#### Normal Distribution

Lecture 10

02/01/2013

(Lecture 10) 02/01/2013 1 / 27

### Cumulative Distribution Function (CDF)

• When  $Z \sim \mathcal{N}(0,1)$ 

$$\mathbb{P}(Z \leq z) = \Phi(z)$$

 We are going to use the letter "Φ" in the case of the standard normal distribution.

(Lecture 10) 02/01/2013 2 / 27

#### Table of Standard Normal Probabilities for Positive Z-scores





-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0,0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-23	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.614
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.722
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.944
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.976
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.995
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.996
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.997
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9999

3/27

Note that the probabilities given in this table represent the area to the LEFT of the z-score.

The area to the RIGHT of a z-score = 1 – the area to the LEFT of the z-score

# Compute $\mathbb{P}(Z < 1.53)$

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
26	0.0052	0.0055	0.0056	0.0057	0.0050	0.0060	0.0061	0.0062	0.0062	0.0064

(Lecture 10) 02/01/2013 4 / 27

# Compute $\mathbb{P}(Z < 1.53)$

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.0	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
26	0.0052	0.0055	0.0056	0.0057	0.0050	0.0060	0.0061	0.0062	0.0062	0.0064

(Lecture 10) 02/01/2013 5 / 27

### Compute P(Z < 1.53) = 0.9370

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9226	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.0	0.9452	0.9463	0.9474	0.7464	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
26	0.0052	0.0055	0.0056	0.000	0.0050	0.0060	0.0061	0.0062	0.0062	0.0064

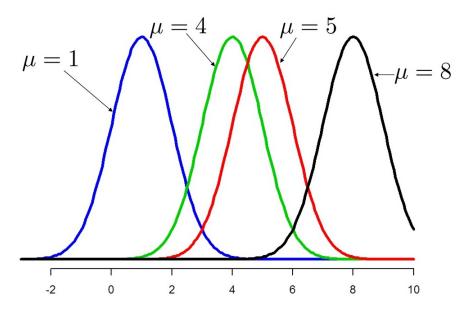
(Lecture 10) 02/01/2013 6 / 27

### Normal Distribution $\mathcal{N}(\mu, \sigma)$

- Take  $Z \sim \mathcal{N}(0,1)$
- Multiply by the standard deviation σ: σ Z
   The new standard deviation becomes σ.
- Add  $\mu$ :  $X = \mu + \sigma Z$ The new expected value is  $\mu$ .
- $X \sim \mathcal{N}(\mu, \sigma)$ 
  - ▶ Mean =  $\mu$
  - Standard Deviation =  $\sigma$

