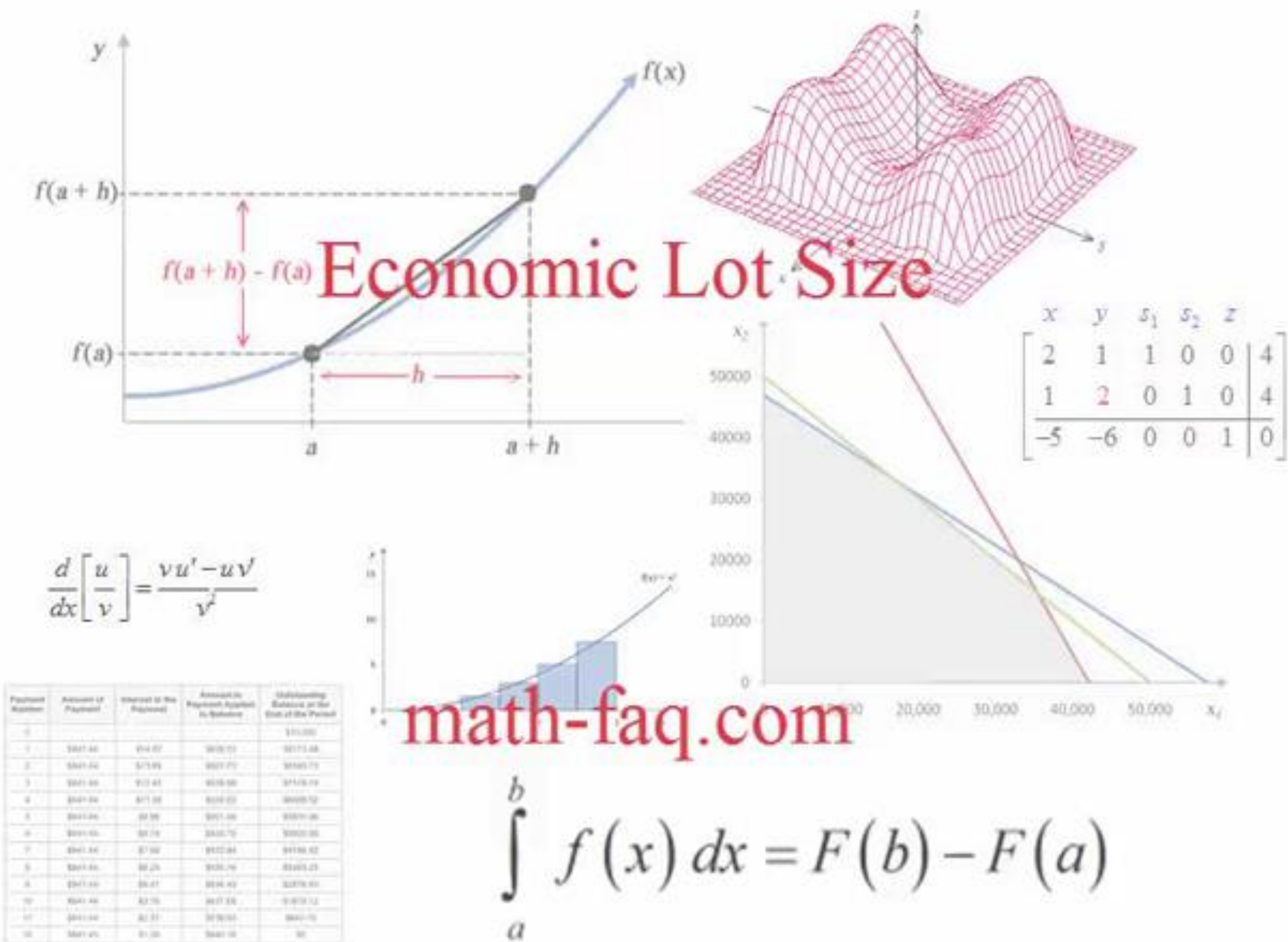
Inventory management: economic lot size

https://www.youtube.com/watch?v=5eQ_3V8hYnE

Economic Lot Size



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Introduction

Inventory?

