Problem Set Problems

Astro C161

Due Friday, April 29, 4:00 pm

0. **TALC** *(5 pts)*: Come to your assigned section of TALC. There will be a sign in sheet to record your attendance.

1. **Initial Condition Problem** *(36 pts)*: In this problem we will explore one of the theoretical problems with current formulations of inflationary theory. To do this we will examine the full Friedmann Equation of our universe:

$$H^2(t) = \left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \left[\frac{\Omega_m^0}{a^3} + \frac{\Omega_r^0}{a^4} + \Omega_\Lambda^0 + \frac{\Omega_k^0}{a^2} + \frac{\Omega_\phi^0}{a^{2\epsilon}}\right]$$

where k denotes curvature, and ϕ denotes an inflationary field. It is convenient (as we will see later) to define a parameter ϵ such that the inflationary energy density scales as $\varepsilon_\phi(a) \propto a^{-2\epsilon}$.

   (a) Consider a universe dominated by an inflationary field. What constraint do we need to put on ϵ to ensure accelerating expansion?

   (b) What is the relation between ϵ and the equation of state $w = p/\varepsilon$? What constraint does accelerating expansion put on w? How does this value of w compare to w for matter, radiation, and Λ?

   (c) Looking at the full Friedmann equation and using your result from part (a), which term would you expect to dominate in the early universe? What (if anything) would you need to do to make inflation dominate in the early universe? Explain whether this is problematic.

2. **The Inflationary Field** *(27 pts)*: Say we have an inflationary field ϕ with a potential $V(\phi)$. This field can be characterized by an energy density

$$\varepsilon_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

and a pressure

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

We can ignore the spatial derivatives because they become negligible soon after inflation begins due to the "no-hair" theorem. Also note that $V(\phi)$ should never be negative as that would imply inflationary contraction.

   (a) What is the equation of state w in terms of ϕ? What constraint do we need to put on $\dot{\phi}^2$ and $V(\phi)$ in order to have accelerating expansion? This is one of the two so-called *slow roll conditions.*

(b) How would you write the inflationary Friedmann equation in terms of $\dot{\phi}$ and $V(\phi)$?

(c) What would the inflationary Friedmann equation look like in the slow-roll approximation?

3. **Constraining Inflation** *(32 pts)*: In order to determine whether a certain inflationary field might describe our universe, we need to compare the theory's predictions to observations. One of the easiest observations to predict is the amplitude of the density perturbations $\delta \equiv (\varepsilon - \bar{\varepsilon})/\bar{\varepsilon}$. This problem will walk through calculating δ for the potential $V(\phi) = \lambda\phi^4$ and then using Planck observations to constrain λ and decide whether this potential is a viable theory.

(a) First we need to find the value of ϕ at the time t_* when the CMB anisotropies were sourced. To do this, we use the *e-fold number* N, which is defined so that $\frac{a}{a_f} = e^N$, where a_f is the scale factor at the end of inflation. Given that $|1 - \Omega_0| \sim 10^{-53}$ at the beginning of inflation (at t_*), and that after inflation $|1 - \Omega_0| \approx 0.005$, what is N_*, the e-folding number when the CMB anisotropies formed? *Hint: Do not use the number from lecture. Recall that curvature goes as* $1/a^2$.

(b) In the slow roll approximation, we can relate the potential to the e-fold number by

$$\frac{V''(\phi_N)}{V(\phi_N)} \approx \frac{1}{N}$$

Use this relation and to find ϕ_*, the value of ϕ at the time when the CMB anisotropies were sourced.

(c) It can be shown that in the slow roll approximation

$$\delta \approx \left| \frac{V^{3/2}(\phi)}{V'(\phi)} \right|$$

Using the Planck observation that $\delta_* \approx 10^{-4}$, find the value of λ that is consistent with observations.

(d) In the slow roll approximation, we can write the tensor-to-scalar ratio r as

$$r \approx 8 \left(\frac{V'(\phi)}{V(\phi)} \right)^2$$

What does the $\lambda\phi^4$ theory predict for r at the time of recombination? Planck has constrained $r_* < 0.01$. Is the $\lambda\phi^4$ consistent with observations?