1. *Counting Rules*

   (a) *(2 points)* How many different arrangements can be made from the letters of the word *ATHENS*?

   $6! = 720$

   (b) *(2 points)* How many ways can a committee of 4 people be selected from a group of 10?

   $${}_{10}C_4 = \frac{10!}{4!6!} = 210$$

   (c) *(6 points)* In a company there are 7 executives: 4 women and 3 men. 3 are selected to attend a management seminar. Compute the probability that 2 men and 1 woman are selected.

   $$\frac{{}_3C_2 \; {}_4C_1}{{}_7C_3} = \frac{12}{35} = 0.34.$$

2. I take a random sample of 7 UCSB students and ask them whether they ride their bike to school or prefer to walk or take the bus. Assume that each response is independent and that with probability 0.7 a student rides her bike. Let X be the number of UCSB students who ride their bike to school.

   (a) *(3 points)* What is the distribution of X. Why? Please explain.

   $X \sim \text{Binomial}(7, 0.7)$

   (i) $n = 7$ fixed
   (ii) independent responses.
   (iii) 2 outcomes: success → ride bike to school
   (iv) fixed prob. of success: $p = 0.7$

   (b) *(3 points)* What is the probability that 2 students choose to ride their bike to school?

   $$P(X=2) = {}_7C_2 \, (0.7)^2 (1-0.7)^{7-2} = 0.357$$

   (c) *(3 points)* What is the probability that at least 1 student chooses to ride his bike to school?

   $$P(X \geq 1) = 1 - P(X=0) = 1 - {}_7C_0 \, (0.7)^0 (1-0.7)^7$$
   $$= 0.99978$$

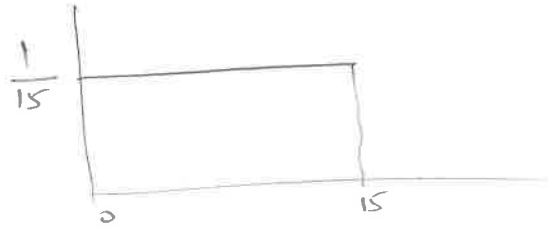
   (d) *(1 point)* What is the mean and the standard deviation of X.

   $E(X) = 4.9$

3. Assume that the bus arrives at the bus stop everyday between 0 and 15 minutes from the time that John gets there. Let U be the time that John waits for the bus. (All waiting times are equally likely.)

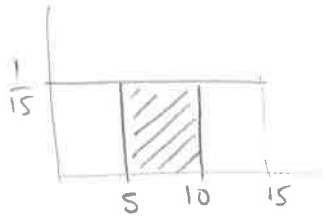
(a) *(3 points)* What is the distribution of U? Draw a graph of the pdf.

$U \sim \text{Uniform}(0, 15)$



(b) *(3 points)* What is the probability that John waits between 5 and 10 minutes?

$$P(5 < U < 10) = (10-5) \cdot \frac{1}{15} = \frac{1}{3}$$



(c) *(4 points)* Given that John has been waiting at the bus stop for 5 minutes, what is the probability that the bus will arrive within 10 minutes from the time that John arrived at the bus stop?

$$P(U \le 10 \mid U \ge 5) = \frac{P(U \le 10 \text{ and } U \ge 5)}{P(U \ge 5)} =$$

$$= \frac{P(5 \le U \le 10)}{P(U \ge 5)} = \frac{(10-5) \cdot \frac{1}{15}}{(15-5) \cdot \frac{1}{15}} = \frac{5}{10} = \frac{1}{2}$$

4. In a physics exam, the scores are normally distributed with average 80 and standard deviation 10.

(a) *(5 points)* What is the probability that a randomly selected student scores above 66?

$$P(X > 66) = 1 - P(X \le 66) = 1 - P\left(Z \le \frac{66-80}{10}\right) =$$

$$= 1 - P(Z \le -1.4) = 1 - 0.0808 = 0.9192$$

(b) *(5 points)* Compute the probability that the student scores 70 or 72.

$$P(X = 70 \text{ or } X = 72) = 0$$

1. *Counting Rules*

   (a) *(2 points)* How many different arrangements can be made from the letters of the word *PLATO*?

