Problem Set 9 (part 1)

1   Great Balls of (Relativistic) Fire

Based on Rybicki & Lightman 4.1.

In astronomy, it is frequently argued that a source of radiation that undergoes a fluctuation of duration Δt must have a physical diameter of order $D \lesssim c\Delta t$. This argument is based on the fact that even if all portions of the source undergo a disturbance at the same instant and for an infinitesimal period of time, the resulting signal at the observer will be smeared out over a tie interval of $\Delta t_{min} \sim D/c$ because of the finite light travel time across the source. Suppose, however, that the source is an optically thick spherical shell of radius $R(t)$ that is expanding with relativistic velocity $\beta \sim 1, \gamma \gg 1$ and energized by a stationary point at its center. By consideration of relativistic beaming effects, show that if the observer sees a fluctaion from the shell of duration Δt at time t, the source may actually be of radius

$$R < 2\gamma^2 c\Delta t \tag{1}$$

rather than the much smaller limit given by the nonrelativistic considerations. In the rest frame of the shell surface, each surface element may be treated as an isotropic emitter.

This latter argument has been used to show that the active regions in quasars may be much larger than $c\Delta t \sim 1$ light month across, and thus avoids much energy being crammed into so small a volume.

2   The Blob

Based on Rybicki & Lightman 4.7.

An object emits a blob of material at speed v at an angle θ to the line-of-sight of a distant observer.

2.1 Show that the apparent tansverse velocity inferred by the observer (i.e. the angular velocity on the sky times the distance to the object) is

$$v_{app} = \frac{v \sin\theta}{1 - (v/c)\cos\theta} \tag{2}$$

2.2 Show that v_{app} can exceed c; find the angle for which v_{app} is maximum, and show that this maximum is $v_{max} = \gamma v$.

2.3 Plot v_{app}/c versus θ for $\gamma = 10^2$. Does the viewing angle θ need to be especially small for superluminal motion to be perceived?

3   Problem Set 9 (part 2)

4   Synchrotron Losses

4.1 Obtain an analytic expression for the energy of a single relativistic electron as a function of time, $E(t)$, taking into account its energy loss by synchrotron radiation. Your expression should

contain only the variables $E(0)$ (the initial energy of the electron), B (the magnetic field, here held fixeId with time, following the rest of the world, though one should worry in general about the field changing with time just as the electron energy spectrum changes with time), time t, and fundamental constants. Assume $\sin \alpha = 1$ (the electron pitch angle is 90 degrees) for simplicity.

For (synchrotron) problems of interest to us, the electron always remains relativistic. It merely evolves from a large $\gamma \gg 1$ to a smaller $\gamma \gg 1$.

4.2 How can you reconcile the loss of energy of the electron with the bald statement of Rybicki & Lightman on page 168 that

$$\frac{d}{dt}(\gamma m c^2) = q\vec{v} \cdot \vec{E} = 0 \tag{3}$$

implies "γ is constant"?

4.3 We have made arguments in class that power-law distributions of electrons in astrophysical sources are maintained against synchrotron losses by continuous energization by central engines (a.k.a. injection). The injection (input) spectrum of electrons is modified by synchrotron losses to produce a steady-state (output) distribution.

Call $\eta(E, t) = dN/dE$ the differential energy spectrum of electrons as discussed repeatedly in class. Continuity of electrons in energy space reads

$$\frac{\partial \eta(E, t)}{\partial t} + \frac{\partial}{\partial E}[\dot{E}\eta(E, t)] = I \tag{4}$$

where I is the rate of injection of electrons with some input distribution and $\dot{E}$ is the rate of energy loss of a single electron by synchrotron radiation. This equation should not mystify you; it merely describes how the number of electrons in a given energy bin changes with time, taking into account a flux divergence (the second term on the left-hand-side) and a source term (the right-hand-side).

We have assumed in class a steady-state distribution of electron energies for which $\eta \propto E^p$. Given p, how must I scale with E? Give on the scaling and forget about the numerical coefficients.

As with most scaling problems, you dont have to solve anything in detail. Ruthlessly work to order of magnitude.

4.4 Electrons having a given energy must wait a characteristic time before synchrotron losses become important. Before this time elapses for all such electrons, how does η scale with E?

4.5 The spectral index ($\alpha = d\ln F_\nu / d\ln \nu$) of radiation in a fixed frequency range from a radio jet flattens with increasing distance from the central galaxy. That is, $\alpha = 0.5$ at the remote edge of the jet (the "hot spot"), and $\alpha = -1$ closer in. Given your understanding in 4.3 and 4.4, where are the "freshest" electrons located, i.e., those newly injected into the energy spectrum? Are they at the end of the jet, or are they closer in? In other words, where is the principal site of particle acceleration?