n)$$
$$x = a + \frac{B\left(x^p\right)}{n^p}$$

Here agents derive utility from both current consumption c and the number of children they choose to have, n. λ_1 is the relative weight on the utility derived from children, and λ_2 controls the curvature of the utility from children.

To generate the negative income-fertility relationship observed in the data, we have to use a $\sigma \in [0,1]$. Specifically, we set the value of σ to be 0.9. We calibrate the values of

Table 8: The Calibration of the Endogenous Fertility Model

Parameter	Value	Source
σ	0.9	Model Specification
β	0.92	annual interest rate: 0.04
ϕ_1	2.9	bequest/wealth ratio: 0.31
ϕ_2	0.07	90 percentile of the bequest distribution: 0.08
λ_1	0.712	Average Fertility Rate: 2.3
λ_2	0.369	Income-Fertility Elasticity: -0.21
σ_ψ^2	1.15	Income Gini: 0.63

Table 9: Wealth Distribution: Benchmark vs. Endogenous Fertility

Percentile	$< 60\%$	60−80 %	$> 80\%$	90−95 %	95−99 %	>99%	Gini Coef.
Benchmark Model	0.05	0.13	0.82	0.18	0.30	0.19	0.79
Endogenous	0.05	0.14	0.81	0.18	0.29	0.19	0.78

λ_1 and λ_2 to match the following two moments: the average fertility rate and the income elasticity of fertility. We calibrate the other parameters using the same moments as in the benchmark model. The fertility rates by each ability group are shown in Table 7. Our calibrated parameters are shown in Table 8.

The wealth distribution from the extended model is shown in Table 9. As can be seen, the main results remain very similar to those in our benchmark model, showing that our results are robust to the assumption of exogenous fertility.

6. Conclusion

This paper pursued three goals. First, to build and run a simple overlapping generations model including differential fertility and intergenerational transfers. We did this using a three period model with childhood, adulthood and retirement, where individuals evinced differential fertility and gave bequests to their children. Second, to represent the negative income-fertility relationship seen in the data. We did this using a combination of child time costs and concave utility from having children and generated an endogenous fertility choice in an extension to our benchmark model that closely matches the data. Third, to match the wealth-income inequality disparity seen in the data, where wealth inequality is higher than income inequality. Although the wealth held by the top 1% from our model does not completely match the data, we come very close and match various other important moments. In fact, we do much better than most macroeconomic models considered in Diaz-Gimenez et al. (1997) at explaining this puzzle. Overall, our results show that allowing differential fertility is crucial in explaining the disparity between the income and wealth inequality. In other words, we show that ignoring the fertility differences between the rich and the poor can only result in an incomplete picture of inequality.

We conclude the paper by drawing attention to a few potentially important issues from which this paper has abstracted. First, this paper contains no welfare analysis. How does the addition of endogenous fertility affect the overall well-being of individuals in this economy, in an absolute sense, and relatively between rich and poor? Second, we do not model government, even though fiscal policies could have interesting implications within this framework. If rich people have fewer children, this could have significant impact on how the estate tax is viewed, and whether an inheritance tax would be more equitable and efficient. Third, it could be important to add discrete education investment by parents into their children, which could have an impact on the debate between private and public education provision, and lead to different levels of investment depending on both wealth and family size. Finally, incorporating accidental fertility, where the chances of an unplanned pregnancy are correlated with education and ability, could help replicate the fertility differences between rich and poor more comprehensively and accurately.

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Appendices

A Markov Matrix and Ability Distribution

In this section we show the Markov chain generated from our Tauchen (1986) process for the ability shock.

	ψ^p_{11}	$\psi^p_1 0$	ψ^p_9	ψ^p_8	ψ^p_7	ψ^p_6	ψ^p_5	ψ^p_4	ψ^p_3	ψ^p_2	ψ^p_1
ψ_{11}	0.05	0.03	0.02	0.01	0.0	0.0	0.0	0.0	0.0	0.0	0.0
ψ_{10}	0.11	0.08	0.05	0.03	0.02	0.01	0.0	0.0	0.0	0.0	0.0
ψ_9	0.21	0.17	0.13	0.09	0.06	0.04	0.02	0.01	0.01	0.0	0.0
ψ_8	0.26	0.25	0.22	0.19	0.15	0.11	0.08	0.05	0.03	0.02	0.01
ψ_7	0.21	0.24	0.25	0.25	0.24	0.21	0.17	0.13	0.09	0.06	0.04
ψ_6	0.11	0.15	0.19	0.22	0.25	0.26	0.25	0.22	0.19	0.15	0.11
ψ_5	0.04	0.06	0.09	0.13	0.17	0.21	0.24	0.25	0.25	0.24	0.21
ψ_4	0.01	0.02	0.03	0.05	0.08	0.11	0.15	0.19	0.22	0.25	0.26
ψ_3	0.0	0.0	0.01	0.01	0.02	0.04	0.06	0.09	0.13	0.17	0.21
ψ_2	0.0	0.03	0.0	0.0	0.0	0.01	0.02	0.03	0.05	0.08	0.11
ψ_1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.01	0.02	0.03	0.05