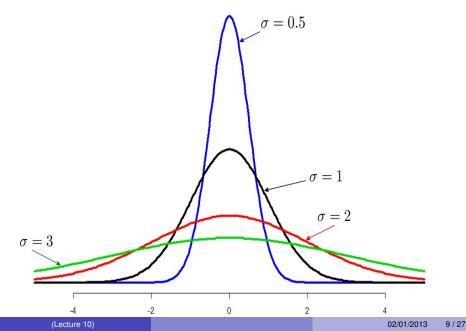
(Lecture 10) 02/01/2013 7 / 27

#### **Different Means**

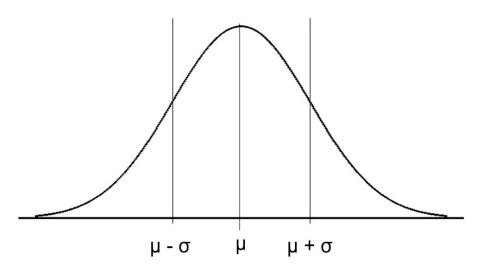


(Lecture 10) 02/01/2013 8 / 27

#### **Different Standard Deviations**

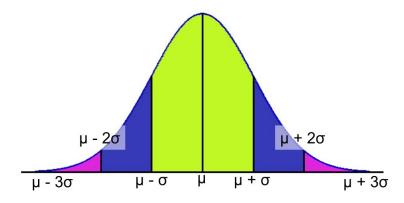


$$m{X} \sim \mathcal{N}(\mu, \sigma)$$



(Lecture 10) 02/01/2013 10 / 27

### $X \sim \mathcal{N}(\mu, \sigma)$



• 
$$\mathbb{P}(\mu - \sigma < X < \mu + \sigma) = 0.68$$

• 
$$\mathbb{P}(\mu - 2\sigma < X < \mu + 2\sigma) = 0.95$$

• 
$$\mathbb{P}(\mu - 3\sigma < X < \mu + 3\sigma) = 0.997$$

(Lecture 10) 02/01/2013

11/27

## How to compute Normal Probabilities for $X \sim \mathcal{N}(\mu, \sigma)$

#### Standardize!

$$Z = \frac{X - \mu}{\sigma}$$

(Lecture 10) 02/01/2013 12 / 27

### Example

$$X \sim \mathcal{N}(300, 5)$$

$$\mathbb{P}(X < 296) = \mathbb{P}\left(\frac{X - \mu}{\sigma} < \frac{296 - \mu}{\sigma}\right)$$
$$= \mathbb{P}\left(Z < \frac{296 - 300}{5}\right)$$
$$= \mathbb{P}\left(Z < -\frac{4}{5}\right) = \mathbb{P}\left(Z < -0.8\right) = 0.2119.$$

