Binomial Distribution

Lecture 6

01/18/2013

Expected Value

$$\mathbb{E}(X) = \sum_{\textit{all } k} k \, \mathbb{P}(X = k)$$

Interpretation

1. A probability-weighted average.
2. Long-run average of X.
3. The fair value of a gamble.
4. The balance point for a probability histogram/bargraph.

Properties of $\mathbb{E}(X)$

1. $\mathbb{E}(X + \alpha) = \mathbb{E}(X) + \alpha$.
2. $E(b\,X) = b\,\mathbb{E}(X)$.
3. $\mathbb{E}(X + Y) = \mathbb{E}(X) + \mathbb{E}(Y)$
4. If X and Y are independent, then

$$\mathbb{E}(XY) = \mathbb{E}(X)\,\mathbb{E}(Y)$$

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Measuring the Spread

- Variance = how spread out the pdf is.
- Variance = average squared distance from the mean:

$$Var(X) = \mathbb{E}(X - \mathbb{E}(X))^2$$

$$Var(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2$$

- Standard Deviation

$$\sigma = \sqrt{Var(X)}$$

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Properties

- $Var(\alpha X) = \alpha^2\ Var(X)$

  For example,

  $$Var(3 \cdot X) = 3^2 \cdot Var(X) = 9 \cdot Var(X)$$

- $Var(\alpha + X) = Var(X)$

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Which one?

The weighted average of the possible values that a variable X can take, where the weights are the probabilities of occurrence, is referred to as the:

(a) variance

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Suppose we have two independent random variables X and Y. Which of the following statements about X and Y are TRUE?

$$I.\ \mathbb{E}(X+Y) = \mathbb{E}(X) + \mathbb{E}(Y)$$
$$II.\ Var(X+Y) = Var(X) + Var(Y)$$
$$III.\ \mathbb{E}(X-Y) = \mathbb{E}(X) - \mathbb{E}(Y)$$
$$IV.\ Var(X-Y) = Var(X) - Var(Y)$$

(a) I, II and III

(b) II and IV

(c) I, II, III and IV

Suppose we have two independent random variables X and Y. Which of the following statements about X and Y are TRUE?

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Independent Events

- If A and B are *independent*

$$\mathbb{P}(A \textit{ and } B) = \mathbb{P}(A)\,\mathbb{P}(B)$$

- If $A_1, A_2, \ldots, A_n$ are *independent*

$$\mathbb{P}(A_1, A_2, \ldots, \textit{ and } A_n) = \mathbb{P}(A_1)\,\mathbb{P}(A_2)\ldots\mathbb{P}(A_n)$$

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Flipping Fair Coins

- Each outcome has probability 1/2

- Outcomes are independent

  If we flip a coin n times

$$\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\cdots\left(\frac{1}{2}\right)=\left(\frac{1}{2}\right)^n$$

- $\mathbb{P}(16 \text{ Heads}) = \frac{1}{2^{16}} = \frac{1}{65536} = 0.00001526$

  $\mathbb{P}(10 \text{ Heads then } 6 \text{ Tails}) = \frac{1}{2^{16}} = \frac{1}{65536} = 0.00001526$

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And Counting Heads

Flip a coin 4 times

- X = number of Heads
- $\mathbb{P}(X = 4) = (\frac{1}{2})(\frac{1}{2})(\frac{1}{2})(\frac{1}{2}) = \frac{1}{16}$
- $\mathbb{P}(X = 0) = \mathbb{P}(4\ \mathit{Tails}) = \frac{1}{16}$
- $\mathbb{P}(X = 1) =$???

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k	0	1	2	3	4
$\mathbb{P}(X = k)$					

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$\mathbb{P}(X = k)$	1/16	1/4	3/8	1/4	1/16

Generally

- Flip a coin n times
- 2^n outcomes in the sample space
- $\mathbb{P}(\text{one sequence}) = (1/2)^n$
- $\mathbb{P}(X = k) = (1/2)^n$ (# of seq. with k Heads)

Question:

How do we count these combinations???

