# Statistics Methods in Finance
# Homework 3

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# Outline (HW3 questions)

- Use the 3 daily-return data you obtained in the second week.

1. (30%) Mean-Variance Portfolio: No Risk-Free Assets
    - Suppose these 3 stocks are the only assets you got. Draw the efficient frontier.
2. (30%) Mean-Variance Portfolio: Risk-Free Assets
    - Suppose now you have a risk-free asset with (annual) return 2%. Construct the capital market line, and determine the market portfolio.
3. (40%) CAPM
    - Based on the previous 2 problems, run the CAPM regression for these three assets. Please report the estimated betas and R-square.
        - Do NOT be afraid of getting a low R-square.

# 1. Mean-Variance Portfolio: No Risk-Free Assets

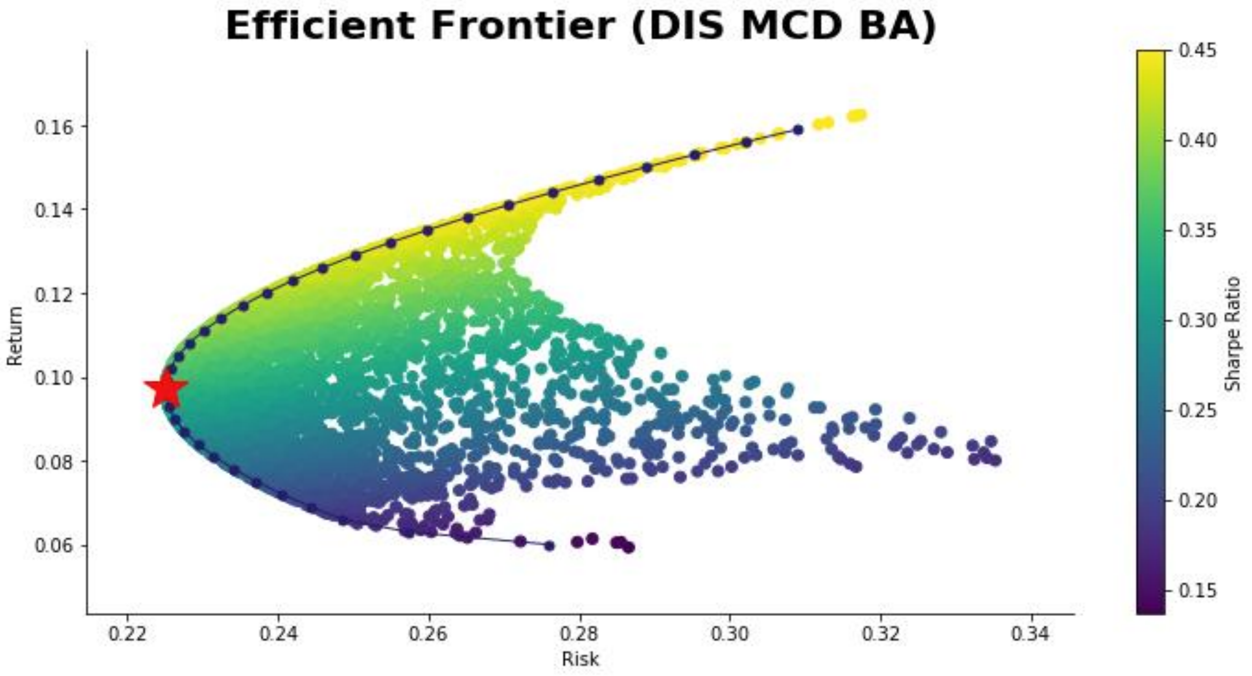
In the following website, I choose DIS, MCD, and BA as the three firms in the historical components of the Dow Jones Industrial Average during 2000/1/1 to 2006/12/31

https://www.wikiwand.com/en/Historical_components_of_the_Dow_Jones_Industrial_Average

```
個股平均收益(6年): DIS:0.081  MCD:0.0583  BA:0.1651
(權重均等)投資組合預期報酬率為: 0.1015
(權重均等)投資組合風險為: 0.2293
```