(Lecture 10) 02/01/2013 13 / 27

z         0.00         0.01         0.02         0.03         0.04         0.05         0.06           -3.4         0.0003         0.0003         0.0003         0.0003         0.0003         0.0003         0.0004           -3.3         0.0005         0.0005         0.0004         0.0004         0.0004         0.0004           -3.2         0.0007         0.0007         0.0006         0.0006         0.0006         0.0006         0.0006           -3.1         0.0010         0.0009         0.0009         0.0008         0.0008         0.0008           -3.0         0.0013         0.0013         0.0012         0.0011         0.0011         0.0011           -2.9         0.0019         0.0018         0.0017         0.0016         0.0016         0.0015           -2.8         0.0026         0.0025         0.0024         0.0023         0.0023         0.0022         0.0021           -2.7         0.0035         0.0034         0.0033         0.0023         0.0023         0.0022         0.0021           -2.4         0.0082         0.0060         0.0059         0.0057         0.0055         0.0054         0.0052           -2.4         0.0082								
-3.3         0.0005         0.0005         0.0005         0.0004         0.0004         0.0004         0.0004         0.0004         0.0004         0.0004         0.0004         0.0004         0.0004         0.0004         0.0004         0.0004         0.0004         0.0004         0.0006         0.0006         0.0006         0.0006         0.0008         0.0011         0.0011         0.0011         0.0011         0.0011         0.0011         0.0011         0.0011         0.0011         0.0011         0.0011         0.0011         0.0011         0.0011         0.0011         0.0011         0.0011         0.0016         0.0016         0.0016         0.0016         0.0016         0.0016         0.0016         0.0016         0.0016         0.0016         0.0016         0.0022         0.0022         0.0022         0.0022         0.0022         0.0022 <th>z</th> <th>0.00</th> <th>0.01</th> <th>0.02</th> <th>0.03</th> <th>0.04</th> <th>0.05</th> <th>0.06</th>	z	0.00	0.01	0.02	0.03	0.04	0.05	0.06
-3.2         0.0007         0.0006         0.0006         0.0006         0.0006         0.0006         0.0006         0.0006         0.0006         0.0006         0.0006         0.0006         0.0006         0.0006         0.0006         0.0006         0.0006         0.0006         0.0008         0.0008         0.0008         0.0008         0.0008         0.0008         0.0008         0.0001         0.0015         0.0012         0.0021         0.0021         0.0022         0.0021         0.0022         0.0021         0.0022         0.0021         0.0022         0.0021         0.0022         0.0021         0.0022         0.0021         0.0022         0.0021         0.0022         0.0022         0.0022         0.0022         0.0022         0.0022 <th>-3.4</th> <th>0.0003</th> <th>0.0003</th> <th>0.0003</th> <th>0.0003</th> <th>0.0003</th> <th>0.0003</th> <th>0.0003</th>	-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003
-3.1         0.0010         0.0009         0.0009         0.0008         0.0008         0.0008           -3.0         0.0013         0.0013         0.0013         0.0012         0.0012         0.0011         0.0011           -2.9         0.0019         0.0018         0.0018         0.0017         0.0016         0.0016         0.0015           -2.8         0.0026         0.0025         0.0024         0.0023         0.0023         0.0022         0.0021           -2.7         0.0035         0.0034         0.0033         0.0032         0.0031         0.0030         0.0029           -2.6         0.0047         0.0045         0.0044         0.0043         0.0041         0.0040         0.0039           -2.5         0.0062         0.0060         0.0059         0.0057         0.0055         0.0054         0.0069           -2.3         0.0107         0.0104         0.0102         0.0099         0.0096         0.0094         0.0091           -2.2         0.0139         0.0136         0.0122         0.0129         0.0122         0.0119           -2.2         0.01199         0.0174         0.0170         0.0166         0.0162         0.0158         0.0154	-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004
-3.0         0.0013         0.0013         0.0013         0.0012         0.0012         0.0011         0.0011           -2.9         0.0019         0.0018         0.0018         0.0017         0.0016         0.0016         0.0016         0.0016         0.0016         0.0016         0.0016         0.0016         0.0016         0.0016         0.0016         0.0012         0.0029         0.0031         0.0030         0.0029         0.0041         0.0040         0.0039         0.0041         0.0040         0.0039         0.0055         0.0055         0.0055         0.0055         0.0055         0.0055         0.0052         0.0071         0.0069         0.0073         0.0071         0.0069         0.0073         0.0071         0.0069         0.0021         0.00793         0.0091         0.0091	-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006
-2.9         0.0019         0.0018         0.0018         0.0017         0.0016         0.0016         0.0016         0.0015           -2.8         0.0026         0.0025         0.0024         0.0023         0.0023         0.0022         0.0021           -2.7         0.0035         0.0034         0.0033         0.0032         0.0031         0.0030         0.0029           -2.6         0.0047         0.0045         0.0044         0.0043         0.0041         0.0040         0.0039           -2.5         0.0062         0.0060         0.0059         0.0057         0.0055         0.0054         0.0062           -2.4         0.0082         0.0080         0.0078         0.0075         0.0073         0.0071         0.0069           -2.3         0.0107         0.0104         0.0102         0.0099         0.0096         0.0094         0.0099           -2.2         0.0139         0.0136         0.0132         0.0129         0.0125         0.0122         0.0119           -2.1         0.0179         0.0174         0.0170         0.0166         0.0162         0.0158         0.0154           -1.9         0.0228         0.0221         0.0212         0.0207	-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008