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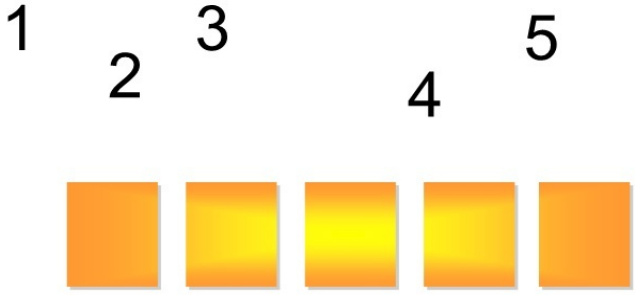

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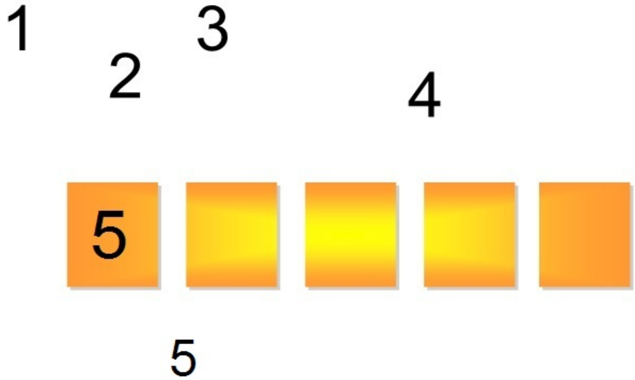

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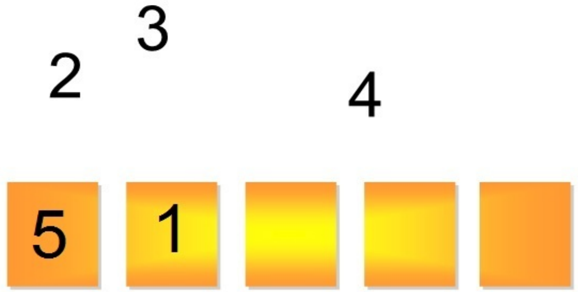

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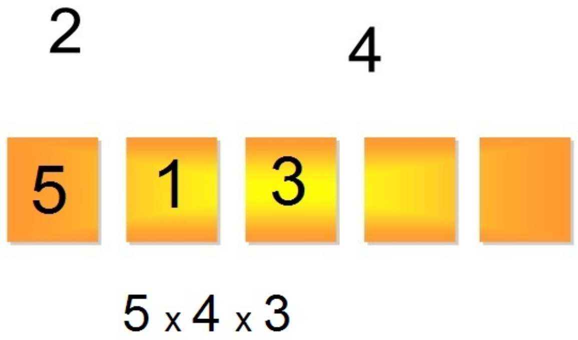


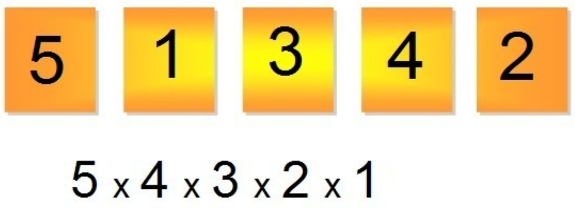
Choose an order

2

5 1 3 4

5 x 4 x 3 x 2

Choose an order



Choose an order

$$5 \times 4 \times 3 \times 2 \times 1 = 120$$

Counting Sequences

- In the above example:

$$(5)\,(4)\,(3)\,(2)\,1 \;=\; 5!$$

- More generally
  - n factorial

$$n! = n\,(n-1)\,(n-2)\,\ldots\,(3)(2)1$$

- For example, $30! = 30\,(29)\,(28)\,\ldots\,(2)1 = 2.65 \times 10^{32}$
- By definition

$$0! = 1$$

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$$0! = 1$$

5 choose 2

5 choose 2




5

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$$5 \times 4 = 20$$

5 choose 2

$$5 \times 4 = 20 \div 2 = 10$$

n choose k

$$ {}_nC_k = \frac{n!}{k!(n-k)!} $$

- $n = 5, \ k = 2$

$$ {}_5C_2 = \frac{5!}{2!\,3!} = \frac{5(4)(3)(2)1}{2(1) \times (3)(2)1} = \frac{120}{2 \times 6} = 10 $$

- $n = 6, \ k = 4$

$$ {}_6C_4 = \frac{6!}{4!\,2!} = \frac{6(5)(4)(3)(2)1}{(4)(3)(2)(1) \times (2)1} = \frac{720}{48} = 15 $$

- $n = 52, \ k = 3$

$$ {}_{52}C_3 = \frac{52(51)50}{(3)(2)1} = \frac{132{,}600}{6} = 22{,}100 $$

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Binomial Coefficients

n=1	1 1
n=2	1 2 1
n=3	1 3 3 1
n=4	1 4 6 4 1
n=5	1 5 10 10 5 1
n=6	1 6 15 20 15 6 1

A coach must choose 3 starters from a team of 6 players. How many different ways can the coach choose the starters?

