Lab 8 – Solution

1. The midterm exam for a physics class contains 20 mutiple-choice questions, each of which has 5 possible answers. We assume that the students answer completely at random. If a student answers correctly, he/she gets 1 point, otherwise he/she loses 0 points.
   X is the score of a student on the exam.

   (a) Write down the probability mass function (pmf) of X and compute the mean and the standard deviation.

   $$X \sim Bin(20, 0.2)$$
   $$E(X) = 4, \quad \sigma = \sqrt{20 \cdot 0.2 \cdot 0.8} = 1.79$$

   (b) This physics class has 100 students with exam scores $X_1, X_2, \ldots, X_{100}$. What do we need to assume about the scores of the students in order to apply the Central Limit Theorem? In that case, what is the approximate distribution of the average midterm score?

   The scores of the students need to be independent and identically distributed, that is to have the same distribution (the one from part 1). In this case,

   $$\bar{X} \sim N(4, 1.79/\sqrt{100})$$

   (c) Compute the approximate probability that the average score on this midterm is between 3.5 an 4.5.

   $$\begin{aligned} \mathbb{P}(3.5 \leq \bar{X} \leq 4.5) &= \mathbb{P}(\bar{X} \leq 5.5) - \mathbb{P}(\bar{X} \leq 3.5) \\ &= \mathbb{P}(Z \leq \frac{4.5 - 4}{0.179}) - \mathbb{P}(Z \leq \frac{3.5 - 4}{0.179}) \\ &= \mathbb{P}(Z \leq 2.79) - \mathbb{P}(Z \leq -2.79) \end{aligned}$$

2. A survey of individuals in New York who passed the seven exams and obtained the rank of Fellow in the actuarial field finds the following results:

(a) For a sample of 20 Fellows the average salary is found to be \$150,000 and the sample standard deviation \$15,000. Construct a 95% confidence interval for the average salary of all Fellows.

Confidence interval for μ when σ is unknown:
$\bar{x} \pm t^* \frac{s}{\sqrt{n}}$, t^* is the value from t-distribution with n-1 df. $150000 \pm 2.093 \frac{15000}{\sqrt{20}}$

(b) For a sample of 1200 Fellows the average salary is found to be \$150,000 and the sample standard deviation \$18,000. Construct a 90% confidence interval for the average salary of all Fellows.

Confidence interval for μ when σ is unknown, but sample size $n > 30$: $\bar{x} \pm Z^* \frac{s}{\sqrt{n}}$, with

C	90%	95%	99%
Z^*	1.645	1.960	2.576

$150,000 \pm 1.645 \frac{18,000}{\sqrt{1,200}}$