   $5! = 120$

   (b) *(2 points)* How many ways can a committee of 3 people be selected from a group of 9?

   $${}_9C_3 = \frac{9!}{3!6!} = 84 .$$

   (c) *(6 points)* In a company there are 7 executives: 4 women and 3 men. 3 are selected to attend a management seminar. Compute the probability that 1 man and 2 women are selected.

   $$\frac{{}_4C_2 \; {}_3C_1}{{}_7C_3} = 0.51$$

2. I take a random sample of 8 UCSB students and ask them whether they have a Science major or not. Assume that each response is independent and that with probability 0.45 a student is Science major. Let X be the number of UCSB students who major in Science.

   (a) *(3 points)* What is the distribution of X. Why? Please explain.

   $X \sim \text{Binomial}(8, 0.45)$

   i) $n = 8$ fixed
   ii) independent responses — major in Science
   iii) 2 outcomes : success
   iv) fixed prob. of success : $p = 0.45$

   (b) *(3 points)* What is the probability that 4 students are Science majors?

   $$P(X=4) = {}_8C_4 \, (0.45)^4 (1-0.45)^{8-4} = 0.26$$

   (c) *(3 points)* What is the probability that at most 7 students are Science majors?

   $$P(X \leq 7) = 1 - P(X=8) = 1 - {}_8C_8 \, (0.45)^8 (0.55)^0 = \ldots$$

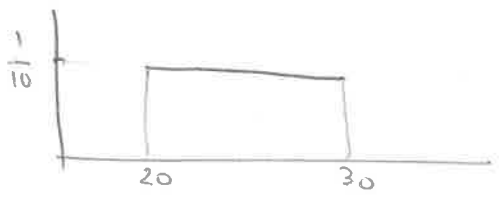
   (d) *(1 point)* What is the mean of X?

   $$E(X) = 8(0.45) = 3.6$$

3. Assume that it takes Mary between 20 and 30 minutes to get to work. Let U be the time that it takes her to arrive at work on a given day. (All arrival times are equally likely.)

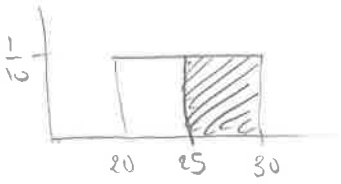
(a) *(3 points)* What is the distribution of U? Draw a graph of the pdf.

$U \sim \text{Uniform}(20,30)$



(b) *(3 points)* What is the probability that Mary needs more than 25 minutes to go to work?

$P(U>25) = (25-20)\cdot\frac{1}{10} = 0.5$



(c) *(4 points)* Given that Mary left 23 minutes ago, what is the probability that she needs more than 25 minutes to get to work?

$$P(U>25 \mid U>23) = \frac{P(U>25 \text{ and } U>23)}{P(U>23)} =$$

$$= \frac{P(U>25)}{P(U>23)} = \frac{5\cdot\frac{1}{10}}{(30-23)\frac{1}{10}} = \frac{5}{7} = 0.71.$$

4. In a history exam, the scores are normally distributed with average 75 and standard deviation 12.

(a) *(5 points)* What is the probability that a randomly selected student scores above 90?

$$P(X>90) = P\left(Z > \frac{90-75}{12}\right) = P(Z>1.25) = 1 - 0.8944$$

$$= 0.1056$$

(b) *(5 points)* Compute the probability that the student scores 70 or 72.

$P(X=70 \text{ or } X=72) = 0$

1. *Counting Rules*

   (a) *(2 points)* How many different arrangements can be made from the letters of the word *HOMER*?

