

Unifying Typing and Subtyping

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Motivation

Dependent types are gaining popularity: Idris (2013), Agda (2007), Coq, etc.

- **Expressiveness** of type system
- **Economy** of concept: less syntax, rules, meta-theory \Rightarrow **our focus**

Theoretical basis of OO is becoming complex:

- Generics in *Java 5* (2004): System $F_{\leq} / F_{\omega}^{\leq}$
- Path-dependent types in *Scala* (2004): Dependent Object Types (DOT, Amin et al., 2012)

Bring the advantages of dependent types to object-oriented programming?

*One thing that makes the study of these systems difficult is that **with dependent types, the typing and subtyping relations become intimately tangled**, which means that tested techniques of examining subtyping in isolation no longer apply.*

— *Aspinall and Compagnoni (1996),
Subtyping Dependent Types*

Challenges (cont.)

- Mutual dependency of **typing** and **subtyping**:

Subsumption rule

$$\frac{\Gamma \vdash e : A \quad \Gamma \vdash A \leq B}{\Gamma \vdash e : B}$$

Subtyping of top type

$$\frac{\Gamma \vdash A : \star}{\Gamma \vdash A \leq \top}$$

- Duplication of concepts:

Abstractions in System F_{ω}^{\leq}

Term abstraction	$\lambda x : A. e$
Type abstraction	$\lambda X \leq A. e$
Operator abstraction	$\lambda X \leq A. \mathbf{B}$

Previous Attempts

- Try to carefully **untangle** typing and subtyping
- More restricted and **lose expressiveness** w.r.t. System F_{\leq} :

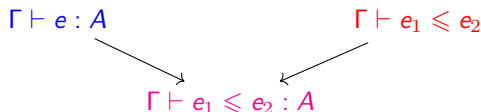
Calculus	Dep. Types	Contravariance	Bounded Quan.	T-type
F_{\leq}	×	✓	✓	✓
PTS^{\leq} (1999)	✓	✓	✓	×
PSS (2010)	✓	×	✓	✓
λP_{\leq} (1996)	✓	✓	×	×
$\lambda \Pi_{\leq}$ (2001)	✓	✓	×	×

Instead of trying to untangle, we embrace tangling!
Key idea: unify **typing** and **subtyping** into **one relation**

- Unified subtyping and λI_{\leq} calculus:
 - ▶ A single relation for both subtyping and typing
 - ▶ Dependent types with unified syntax
 - ▶ Fully subsume System F_{\leq}
- Mechanized metatheory in Coq
- Object encodings in λI_{\leq} calculus

Unified Subtyping

- Unified subtyping combines typing and subtyping:



Explanation: e_1 is a subtype of e_2 and both of them have type A .

- Typing and type well-formedness are now **syntactic sugar**:

$$\Gamma \vdash e : A \triangleq \Gamma \vdash e \leq e : A$$

$$\Gamma \vdash A : \star \triangleq \Gamma \vdash A \leq A : \star$$

Unified Subtyping (Cont.)

Simplify the combination of typing and subtyping:

- No mutual dependency, trivially:

Subsumption rule

$$\frac{\Gamma \vdash e \leq e : A \quad \Gamma \vdash A \leq B : \star}{\Gamma \vdash e \leq e : B}$$

Subtyping of top type

$$\frac{\Gamma \vdash A \leq A : \star}{\Gamma \vdash A \leq \top : \star}$$

- Only one single form of abstraction $\lambda x \leq e_1 : A. e_2$:

$$\lambda x : A. e \triangleq \lambda x \leq \top : A. e \quad (*)$$

$$\lambda x \leq A. B \triangleq \lambda x \leq A : \star. B$$

**Note: we generalize \top to be any type A .*

Dependent Types with Casts

- Traditional approach: mix up **conversion rule** and **subsumption rule**

Subsumption

$$\frac{\Gamma \vdash e : A \quad \Gamma \vdash A \leq B}{\Gamma \vdash e : B}$$

Conversion

$$\frac{\Gamma \vdash e : A \quad A =_{\beta} B}{\Gamma \vdash e : B}$$

$\xRightarrow{\text{subsumes}}$

- ▶ *Consequence*: **strong normalization** is entangled with other proofs (transitivity, decidable type checking, etc.)
- Our approach: replace conversion rule with **explicit type casts**
 - ▶ Well-studied approach to *decouple* strong normalization (Sjöberg et al.; Stump et al.; Yang et al.; Casinghino et al.)
 - ▶ Only up to alpha equality, inconvenience for *heavy* type conversion
 - ▶ Still sufficient for object encodings

Summary

To summarize, we simplify the design by two means:

- **Unified subtyping**: avoid circularity
- **Explicit type casts**: decouple strong normalization

Is it oversimplified?

No, still expressive enough to encode objects that require
high-order subtyping.

