Practice Problems 1: *Introduction to Probability*

A. From your Book:

- Chapter 1 (all examples and exercises)

B. Additional Problems:

1. A poll was conducted a month before the 2008 presidential elections in a CA county. The results are summarized in the following contingency table:

	McCain	Obama	Other	
Male	23.5%	21%	2%	
Female	21.5%	30%	2%	

An individual is chosen at random.

(a) Compute the joint probabilities:

$$\mathbb{P}(\text{Male and Vote Obama}), \quad \mathbb{P}(\text{Female and not vote for Obama})$$

(b) Compute the marginal probabilities. What is the probability to vote for McCain?

(c) Compute the conditional probabilities:

$$\mathbb{P}(\text{Vote Obama}|\text{Female}), \quad \mathbb{P}(\text{Male}|\text{Vote McCain})$$

2. In a certain population, 1% of the individuals are infected with a certain virus. A test is applied to the whole population and

   - 29.7% of individuals test positive
   - Out of the infected individuals, 99% test positive
   - Out of the non infected individuals, 2% test positive

An individual is chosen at random. Consider the following events:

$$A = \{\text{the individual \textbf{is infected}}\} \quad B = \{\text{the individual \textbf{tests positive} }\}$$

(a) Construct a probability tree diagram describing this situation.

(b) What is $P(B|A)$ and $P(A|B)$?

(c) Are the events A and B independent?

3. A box contains 8 white marbles and 6 black marbles. Marbles are chosen from the box at random and without replacement.

(a) What is the probability that the first marble drawn is black?

(b) What is the conditional probability that the second marble drawn is white given that the first marble drawn is black?

(c) What is the probability that the first marble is black and the second is white?

(d) What is the probability that, of the two marbles drawn from the box, one is white and one is black? (How is this event different from the event described in part (c)?)

4. A standard deck of cards consists of: 4 suits (clubs, diamonds, hearts and spades) and each suit has 13 cards (numbers 1-10, and faces Jack [J], Queen [Q], King [K]).
Suppose that you are playing poker with a standard deck of 52 cards. The first three cards dealt are Jack of clubs, Jack of diamonds, and eight of hearts.

(a) What is the probability that the next card is the King of clubs?

(b) What is the probability that the next card is a spade?

(c) What is the probability that the next card is a Jack?

(d) What is the probability that the next card is greater than 8? (ie. 9, 10, J, Q, K, or A (ace).)

5. Suppose that Bob can decide to go to work by bus or commuter train. Because of high traffic, if he decides to go by bus, the probability of being late is 20%. The commuter train is almost never late, with a probability of only 1%, but is more expensive than the bus.

Bob takes commuter train to work 90% of the time, but sometimes (10% of the time) takes the bus. What is the probability that Bob got the train to work that day, given that he was late?

6. A die is rolled 10 times. What is the probability of the following events:

(a) $A = \{\text{get 10 sixes}\}$

(b) $B = \{\text{not get 10 sixes}\}$

(c) $C = \{\text{all the rolls show 5 or less}\}$

7. If you pick a number completely at random from the natural numbers between 1 and 42, find the probability that the number is a multiple of 3. Then, show that the events

$$A = \{\text{the number is a multiple of 3}\}$$

$$B = \{\text{the number is a multiple of 7}\}$$

are independent.

8. $\mathbb{P}(A) = 1/3$ and $\mathbb{P}(B) = 1/10$. True or False and explain:

(a) If A and B are independent, they must also be mutually exclusive.

(b) If A and B are mutually exclusive, then they are also independent.

9. Four cards will be dealt off the top of a well-shuffled deck. There are two options:

(a) To win $1 if the first card is a club and the second a diamond and the third is a heart and the fourth is a spade.

(b) To win $1 if the four cards are of four different suits.

Which option is better? Or are they the same? Explain.