Continuous Random Variables

Uniform Distribution

Lecture 8

01/28/2013

Binomial Distribution

- Binomial Experiment
  1. n trials
  2. Trials are independent
  3. Two possible outcomes
  4. $\mathbb{P}($ "Success" $) = p$

- Binomial Probability

$$\begin{aligned} \mathbb{P}(X = k) &= {}_nC_k \, p^k \, (1-p)^{n-k} \\ &= \frac{n!}{k! \, (n-k)!} \, p^k \, (1-p)^{n-k} \end{aligned}$$

Binomial Distribution ($n = 1$)

$$X = \begin{cases} 1, & \text{with probability } p \\ 0, & \text{with probability } (1-p) \end{cases}$$

$X \sim Bin(1,\ p)$

k	0	1
$\mathbb{P}(X = k)$	$1-p$	p

$$\mathbb{E}(X) = 1\ p + 0\ (1-p) = p$$

$$\mathbb{E}(X^2) = 1^2\ p + 0\ (1-p) = p$$

$$Var(X) = \mathbb{E}(X^2) - \Big(\mathbb{E}(X)\Big)^2 = p - p^2 = p(1-p)$$

Mean & Variance of S

$$S = X_1 + X_2 + \ldots + X_n$$

- If $X_1, X_2, \ldots, X_n$ are independent with pdf $X \sim Bin(1, p)$, then

$$S \sim Bin(n, p)$$

- Expected value of S

$$\begin{aligned}\mathbb{E}(S) &= \mathbb{E}(X_1) + \mathbb{E}(X_2) + \ldots + \mathbb{E}(X_n) \\ &= n\,\mathbb{E}(X) = n\,p\end{aligned}$$

- Variance of S

$$\begin{aligned}Var(S) &= Var(X_1) + Var(X_2) + \ldots + Var(X_n) \\ &= n\,Var(X) = n\,p\,(1-p)\end{aligned}$$

Mean and Variance of the Binomial

$X \sim Bin(n,\ p)$

- PDF

$$\mathbb{P}(X = k) =_n C_k\, p^k\, (1-p)^{n-k}$$

- Expected value

$$\mathbb{E}(X) = n\, p$$

- Variance

$$Var(X) = n\, p\, (1-p)$$

- Standard Deviation

$$\sigma = \sqrt{Var(X)} = \sqrt{n\, p\, (1-p)}$$

Continuous vs. Discrete Random Variables

Discrete Random Variable

- X takes values at random

k	1	2	3
$\mathbb{P}(X = k)$	0.2	0.6	0.2

Measurements

- X = height
- Y = waiting time
- Z = exam score
- U = random number between 0 and 1

Continuous vs. Discrete

- Discrete Random Variables
  - Take values from a list of distinct numbers
  - Are counts
- Continuous Random Variables
  - Take values on an interval
  - Are physical measurements

Turn on your clickers!

Discrete or Continuous?

- X = Time you wait for the bus

(a) Discrete

(b) Continuous

Turn on your clickers!

Discrete or Continuous?

- X = # of red Skittles in a bag

(a) Discrete

(b) Continuous

Turn on your clickers!

Discrete or Continuous?

- X = hours of sleep

(a) Discrete

(b) Continuous

Random Variable U

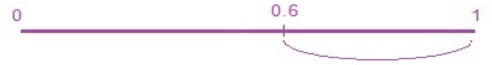
- U is any number between 0 and 1

- $\mathbb{P}(U < 1/2) = 1/2$

- $\mathbb{P}(0.5 < U < 0.6) = 0.1$

Interval Length

- $\mathbb{P}(U \;\; in \;\; A)$ = Length of A
- $\mathbb{P}(U > 0.6) = 1 - 0.6 = 0.4$



- $\mathbb{P}(U = 0.3) = ??$

$$\mathbb{P}(U = 1 \text{ distinct number }) = 0$$

Example

- $A = \{0.15 < U < 0.55\}$

$$\mathbb{P}(A) = \mathbb{P}(0.15 < U < 0.55) = 0.55 - 0.15 = 0.40$$

- $B = \{0.2 < U < 0.85\}$

$$\mathbb{P}(B) = \mathbb{P}(0.2 < U < 0.85) = 0.85 - 0.2 = 0.65$$

- A or B

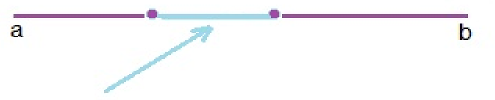
$$\mathbb{P}(A \;\; or \;\; B) = \mathbb{P}(0.15 < U < 0.85) = 0.85 - 0.15 = 0.7$$

- A and B

$$\mathbb{P}(A \;\; and \;\; B) = \mathbb{P}(0.2 < U < 0.55) = 0.55 - 0.2 = 0.35$$

Generally

- Uniform on the interval from *a* to *b*



$$\mathbb{P}(\text{ interval }) = \frac{\text{length of event interval}}{\text{length of whole interval}}$$

Continuous Random Variable

- $\mathbb{P}(X = 6) = 0$

  One number has negligible probability.

- Intervals have probability > 0

- Uniform:

$$\mathbb{P}(x < U < y) = \frac{y - x}{\text{length of the whole interval}}$$

Uniform pdf: $U(a, b)$

$$f(x) = \frac{1}{b - a}$$

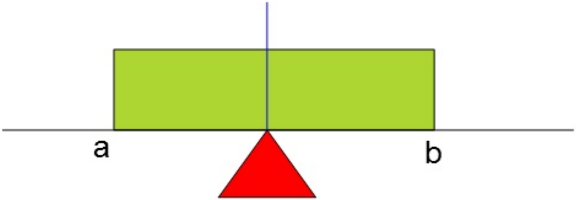
Uniform pdf: $U(a, b)$

Computing Probabilities

$$
\begin{aligned}
\mathbb{P}(x < U < y) &= \text{ Area} \\
&= \text{ width} \times \text{ height} \\
&= (y - x) \cdot \Big(\frac{1}{b - a}\Big)
\end{aligned}
$$

Expected Value of a Uniform

Center of Gravity of the Density



$$\mathbb{E}(U) = \frac{b+a}{2}$$