-2.8         0.0026         0.0025         0.0024         0.0023         0.0023         0.0022         0.0021           -2.7         0.0035         0.0034         0.0033         0.0031         0.0030         0.0029           -2.6         0.0047         0.0045         0.0044         0.0043         0.0041         0.0040         0.0039           -2.5         0.0062         0.0060         0.0059         0.0057         0.0055         0.0054         0.0052           -2.4         0.0082         0.0080         0.0078         0.0075         0.0073         0.0071         0.0069           -2.3         0.0107         0.0104         0.0102         0.0099         0.0096         0.0094         0.0091           -2.2         0.0139         0.0136         0.0132         0.0129         0.0125         0.0122         0.0119           -2.1         0.0179         0.0174         0.0170         0.0166         0.0162         0.0158         0.0154           -2.0         0.0228         0.0222         0.0217         0.0212         0.0207         0.0202         0.0197           -1.9         0.02287         0.0281         0.0274         0.0268         0.0262         0.0256 <td< th=""><th>-3.0</th><th>0.0013</th><th>0.0013</th><th>0.0013</th><th>0.0012</th><th>0.0012</th><th>0.0011</th><th>0.0011</th></td<>	-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011
-2.7         0.0035         0.0034         0.0033         0.0032         0.0031         0.0030         0.0029           -2.6         0.0047         0.0045         0.0044         0.0043         0.0041         0.0040         0.0039           -2.5         0.0062         0.0060         0.0059         0.0057         0.0055         0.0054         0.0052           -2.4         0.0082         0.0080         0.0078         0.0075         0.0073         0.0071         0.0069           -2.3         0.0107         0.0104         0.0102         0.0099         0.0096         0.0094         0.0091           -2.2         0.0139         0.0136         0.0132         0.0129         0.0125         0.0122         0.0119           -2.1         0.0179         0.0174         0.0170         0.0166         0.0162         0.0158         0.0154           -2.0         0.0228         0.0222         0.0217         0.0212         0.0207         0.0202         0.0197           -1.9         0.0228         0.0221         0.0268         0.0262         0.0256         0.0250           -1.8         0.0359         0.0351         0.0344         0.0336         0.0329         0.0322	-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015
-2.6         0.0047         0.0045         0.0044         0.0043         0.0041         0.0040         0.0039           -2.5         0.0062         0.0060         0.0059         0.0057         0.0055         0.0054         0.0052           -2.4         0.0082         0.0080         0.0078         0.0075         0.0073         0.0071         0.0069           -2.3         0.0107         0.0104         0.0102         0.0099         0.0096         0.0094         0.0091           -2.2         0.0139         0.0136         0.0132         0.0129         0.0125         0.0122         0.0119           -2.1         0.0179         0.0174         0.0170         0.0166         0.0162         0.0158         0.0154           -2.0         0.0228         0.0222         0.0217         0.0212         0.0207         0.0202         0.0197           -1.9         0.0287         0.0281         0.0274         0.0268         0.0262         0.0256         0.0250           -1.8         0.0359         0.0351         0.0344         0.0336         0.0329         0.0322         0.0314           -1.7         0.0446         0.0436         0.0427         0.0418         0.0409	-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021
-2.5         0.0062         0.0060         0.0059         0.0057         0.0055         0.0054         0.0052           -2.4         0.0082         0.0080         0.0078         0.0075         0.0073         0.0071         0.0069           -2.3         0.0107         0.0104         0.0102         0.0099         0.0096         0.0094         0.0091           -2.2         0.0139         0.0136         0.0132         0.0129         0.0125         0.0122         0.0119           -2.1         0.0179         0.0174         0.0170         0.0166         0.0162         0.0158         0.0154           -2.0         0.0228         0.0222         0.0217         0.0212         0.0207         0.0202         0.0197           -1.9         0.0287         0.0281         0.0274         0.0268         0.0262         0.0256         0.0250           -1.8         0.0359         0.0351         0.0344         0.0336         0.0329         0.0322         0.0314           -1.7         0.0446         0.0436         0.0427         0.0418         0.0409         0.0401         0.0392           -1.5         0.0668         0.0655         0.0643         0.0630         0.0618	-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029