## Constraint optimization for finding efficient frontier

```
def standard_deviation(weights):
    return np.sqrt(reduce(np.dot, [weights, covariance_matrix, weights.T]))
'''效率前緣(Efficient Frontier)'''
x0 = stocks_weights #变量的初始猜测值
bounds = tuple((0, 1) for x in range(total_stock))

efficient_fronter_return_range = np.arange(0.06, 0.16, .003)
efficient_fronter_risk_list = []

for i in efficient_fronter_return_range:
    constraints = [{'type': 'eq', 'fun': lambda x: sum(x) - 1},
                   {'type': 'eq', 'fun': lambda x: sum(x * stocks_expected_return) - i}]
    efficient_fronter = solver.minimize(standard_deviation, x0=x0,
                                        constraints=constraints, bounds=bounds)
    efficient_fronter_risk_list.append(efficient_fronter.fun)
```



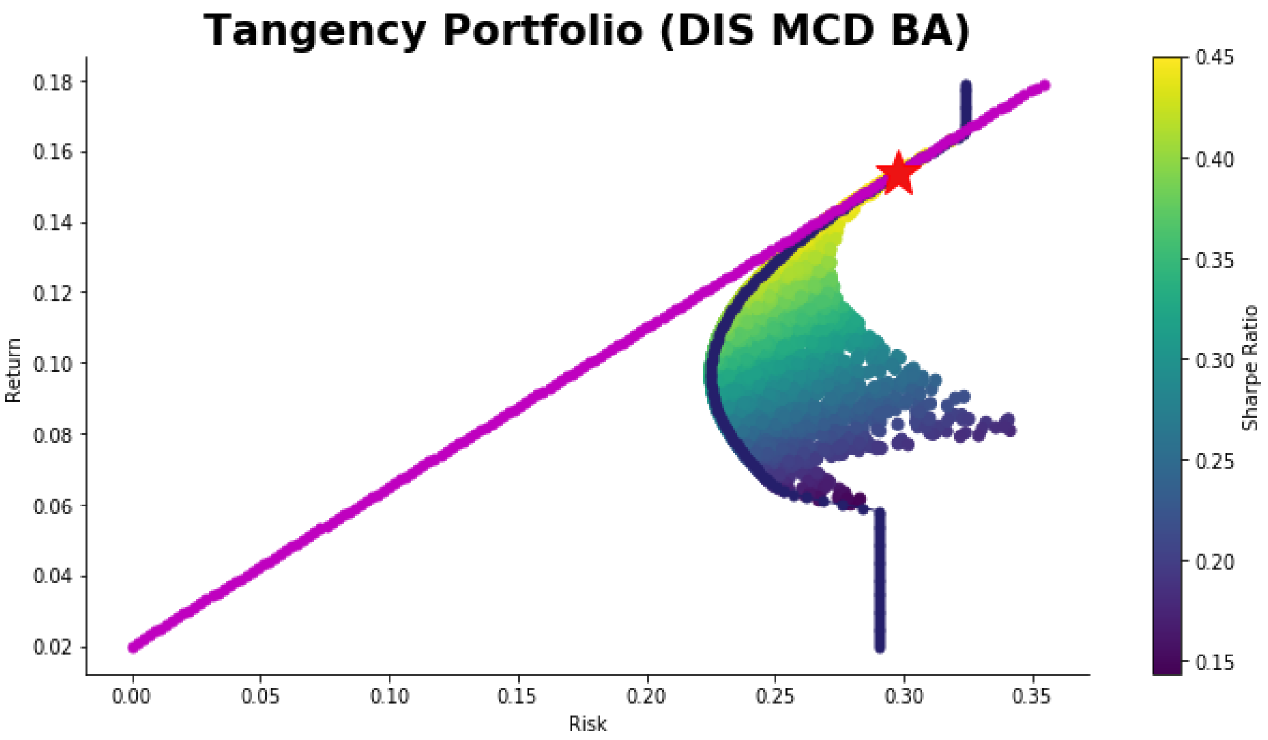

## The star point means:

```
風險最小化投資組合預期報酬率為:0.097
風險最小化投資組合風險為:0.225
DIS 佔投資組合權重 : 0.2340
MCD 佔投資組合權重 : 0.4530
BA 佔投資組合權重 : 0.3130
```

