Practice Problems 3: *The Binomial Distribution*

A. From your Book:

- Chapter 3 (all examples and exercises)

B. Additional Problems:

1. We have two random variables X and Y and we know that $\mathbb{E}(X) = 2$ and $\mathbb{E}(Y) = -3$ and $Var(X) = 1$ and $Var(Y) = 0.5$.
   (a) Compute $\mathbb{E}(2X - 3Y)$.
   (b) If X and Y are independent, compute $Var(2X - 3Y)$.

2. I take a random sample of 16 students and ask them whether they are voting for the Democratic or Republican candidate. Suppose that each response is independent, and the probability the student is supporting the Democratic candidate is p = 0.6.
   (a) Define the random variable of interest and determine its distribution.
   (b) Calculate the probability that exactly 8 students respond that they are going to support the Republican candidate.
   (c) Calculate the probability that more than 12 students respond that they are going to support the Democratic candidate.
   (d) Calculate the probability that the number who say they are going to support the Republican candidate is between 5.7 and 11.5.

3. Samantha plans to attend a volleyball game and want to get some of her friends to go with her. She calls seven friends and asks them individually if they are interested. The probability that one of them will say 'yes' is 0.82. Assume that their responses are independent. Answer the following questions:
   (a) What is the distribution of X and what are its parameter(s)?
   (b) What is the probability that exactly four of her friends say 'yes'?
   (c) Given that at least three friends have said yes, what is the probability that at least six have said 'yes'?

4. You roll a (not necessarily fair) coin once, thus the probability to show Heads is p, where p is some number between 0 and 1. If the outcome is Heads you win \$1, otherwise you neither win nor loose anything.
   (a) Find your expected gain. (This will obviously depend on p)
   (b) What is the standard deviation of your gain?

5. Our unreliable computer system breaks down from time to time, and we are certain that when it breaks down next is independent of when it last broke down.

   (a) Let X = the number of times the system broke down this week. Is X a binomial random variable? Why or why not?

   (b) Let Y = the number of days this week that the computer broke down at least once. Is Y a binomial random variable? Why or why not?

6. (*) Suppose that the US Supreme Court has to rule on a case and there are two choices, in favor of or against something (think about your favorite case). Assume that you are one of the nine judges and that the eight other judges are *equally likely* to vote in favor or against.

   (a) Can you compute (using the binomial formula) the probability that your vote is decisive ?

   (b) What additional assumption do you have to make in order to use the binomial formula?