

Introduction to Cryptography

Alessandro Armando

Computer Security Laboratory (CSec)
DIBRIS, Università di Genova

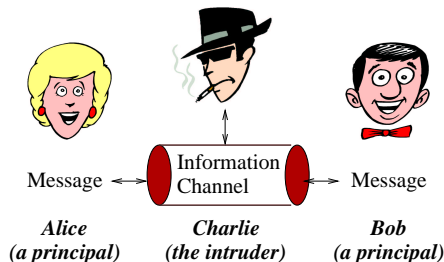
Computer Security



- 1 **Basic Concepts**
- 2 A Mathematical Formalization
- 3 Symmetric-key encryption
- 4 Substitution Techniques
- 5 Transposition ciphers
- 6 Composite Ciphers



What's it all about?



- How do we turn **untrustworthy channels** into **trustworthy** ones?

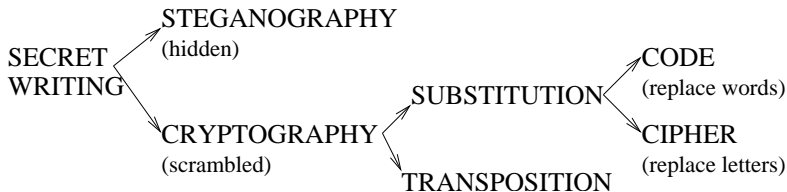
Confidentiality: Transmitted information remains secret.

Integrity: Information not corrupted (or alterations detected).

Authentication: Principals know who they are speaking to.

- Other goals desirable. E.g., non-repudiation.
- Cryptography is the enabling technology.**

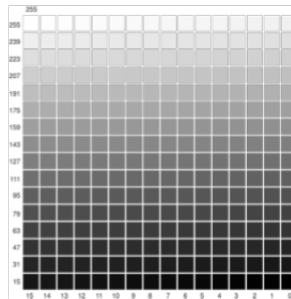




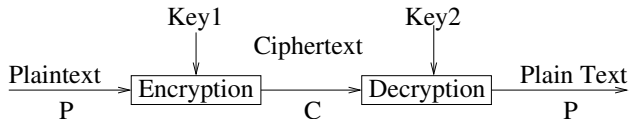
- **Cryptology**: the study of secret writing.
- **Steganography**: the science of hiding messages in other messages.
- **Cryptography**: the science of secret writing.



Steganography



General cryptographic schema



where $E_{key_1}(P) = C$, $D_{key_2}(C) = P$

- **Security depends on secrecy of the key, not of the algorithm**
- **Symmetric** algorithms
 - Key1 = Key2, or are easily derived from each other.
- **Asymmetric** or **public key** algorithms
 - Different keys, which cannot be derived from each other.
 - **Public key** can be published without compromising **private key**.
- Encryption and decryption should be easy, if keys are known.



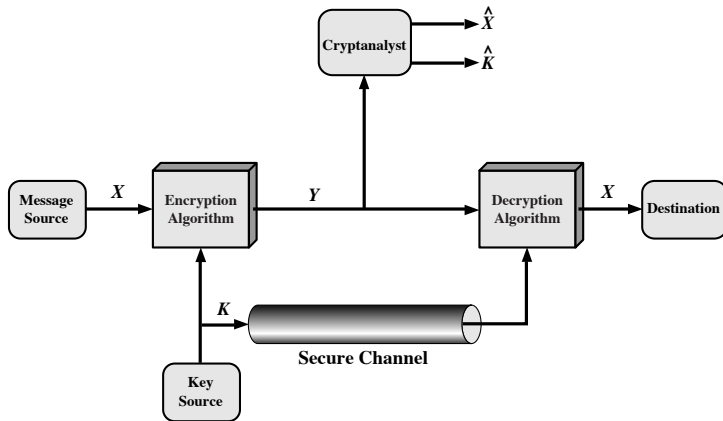
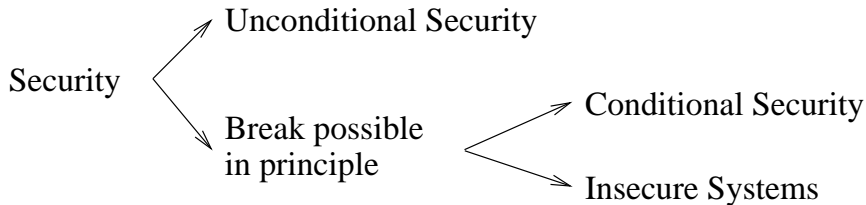


Figure 2.2 Model of Symmetric Cryptosystem

Classification of security



Unconditional Security: System is secure even if adversary has unbounded computing power since the ciphertext provides insufficient information to uniquely determine the corresponding plaintext. Security measured using **information theory**.