# 2. Mean-Variance Portfolio: Risk-Free Assets

## Constraint optimization for finding CML and tangency portfolio

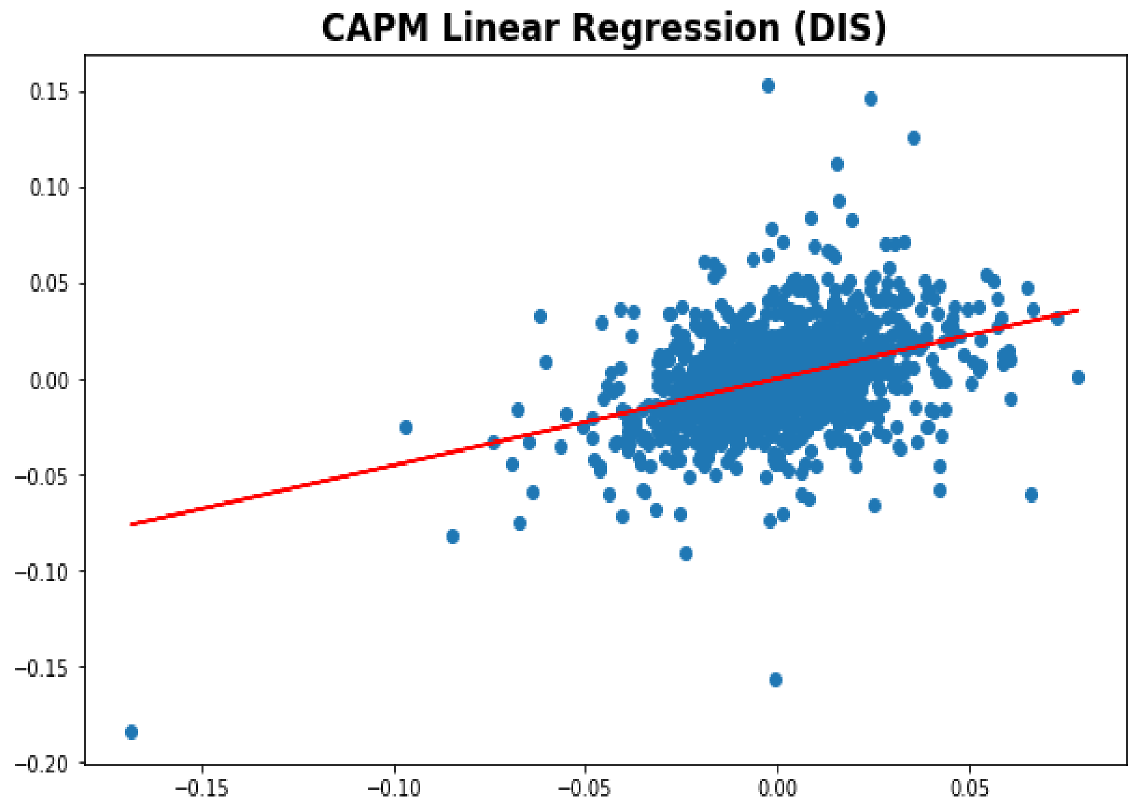
```python
'''效率前緣(Efficient Frontier)'''
x0 = stocks_weights
bounds = tuple((0, 1) for x in range(total_stock))

efficient_fronter_return_range = np.arange(0.02, 0.18, .001)
efficient_fronter_risk_list = []
portfolio_weights = []
for i in efficient_fronter_return_range:
    constraints1 = [{'type': 'eq', 'fun': lambda x: sum(x) - 1},
                    {'type': 'eq', 'fun': lambda x: sum(x * stocks_expected_return) - i}]
    efficient_fronter = solver.minimize(standard_deviation, x0=x0,
                                        constraints=constraints1, bounds=bounds)
    efficient_fronter_risk_list.append(efficient_fronter.fun)
    portfolio_weights.append(efficient_fronter.x)

'''資本市場線(Capital Market Line)'''
x0 = stocks_weights
bounds = tuple((0, 1) for x in range(total_stock))
CML_risk_list = []
for i in efficient_fronter_return_range:
    constraints2 = [{'type': 'eq',
                     'fun': lambda x: 0.02 + sum(x * (stocks_expected_return-0.02)) - i}]
    efficient_fronter = solver.minimize(standard_deviation, x0=x0,
                                        constraints=constraints2, bounds=bounds)
    CML_risk_list.append(efficient_fronter.fun)


'''切點(Tangency Portfolio)'''
TP_idx = np.argmin(abs(np.array(CML_risk_list) - np.array(efficient_fronter_risk_list)))
[TP_x, TP_y] = [CML_risk_list[TP_idx], efficient_fronter_return_range[TP_idx]]

print('切點投資組合預期報酬率為:' + str(round(TP_y,3)))
print('切點投資組合風險為:' + str(round(TP_x,3)))
for i in range(total_stock):
    print(str(returns.columns[i])+' 佔投資組合權重 : ' + str(round(portfolio_weights[TP_idx][i],4)))
```