Object Encodings

- **Existential object encodings** by Pierce and Turner (1994):

$$\text{Obj} = \lambda I : \star \rightarrow \star. \quad \exists X. \quad X \quad \times \quad (X \rightarrow I X)$$

interface *state type* *state* *methods*

- Dependency: Church encoding of (weak) dependent sum types:

$$\Sigma x : A. B \triangleq \Pi z : \star. (\Pi x : A. B \rightarrow z) \rightarrow z$$

Useful to encode OO concepts, e.g. type member:

Abstract Set Interface (Scala)

```
trait Set {  
  type T  
  def empty(): T  
  def member(x: Int, s: T): Boolean  
  def insert(x: Int, s: T): T  
}
```

Encoding in λ_{\leq}

```
Set =  $\Sigma T : \star. \{$   
  empty : T,  
  member : Int  $\rightarrow$  T  $\rightarrow$  Bool,  
  insert : Int  $\rightarrow$  T  $\rightarrow$  T  
}
```

Proof Strategy

High-level view of methodology:

- Two main targets: **transitivity** and **type safety**
- Figure out the correct **form** and **proof order** of lemmas ([Tricky!](#))
- Use standard induction techniques to finish proofs

Circularity of Lemmas

- One normally tries to first prove the fundamental **reflexivity** lemma (also called regularity):

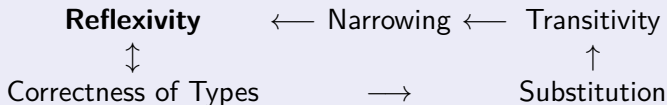
Lemma (Reflexivity)

If $\Gamma \vdash e_1 \leq e_2 : A$, then both $\Gamma \vdash e_1 : A$ and $\Gamma \vdash e_2 : A$ hold.

- However, it is mutually dependent on **correctness of types**:

Lemma (Correctness of Types)

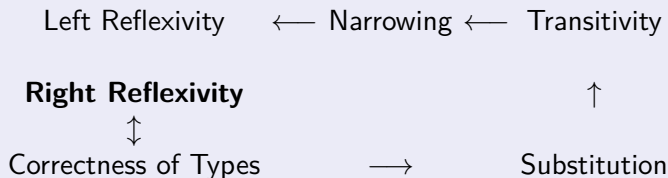
If $\Gamma \vdash e_1 \leq e_2 : A$, then $\Gamma \vdash A : \star$.



Break the Circularity

Observation: correctness of types only depends on **right** reflexivity. We divide reflexivity into two sub-lemmas:

- *Left reflexivity:* $\Gamma \vdash e_1 \leq e_2 : A \Rightarrow \Gamma \vdash e_1 : A$
- *Right reflexivity:* $\Gamma \vdash e_1 \leq e_2 : A \Rightarrow \Gamma \vdash e_2 : A$



Related Work

Subtyping w/ traditional dependent types:

- Summary of calculi:

Name	D.T.	β -eq.	Unified Syn.	Contravar.	B. Quan.	T-type
F_{\leq}	×	×	×	✓	✓	✓
λI_{\leq}	✓	×	✓	✓	✓	✓
PTS^{\leq} (1999)	✓	✓	✓	✓	✓*	×
PSS (2010)	✓	✓	✓	×	✓	✓
λP_{\leq} (1996)	✓	✓	×	✓	×	×
$\lambda \Pi_{\leq}$ (2001)	✓	✓	×	✓	×	×

**No subtyping on bounded quantified abstractions.*

- Pure Subtype Systems (Hutchins, 2010):
 - ▶ Purely based on subtyping; no concept of typing
 - ▶ Function and function type are merged; no contravariance
 - ▶ Metatheory proof is complex and incomplete

Related Work (Cont.)

Subtyping w/ *restricted* dependent types: path-dependent types (Amin et al., 2014)

- DOT (Amin et al., 2012): theoretic foundation for Scala
- Both lower and upper bounded quantification; subsume System F_{\leq}
- Complex soundness proof (Rompf and Amin, 2016); no transitivity elimination
- Compared to λI_{\leq} : non-unified **stratified** syntax; non-traditional dependent types based on **path selection**

More in the Paper...

- **Object encodings:** generic message passing; cell object encoding and evaluation
- **Formal presentation of λI_{\leq} :** syntax, static and dynamic semantics
- **Proof details:** generalization of transitivity and type preservation; Coq scripts available online
- **Subsumption of F_{\leq} :** translation to F_{\leq} and manual proofs
- **Discussion of variants:** bidirectional system, recursion, call-by-value reduction, full reduction, full contravariance, etc.

Conclusion and Future Work

- **Unified subtyping**: simplify combining dependent types and subtyping
- **The λ/\leq calculus**: application of unified subtyping with well-developed metatheory
- **Future work**: decidability; different variants; extended with lower bounds like DOT

Thanks for listening!

Any questions?

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