Conditional Security: System can be broken in principle, but this requires more computing power than a realistic adversary would have. Security measured using **complexity theory**.



- **Cryptanalysis:** science of recovering the plaintext from ciphertext without the key.
- But typical objective is to recover key not just message
- General approaches:
 - brute-force attack
 - cryptanalytic attack



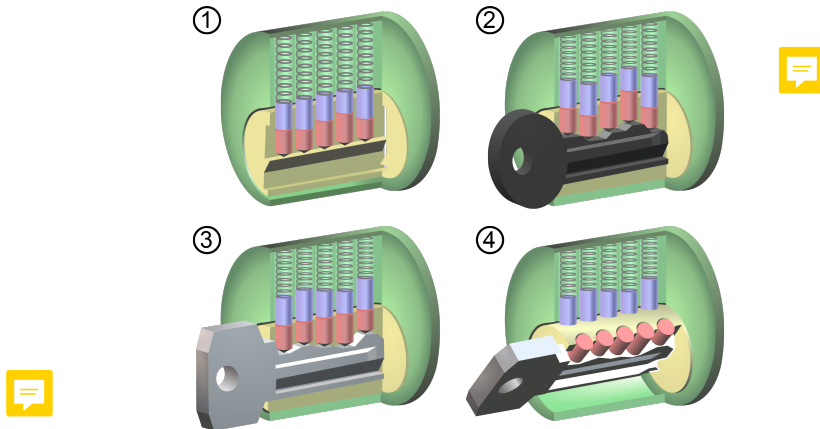
Brute-force Attack

- It is always possible: simply try every key
- Its cost (heavily) depends on key size
- It assumes that plaintext is known or recognisable

Key size (bits)	Number of keys	Time required at 1 decryption/ μ s	Time required at 10^6 decryptions/ μ s
32	$2^{32} = 4.3 \times 10^9$	$2^{31} \mu$ s=35.8 min	2.15 ms
56	$2^{56} = 7.2 \times 10^{16}$	$2^{55} \mu$ s=1142 years	10.01 hours
128	$2^{128} = 3.4 \times 10^{38}$	$2^{127} \mu$ s= 5.4×10^{24} years	5.4×10^{18} years
168	$2^{168} = 3.7 \times 10^{50}$	$2^{167} \mu$ s= 5.9×10^{36} years	5.9×10^{30} years
26 chars (permut.)	$26! = 4 \times 10^{26}$	$2 \times 10^{26} \mu$ s= 6.4×10^{12} years	6.4×10^6 years



Analogy with Physical Security: Tumbler Key Lock



Analogy with Physical Security: Safes

- Security of safes is rated according to the degree of protection provided against an attempted burglary attack.
- Class TL-X (for $X = 15, 30, 40$):¹

The door successfully resist entry for [...] X minutes when attacked with common hand tools, picking tools, mechanical or portable electric tools, grinding points, carbide drills and pressure applying devices or mechanisms.

- assuming that²

the laboratory receives the plans of the samples to be tested in advance so that the vault, its structure, its composition, its assembly and any weak points can be examined in depth beforehand

¹<https://www.safeandvaultstore.com/pages/burglary-ratings>

²<https://www.bunkerkit.com/en/performances/vault/>



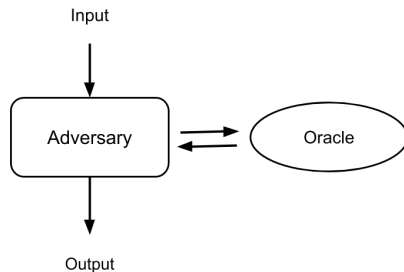
Model of Attack

We can think of the adversary as playing a game:

Input: Whatever adversary necessarily knows from the beginning, e.g., public key, distribution of plain texts, etc.

Oracle: Models information adversary can obtain during an attack. Different kinds of information characterize different types of attacks.

Output: Whatever the adversary wants to compute, e.g., secret key, partial information on plain text, etc. He wins if he succeeds.



