

## Practice Problems 2: *Discrete Distributions*

### A. From your Book:

- Chapter 2 (all examples and exercises)

### B. Additional Problems:

1. Imagine flipping a coin four times and recording the number of times a Head comes up.

- (a) Fill in this pdf for the random variable  $X = \text{number of Heads}$

$k$	0	1	2	3	4
$\mathbb{P}(X = k)$					

- (b) What is the expected number of Heads?
  - (c) What is the probability that we get exactly the expected number of Heads when we have flipped the coin 4 times?
  - (d) Calculate the standard deviation of  $X$ .
2. We are doing a study of brood sizes for arctic penguins, and let  $X$  be a random variable equal to the number of eggs in a randomly chosen nest.

$k$	0	1	2	3	4
$\mathbb{P}(X = k)$	0.25	0.25	0.3	0.15	0.05

- (a) What is the probability of at least two eggs being in the nest?
  - (b) Calculate  $\mathbb{E}(X)$ .
  - (c) Calculate the standard deviation of  $X$ .
3. Suppose that  $Z$  is a random variable equal to -1 with probability 0.25 and equal to 1 with probability 0.75.
    - (a) Calculate the expected value of  $Z$ .
    - (b) Calculate the variance of  $Z$ .
    - (c) Calculate the standard deviation of  $Z$ .
  4. You roll a (not necessarily fair) coin once, thus the probability to show Heads is  $p$ , where  $p$  is some number between 0 and 1. If the outcome is Heads you win \$1, otherwise you neither win nor lose anything.
    - (a) Find your expected gain. (This will obviously depend on  $p$ )
    - (b) What is the standard deviation of your gain?

5. You are a contestant on “Who Wants to Be a Millionaire?” and you have already answered the \$125,000 question correctly, but you have no life lines left. You see the \$250,000 question and you are not sure of the answer. If you quit, you keep \$125,000. If you guess and get the question wrong you get only \$64,000, but if you get it right you get \$250,000. How confident in your answer should you be before you offer a guess? What if you thought you had 50-50 chance of getting it right? What if you were just wildly guessing and only had a 1 in 4 chance?