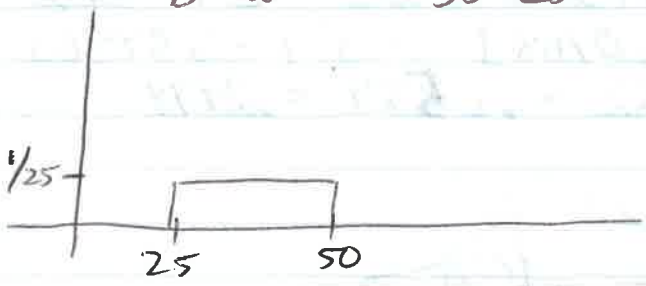
Practice Problems 4 – Solutions

(1) a. height $= \frac{1}{b-a} = \frac{1}{50-25} = \frac{1}{25}$

pdf:



$U \sim unif(25, 50)$

b. $E(U) = \frac{a+b}{2} = 37.5$

$Var(U) = \frac{(b-a)^2}{12} = \frac{25^2}{12} \approx 52.08$

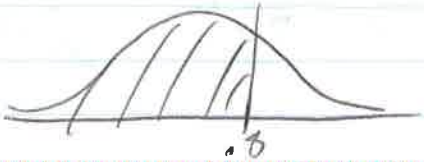
c. $P(32 < U < 34) = \frac{34-32}{25} = \frac{2}{25}$

d. $P(32 < U < 34 \mid 30 < U < 40)$
$= \frac{P(32 < U < 34)}{P(30 < U < 40)} = \frac{2/25}{10/25} = 1/5$

(2) a. $P(Z > 1.25) = 1 - P(Z < 1.25) = P(Z < -1.25)$
$= .1056$

b. $P(Z < -.40) = .3446$

c. $P(Z < 0.80) = 1 - P(Z < -.8) = 1 - .2119 = .7881$



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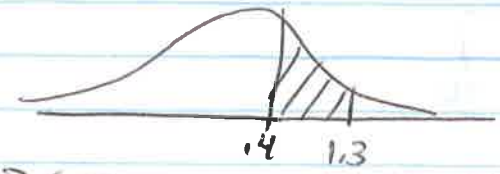
d. $P(.4 < Z < 1.30) = P(Z < 1.3) - P(Z < .4)$
$= [1 - P(Z < -1.3)] - [1 - P(Z < -.4)]$
$= [1 - .0968] - [1 - .3446]$
$= .9032 - .6554 = .2478$



e. $P(Z < -1.5 \text{ and } Z < 1.5) = P(Z < -1.5)$
$= 0.0668$

(3) $X \sim N(64, 2.5^2)$

a. $P(X > 72) = P(Z > \frac{72-64}{2.5})$
$= P(Z > 3.2)$
$= P(Z < -3.2) = 0.0007$

b. $P(X > 64) = 0.5$
$[P(X > u) = P(X < u) = .5]$

c. $P(X < 60) = P(Z < \frac{60-64}{2.5})$
$= P(Z < -1.6) = 0.0548$

d. $P(62 < X < 68) = P(\frac{62-64}{2.5} < Z < \frac{68-64}{2.5})$
$= P(-.8 < Z < 1.6)$
$= P(Z < 1.6) - P(Z < -.8)$
$= (1 - P(Z < -1.6)) - P(Z < -.8)$
$= (1 - .0548) - .2119$
$= .9452 - .2119 = .7333$

(4) $X \sim N(50, 10^2)$

a. Z-scores – 60: $Z = (60-50)/10 = 1$
45: $Z = (45-50)/10 = -.5$
75: $Z = (75-50)/10 = 2.5$

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b. Unstandardize: $0: X = 10\times(0) + 50 = 50$

$1.5: X = 10\times(1.5) + 50 = 65$

$-2.8: X = 10\times(-2.8) + 50 = -28 + 50 = 22$

(5) $X \sim N(160, 10)$

a. $P(X < 166) = P\left(Z < \frac{166-160}{10}\right)$

$= P(Z < .6) = 1 - P(Z < -.6)$

$= 1 - .2743 = .7257$

b. $x = 160 + (.5)(10) = 165$

$P(X \geq 165) = P\left(Z < \frac{165-160}{10}\right)$

$= P(Z < .5)$

$= 1 - P(Z < -.5) = 1 - .3085$

$= .6915$

(6) $X \sim N(3.0, .5^2)$

30th percentile: Find x such that

$P(X < x) = .30$

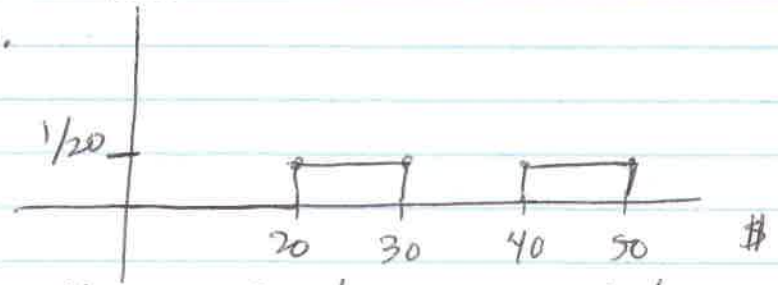
We know from Z-table that

$P(Z < -.5) \approx .30$

$\rightarrow X = (.5)(-.5) + 3.0 = -.25 + 3.0 = 2.75$

(7) a. continuous

b.



c. No - we have a mixture of two uniform distributions