- **Ciphertext only**



- **Given:** $C_1 = E_K(M_1), \dots, C_n = E_K(M_n)$
- **Deduce:** M_1, \dots, M_n or algorithm to compute M_{n+1} from $C_{n+1} = E_K(M_{n+1})$

- **Known plaintext**

- **Given:** $M_1, C_1 = E_K(M_1), \dots, M_n, C_n = E_K(M_n)$
- **Deduce:** Inverse key or algorithm to compute M_{n+1} from $C_{n+1} = E_K(M_{n+1})$

- **Chosen plaintext** Same as above but cryptanalyst may choose M_1, \dots, M_n .

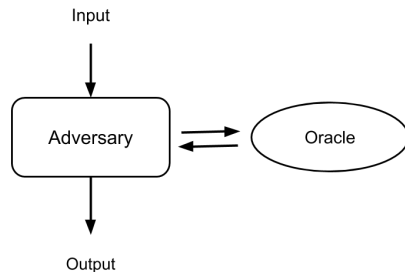
- **Adaptive chosen plaintext** Cryptanalyst can not only choose plaintext, but he can modify the plaintext based on encryption results.

- **Chosen ciphertext** Cryptanalyst can choose different ciphertexts to be decrypted and gets access to the decrypted plaintext.



How to build a definition of security

- 1 Specify an oracle (a type of attack).
- 2 Define what the adversary needs to do to win the game, i.e., a condition on his output.
- 3 The system is secure under the definition, if any **efficient** adversary wins the game with only **negligible** probability.



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Encryption/decryption

- \mathcal{A} , the **alphabet**, is a finite set.
- $\mathcal{M} \subseteq \mathcal{A}^*$ is the **message space**. $M \in \mathcal{M}$ is a **plaintext (message)**.
- \mathcal{C} is the **ciphertext space**, whose alphabet may differ from \mathcal{M} .
- \mathcal{K} denotes the **key space** of **keys**.
- Each $e \in \mathcal{K}$ determines a bijective function from \mathcal{M} to \mathcal{C} , denoted by E_e . E_e is the **encryption function** (or **transformation**).
- For each $d \in \mathcal{K}$, D_d denotes a bijection from \mathcal{C} to \mathcal{M} . D_d is the **decryption function**.
- Applying E_e (or D_d) is called **encryption** (or **decryption**).



- An **encryption scheme** (or **cipher**) consists of a set $\{E_e : e \in \mathcal{K}\}$ and a corresponding set $\{D_d : d \in \mathcal{K}\}$ with the property that for each $e \in \mathcal{K}$ there is a unique $d \in \mathcal{K}$ such that $D_d = E_e^{-1}$; i.e.,

$$D_d(E_e(m)) = m \quad \text{for all } m \in \mathcal{M}.$$

- The keys e and d above form a **key pair**, sometimes denoted by (e, d) . They can be identical (i.e., **the** symmetric key).
- To **construct** an encryption scheme requires fixing a message space \mathcal{M} , a ciphertext space \mathcal{C} , and a key space \mathcal{K} , as well as encryption transformations $\{E_e : e \in \mathcal{K}\}$ and corresponding decryption transformations $\{D_d : d \in \mathcal{K}\}$.



An example

Let $\mathcal{M} = \{m_1, m_2, m_3\}$ and $\mathcal{C} = \{c_1, c_2, c_3\}$. There are $3! = 6$ bijections from \mathcal{M} to \mathcal{C} . The key space $\mathcal{K} = \{1, 2, 3, 4, 5, 6\}$ specifies these transformations.

	E_1		E_2		E_3
m_1	$\rightarrow c_1$	m_1	$\rightarrow c_1$	m_1	$\rightarrow c_2$
m_2	$\rightarrow c_2$	m_2	$\rightarrow c_3$	m_2	$\rightarrow c_1$
m_3	$\rightarrow c_3$	m_3	$\rightarrow c_2$	m_3	$\rightarrow c_3$

	E_4		E_5		E_6
m_1	$\rightarrow c_2$	m_1	$\rightarrow c_3$	m_1	$\rightarrow c_3$
m_2	$\rightarrow c_3$	m_2	$\rightarrow c_1$	m_2	$\rightarrow c_2$
m_3	$\rightarrow c_1$	m_3	$\rightarrow c_2$	m_3	$\rightarrow c_1$

Suppose Alice and Bob agree on the transformation E_6 . To encrypt m_1 , Alice computes $E_6(m_1) = c_3$. Bob decrypts c_3 by reversing the arrows on the diagram for E_6 and observing that c_3 points to m_1 .



