

The table below shows utility patents granted for a specific year.

	Corporation	Government	SUM
United States	0.50	0.07	0.57
Foreign	0.40	0.03	0.43
SUM	0.9	0.1	1

Select one patent at random.

1. What is the probability that it is a foreign patent? $P(\text{foreign}) = 0.4 + 0.03 = 0.43$

(a) 0.40

(b) 0.43

(c) 0.03

(d) 0.90

2. What is the probability that the patent was issued to a government, given that it was a U.S. patent?

(a) 0.12

(b) 0.07

(c) 0.57

(d) 0.70

$$P(\text{Gov} | \text{US}) = \frac{P(\text{Gov} \& \text{US})}{P(\text{US})} = \frac{0.07}{0.57}$$

3. What is the probability that the patent was a U.S. patent, given that was issued to a government?

(a) 0.12

(b) 0.07

(c) 0.57

(d) 0.70

$$P(\text{US} | \text{Gov}) = \frac{P(\text{Gov} \& \text{US})}{P(\text{Gov})} = \frac{0.07}{0.1}$$

4. What is the probability that the patent was a U.S. patent or a patent issued to a government?

(a) 0.60

(b) 0.67

(c) 0.74

(d) 0.07

$$P(\text{US or Gov}) = P(\text{US}) + P(\text{Gov}) - P(\text{US and Gov}) = 0.57 + 0.1 - 0.07$$

5. In a jar there are 8 blue, 5 yellow and 3 red marbles. If two marbles are selected one at a time without replacement, what is the probability to select two blue marbles?

(a) 0.50

(b) 0.23

(c) 0.25

(d) 0.97

$$\frac{8}{16} \times \frac{7}{15} = 0.23$$

6. In a jar there are 8 blue, 5 yellow and 3 red marbles. If two marbles are selected one at a time with replacement, what is the probability to select 2 blue marbles?

(a) 0.50

(b) 0.23

(c) 0.25

(d) 0.97

$$\frac{8}{16} \times \frac{8}{16} = 0.25$$

7. An event A occurs with probability 0.3. Event B occurs with probability 0.5. If A and B are mutually exclusive, then $P(A \text{ or } B) = P(A) + P(B) = 0.8$ since $P(A \& B) = 0$

(a) 0.80

(b) 0.15

(c) 0.20

(d) Not enough info to compute.

80 students in a school cafeteria were asked if they favored a ban on smoking in the cafeteria. The results of the survey are shown below

Students	Favor	Oppose	No opinion	SUM
Freshman	15	27	8	50
Sophomore	23	5	2	30
SUM	38	32	10	80

If a student is selected at random:

8. What is the probability that the student is a freshman and in favor?

(a) 15

(b) 0.1875

(c) 0.475

(d) 0.625

$$\frac{15}{80} = 0.1875$$

$$P(\text{Oppose} | \text{Fresh}) =$$

9. What is the probability that given that a student is a freshman, he or she opposes the ban? $\frac{P(\text{Oppose} \& \text{Fresh})}{P(\text{Fresh})} = \frac{27\%}{50\%}$
- (a) 0.54 (b) 0.84 (c) 0.3375 (d) 0.625

10. X and Y are two random variables such that $E(X) = 3$ and $E(Y) = -2$. Compute $E(Y - 2X) = E(Y) - 2E(X) = -2 - 2 \cdot 3 = -8$
- (a) 4 (b) -6 (c) 8 (d) -8

11. X and Y are two independent random variables such that $\text{Var}(X) = 4$, $\text{Var}(Y) = 1$ Compute $\text{Var}(Y - 2X) = \text{Var}(Y) + 4 \text{Var}(X) = 1 + 4 \cdot 4 = 17$
- (a) 17 (b) -15 (c) 9 (d) -7

Consider the following pdf table:

k	0	1	2	3	4
Probability	0.06	0.7	0.2	0.03	x

12. Compute the value of x . $0.06 + 0.7 + 0.2 + 0.03 + x = 1 \Rightarrow x = 0.01$
- (a) 1 (b) 0.99 (c) 0.01 (d) We do not have enough info.

13. Compute the probability that X is at most 2. $P(X \leq 2) = P(X=0) + P(X=1) + P(X=2) = 0.06 + 0.7 + 0.2 = 0.96$
- (a) 0.96 (b) 0.76 (c) 0.20 (d) 1

14. A multiple choice question has 4 possible answers. If you choose the correct answer, you get 2 points. How many points should I deduct if you choose a wrong answer for the grade to be fair? (Each answer is equally likely to be selected.)

- (a) 2/3 (b) 1/2 (c) 1/4 (d) 0

15. (I) If two events are dependent, they must have the same probability of occurring. F
(II) An event and its complement can occur at the same time. F

- (a) Both (I) and (II) are True. (b) Only (I) is True.
(c) Only (II) is True. (d) Both (I) and (II) are False.

k	2	$-x$
Prob	$1/4$	$3/4$

$$E(X) = 0 \Rightarrow 2 \cdot \frac{1}{4} - x \cdot \frac{3}{4} = 0$$

$$x = \frac{2}{3}$$

↳ An event A and its complement A^c cannot occur at the same time.

E.g. $A = \{\text{rain}\} \rightarrow A^c = \{\text{no rain}\}$.

↳ If two events are dependent we just know that $P(A|B) \neq P(A)$. They don't have the same probability.