Practice Problems - Solutions

1. a. HHHH HTHH THHH TTHH,
HHHT HTHT, THHT, TTHT,
HHTH HTTH, THTH, TTTH,
HHTT, HTTT THTT, TTTT

k	0	1	2	3	4
$\mathbb{P}(X=k)$	$1/16$	$4/16$	$6/16$	$4/16$	$1/16$

b. $\mathbb{E}(X) = 0\cdot\frac{1}{16} + 1\cdot\frac{4}{16} + 2\cdot\frac{6}{16} + 3\cdot\frac{4}{16} + 4\cdot\frac{1}{16}$
$= 0 + \frac{1}{4} + \frac{3}{4} + \frac{3}{4} + \frac{1}{4}$
$= 8/4 = 2$

c. $\mathbb{P}(X = \mathbb{E}(X)) = \mathbb{P}(X=2) = 3/8$

d. $SD(X) = \sqrt{Var(X)}$
$Var(X) = \mathbb{E}(X^2) - [\mathbb{E}(X)]^2$
$\mathbb{E}(X^2) = 0^2\cdot\frac{1}{16} + 1^2\cdot\frac{4}{16} + 2^2\cdot\frac{6}{16} + 3^2\cdot\frac{4}{16} + 4^2\cdot\frac{1}{16}$
$= 0 + \frac{1}{4} + \frac{3}{2} + \frac{9}{4} + 1$
$= 20/4 = 5$
$Var(X) = 5 - (2)^2 = \underline{1}$
$\Rightarrow SD(X) = 1$

2. a. $\mathbb{P}(X \geq 2) = 1 - \mathbb{P}(X=0) - \mathbb{P}(X=1)$
$= 1 - \frac{1}{16} - \frac{4}{16} = 11/16$

b. $\mathbb{E}(X) = 0\times 0.25 + 1\times .25 + 2\times .3$
$+ 3\times .15 + 4\times .05$
$= 0 + .25 + .6 + .45 + .2 = 1.5$

→

c. $Var(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2$

$\mathbb{E}(X^2) = 0^2(.25) + 1^2(.25) + 2^2(.3) + 3^2(.15) + 4^2(.05)$

$= .0 + .25 + 1.2 + 1.35 + .8$

$= 3.6$

$Var(X) = 3.6 - (1.5)^2 = 3.6 - 2.25$

$= 1.35$

$\Rightarrow SD(X) = \sqrt{1.35} \approx 1.162$

3. a. $\mathbb{E}(Z) = (-1)(.25) + (1)(.75)$

$-.25 + .75 = 0.5$

b. $Var(Z) = \mathbb{E}(Z^2) - (\mathbb{E}(Z))^2$

$\mathbb{E}(Z^2) = (-1)^2(.25) + (1)^2(.75)$

$= .25 + .75 = 1$

$\Rightarrow Var(Z) = 1 - (.5)^2 = 0.75$

c. $SD(Z) = \sqrt{0.75} \approx 0.866$

4. a. Let X = \$ you gain

$\mathbb{E}(X) = (1)p + (0)(1-p) = p$

b. $SD(X) = \sqrt{Var(X)}$

$Var(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2$

$\mathbb{E}(X^2) = (1)^2 p + (0)^2(1-p) = p$

$Var(X) = p - p^2 = p(1-p)$

$SD(X) = \sqrt{p(1-p)}$

$\rightarrow$

5. Case 1: Your probability of guessing right is p

Let X = \$ you win after you answer

$\mathbb{E}(X) = (250,000)p + (64,000)(1-p)$

We want to find the probability p such that $\mathbb{E}(X) \geq 125,000$:

$250,000p + (64,000)(1-p) \geq 125,000$

$\Rightarrow 186,000p + 64,000 \geq 125,000$

$\Rightarrow 186,000p \geq 61,000$

$\Rightarrow p \geq 61,000/186,000 = 61/186 \approx 0.328$

You should be at least 32.8 % sure you have the right answer before you guess.

Case 2: You have 50-50 chance

$\mathbb{E}(X) = (250,000)(\frac{1}{2}) + (64,000)(\frac{1}{2})$

$= 125,000 + 32,000 = 157,000$

Your expected gain is \$157,000, so you should guess.

Case 3: You have 25% chance

$\mathbb{E}(X) = (250,000)(\frac{1}{4}) + (64,000)(\frac{3}{4})$

$= 62,500 + 48,000 = 110,000$

Your expected gain is \$110,000, so you should NOT guess.