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Symmetric key encryption

- Consider an encryption scheme $\{E_e : e \in \mathcal{K}\}$ and $\{D_d : d \in \mathcal{K}\}$. The scheme is **symmetric-key** if for each associated pair (e, d) it is computationally “easy” to determine d knowing only e and to determine e from d . In practice $e = d$.
- Other terms: **single-key**, **one-key**, **shared-key**, and **conventional encryption**.
- sender and recipient share a common key
- all classical encryption algorithms are symmetric-key (it was the only type of encryption prior to invention of public-key in 1970's)
- by far most widely used



Block ciphers, stream ciphers, and codes

- A **block cipher** is an encryption scheme that breaks up the plaintext message into strings (**blocks**) of a fixed length t and encrypts one block at a time.
- A **stream cipher** is one where the block-length is 1.
- In contrast, **codes** work on words of varying length.



- Translation given by a 'code-book'.

Word	Code
...	...
The	1701
secret	5603
mischiefs	4008
that	3790
I	2879
set	0524
...	...

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Word	Code
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2327 6605 1702 9853 0001 0970 3190 8817 1320 0000 =
1701 5603 4008 3790 2879 0524 7946 =
2879 2870 6699 1702 3982 5550 8102 7354 0000 =



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2327 6605 1702 9853 0001 0970 3190 8817 1320 0000 = I do the wrong, and first begin to brawl.
1701 5603 4008 3790 2879 0524 7946 = The secret mischiefs that I set abroad
2879 2870 6699 1702 3982 5550 8102 7354 0000 = I lay unto the grievous charge of others.
(Richard III, Act I, Scene 3)



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Simple Substitution ciphers

- **KHOOR ZRUOG** = **HELLO WORLD**

Caesar cipher: each plaintext character is replaced by the character three to the right modulo 26.

- **Zl anzr vf Nqnz** = **My name is Adam**

ROT13: shift each letter by 13 places.

Under Unix:

```
tr a-zA-Z n-za-mN-ZA-M
```

- **2-25-5 2-25-5** = **BYE BYE**

Alphanumeric: substitute numbers for letters.

How hard are these to cryptanalyze? Caesar? General?



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KEY		PHHW	PH	DIWHU	WKH	WRJD	SDUWB
1		oggv	og	chvgt	vjg	vqic	rctva
2		nffu	nf	bgufs	uif	uphb	qbsuz
3		meet	me	after	the	toga	party
4		ldds	ld	zesdq	sgd	snfz	ozqsx
5		kccr	kc	ydrpc	rfc	rmey	nyprw
6		jbbq	jb	xcqbo	qeb	qldx	mxoqv
7		iaap	ia	wbpan	pda	pkcw	lwnpu
8		hzzo	hz	vaozm	ocz	objv	kvmot
9		gyyn	gy	uznyl	nby	niau	julns
10		fxxm	fx	tymxk	max	mhzt	itkmr
11		ewwl	ew	sxlwj	lzw	lgys	hsjlq
12		dvvk	dv	rwkvi	kyv	kfxr	grikp
13		cuuj	cu	qvjuh	jxu	jewq	fqhjo
14		btti	bt	puitg	iwt	idvp	epgin
15		assh	as	othsf	hvs	hcuo	dofhm
16		zrrg	zr	nsgre	gur	gbtn	cnegl
17		yqqf	yq	mrfqd	ftq	fasm	bmdfk
18		xppe	xp	lqepc	esp	ezrl	alcej
19		wood	wo	kpdob	dro	dyqk	zkbdi
20		vnnc	vn	jocna	cqn	cxpj	yjach
21		ummb	um	inbmz	bpm	bwoi	xizbg
22		tlla	tl	hmaly	aol	avnh	whyaf
23		skkz	sk	glzcx	znk	zumg	vgxze
24		rjyy	rj	fkyjw	ymj	ytlf	ufwyd
25		qiix	qi	ejxiv	xli	xske	tevxc



Mono-alphabetic substitution ciphers

- Key idea: generalise Caesar cipher by allowing an arbitrary substitution.
- Let \mathcal{K} be the set of all permutations on the alphabet \mathcal{A} . Define for each $e \in \mathcal{K}$ an encryption transformation E_e on strings $m = m_1 m_2 \cdots m_n \in \mathcal{M}$ as

$$E_e(m) = e(m_1)e(m_2) \cdots e(m_n) = c_1 c_2 \cdots c_n = c$$

- To decrypt c , compute the inverse permutation $d = e^{-1}$ and

$$D_d(c) = d(c_1)d(c_2) \cdots d(c_n) = m$$

- E_e is a **simple substitution cipher** or a **mono-alphabetic substitution cipher**.

