

## Practice Problems 6: *Central Limit Theorem*

### A. From your Book:

- Sections 5.2, 5.3

### B. Additional Problems:

- (a) State the assumptions needed in order for the CLT to hold for the sample mean.
  - (b) Under the above assumptions, what is (approximately) the distribution of the sample mean of  $n$  observations?
2. The national scores of individual students on ACT have mean 18.6 and standard deviation 5.9. At Northside High, 76 seniors take the test. If the scores at this school have the same distribution as national scores, what are the mean and standard deviation of the sample mean score for the 76 students?
3. The number of flaws per square yard in a type of carpet material varies with mean 1.6 flaws per square yard and standard deviation 1.2 flaws per square yard. The distribution is not normal—in fact, it is discrete. An inspector studies 200 square yards of the material, records the number of flaws found in each square yard inspected. Use the CLT to find (approximately) the probability that the mean number of flaws exceeds 2 per square yard.
4. Children in kindergarten are sometimes given the Raven Progressive Matrices Test (RPMT) to assess their readiness for learning. Experience at a certain elementary school suggests that the RPMT scores for that school have mean 13.6 and standard deviation 3.1. Mr Smith has 45 children in his kindergarten class this year. He suspects that their RPMT scores will be unusually low because the test was interrupted by a fire drill. To check his suspicion, he wants to find the level  $L$  such that the probability is only 0.05 that the **mean** score of his 22 children falls below  $L$  when the usual distribution (for that school) remains true. What is the value of  $L$ ?
5.  $Y$  is a random variable that takes the value 1 if we get a 3 and 0 otherwise. We draw again 400 numbers from the box with replacement and  $W$  is the number of 3's that we get (in other words the number of successes).
  - (a) What is the exact distribution of  $Y$ ? What is the exact distribution of  $W$ ? Compute its mean and standard deviation.
  - (b) What is the approximately distribution of  $W$ .
  - (c) Compute approximately the probability that there will be less than 90 “3”s.

6. The amount of time (in minutes),  $U$ , that Mary has to wait for the bus a continuous random variable, uniformly distributed from 0 to 30.
- (a) Sketch its pdf of  $U$  and compute the mean and the standard deviation of  $U$ .
  - (b) Mary walks the bus stop at a random time between 8am and 8.30am and she has been doing so for the last year (365 days). According to the Central Limit Theorem(CLT), what is the distribution of the total amount of time that she has been waiting for the bus for the last year? What assumptions did you make in order use the CLT?
  - (c) Compute the approximate probability that during the last year the total amount of time she had to wait was more than 5645 minutes.