Inventory is the material within the logistics process (waiting to be used, processed, assembled, distributed, sold, used, consumed, etc.).

Examples:

- raw material;
- semi-finished products, components, subassembled;
- work in process (wip: stocks present / loaded in the production departments awaiting processing or in progress, not loaded in the warehouse stocks);
- finished products.

Introduction

Inventory cost may be due to:

- financial charges;
- opportunity costs (cost of capitalization with sacrifice of alternative investments);
- risk charges (obsolescence, theft, damage, ...);
- insurance costs;
- storage costs (spaces, handling);
- management costs (structures, security, personnel, administration).

Introduction

Inventory cost can be classified in:

- fixed costs: costs almost independent of the volume of activities;
- variable costs: costs as a function of the volume of activities, proportionally as a first approximation;
- direct costs: costs directly associated with a specific product
- indirect costs: costs not directly associated with a single product

Introduction

Inventory cost can be modelled in 4 main categories:

- maintenance or storage cost;
- cost of restocking or procurement;
- shortage cost;
- cost of obsolescence.

Storage cost

The storage cost models all those costs (fixed and variable) related to the presence of goods in the warehouse:

- Financial charges ;
- opportunity costs;
- risk charges;
- insurance costs;
- storage costs;
- management costs.

In general, the storage cost depends on the level of the stocks and their time in the warehouse: variable cost (generally estimated around 10-20% of the value of the goods per year)

Restocking cost

The cost of replenishment or restocking models all the costs related to the issue and execution of a replenishment order:

- costs of issuing and processing orders;
- purchase / production costs;
- ancillary costs (printed, used plant depreciation);
- transportation costs;
- staff costs.

In general, the cost of replenishment is deemed to have a fixed component and a variable proportional to the size of the order (if purchase costs are considered)

Shortage cost

The cost of shortage is the cost incurred if, due to the absence of goods in the warehouses, it is impossible to immediately fulfil the customer's order. This cost economically quantifies the reasons for maintaining stocks and is classified in:

- ➔ costs due to the risk of stock breakage;
- ➔ costs due to production freezing, non-sale, customer disaffection, etc.

The cost of shortage is considered proportional to:

- quantity not satisfied if there is lost-sales;
- quantity not met and waiting time if late delivery is possible (back-order).

Inventory management problems

Inventory management problems can be classified according to several criteria:

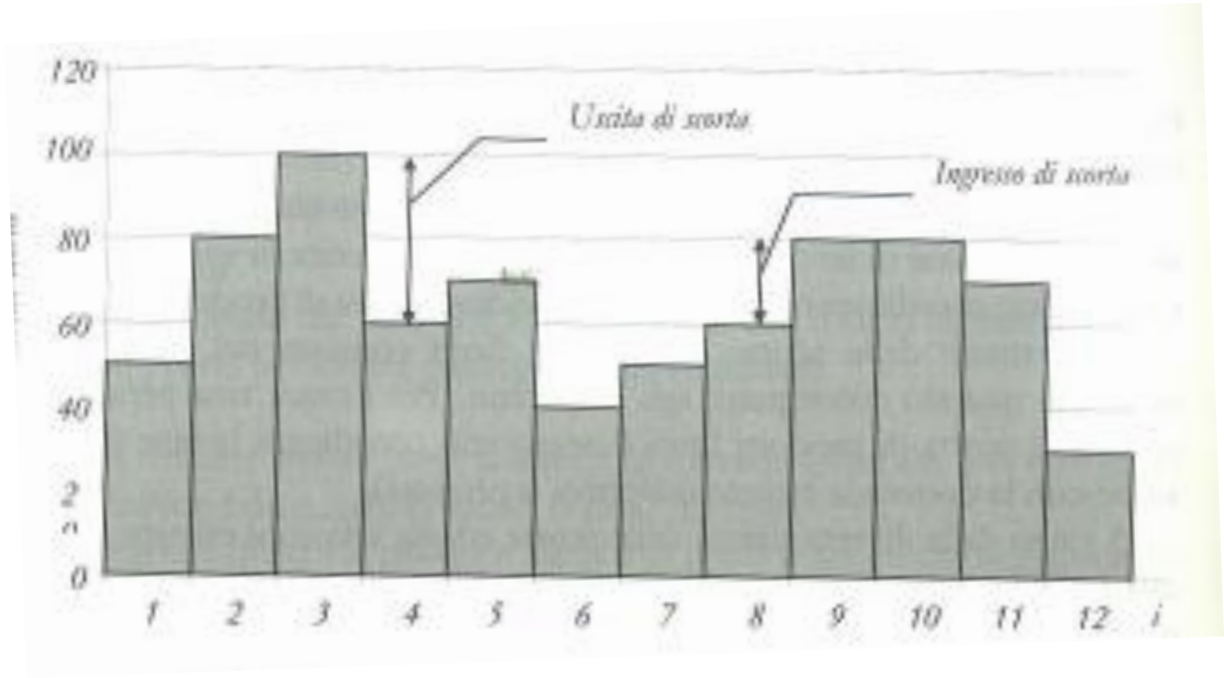
- Number of storage points;
- Number of products;
- Supply demand and times
  - Deterministic or stochastic demand
  - Deterministic or stochastic supply time
  - Products with high demand (fast moving items) or products with slow request (slow moving items)
- Supply methods
  - Continuous supply
  - Batch supply