B Relationship between savings and bequests

In this section we explore the relationship between saving and expected bequests. To explicitly show the relationship between expected bequests and saving we make use of the second period value function and the budget constraints. We will simplify to get to a solution for savings, then find the marginal effect of a change in the amount of expected bequest on the amount of optimal savings.

$$V_2(\psi, n^p, x^p) = \max_{c,a,n} \left[\frac{c^{1-\sigma}}{1-\sigma} + \eta_1 n^{\eta_2} + \beta V_3(x) \right]$$

$$c + a \leq \psi w(1 - \gamma n)$$

$$x = a(1 + r) + b^p$$

Using non-satiation to allow us to assume all income with be either saved or consumed and substituting:

$$V_2(\psi, n^p, x^p) = \max_{c,a,n} \left[\frac{(\psi w(1 - n\gamma) - a)^{1-\sigma}}{1-\sigma} + \eta_1 n^{\eta_2} + \beta V_3(x) \right]$$

Solving for optimal savings (a*) by rearranging, and using concavity to ensure uniqueness:

$$\frac{\partial V_2(\psi, n^p, x^p)}{\partial a} = -(\psi w(1 - n\gamma) - a)^{-\sigma} + \beta((1 + r)^T a + b^p)^{-\sigma}] = 0$$

$$a* = \frac{\psi w(1 - n\gamma) - \beta^\sigma (b^p)}{(1 + r)^T (1 + \beta^\sigma)}$$

Comparative statics:

$$\frac{\partial a*}{\partial b^p} = -\frac{\beta^\sigma}{(1 + r)^T (1 + \beta^\sigma)} \leq 0$$

We explore the predicted effect empirically by estimation. To do so, we first use the 2013 Survey of Consumer Finance to estimate how optimal savings a^* changes as a function of expected bequest $b^p \equiv \frac{B(x^p)}{n^p}$. The Survey of Consumer Finance has a survey question that asks respondents about how much they believe they should have saved. We use the responses from that question to estimate the following regression equation

$$a_i^* = \beta_1 \mathbb{E}\left[b_i^p\right] + \beta_{iq} \chi_{iq} + \epsilon_i \tag{1}$$

Table 10: SCF Regression Results

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β_1	$-.023^{***}$	$-.028^{*}$	-.034	-.035	-.023	-.028	-.033	-.035
S.E.	(.008)	(.015)	(.027)	(.045)	(.060)	(.077)	(.087)	(.159)
Bootstrapped S.E.	No	No	No	No	Yes	Yes	Yes	Yes
Top	100%	30%	10%	5%	100%	30%	10%	5%

***-Significant at the 1% level

where a_i^* is the individual i's response to the aforementioned survey question—which we consider to be the proxy for the optimal amount of savings—and $[b_i^p]$ represents how much they expect to receive from a substantial inheritance or transfer of assets in the future from their parents—which we consider to be the proxy for expected bequest in the model. Notice that a_i^* and $[b_i^p]$ from the regression equation (1) correspond to their model counterparts from the utility maximization problem. In the regression, we also control for age, partners age, mother's age, partner's mother's age (second order polynomials), liquid assets, retirement accounts (IRAs, Pensions, etc), saving accounts, bonds, equity, total income (adjusted if an "abnormal year"), received inheritance in the past, race and education. These control variables are represented by the vector χ_q.

Regression results estimating Equation (1) are shown in Table 10. Specification (1-4) of Table 10 is a basic reduced form OLS. Specification (5-8) uses a bootstrapping standard error technique with a correction for multiple imputation on our entire sample. Specifications (2-4) and (6-8) run on a subsample of top 30%, 10% and top 5% of the income distribution. The reason why we restrict ourselves to these subsamples is because almost all the bequests observed in both the data and in our model occur within these subsamples. Even when the standard error is calculated correctly via bootstrapping, the point estimate is unchanged even though the statistical significance goes away.[11]

[11]Even though β_1 is not statistically significant when standard errors are calculated by bootstrapping, we believe that the estimated coefficients are still indicative of the effect that we expect.

C Graphical Representation of Wealth

Figure 2: Wealth under the Benchmark Model (solid) and the Identical Fertility Model (dashed)