-2.4         0.0082         0.0080         0.0078         0.0075         0.0073         0.0071         0.0069           -2.3         0.0107         0.0104         0.0102         0.0099         0.0096         0.0094         0.0091           -2.2         0.0139         0.0136         0.0132         0.0129         0.0125         0.0122         0.0119           -2.1         0.0179         0.0174         0.0170         0.0166         0.0162         0.0158         0.0154           -2.0         0.0228         0.0222         0.0217         0.0212         0.0207         0.0202         0.0197           -1.9         0.0287         0.0281         0.0274         0.0268         0.0262         0.0256         0.0256           -1.8         0.0359         0.0351         0.0344         0.0336         0.0329         0.0322         0.0314           -1.7         0.0446         0.0436         0.0427         0.0418         0.0409         0.0401         0.0392           -1.5         0.0668         0.0655         0.0643         0.0630         0.0618         0.0606         0.0554           -1.3         0.0968         0.0951         0.0934         0.0918         0.0901	-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039
-2.3         0.0107         0.0104         0.0102         0.0099         0.0096         0.0094         0.0091           -2.2         0.0139         0.0136         0.0132         0.0129         0.0125         0.0122         0.0119           -2.1         0.0179         0.0174         0.0170         0.0166         0.0162         0.0158         0.0154           -2.0         0.0228         0.0222         0.0217         0.0212         0.0207         0.0202         0.0197           -1.9         0.0287         0.0281         0.0274         0.0268         0.0262         0.0256         0.0250           -1.8         0.0359         0.0351         0.0344         0.0336         0.0329         0.0322         0.0314           -1.7         0.0446         0.0436         0.0427         0.0418         0.0409         0.0401         0.0392           -1.6         0.0548         0.0537         0.0526         0.0516         0.0505         0.0495         0.0485           -1.5         0.0668         0.0655         0.0643         0.0630         0.0618         0.0606         0.0594           -1.3         0.0968         0.0793         0.0778         0.0764         0.0749	-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052
-2.2         0.0139         0.0136         0.0132         0.0129         0.0125         0.0122         0.0119           -2.1         0.0179         0.0174         0.0170         0.0166         0.0162         0.0158         0.0154           -2.0         0.0228         0.0222         0.0217         0.0212         0.0207         0.0202         0.0197           -1.9         0.0287         0.0281         0.0274         0.0268         0.0262         0.0256         0.0250           -1.8         0.0359         0.0351         0.0344         0.0336         0.0329         0.0322         0.0314           -1.7         0.0446         0.0436         0.0427         0.0418         0.0409         0.0401         0.0392           -1.6         0.0548         0.0537         0.0526         0.0516         0.0505         0.0495         0.0485           -1.5         0.0668         0.0655         0.0643         0.0630         0.0618         0.0606         0.0594           -1.4         0.0808         0.0793         0.0778         0.0764         0.0749         0.0735         0.0721           -1.3         0.0968         0.0951         0.0934         0.0918         0.0901	-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069
-2.1         0.0179         0.0174         0.0170         0.0166         0.0162         0.0158         0.0154           -2.0         0.0228         0.0222         0.0217         0.0212         0.0207         0.0202         0.0197           -1.9         0.0287         0.0281         0.0274         0.0268         0.0262         0.0256         0.0250           -1.8         0.0359         0.0351         0.0344         0.0336         0.0329         0.0322         0.0314           -1.7         0.0446         0.0436         0.0427         0.0418         0.0409         0.0401         0.0392           -1.6         0.0548         0.0537         0.0526         0.0516         0.0505         0.0495         0.0485           -1.5         0.0668         0.0655         0.0643         0.0630         0.0618         0.0606         0.0594           -1.4         0.0808         0.0793         0.0778         0.0764         0.0749         0.0735         0.0721           -1.3         0.0968         0.0951         0.0934         0.0918         0.0901         0.0885         0.0869           -1.2         0.1151         0.1131         0.1112         0.1093         0.1075	-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091
-2.0         0.0228         0.0222         0.0217         0.0212         0.0207         0.0202         0.0197           -1.9         0.0287         0.0281         0.0274         0.0268         0.0262         0.0256         0.0250           -1.8         0.0359         0.0351         0.0344         0.0336         0.0329         0.0322         0.0314           -1.7         0.0446         0.0436         0.0427         0.0418         0.0409         0.0401         0.0392           -1.6         0.0548         0.0537         0.0526         0.0516         0.0505         0.0495         0.0485           -1.5         0.0668         0.0655         0.0643         0.0630         0.0618         0.0606         0.0594           -1.4         0.0808         0.0793         0.0778         0.0764         0.0749         0.0735         0.0721           -1.3         0.0968         0.0951         0.0934         0.0918         0.0901         0.0885         0.0869           -1.2         0.1151         0.1131         0.1112         0.1093         0.1075         0.1056         0.1038           -1.1         0.1357         0.1355         0.1349         0.1515         0.1492	-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119
