Worksheet 4 – Key

1. Suppose that U is a *continuous* random variable that is uniformly distributed between 0 and 6.

   (a) Sketch the probability density function for U.

   (b) What is the expected value of U?

   $$\mathbb{E}(U) = \frac{6+0}{2} = 3$$

   (c) Compute the probability that U is between 3 and 5.

   $$\mathbb{P}(3 < U < 5) = 2 \cdot \frac{1}{6}$$

   (d) What is the probability that U is less than 2 or greater than 5?

   $$\mathbb{P}(U < 2\ OR\ U > 5) = \mathbb{P}(U < 2) + \mathbb{P}(U > 5) = 2 \cdot \frac{1}{6} + 1 \cdot \frac{1}{6} = \frac{3}{6}.$$

   (e) What is the probability that U is greater than 5 given that it is between 4.5 and 6?

   $$\begin{aligned} \mathbb{P}(U > 5 | 4.5 < U < 6) &= \frac{\mathbb{P}(U > 5\ AND\ 4.5 < U < 6)}{\mathbb{P}(4.5 < U < 6)} \\ &= \frac{\mathbb{P}(5 < U < 6)}{\mathbb{P}(4.5 < U < 6)} \\ &= \frac{1 \cdot \frac{1}{6}}{1.5 \cdot \frac{1}{6}} = \frac{1}{1.5}. \end{aligned}$$

   (f) Compute $\mathbb{P}(U = 2)$ and $\mathbb{P}(7 < U < 8)$.

   $$\mathbb{P}(U = 2) = 0 \quad and \quad \mathbb{P}(7 < U < 8) = 0.$$

   (g) Calculate E(U).

   $$\mathbb{E}(U) = \frac{0+6}{2} = 3$$

2. The scores of children on the Wechsler Intelligence Scale for Children (WISC) follow the normal distribution with mean 100 and standard deviation 15. What score has a child achieved on the WISC in order to fall in the top 5% of the sample? In the top 1%?

   To find the score that a child achieved in order to be in the top 5%, we need to compute the 95th percentile. In order to do so, we have the following steps: We find the z-score that corresponds to 0.95, i.e. we look inside the table for 0.95 and then we observe that this corresponds to 1.65. Now, in order to answer the question, we have to find to which number 1.65 corresponds to in the WISC scale:

   $$value = \ z - score \cdot \sigma + \mu = 1.65 \cdot 15 + 100 = 124.75$$

   Similarly to find the WISC score that placed a student in the top 1%, we need to compute the 99th percentile. The z-score that corresponds to 0.99 is 2.33. And if we scale 2.33 back to the WISC scale, we have that

   $$value = \ z - score \cdot \sigma + \mu = 2.33 \cdot 15 + 100 = 134.95.$$