Strategic planning

Strategic plan in logistics

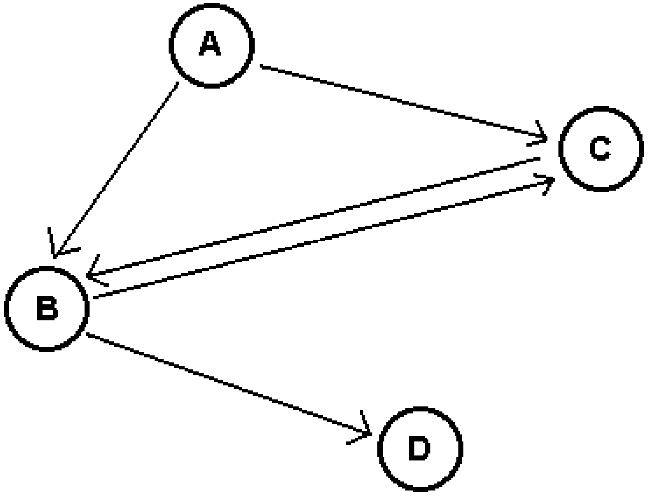
The definition of the set of connections between logistic nodes is attributable to the resolution of a particular class of Mathematical Programming problems: the problems of network design (Network Design).

In Network Design problems the network is typically represented by a graph that is at least weakly connected in which:

- Node: logistics center where assembly, transformation, packaging, storage or sales activities can be carried out on the product.
- Link: connection between two nodes.

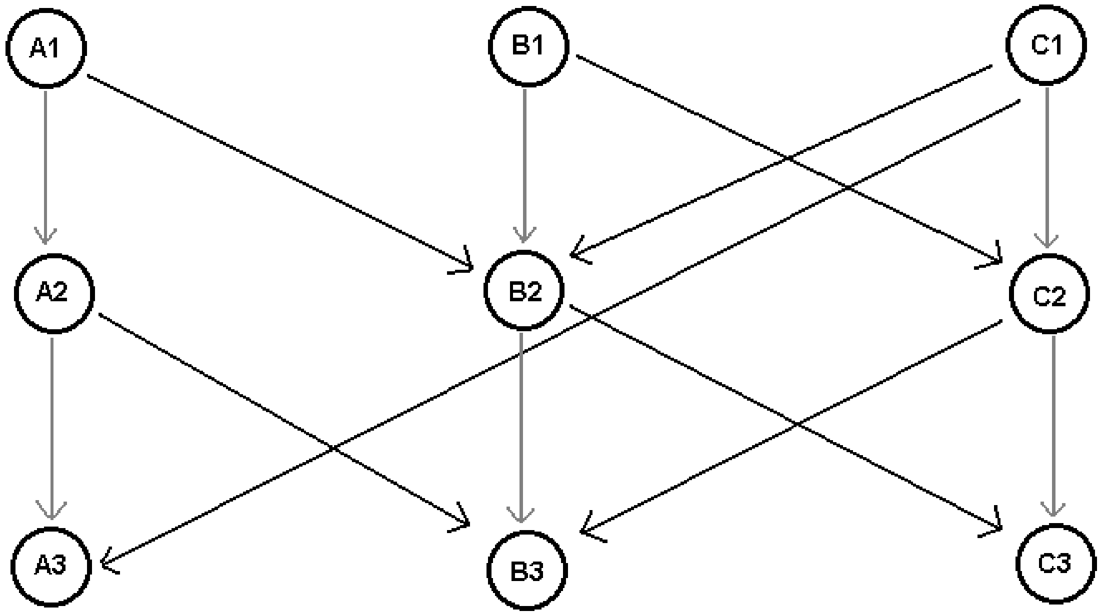
Graph

The graph can be static (the characteristics of the model do not depend on time)



Introduction

or dynamic (some features of the model are explicitly time dependent)



Each logistic node in the system is associated with several vertices, each characterized by its own demand, offer, constraints, costs ...