Example:

Plain:	ABCDEFGHIJKLMNOPQRSTUVWXYZ
Cipher:	DKVQFIBJWPESCXHTMYAUOLRGZN
Plaintext:	IFWEWISHTOREPLACELETTERS
Ciphertext:	WIRFRWAJUH YFTSDVFSFUUFYA



Example: Affine Ciphers

- An *affine cipher* is a monoalphabetic substitution cipher such that

$$e(m) = (a \cdot m + b) \bmod |\mathcal{A}|$$

where a and b are positive integers and are the key of the cipher.

- For the cipher to be invertible, a and $|\mathcal{A}|$ must be *relatively prime*, i.e. the only positive integer that divides both of them must be 1.
- The decryption function is



$$D(c) = a^{-1}(c - b) \bmod |\mathcal{A}|,$$

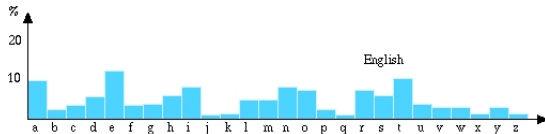
where a^{-1} is the modular multiplicative inverse of a modulo $|\mathcal{A}|$, i.e. it satisfies the equation

$$1 = a \cdot a^{-1} \bmod |\mathcal{A}|$$



(In)security of substitution ciphers

- Key spaces are typically huge. 26 letters \rightsquigarrow $26!$ possible keys.
- Yet, they can be easily cracked using frequency analysis (letters, digraphs, etc.).
- Frequencies for English based on data-mining books/articles.



⇒ Serdhrapvrf sbe Ratyvfu onfrq ba qngn-zvavat obbxf/negvpyrf.

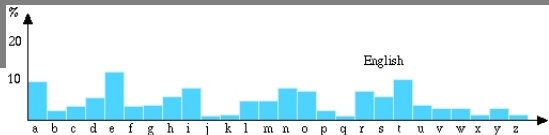
- Easy to apply, except for short, atypical texts

From Zanzibar to Zambia and Zaire, ozone zones make zebras run zany zigzags.

⇒ More sophistication required to mask statistical regularities.



Example



Given ciphertext:

UZQSOVUOHXMOPVGPOZPEVSGZWSZOPFPESXUDBMETSXAI Z

VUEPHZHMDZSHZOWSFPAPDTSVPQUZWYMXUZUHSX

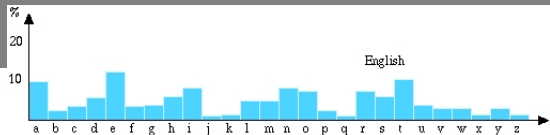
EPYEPOPDZSZUFFPOMBZWPFUPZHMDJUDTMOHMQ

- Count relative letter frequencies
- Since **P** and **Z** occur most frequently, guess they correspond to **E** and **T** respectively.
- Count relative digram frequencies.
- Since **ZW** occurs most frequently, guess it corresponds to **TH** (which is the digram occurring most frequently in English)
- Hence **ZWP** is **THE**
- By proceeding with trial and error finally get:

IT WAS DISCLOSED YESTERDAY THAT SEVERAL INFORMAL BUT DIRECT
CONTACTS HAVE BEEN MADE WITH POLITICAL REPRESENTATIVES OF THE
VIET CONG IN MOSCOW



Example



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UZQSOVUOHXMOPVGPOZPEVSGZWSZOPFPESXUDBMETSXAI Z

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Homophonic substitution ciphers

- To each $a \in \mathcal{A}$ associate a set $H(a)$ of strings of t symbols, where $H(a), a \in \mathcal{A}$ are pairwise disjoint. A **homophonic substitution cipher** replaces each a with a randomly chosen string from $H(a)$. To decrypt a string c of t symbols, one must determine an $a \in \mathcal{A}$ such that $c \in H(a)$. The key for the cipher is the sets $H(a)$.
- **Example:** $\mathcal{A} = \{x, y\}$, $H(x) = \{00, 10\}$, and $H(y) = \{01, 11\}$. The plaintext xy encrypts to one of 0001, 0011, 1001, 1011.
- Rational: makes frequency analysis more difficult.
Cost: data expansion and more work for decryption.



