Worksheet 8 – Solution

1. The sample size n is greater than 30, so can use the z-scores to construct a confidence interval:

$$\hat{\mu} - z_{\alpha/2} \cdot \frac{\hat{\sigma}}{\sqrt{n}} < \mu < \hat{\mu} + z_{\alpha/2} \cdot \frac{\hat{\sigma}}{\sqrt{n}}.$$

$4150 - 0.93\frac{480}{40} < \mu < 4150 + 0.93\frac{480}{40}$

2. A sample of reading scores of 35 fifth-graders has a mean of 82. The reading standard deviation of the sample is 15.

   (a) Find the best point estimate for the mean.

   $\hat{\mu} = \bar{x} = 82$

   (b) Find the 95% confidence interval for the mean reading scores of all fifth-graders.

   The sample size n is greater than 30, so can use the z-scores to construct a confidence interval:

$$\hat{\mu} - z_{\alpha/2} \cdot \frac{\hat{\sigma}}{\sqrt{n}} < \mu < \hat{\mu} + z_{\alpha/2} \cdot \frac{\hat{\sigma}}{\sqrt{n}}.$$

   We know the following:

$$\hat{\mu} = 82, \hat{\sigma} = 15, n = 35$$

   We want a 95% confidence interval, so the significance level is 1-0.95 = 0.05. So now, we need to compute the z-scores that correspond to $\alpha/2 = 0.025$. We look inside the table for 0.025 and we get the value $z_{\alpha/2} = 1.96$ (we drop the $-$ sign). [Here you can also use 2 instead of 1.96, since we are approximately 2 standard deviations away from the mean]. Now that we have all the information we need, we plug-in everything into the above formula

$$\begin{aligned}
\hat{\mu} - z_{\alpha/2} \cdot \frac{\hat{\sigma}}{\sqrt{n}} < & \quad \mu \quad < \hat{\mu} + z_{\alpha/2} \cdot \frac{\hat{\sigma}}{\sqrt{n}} \\
82 - 1.96 \cdot \frac{15}{\sqrt{35}} < & \quad \mu \quad < 82 + 1.96 \cdot \frac{15}{\sqrt{35}} \\
82 - 4.97 < & \quad \mu \quad < 82 + 4.97 \\
77.03 < & \quad \mu \quad < 86.97
\end{aligned}$$

   (c) Find the 99% confidence interval for the mean reading scores of all fifth-graders.

   the only thing that changes from before is the value of $z_{\alpha/2}$. In this case

$$\alpha = 1 - 0.99 = 0.01$$

   So, we need to compute $z_{\alpha/2}$ which in this case is 2.33. So, the interval now becomes

$$\begin{aligned}
\hat{\mu} - z_{\alpha/2} \cdot \frac{\hat{\sigma}}{\sqrt{n}} < & \quad \mu \quad < \hat{\mu} + z_{\alpha/2} \cdot \frac{\hat{\sigma}}{\sqrt{n}} \\
82 - 2.57 \cdot \frac{15}{\sqrt{35}} < & \quad \mu \quad < 82 + 2.57 \cdot \frac{15}{\sqrt{35}}
\end{aligned}$$

   (d) Which interval is larger? Explain why.

   The 99% confidence interval should be larger, because we are more certain that the true mean will be included in this interval.