Problem Set 3

Astro C161
Due Friday, February 12, 4:00 pm

0. **TALC** *(5 pts)*: Come to your assigned section of TALC. There will be a quiz/worksheet to record your attendance.

1. **Proper Distance** *(24pts)*:

   (a) In a flat universe with $H_0 = 70$ km s^{-1}Mpc^{-1}, you observe a galaxy at a redshift $z = 7$. What is the current proper distance to the galaxy, $d_p(t_0)$, if the universe contains only radiation? Repeat for universes with only matter and only a cosmological constant. (Note: Assume you measure the distance to the galaxy using light that was emitted at some time t_1 and arrived at time t_0.)

   (b) What was the proper distance at the time the light from part (a) was emitted, $d_p(t_1)$, if the universe contains only radiation? Repeat for universes with only matter and only a cosmological constant. (Note: Assume you measure the distance to the galaxy using light that was emitted at some time t_2 and arrived at time t_1.)

2. **Elbbuh Niwde** *(23pts)*: Consider a positively curved universe containing only matter (the "Big Crunch" model). At some time $t_0 > t_{\rm Crunch}/2$, during the contraction phase of this universe, an astronomer named Elbbuh Niwde discovers that nearby galaxies have blueshifts ($-1 \leq z < 0$) proportional to their distance. She then measures H_0 and Ω_0, finding $H_0 < 0$ and $\Omega_0 > 1$. Given H_0 and Ω_0, how long a time will elapse between Dr. Niwde's observations at $t = t_0$ and the final Big Crunch at $t = t_{\rm Crunch}$? What is the minimum blueshift that Dr. Niwde is able to observe? What is the lookback time to an object with this blueshift?

3. **Equation of State** *(21pts)*: For the following, assume a spatially flat universe.

   (a) The state parameter w Express the state parameter $w = P/\epsilon$ in terms of the Hubble parameter and its time derivative.

   (b) Evaluate the derivative of w with respect to time. Express your answer in terms of the speed of sound in the medium, which is given by $c_s^2 = c^2 \frac{dP}{d\epsilon}$. The speed of sound is determined by the medium's compressibility and density.

   (c) Show that for a spatially flat universe, the Friedman equation and the fluid equation are invariant under the transformation:

$$\begin{cases} a \to \alpha = \frac{1}{a} \\ (w+1) \to (u+1) = -(w+1) \end{cases}$$

4. **Angular Diameter Distance** *(27pts)*:

(a) The *angular diameter distance* d_A is defined to be $d_A = D/\theta$, where D is the physical size of an object and θ is its angular diameter (i.e. the angle subtended in the sky). Explain why d_A is related to the comoving radial distance r by $d_A = r/(1+z)$.

(b) For an object of physical size D at redshift z, write down a general expression for its angular diameter as a function of D, z, $\Omega_{0,m}$, and H_0 (assume $\Omega_\Lambda = 0$ for simplicity). Rewrite the formula so that the angular diameter is in units of h arcsec (where $H_0 = 100\,h$ km/s/Mpc) and D in units of kpc.

(c) The physical size of the luminous part of a Milky-Way-like galaxy is about 20 kpc. On log-log scales, plot the angular diameter of such a galaxy versus redshift ($0.01 \leq z \leq 10$) for three values of $\Omega_{0,m}$: 0.27, 1, and 2.7. Comment on any interesting features in your curves. (Again assume $\Omega_\Lambda = 0$.)