Polyalphabetic substitution ciphers



- Idea (Leon Alberti): conceal distribution using family of mappings.
- A **polyalphabetic substitution cipher** is a block cipher with block length t over alphabet \mathcal{A} where:
 - the key space \mathcal{K} consists of sequences of permutations over \mathcal{A} of the form (e_1, \dots, e_t) .
 - Encryption of $m = m_1 m_2 \dots$ under key $e = (e_1, \dots, e_t)$ is $E_e(m) = c_1 c_2 \dots$, where $c_i = e_{(i \bmod t)}(m_i)$ for $i = 1, 2, \dots$.
 - Decryption key for e is $d = (e_1^{-1}, \dots, e_t^{-1})$.



Example: Vigenère ciphers

- Permutations are defined in terms of a sequence of numbers k_1, \dots, k_t in the following way:

$$e_i(b) = (b + k_i) \bmod |\mathcal{A}| \quad \text{for all } b \in \mathcal{A} \text{ and } i = 1, \dots, t$$

- Example: English ($n = 26$), with $k = k_1, k_2, k_3$ with $k_1 = 3$, $k_2 = 7$, and $k_3 = 10$:

m	=	THI	SCI	PHE	RIS	CER	TAI	NLY	NOT	SEC	URE
$E_e(m)$	=	WOS	VJS	SOO	UPC	FLB	WHS	QSI	QVD	VLM	XYO



One-time pads (Vernam cipher)

- A **one-time pad** is a stream cipher defined on $\mathcal{A} = \{0, 1\}$. Message $m_1 \cdots m_n$ is encrypted by a *randomly chosen* binary key string $k_1 \cdots k_n$.

$$E_{k_1 \cdots k_n}(m_1 \cdots m_n) = (m_1 \oplus k_1) \cdots (m_n \oplus k_n)$$

$$D_{k_1 \cdots k_n}(c_1 \cdots c_n) = (c_1 \oplus k_1) \cdots (c_n \oplus k_n)$$

- Example:

$$\begin{array}{rcl} m & = & 010111 \\ k & = & 110010 \\ \hline c & = & 100101 \end{array}$$

- Since every key sequence is equally likely, so is every plaintext!
Unconditional (information theoretic) security, if key isn't reused!
- Moscow–Washington communication previously secured this way.
- Problem? Securely exchanging and synchronizing long keys.



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Stream Cipher Limitations

- Keys must not be reused! (Hence the name “one-time” pad.)
- Suppose Alice wishes to encrypt and send Bob two messages M_1 and M_2 .
- Alice decides to encrypt both messages using the same stream cipher key S , i.e.

$$C_1 = S \oplus M_1 \qquad C_2 = S \oplus M_2$$

- An adversary, who intercepts C_1 and C_2 , can compute

$$C_1 \oplus C_2 = (S \oplus M_1) \oplus (S \oplus M_2) = M_1 \oplus M_2$$

- Since English text contains redundancy, given $M_1 \oplus M_2$ the adversary can recover both M_1 and M_2 in the clear (for sufficiently long M_1 and M_2).



During WWII the Soviet Union could not produce enough one-time pads... to keep up with the enormous demand... So, they used a number of one-time pads twice, thinking it would not compromise their system. American counter-intelligence during WWII collected all incoming and outgoing international cables. Beginning in 1946, it began an intensive effort to break into the Soviet messages with the cooperation of the British and by... the Soviet error of using some one-time pads as two-time pads, was able, over the next 25 years, to break some 2900 messages, containing 5000 pages of the hundreds of thousands of messages that been sent between 1941 and 1946 (when the Soviets switched to a different system).³

³J. Haynes and H. Klehr. *Venona: Decoding Soviet Espionage in America*. Yale University Press, 1999.