-1.9         0.0287         0.0281         0.0274         0.0268         0.0262         0.0256         0.0250           -1.8         0.0359         0.0351         0.0344         0.0336         0.0329         0.0322         0.0314           -1.7         0.0446         0.0436         0.0427         0.0418         0.0409         0.0401         0.0392           -1.6         0.0548         0.0537         0.0526         0.0516         0.0505         0.0495         0.0485           -1.5         0.0668         0.0655         0.0643         0.0630         0.0618         0.0606         0.0594           -1.4         0.0808         0.0793         0.0778         0.0764         0.0749         0.0735         0.0721           -1.3         0.0968         0.0951         0.0934         0.0918         0.0901         0.0885         0.0869           -1.2         0.1151         0.1131         0.1112         0.1093         0.1075         0.1056         0.1038           -1.1         0.1357         0.1350         0.1515         0.1492         0.1469         0.1446           0.9         0.1841         0.1814         0.1788         0.1762         0.1736         0.1711         0	-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154
-1.8         0.0359         0.0351         0.0344         0.0336         0.0329         0.0322         0.0314           -1.7         0.0446         0.0436         0.0427         0.0418         0.0409         0.0401         0.0392           -1.6         0.0548         0.0537         0.0526         0.0516         0.0505         0.0495         0.0485           -1.5         0.0668         0.0655         0.0643         0.0630         0.0618         0.0606         0.0594           -1.4         0.0808         0.0793         0.0778         0.0764         0.0749         0.0735         0.0721           -1.3         0.0968         0.0951         0.0934         0.0918         0.0901         0.0885         0.0869           -1.2         0.1151         0.1131         0.1112         0.1093         0.1075         0.1056         0.1038           -1.1         0.1357         0.1335         0.1314         0.1292         0.1271         0.1251         0.1230           -1.0         0.1587         0.1562         0.1539         0.1515         0.1492         0.1469         0.1446           0.9         0.1841         0.1814         0.17188         0.1762         0.1736	-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197
-1.7         0.0446         0.0436         0.0427         0.0418         0.0409         0.0401         0.0392           -1.6         0.0548         0.0537         0.0526         0.0516         0.0505         0.0495         0.0485           -1.5         0.0668         0.0655         0.0643         0.0630         0.0618         0.0606         0.0594           -1.4         0.0808         0.0793         0.0778         0.0764         0.0749         0.0735         0.0721           -1.3         0.0968         0.0951         0.0934         0.0918         0.0901         0.0885         0.0869           -1.2         0.1151         0.1131         0.1112         0.1093         0.1075         0.1056         0.1038           -1.1         0.1357         0.1355         0.1314         0.1292         0.1271         0.1251         0.1230           -1.0         0.1587         0.1562         0.1539         0.1515         0.1492         0.1469         0.1446           0.9         0.1841         0.1814         0.1738         0.1762         0.1736         0.1711         0.1688           -0.8         0.2119         0.2090         0.2061         0.2033         0.2005         0	-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250
-1.6         0.0548         0.0537         0.0526         0.0516         0.0505         0.0495         0.0485           -1.5         0.0668         0.0655         0.0643         0.0630         0.0618         0.0606         0.0594           -1.4         0.0808         0.0793         0.0778         0.0764         0.0749         0.0735         0.0721           -1.3         0.0968         0.0951         0.0934         0.0918         0.0901         0.0885         0.0869           -1.2         0.1151         0.1131         0.1112         0.1093         0.1075         0.1056         0.1038           -1.1         0.1357         0.1335         0.1314         0.1292         0.1271         0.1251         0.1230           -1.0         0.1587         0.1562         0.1539         0.1515         0.1492         0.1469         0.1446           0.9         0.1841         0.1814         0.1788         0.1762         0.1736         0.1711         0.1685           -0.8         0.2119         0.2090         0.2061         0.2033         0.2005         0.1977         0.1949           -0.7         0.2420         0.2389         0.2358         0.2327         0.2296         0	-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314
-1.5         0.0668         0.0655         0.0643         0.0630         0.0618         0.0606         0.0594           -1.4         0.0808         0.0793         0.0778         0.0764         0.0749         0.0735         0.0721           -1.3         0.0968         0.0951         0.0934         0.0918         0.0901         0.0885         0.0869           -1.2         0.1151         0.1131         0.1112         0.1093         0.1075         0.1056         0.1038           -1.1         0.1357         0.1335         0.1314         0.1292         0.1271         0.1251         0.1230           -1.0         0.1587         0.1562         0.1539         0.1515         0.1492         0.1469         0.1446           0.9         0.1841         0.1814         0.1788         0.1762         0.1736         0.1711         0.1688           -0.8         0.2119         0.2090         0.2061         0.2033         0.2005         0.1977         0.1949           -0.7         0.2420         0.2389         0.2358         0.2327         0.2296         0.2266         0.2236           -0.6         0.2743         0.2709         0.2676         0.2643         0.2611         0	-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392
