## Multiallelic calling model in bcftools (-m) Petr Danecek, Richard Durbin Version: December 5, 2013

Let  $f_A$  be an estimate of the allele frequency of the base A across all samples

$$f_A = \frac{\sum_k q_k \delta_{b_k, A}}{\sum_k q_k},$$

where  $b_k \in \{A, C, G, T\}$  is the base at k-th read and  $q_k$  the corresponding quality and

$$f_{A|AC} = \frac{f_A}{f_A + f_C}$$

$$f_{A|ACG} = \frac{f_A}{f_A + f_C + f_G}.$$

Calculate likelihoods of all possible combinations of alleles  $^2$  across all samples i as

$$L_A = \prod_i L_A^i$$
 
$$L_{AC} = \prod_i L_{AC}^i$$
 
$$L_{ACG} = \prod_i L_{ACG}^i$$
 
$$L_{ACGT} = \prod_i L_{ACGT}^i$$

where

$$\begin{split} L_{A}^{i} = & PL^{i}(AA) = P(data|AA) \\ L_{AC}^{i} = & f_{A|AC}^{2}PL^{i}(AA) + f_{C|AC}^{2}PL^{i}(CC) + 2f_{A|AC}f_{C|AC}PL^{i}(AC) \\ L_{ACG}^{i} = & f_{A|ACG}^{2}PL^{i}(AA) + f_{C|ACG}^{2}PL^{i}(CC) + f_{G|ACG}^{2}PL^{i}(GG) \\ & + 2(f_{A|ACG}f_{C|ACG}PL^{i}(AC) + f_{A|ACG}f_{G|ACG}PL^{i}(AG) + f_{C|ACG}f_{G|ACG}PL^{i}(CG)) \\ L_{a_{1},...,a_{n}}^{i} = & \sum_{j} f_{a_{j}|a_{1},...,a_{n}}^{2}PL^{i}(a_{j}a_{j}) + 2\sum_{j < k} f_{a_{j}|a_{1},...,a_{n}}f_{a_{k}|a_{1},...,a_{n}}PL^{i}(a_{j}a_{k}). \end{split}$$

 $<sup>^1\</sup>mathrm{See}$  bcf\_call\_glfgen in bam2bcf.c for calculation of  $q_k.$ 

<sup>&</sup>lt;sup>2</sup>In the current implementation, at most tri-allelic sites are considered.

Select the most likely set of alleles  $\{a\}$  with likelihood  $L_{\{a\}}$  and the second most likely set of alleles  $\{b\}$  with likelihood  $L_{\{b\}}$ . Accept  $\{a\}$  either if the number of alleles in  $\{a\}$  is smaller than in  $\{b\}$  or if the significance level for a 1 degree of freedom likelihood ratio test

$$\chi^2 = 2\log \frac{L_{\{b\}}}{L_{\{a\}}}$$

exceeds a given threshold. Assuming HWE, for *i*-th sample we select the genotype  $\{a\} = a_1 a_2$  which maximizes the likelihood

$$P_{\{a\}}^{i} = (2 - \delta_{a_1, a_2}) f_{a_1 | \{a\}} f_{a_2 | \{a\}} PL^{i}(a_1 a_2).$$

The corresponding genotype quality is

$$GQ_{\{a\}}^{i} = -10\log\left[1 - \frac{P_{\{a\}}^{i}}{\sum_{\{b\}} P_{\{b\}}^{i}}\right]$$

and the call quality is

$$\text{QUAL} = -10 \log \frac{L_{\{ref\}}}{\sum_{\{b\}} L_{\{b\}}}$$

for variant calls and

$$\mathrm{QUAL} = -10\log\left[1 - \frac{L_{\{ref\}}}{\sum_{\{b\}} L_{\{b\}}}\right]$$

for non-variant calls, where the sum iterates over all possible genotypes  $\{b\}$ .