Malleability of One-time Pads

- A cryptographic function $E(K, M)$ is *malleable* iff there exist two functions $F(X)$ and $G(X)$ such that

$$F(E(K, M)) = E(K, G(M)) \quad \text{for all keys } K \text{ and messages } M$$

- $E(K, M) = K \oplus M$ is clearly malleable.
Let $F(X) = G(X) = N \oplus X$ for any given N of appropriate size.

$$F(E(K, M)) = N \oplus (K \oplus M) = K \oplus (N \oplus M) = E(K, G(M))$$

- Corollary: You can turn the ciphertext $C_1 = K \oplus M_1$ of a given known message M_1 into the ciphertext $C_2 = K \oplus M_2$ of any M_2 of choice even if you do not know K ! How? It suffices to compute the \oplus of C_1 and $M_1 \oplus M_2$. In fact,

$$C_1 \oplus (M_1 \oplus M_2) = (K \oplus M_1) \oplus (M_1 \oplus M_2) = K \oplus M_2$$



Outline

- 1 Basic Concepts
- 2 A Mathematical Formalization
- 3 Symmetric-key encryption
- 4 Substitution Techniques
- 5 Transposition ciphers**
- 6 Composite Ciphers



Transposition ciphers

- For block length t , let \mathcal{K} be the set of permutations on $\{1, \dots, t\}$. For each $e \in \mathcal{K}$ and $m \in \mathcal{M}$

$$E_e(m) = m_{e(1)}m_{e(2)} \cdots m_{e(t)}$$

- The set of all such transformations is called a **transposition cipher**.
- To decrypt $c = c_1 c_2 \cdots c_t$ compute $D_d(c) = c_{d(1)}c_{d(2)} \cdots c_{d(t)}$, where d is inverse permutation.
- Letters unchanged so one can exploit frequency analysis for dipthongs, triphongs, words, etc.



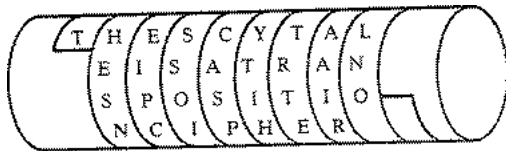
Example: transposition ciphers

- $C = \text{Aduaenttlydhatoiekounletmtoihahvsekeeeleeyqonouv}$

A	n	d	i	n	t	h	e	e	n
d	t	h	e	l	o	v	e	y	o
u	t	a	k	e	i	s	e	q	u
a	l	t	o	t	h	e	l	o	v
e	y	o	u	m	a	k	e		

Table defines a permutation on $1, \dots, 50$.

- Idea goes back to Greek **Scytale**: wrap belt spirally around baton and write plaintext lengthwise on it.



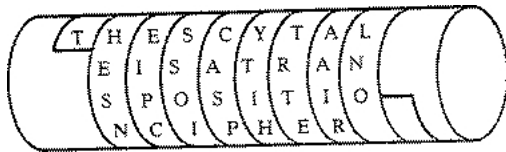
Example: transposition ciphers

- $C = \text{Aduaenttlydhatoiekounletmtoihahvsekeeeleyqonouv}$

A	n	d	i	n	t	h	e	e	n
d	t	h	e	l	o	v	e	y	o
u	t	a	k	e	i	s	e	q	u
a	l	t	o	t	h	e	l	o	v
e	y	o	u	m	a	k	e		

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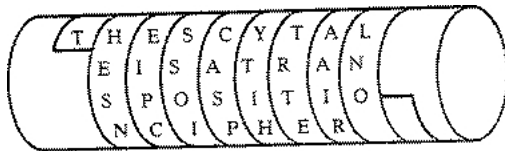
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u	t	a	k	e	i	s	e	q	u
a	l	t	o	t	h	e	l	o	v
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Composite ciphers

- Ciphers based on just substitutions or transpositions are not secure
- Ciphers can be combined. However ...
 - two substitutions are really only one more complex substitution,
 - two transpositions are really only one transposition,
 - but a substitution followed by a transposition makes a new harder cipher.
- Product ciphers chain substitution-transposition combinations.
- Difficult to do by hand \rightsquigarrow invention of cipher machines.



- William Stallings. *Cryptography and Network Security*. Fourth Edition, Prentice Hall, 2006.
- Dieter Gollmann. *Computer Security*. Wiley, 2000.
- Bruce Schneier. *Applied Cryptography*. John Wiley & Sons, New York, 1996.
- Alfred J. Menezes, Paul C. van Oorschot, Scott A. Vanstone. *Handbook of Applied Cryptography*. CRC Press, 1996.
Available online at <http://cacr.math.uwaterloo.ca/hac/>
- Arthur E. Hutt, Seymour Bosworth, Douglas B. Hoyt. *Computer Security Handbook*. John Wiley & Sons, 1995.