Inventory control key performance indicators (KPIs)

Average consistency in period T

$$C_m = \frac{\int_{t \in T} S(t)dt}{T}$$

with *S(t)* = inventory level at instant t



Inventory control KPIs

Physical rotation or rotation frequency

$$IR = \frac{Q_u}{C_m}$$

Average stock

$$S_m = \frac{1}{IR}$$

with Q_u = total output flows

IR is usually measured on an annual basis: for example if an article has an IR=15 it means that in one year the average stock has been replaced 15 times. The definition provided with a rotation index does not allow easy measurement since, in order to correctly evaluate the average stock, information related to the trend of the inventory should be available through fairly frequent surveys.

Basic inventory management problem

We assume that:

- the demand is deterministic and constant;
- the reorder time (time interval between order and arrival of goods) is deterministic and constant.

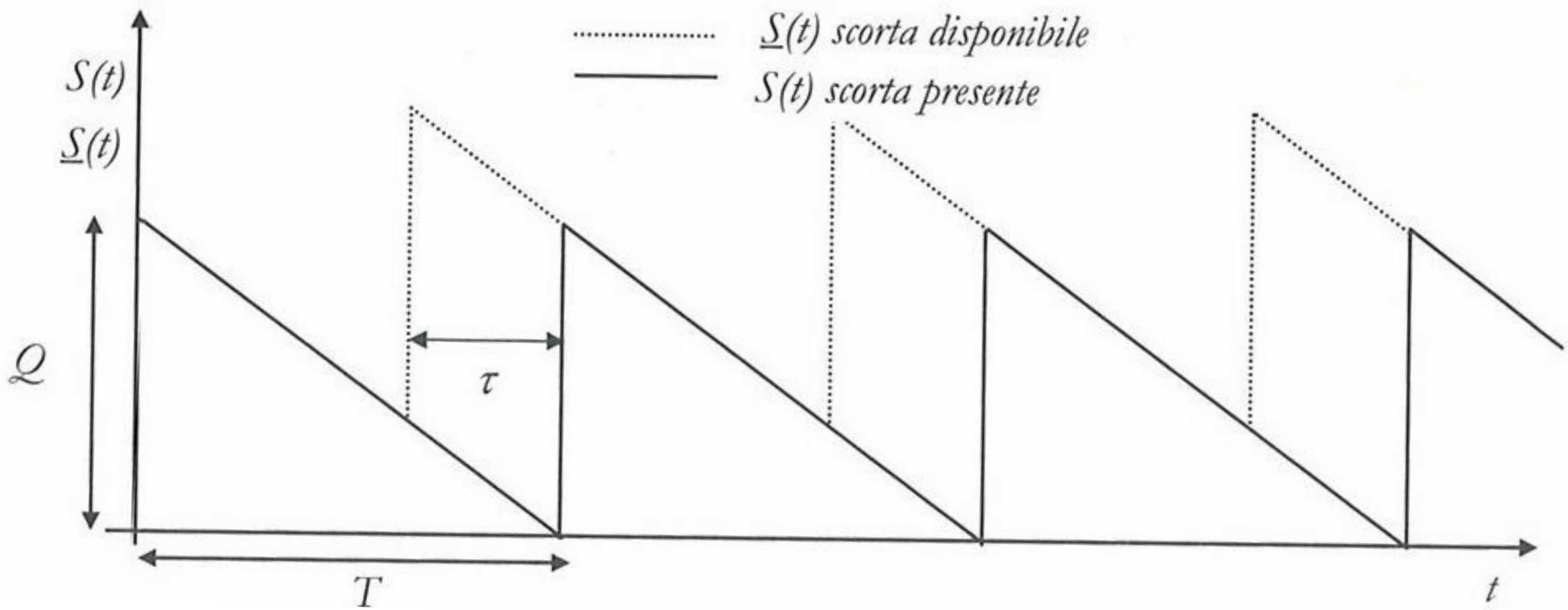
⬇

- supplies are periodic
- the objective of the inventory management problem is the minimization of the overall average cost in the period

Notation

- T: duration of the supply cycles (between the arrival of two lots);
- λ: demand rate (quantity of product taken in the unit of time);
- D: total demand during the reporting period
- Q: quantity of product ordered at the beginning of each cycle ($Q = \lambda T$);
- r: replenishment rate (quantity of product received in the unit of time during the supply period);
- τ: make-up time or re-order time or lead time (time between the re-order decision and the availability of the product in the warehouse). The lead time is constant and initially assumed to be zero.