-1.4         0.0808         0.0793         0.0778         0.0764         0.0749         0.0735         0.0721           -1.3         0.0968         0.0951         0.0934         0.0918         0.0901         0.0885         0.0869           -1.2         0.1151         0.1131         0.1112         0.1093         0.1075         0.1056         0.1038           -1.1         0.1357         0.1335         0.1314         0.1292         0.1271         0.1251         0.1230           -1.0         0.1587         0.1562         0.1539         0.1515         0.1492         0.1469         0.1446           0.9         0.1841         0.1814         0.1788         0.1762         0.1736         0.1711         0.1685           -0.8         0.2119         0.2090         0.2061         0.2033         0.2005         0.1977         0.1949           -0.7         0.2420         0.2339         0.2358         0.2327         0.2296         0.2266         0.2236           -0.6         0.2743         0.2709         0.2676         0.2643         0.2611         0.2578         0.2546	-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485
-1.3         0.0968         0.0951         0.0934         0.0918         0.0901         0.0885         0.0869           -1.2         0.1151         0.1131         0.1112         0.1093         0.1075         0.1056         0.1038           -1.1         0.1357         0.1335         0.1314         0.1292         0.1271         0.1251         0.1230           -1.0         0.1587         0.1562         0.1539         0.1515         0.1492         0.1469         0.1446           0.9         0.1841         0.1814         0.1788         0.1762         0.1736         0.1711         0.1685           -0.8         0.2119         0.2090         0.2061         0.2033         0.2005         0.1977         0.1949           -0.7         0.2420         0.2389         0.2358         0.2327         0.2296         0.2266         0.2236           -0.6         0.2743         0.2709         0.2676         0.2643         0.2611         0.2578         0.2546	-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594
-1.2         0.1151         0.1131         0.1112         0.1093         0.1075         0.1056         0.1038           -1.1         0.1357         0.1335         0.1314         0.1292         0.1271         0.1251         0.1230           -1.0         0.1587         0.1562         0.1539         0.1515         0.1492         0.1469         0.1469         0.1446           0.9         0.1841         0.1814         0.1788         0.1762         0.1736         0.1711         0.1685           -0.8         0.2119         0.2090         0.2061         0.2033         0.2005         0.1977         0.1949           -0.7         0.2420         0.2389         0.2358         0.2327         0.2296         0.2266         0.2236           -0.6         0.2743         0.2709         0.2676         0.2643         0.2611         0.2578         0.2546	-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721
-1.1         0.1357         0.1335         0.1314         0.1292         0.1271         0.1251         0.1230           -1.0         0.1587         0.1562         0.1539         0.1515         0.1492         0.1469         0.1446           0.9         0.1841         0.1814         0.1788         0.1762         0.1736         0.1711         0.1685           -0.8         0.2119         0.2090         0.2061         0.2033         0.2005         0.1977         0.1949           -0.7         0.2420         0.2338         0.2327         0.2296         0.2266         0.2236           -0.6         0.2743         0.2709         0.2676         0.2643         0.2611         0.2578         0.2546	-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869
-1.0         0.1587         0.1562         0.1539         0.1515         0.1492         0.1469         0.1446           0.9         0.1841         0.1814         0.1788         0.1762         0.1736         0.1711         0.1685           -0.8         0.2119         0.2030         0.2061         0.2033         0.2005         0.1977         0.1949           -0.7         0.2420         0.2389         0.2358         0.2327         0.2296         0.2266         0.2236           -0.6         0.2743         0.2709         0.2676         0.2643         0.2611         0.2578         0.2546	-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038
0.9         0.1841         0.1814         0.1788         0.1762         0.1736         0.1711         0.1685           -0.8         0.2119         0.2090         0.2061         0.2033         0.2005         0.1977         0.1949           -0.7         0.2420         0.2389         0.2358         0.2327         0.2296         0.2266         0.2236           -0.6         0.2743         0.2709         0.2676         0.2643         0.2611         0.2578         0.2546	-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230
-0.8         0.2119         0.2090         0.2061         0.2033         0.2005         0.1977         0.1949           -0.7         0.2420         0.2389         0.2358         0.2327         0.2296         0.2266         0.2236           -0.6         0.2743         0.2709         0.2676         0.2643         0.2611         0.2578         0.2546			0.1562	0.1539		0.1492	0.1469	0.1446
-0.7         0.2420         0.2389         0.2358         0.2327         0.2296         0.2266         0.2236           -0.6         0.2743         0.2709         0.2676         0.2643         0.2611         0.2578         0.2546	0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685
-0.6 0.2743 0.2709 0.2676 0.2643 0.2611 0.2578 0.2546								
-0.5 <b>(</b> 0.3085 <b>)</b> 0.3050 0.3015 0.2981 0.2946 0.2912 0.2877								
	-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877