The star point shows the determined market portfolio:

```
切點投資組合預期報酬率為:0.154
切點投資組合風險為:0.298
DIS 佔投資組合權重 : 0.0642
MCD 佔投資組合權重 : 0.0534
BA 佔投資組合權重 : 0.8824
```

# 3. Run the CAPM regression for the asset DIS



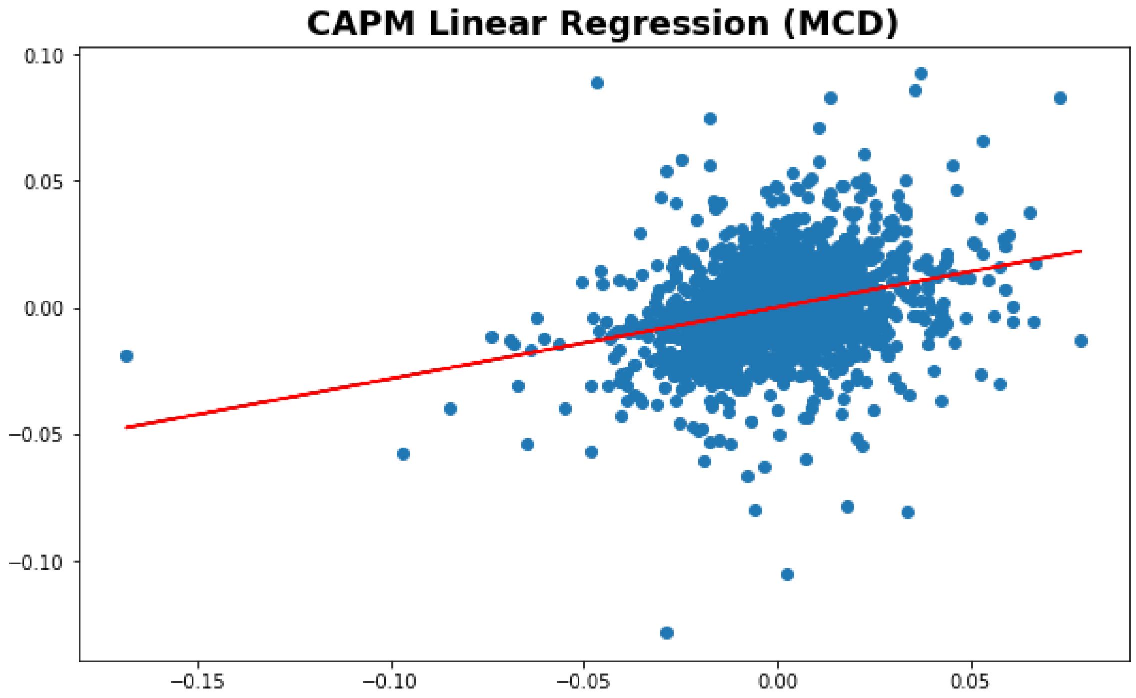

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    DIS   R-squared:                       0.148
Model:                            OLS   Adj. R-squared:                  0.147
Method:                 Least Squares   F-statistic:                     304.7
Date:                Mon, 12 Oct 2020   Prob (F-statistic):           4.77e-63
Time:                        23:12:16   Log-Likelihood:                 4351.7
No. Observations:                1758   AIC:                            -8699.
Df Residuals:                    1756   BIC:                            -8689.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const        1.46e-06      0.000      0.003      0.998      -0.001       0.001
0              0.4525      0.026     17.456      0.000       0.402       0.503
==============================================================================
Omnibus:                      290.596   Durbin-Watson:                   2.062
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             3846.910
Skew:                           0.328   Prob(JB):                         0.00
Kurtosis:                      10.217   Cond. No.                         53.4
==============================================================================
```

Therefore the β=0.4525 and the R-square=0.148

Therefore the linear regression for CAPM model is:

$$R_i - R_f \approx \beta_i\left(R_m - R_f\right) \;\rightarrow\; R_i - R_f \approx 0.4525\left(R_m - R_f\right)$$

# 3. Run the CAPM regression for the asset MCD



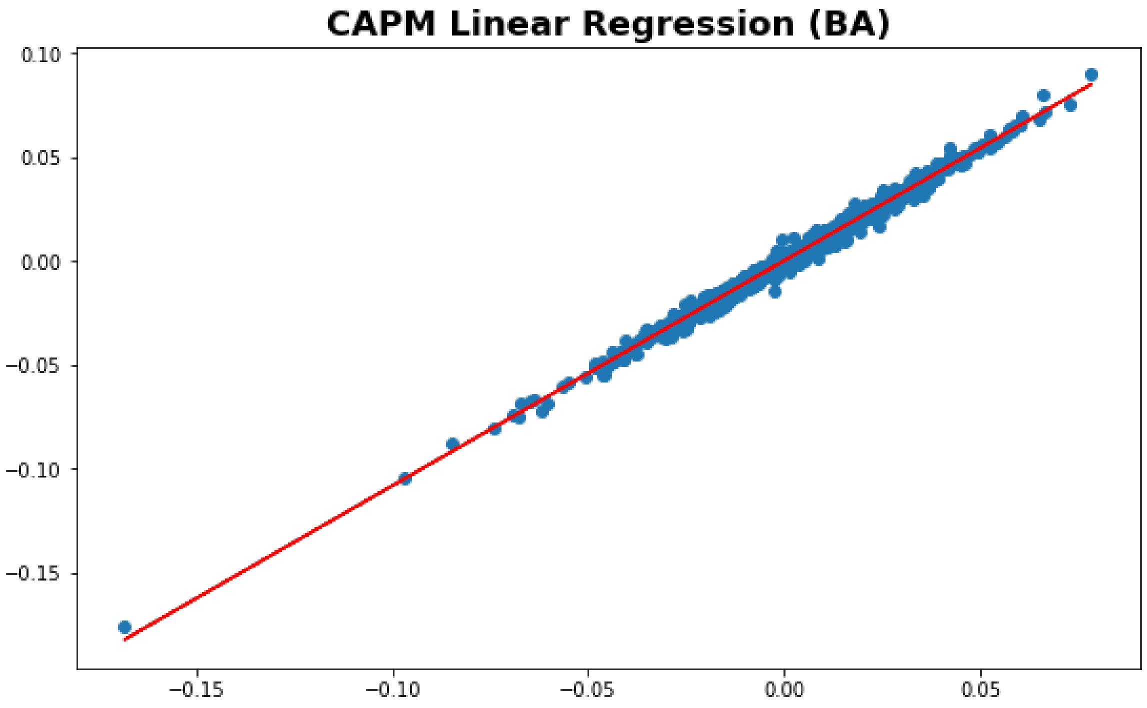

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    MCD   R-squared:                       0.084
Model:                            OLS   Adj. R-squared:                  0.083
Method:                 Least Squares   F-statistic:                     160.6
Date:                Mon, 12 Oct 2020   Prob (F-statistic):           2.80e-35
Time:                        23:25:17   Log-Likelihood:                 4617.4
No. Observations:                1758   AIC:                            -9231.
Df Residuals:                    1756   BIC:                            -9220.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const       1.702e-06      0.000      0.004      0.997      -0.001       0.001