Network design

define the best combination of volumes and paths for satisfying, at minimum cost, the requests and offers of all the nodes that make up the network, respecting capacity constraints and any other restrictions.

e.g. location of facilities

Network design: formalisation

- ***G (V,A)*** a directed graph at least weakly connected;
- ***V*** set of vertexes (nodes);
- ***A*** set of edges (links);
- ***K*** set of products
- $O(k),\ k \in K$, offer sites for product k;
- $D(k),\ k \in K$, demand site for product k;
- $T(k),\ k \in K$, transit nodes for product k;
- $O_i^k,\ k \in K,\ i \in O(k)$, quantity of product k as offer in node i;
- $D_i^k,\ k \in K,\ i \in D(k)$, quantity of product k as demand in node i;
- $u_{ij},\ (i,j) \in A$, capacity of link (i,j);
- $u_{ij}^k,\ (i,j) \in A,\ k \in K$, maximum flow of product k on link (i,j).

Decisional variables:

- ➔ ***x_{ij}^k , (i,j) $\in$ A, k $\in$ K***, flow of product k on link (i,j);
- ➔ ***y_{ij} , (i,j) $\in$ A***, binary value related to communication link (i,j) if it planned (y_{ij} = 1) or not (y_{ij} = 0);

Costs

- ➔ ***C_{ij}^k (x_{ij}^k), (i,j) $\in$ A, k $\in$ K***, transportation cost of a quantity x_{ij}^k of product k along (i,j);
- ➔ ***f_{ij}, (i,j) $\in$ A***, cost to build link (i,j).

Minimum Cost Flow Problem

- All link are built: $\boldsymbol{y_{ij}=1}$ for all (i,j) $\in$ A.
- 1 product

$$\min \sum_{(i,j)\in A} c_{ij}\cdot x_{ij}$$

subject to

$$\sum_{j\in V,(i,j)\in A} x_{ij} - \sum_{j\in V,(i,j)\in A} x_{ji} = \begin{cases} o_i & \text{se } i\in O \\ -d_i & \text{se } i\in D, \\ 0 & \text{se } i\in T \end{cases} \qquad i\in V,$$

$$x_{ij} \le u_{ij}, \qquad (i,j)\in A,$$

$$x_{ij} \ge 0, \qquad (i,j)\in A$$

MCFP k products

$$\min \sum_{k\in K} \sum_{(i,j)\in A} c_{ij}^k \cdot x_{ij}^k$$

subject to

$$\sum_{j\in V,(i,j)\in A} x_{ij}^k - \sum_{j\in V,(i,j)\in A} x_{ji}^k = \begin{cases} o_i^k & \text{se } i\in O(k) \\ -d_i^k & \text{se } i\in D(k), \\ 0 & \text{se } i\in T(k) \end{cases} \qquad i\in V, k\in K$$

$$x_{ij}^k \le u_{ij}^k, \qquad (i,j)\in A, k\in K$$

$$\sum_{k\in K} x_{ij}^k \le u_{ij}, \qquad (i,j)\in A$$

$$x_{ij}^k \ge 0, \qquad (i,j)\in A, k\in K$$

Fixed Charge Network Design Problem

The FCNDP problem is a network design problem with a linear objective function and with the possibility of having both active and inactive connections.

This model is also used when there are transport alternatives between the same pair of nodes. In this case, parallel links are introduced that connect the same pair of nodes with different costs. The problem becomes a problem on the multigraph.

FCNDP

$$\min \sum_{k\in K}\sum_{(i,j)\in A} c_{ij}^k \cdot x_{ij}^k + \sum_{(i,j)\in A} f_{ij} y_{ij}$$

subject to

$$\sum_{j\in V,(i,j)\in A} x_{ij}^k - \sum_{j\in V,(i,j)\in A} x_{ji}^k = \begin{cases} o_i^k & \text{se } i \in O(k) \\ -d_i^k & \text{se } i \in D(k), \\ 0 & \text{se } i \in T(k) \end{cases} \qquad i \in V, k \in K$$

$$x_{ij}^k \le u_{ij}^k, \qquad (i,j) \in A, k \in K$$

$$\sum_{k\in K} x_{ij}^k \le u_{ij} y_{ij}, \qquad (i,j) \in A$$

$$x_{ij}^k \ge 0, \qquad (i,j) \in A, k \in K$$

$$y_{ij} \in \{0,1\}, \qquad (i,j) \in A$$