

## Problem Set 3

Astro C161

Due Friday, February 12, 4:00 pm

0. **TALC** (5 pts): Come to your assigned section of TALC. There will be a quiz/worksheet to record your attendance.

1. **Proper Distance** (24pts):

- (a) In a flat universe with  $H_0 = 70 \text{ km s}^{-1}\text{Mpc}^{-1}$ , you observe a galaxy at a redshift  $z = 7$ . What is the current proper distance to the galaxy,  $d_p(t_0)$ , if the universe contains only radiation? Repeat for universes with only matter and only a cosmological constant. (Note: Assume you measure the distance to the galaxy using light that was emitted at some time  $t_1$  and arrived at time  $t_0$ .)
- (b) What was the proper distance at the time the light from part (a) was emitted,  $d_p(t_1)$ , if the universe contains only radiation? Repeat for universes with only matter and only a cosmological constant. (Note: Assume you measure the distance to the galaxy using light that was emitted at some time  $t_2$  and arrived at time  $t_1$ .)

2. **Elbbuh Niwde** (23pts): Consider a positively curved universe containing only matter (the “Big Crunch” model). At some time  $t_0 > t_{\text{Crunch}}/2$ , during the contraction phase of this universe, an astronomer named Elbbuh Niwde discovers that nearby galaxies have blueshifts ( $-1 \leq z < 0$ ) proportional to their distance. She then measures  $H_0$  and  $\Omega_0$ , finding  $H_0 < 0$  and  $\Omega_0 > 1$ . Given  $H_0$  and  $\Omega_0$ , how long a time will elapse between Dr. Niwde’s observations at  $t = t_0$  and the final Big Crunch at  $t = t_{\text{Crunch}}$ ? What is the minimum blueshift that Dr. Niwde is able to observe? What is the lookback time to an object with this blueshift?

3. **Equation of State** (21pts):

- (a) The state parameter  $w$  Express the state parameter  $w = P/\epsilon$  in terms of the Hubble parameter and its time derivative.
- (b) Evaluate the derivative of  $w$  with respect to time. Express your answer in terms of the speed of sound in the medium, which is given by  $c_s^2 = c^2 \frac{dP}{d\epsilon}$ . The speed of sound is determined by the medium’s compressibility and density.
- (c) Show that for a spatially flat universe, the Friedman equation and the fluid equation are invariant under the transformation:

$$\begin{cases} a \rightarrow \alpha = \frac{1}{a} \\ (w + 1) \rightarrow (u + 1) = -(w + 1) \end{cases}$$

4. **Angular Diameter Distance** (27pts):

- (a) The *angular diameter distance*  $d_A$  is defined to be  $d_A = D/\theta$ , where  $D$  is the physical size of an object and  $\theta$  is its angular diameter (i.e. the angle subtended in the sky). Explain why  $d_A$  is related to the comoving radial distance  $r$  by  $d_A = r/(1+z)$ .
- (b) For an object of physical size  $D$  at redshift  $z$ , write down a general expression for its angular diameter as a function of  $D$ ,  $z$ ,  $\Omega_{0,m}$ , and  $H_0$  (assume  $\Omega_\Lambda = 0$  for simplicity). Rewrite the formula so that the angular diameter is in units of  $h$  arcsec (where  $H_0 = 100 h$  km/s/Mpc) and  $D$  in units of kpc.
- (c) The physical size of the luminous part of a Milky-Way-like galaxy is about 20 kpc. On log-log scales, plot the angular diameter of such a galaxy versus redshift ( $0.01 \leq z \leq 10$ ) for three values of  $\Omega_{0,m}$ : 0.27, 1, and 2.7. Comment on any interesting features in your curves. (Again assume  $\Omega_\Lambda = 0$ .)