NOTE: *Q (or equivalently T) is a decision variable*

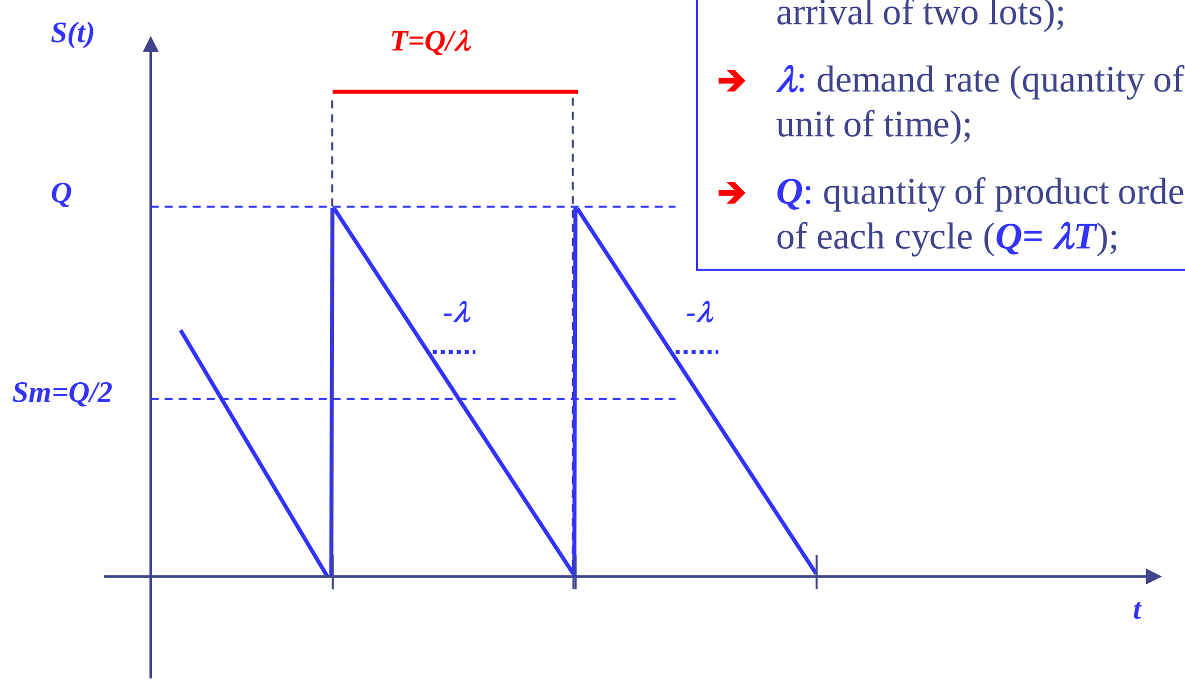

S(t) scorta disponibile
S(t) scorta presente
S(t)
S(t)
Q
τ
T
t

Economic lot model

- Purchase price known and independent of the quantity purchased
- No limit on the size of the batch
- Known and constant demand
- High shortage costs
- Maintenance costs proportional to the stock and time of presence in the warehouse

Economic batch model

In the event that the presence of a shortage is not allowed (Wilson model):



- ***T***: duration of the supply cycles (between the arrival of two lots);
- ***λ***: demand rate (quantity of product taken in the unit of time);
- ***Q***: quantity of product ordered at the beginning of each cycle (***Q= λT***);

Economic batch model

Objective:

decide the optimal size of the economic order quantity Q* (equivalently of T*) to minimize the sum:

- of maintenance costs
- and ordering costs

Economic batch model: storage cost

- Let's define with P the price of one unit of the product and with c% the storage cost rate in the reference period expressed as a percentage of price (ex: 12% per year), the storage cost per unit of product is given by cm= c%*P.
- The total storage cost is therefore given by:

$$C_i(Q) = (c\%P)\, S_m = c_m\, Q/2$$

Economic batch model: ordering cost

The ordering cost $C_o(Q)$ is obtained as a product of the cost of the single order number (f_0) for the number of orders. Specifying with D the total demand during the reference period which, by hypothesis, is constant (ex: 3000 units per year), considering that the quantity of goods requested in period T is equal to the size of lot Q, the number of orders (n_0) in the reference period is given by D / Q; therefore the ordering cost is

$$f_0\,(D/Q)$$

Economic batch formula

The total cost of managing the inventory in the reference period is:

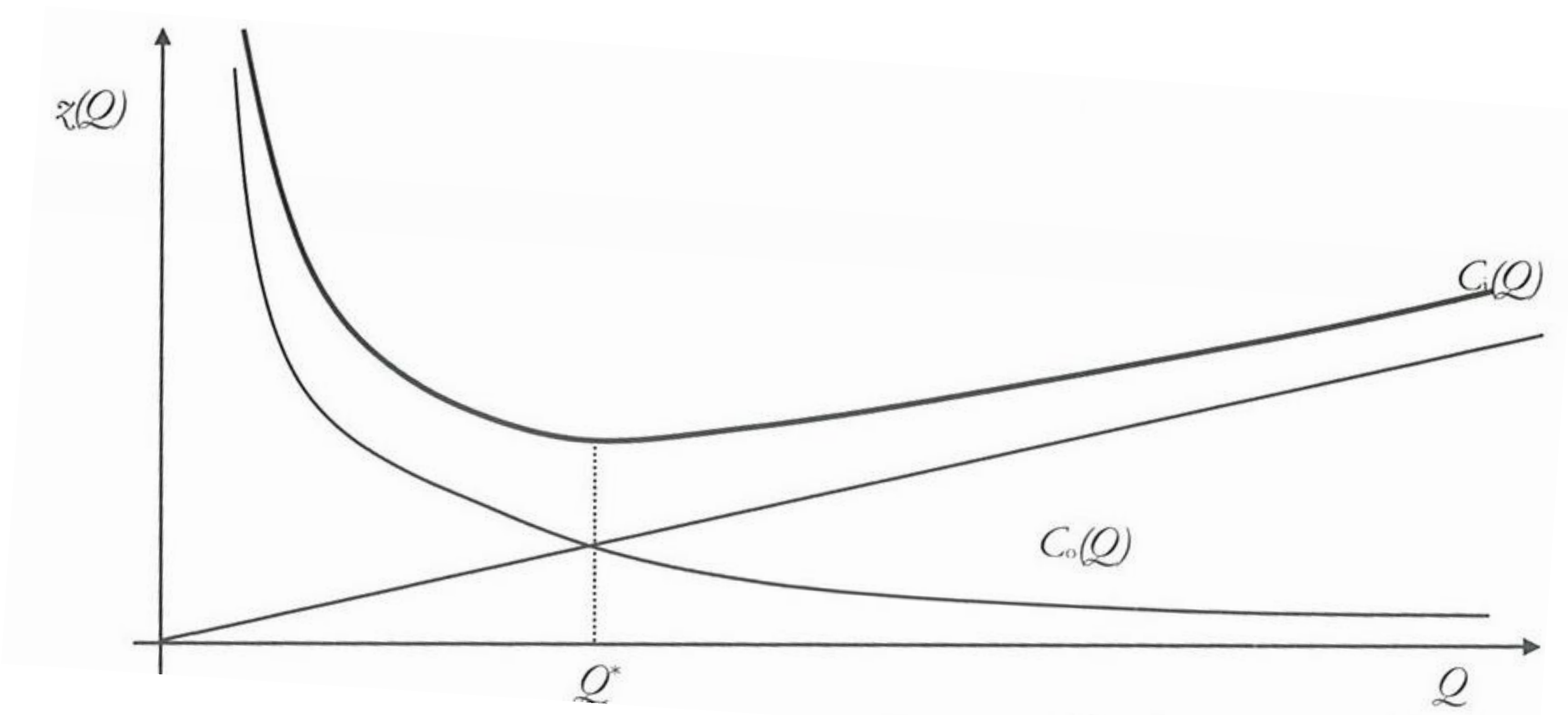
$$z(Q) = C_i(Q) + C_o(Q) = c_m\, Q/2 + f_o\,(D/Q)$$

Deriving and equalling to zero we have

$$z'(Q) = c_m/2 - f_o D/Q^2 = 0 \quad \Longrightarrow \quad Q^* = \sqrt{\frac{2 f_o D}{c_m}}$$

which is the minimum of *z(Q)*. The expression of *Q** represents the so-called formula of the economic lot which indicates the quantity of goods to be purchased or produced to minimize costs.
From the formula it is possible to obtain the optimal supply time *T** and the number of orders *n**:

$$T^* = \frac{Q^*}{D} = \sqrt{\frac{2 f_o}{c_m D}} \qquad n^* = \frac{1}{T^*} = \frac{D}{Q^*} = \sqrt{\frac{c_m D}{2 f_o}}$$



z(Q)
C(Q)
C₀(Q)
Q*
Q

Elements to evaluate

- Sensitivity of the cost to the size of the lot
- Sensitivity of the lot size to total demand
- Evaluate reorder time

O Example

Consider a product with a cost of P = 4, with an annual demand of 8000 units; the cost order is 25 while the storage cost is 10% per year per unit of product. Being cm=0.10*4=0.4, from the formula of the economic batch we obtain:

$$Q^* = ((2 \cdot 25 \cdot 8000)/0{,}4)^{1/2} = 1000$$

and, therefore, the number of orders

$$n^* = 8000/1000 = 8.$$

The total cost of management results

$$z(Q^*) = c_m(Q^*/2) + f_o n^* = 0{,}4 \cdot 500 + 25 \cdot 8 = 200 + 200 = 400.$$

The case of batch feeding

Example:

A company manufactures spare parts for vehicles. One of the components made has a demand of 20000 units per year, each component costs 10 Euros, the order cost is 50 Euros, while the unit cost of storing the products is 0.75 Euros.

Define:

- economic lot size;
- optimal number of orders per year;
- loss in the case of monthly orders;
- loss with minimum lot size of 2000 units.