   $5! = 120$

   (b) *(2 points)* How many ways can a committee of 5 people be selected from a group of 10?

   $${}_{10}C_5 = \frac{10!}{5!(10-5)!} = 252$$

   (c) *(6 points)* In a company there are 8 executives: 4 women and 3 men. 4 are selected to attend a management seminar. Compute the probability that 2 men and 2 women are selected.

   $$\frac{{}_4C_2 \; {}_3C_2}{{}_8C_4} = \frac{\frac{4!}{2!2!} \cdot \frac{3!}{2!1!}}{\frac{8!}{4!4!}} = \frac{(12)(3)}{70} = 0.51$$

2. I take a random sample of 8 UCSB students and ask them whether they are from California or not. Assume that each response is independent and that with probability 0.25 a student is not from California. Let X be the number of UCSB students who are not from California.

   (a) *(3 points)* What is the distribution of X. Why? Please explain.

   $X \sim \text{Binomial}(8, 0.25)$

   (i) Fixed number of trials ($n=8$)
   (ii) Independent responses.
   (iii) 2 outcomes → success: "not from Cali", failure: "from Cali"
   (iv) fixed prob of success: $p = 0.25$.

   (b) *(3 points)* What is the probability that 6 students are not from California?

   $$P(X=6) = {}_8C_6 (0.25)^6 (1-0.25)^{8-6} = \frac{8!}{6!2!} (0.25)^6 (0.75)^2 = 0.0038$$

   (c) *(3 points)* What is the probability that at least 1 student is not from California?

   $$P(X \geq 1) = 1 - P(X=0) = 1 - {}_8C_0 (0.25)^0 (1-0.25)^8$$
   $$= 1 - \frac{8!}{8!0!} (0.75)^8 = 0.8999.$$

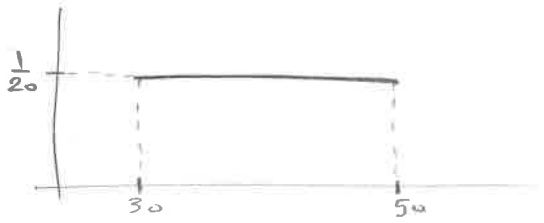
   (d) *(1 point)* What is the mean of X?

   $E(X) = np = 8(0.25) = 1.25$

3. Assume that Jennifer is paying between \$30 and \$50 per month at SoCal Edison. Let U be the amount of money she pays on January. (Assume that all amounts are equally likely.)

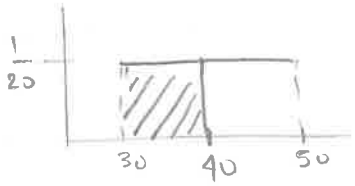
(a) *(3 points)* What is the distribution of U? Draw a graph of the pdf.

$U \sim \text{Uniform}(30, 50)$



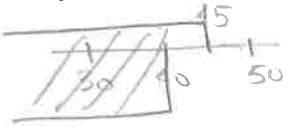
(b) *(3 points)* What is the probability that Jennifer pays less than \$40 this month?

$$P(U<40) = (40-30)\cdot\frac{1}{20} = \frac{1}{2}$$



(c) *(4 points)* Given that Jennifer paid no more than 45\$, what is the probability that she will not pay more than 40\$?

$$P(U\le 40 \mid U\le 45) = \frac{P(U\le 40 \text{ and } U\le 45)}{P(U\le 45)} =$$



$$= \frac{P(U\le 40)}{P(U\le 45)} = \frac{(40-30)\cdot\frac{1}{20}}{(45-30)\cdot\frac{1}{20}} = \frac{10}{15} = 0.667$$

4. In a geography exam, the scores are normally distributed with average 82 and standard deviation 8.

(a) *(5 points)* What is the probability that a randomly selected student scores above 75?

$$P(X>75) = P\left(Z > \frac{75-82}{8}\right) = P(Z > -0.875) = 0.1894$$

$-0.875 \approx -0.88$

(b) *(5 points)* Compute the probability that the student scores 78 or 82.

$$P(X=78 \text{ or } X=82) = 0$$

1. *Counting Rules*

   (a) *(2 points)* How many different arrangements can be made from the letters of the word *VENUS*?