0              0.2824      0.022     12.672      0.000       0.239       0.326
==============================================================================
Omnibus:                      201.132   Durbin-Watson:                   2.015
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             1795.542
Skew:                          -0.089   Prob(JB):                         0.00
Kurtosis:                       7.948   Cond. No.                         53.4
==============================================================================
```

The β=0.2824 and the R-square=0.084

Therefore the linear regression for CAPM model is:

$$R_i - R_f \approx \beta_i(R_m - R_f) \;\rightarrow\; R_i - R_f \approx 0.2824(R_m - R_f)$$

# 3. Run the CAPM regression for the asset BA




```
                            OLS Regression Results
==============================================================================
Dep. Variable:                     BA   R-squared:                       0.991
Model:                            OLS   Adj. R-squared:                  0.991
Method:                 Least Squares   F-statistic:                 1.922e+05
Date:                Mon, 12 Oct 2020   Prob (F-statistic):               0.00
Time:                        23:26:59   Log-Likelihood:                 8484.2
No. Observations:                1758   AIC:                        -1.696e+04
Df Residuals:                    1756   BIC:                        -1.695e+04
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const      -2.091e-07   4.63e-05     -0.005      0.996   -9.11e-05    9.06e-05
0              1.0832      0.002    438.461      0.000       1.078       1.088
==============================================================================
Omnibus:                      150.281   Durbin-Watson:                   2.002
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              840.164
Skew:                          -0.148   Prob(JB):                    3.64e-183
Kurtosis:                       6.374   Cond. No.                         53.4
==============================================================================
```

Therefore the β=1.0832 and the R-square=0.991

Therefore the linear regression for CAPM model is:

$$R_i - R_f \approx \beta_i(R_m - R_f) \;\rightarrow\; R_i - R_f \approx 1.0832(R_m - R_f)$$