(a) 18

(b) 20

(c) 720

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(a) 18

(b) ${}_6C_3 = 6!/(3!\,3!) = 20$

(c) 720

Probability of Flipping 6 of 10

$$\mathbb{P}(\text{6 Heads in 10 Tries}) = {}_{10}C_6 \left(\frac{1}{2}\right)^{10}$$

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# of orderings

Probability of each one

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# of orderings

Probability of each one

$$\frac{10!}{6!\,4!}\left(\frac{1}{1024}\right) = \frac{10(9)(8)(7)}{4(3)(2)1}\left(\frac{1}{1024}\right) = \frac{210}{1024} = 0.2051$$

Other Probabilities

- What if

$$\mathbb{P}(H) = 0.2 \text{ and } \mathbb{P}(T) = 0.8$$

$$\text{Then, } \mathbb{P}(HT) = 0.2\ (0.8) = 0.16$$

$$\mathbb{P}(HTTHT) = 0.2(0.8)(0.8)(0.2)(0.8) = (0.2)^2(0.8)^3 = 0.02048$$
$$\mathbb{P}(TTHTH) = 0.8(0.8)(0.2)(0.8)(0.2) = (0.2)^2(0.8)^3 = 0.02048$$
$$\mathbb{P}(TTTHH) = 0.8(0.8)(0.8)(0.2)(0.2) = (0.2)^2(0.8)^3 = 0.02048$$

$$\mathbb{P}(\text{ 2 Heads }) = {}_5C_2(0.2)^2(0.8)^3 = 10(0.02048) = 0.2048$$

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Sequence of Independent Events

$A_1, A_2, \ldots, A_n$ are independent with

$$\mathbb{P}(A_i) = p$$
$$\mathbb{P}(A_i^c) = 1 - p$$

Then...

$$\mathbb{P}(A_1, A_2^c, \text{ and } A_3) = p\,(1-p)\,p = p^2\,(1-p)$$

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Example

- $\mathbb{P}($ Dodgers win $) = 0.58$
- $\mathbb{P}($ Dodgers win twice $) = (0.58)(0.58) = 0.3364$
- The probability to lose 2 games and then win 4:

$$\begin{aligned}\mathbb{P}(\text{ Lose 2 then Win 4 }) &= (0.42)(0.42)(0.58)(0.58)(0.58)(0.58) \\ &= (0.42)^2(0.58)^4 = 0.01996\end{aligned}$$

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- $\mathbb{P}($ Dodgers win twice $) = (0.58)(0.58) = 0.3364$
- The probability to lose 2 games and then win 4:

$$\begin{aligned}\mathbb{P}(\text{ Lose 2 then Win 4 }) &= (0.42)(0.42)(0.58)(0.58)(0.58)(0.58)\\ &= (0.42)^2(0.58)^4 = 0.01996\end{aligned}$$

Binomial Experiment

1. n trials
2. Trials are independent
3. Two possible outcomes
   - "Success": 1
   - "Failure": 0
4. $\mathbb{P}(\text{ "Success" }) = p$

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Example: Proposition 8

Sample of 100 California voters

1. $n = 100$
2. Assume they are Independent (?)
3. “Support” = “success”
4. $p = 0.44$

$X =$ # of supporters

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Binomial Probability

$$\mathbb{P}(X = k) = {}_n C_k \, p^k \, (1 - p)^{n-k}$$

- n = total number of trials
- p = probability of success

$$\mathbb{P}(\text{ 40 supporters }) = \mathbb{P}(X = 40)$$

$$= {}_{100} C_{40} (0.44)^{40} (0.56)^{60}$$

$$= (1.3746 \times 10^{28})(5.47151 \times 10^{-15})(7.78541 \times 10^{-16})$$

$$= 0.0586$$

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Binomial Distribution

- Binomial Experiment
  1. n trials
  2. Trials are independent
  3. Two possible outcomes
  4. $\mathbb{P}($ "Success" $) = p$
- Binomial Probability

$$\begin{aligned} \mathbb{P}(X = k) &= {}_n C_k \, p^k \, (1-p)^{n-k} \\ &= \frac{n!}{k! \, (n-k)!} \, p^k \, (1-p)^{n-k} \end{aligned}$$