(Lecture 10) 02/01/2013 14 / 27

### Example

$$X \sim \mathcal{N}(300, 5)$$

$$\mathbb{P}(X > 296) = \mathbb{P}(Z > -0.8)$$
  
= 1 - \mathbb{P}(Z < -0.8)  
= 1 - 0.2119 = 0.7881

$$\mathbb{P}(293 < X < 305) = \mathbb{P}\left(-\frac{7}{5} < Z < \frac{5}{5}\right)$$

$$= \mathbb{P}(-1.4 < Z < 1)$$

$$= \mathbb{P}(Z < 1) - \mathbb{P}(Z < -1.4)$$

$$= 0.8413 - 0.0808 = 0.7605$$

(Lecture 10) 02/01/2013 15 / 27

#### Values not on the table

$$\mathbb{P}(Z < -5) = 0.00000028711 \approx 0$$

$$\mathbb{P}(Z < 4) = 0.99997 \approx 1$$

Numbers off the table are either 0 or 1.

(Lecture 10) 02/01/2013 16 / 27

### **Steps**

Standardize

$$Z = \frac{X - \mu}{\sigma}$$

Manipulate (draw a picture)
P(Z < a)</p>

Look up in the table

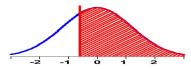
(Lecture 10) 02/01/2013 17 / 27

$$X \sim \mathcal{N}(150, 12), \mathbb{P}(X > 144) = ??$$

• Standardize:  $Z = \frac{X - \mu}{\sigma}$ .

$$\mathbb{P}(X > 144) = \mathbb{P}\left(\frac{X - 150}{12} > \frac{144 - 150}{12}\right) = \mathbb{P}(Z > -0.5)$$

2 Draw a picture:  $\mathbb{P}(Z > -0.5)$ 

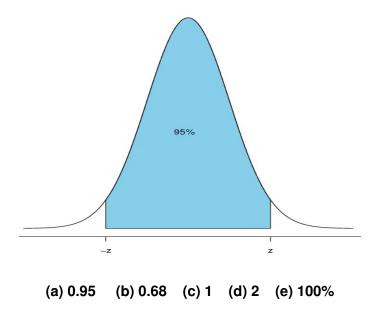


Sook up in the table

$$\mathbb{P}(Z > -0.5) = 1 - \mathbb{P}(Z < -0.5) = 1 - 0.3085 = 0.6915$$

(Lecture 10) 02/01/2013 18 / 27

#### What is the value of z?



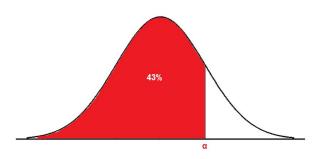
(Lecture 10) 02/01/2013 19 / 27

#### **Percentiles**

 $\bullet$   $\alpha$  is the 43rd percentile when

$$\mathbb{P}(X \leq \alpha) = 0.43$$

• This means that 43% of the time X is less than  $\alpha$ .



(Lecture 10) 02/01/2013 20 / 27

#### **Percentiles**

#### SAT Example

How high an SAT score do you need, to be in the 60th percentile?

$$P(X < score) = 0.60$$

Work the whole problem backwards.

(Lecture 10) 02/01/2013 21 / 27

### Look Inside the Table

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916

(Lecture 10) 02/01/2013 22 / 27

### Look Inside the Table

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916

(Lecture 10) 02/01/2013 23 / 27

### Look Inside the Table

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5 99	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5396	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5048	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916

(Lecture 10) 02/01/2013 24 / 27

#### **Un-Standardize**

$$Z = \frac{X - \mu}{\sigma}$$

 $X = \mu + \sigma Z$ 

$$\mathbb{P}(X < SATscore) = 0.60$$

0.60 corresponds to a z-score equal to 0.25:

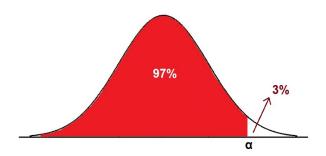
$$Z \le 0.25 \Rightarrow X \le 500 + 100 \ (0.25) = 525$$

60th percentile of X is 525

(Lecture 10) 02/01/2013 25 / 27

### **Top 3%**

• The SAT scores are normally distributed with mean 500 and standard deviation 100. A student wants to be in "Top 3%" of the SAT scores. How much does she need to get?



(Lecture 10) 02/01/2013 26 / 27

### **Top 3%**

- A student wants to be in "Top 3%" of the SAT scores. How much does she need to get?
  - ► Look for 97th percentile (1-0.03 = 0.97)
  - i.e. look inside the table for 0.97:

$$\mathbb{P}(Z \le 1.88) = 0.9699$$

Unstandardize:

$$X = \sigma Z + \mu$$
  
 $X = 500 + 100(1.88) = 688$ 

The student need to score at least 688 at the SAT.

$$\mathbb{P}(X \le 688) = 0.9699$$

(Lecture 10) 02/01/2013 27 / 27