   $5! = 120$

   (b) *(2 points)* How many ways can a committee of 2 people be selected from a group of 9?

   $${}_9C_2 = \frac{4!}{2!7!} = 36$$

   (c) *(6 points)* In a company there are 7 executives: 4 women and 3 men. 4 are selected to attend a management seminar. Compute the probability that 1 man and 3 women are selected.

   $$\frac{{}_4C_3\ {}_3C_1}{{}_7C_4} = \frac{\frac{4!}{3!\,1!}\ \frac{3!}{1!\,2!}}{\frac{7!}{4!\,3!}} = \frac{12}{35} = 0.34$$

2. I take a random sample of 9 UCSB students and ask them whether they like to surf. Assume that each response is independent and that with probability 0.6 a student enjoys surfing. Let X be the number of UCSB students who like surfing.

   (a) *(3 points)* What is the distribution of X. Why? Please explain.

   $X \sim \text{Binomial}(9, 0.6)$

   i) $n = 9$ fixed
   ii) independent responses
   iii) 2 outcomes → success: enjoy surfing
   iv) fixed prob of success: $p = 0.6$

   (b) *(3 points)* What is the probability that 3 students like to surf?

   $$P(X=3) = {}_9C_3\ (0.6)^3\ (1-0.6)^{9-3} = 0.074$$

   (c) *(3 points)* What is the probability that at most 8 students like to surf?

   $$P(X \leq 8) = 1 - P(X=9) = 1 - {}_9C_9\ (0.6)^9\ (1-0.6)^{9-9} = 0.9997$$

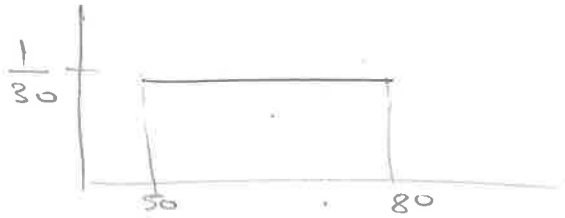
   (d) *(1 point)* What is the mean of X?

   $E(X) = 5.4$

3. Assume that George is paying between \$50 and \$80 per month for gas. Let U be the amount of money he pays on January. (Assume that all amounts are equally likely.)

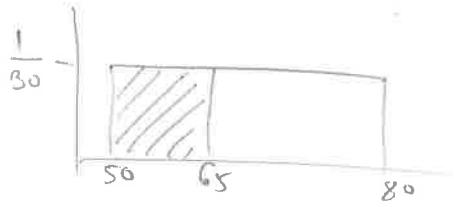
(a) *(3 points)* What is the distribution of U? Draw a graph of the pdf.

$U \sim \text{Uniform}(50, 80)$



(b) *(3 points)* What is the probability that Georges pays less than \$65 this month?

$$P(U<65) = (65-50)\cdot\frac{1}{30} = \frac{1}{2}$$



(c) *(4 points)* Given that George paid 60\$ already, what is the probability that he will spend less than \$65?

$$P(U<65 \mid U>60) = \frac{P(60<U<65)}{P(U>60)} = \frac{5/30}{20/30} = 0.25$$

4. In a linguistics exam, the scores are normally distributed with average 65 and standard deviation 18.

(a) *(5 points)* What is the probability that a randomly selected student scores above 80?

$$P(X>80) = P\left(Z > \frac{80-65}{18}\right) = P(Z>0.83) = 1 - P(Z \le 0.83) = 1 - .7967 = 0.2033.$$

(b) *(5 points)* Compute the probability that the student scores 75 or 85.

$$P(X=75 \text{ or } X=85) = 0.$$