Problem Set 11 (part 1)

1 Dust Temperatures and Opacity

The bulk of the interstellar radiation field in the Galaxy is from the light of O stars, which have mass $\sim 100 M_{\odot}$. There are $\sim 5 \times 10^4$ O stars in the galaxy, distributed over a cylindrical disk with radius 50 kpc and height 200 pc.

1.1 Estimate the total energy density of starlight in the Galaxy in units of eV cm^{-3}. Use the fact that O stars radiate near the Eddington luminosity.

1.2 This interstellar radiation field heats dust grains in the interstellar medium. Estimate the temperature t of the largest grains in the ISM, which have a radius of $a \sim 0.1$ microns. Assume $Q_{abs} \sim 1$ for wavelengths shorter than $2\pi a$, and for longer wavelengths, take $m = 1 + 0.25i$. Is the grain hotter, cooler, or equal to the temperature of an ideal blackbody placed in the interstellar radiation field.

1.3 At what wavelength, λ_{peak}, does the energy density (νF_ν) of the grains peak?

1.4 The largest grains carry most of the mass in the interstellar grain distribution. Given a dust-to-gas ratio of 0.01 and an average hydrogen number density $n_H = 0.1\text{cm}^{-1}$, calculate the specific intensity of the Milky Way at λ_{peak}.

1.5 Typically, the number density of dust grains scales with size as $dn/da \propto a^{-3.5}$ in the ISM, where dn is the number density of grains with radii between a and $a + da$. The law holds over $a_{min} = 10^{-3}$ microns to $a_{max} = 0.1$ microns. Plot the grain opacity $\kappa(\lambda)$ contributed by all dust grains over the wavelength range $\lambda = 0.1$ to 10 microns. Consider only absorption and neglect scattering. Indicate over each decade of wavelength which grain sizes dominate the opacity.

Problem Set 10 (part 2)

2 Diatomic

Based on Rybicki & Lightman 11.1.

Consider an electrically neutral medium of diatomic molecules in thermal equilibrium at temperature T. Each molecule contains one nucleus of mass m_p and one of mass $2m_p$ at an equilibrium separation r_0.

2.1 Estimate r_0 in terms of fundamental constants.

2.2 Estimate the cross-section σ_c for collisions between molecules.

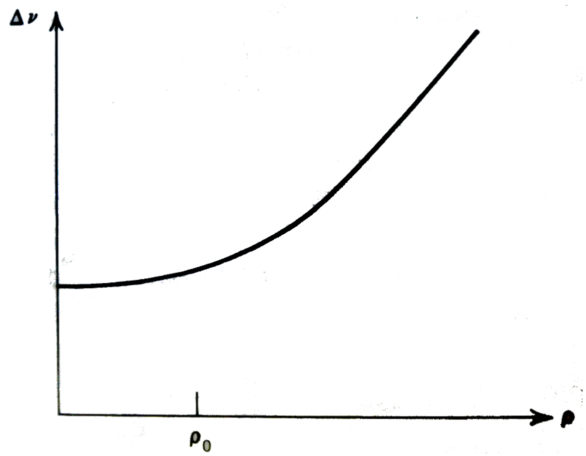

Figure 1: Line width as a function of denisty for emission from a medium of diatomic molecules.

2.3 It is experimentally observed that , as a function of mass density ρ of the medium, the line width of the rotational lines has the form shown in Figure 1. If only Doppler and collisional broadening are present, estimate ρ_0 and show that it may be written completely in terms of fundamental constants, independent of m_p.

We derived in class that for a synchrotron self-Compton (SSC) emitting source that is static in bulk, the ratio of the luminosity due to first-generation inverse Compton scattering of synchrotron photons to the luminosity due to synchrotron processes is

$$f \equiv \frac{L_{IC,1}}{L_{sync}} = C\nu_m T_{bm}^5 \tag{1}$$

where ν_m is the frequency at which the synchrotron spectrum beaks, and T_{bm} is the brightness temperature of the synchrotron radiation at that frequency.

2.4 Derive an estimate for the constant C in terms of fundamental constants. Assume whatever geometry you like for the source. I found I had to use the (usual) constants e, k, c, and m_e.

2.5 If $\nu_m \sim 1$ GHz (as it typically is for compact radio sources), what is the maximum T_{bm} above which $f > 1$? Express in Kelvin.

2.6 Given f, what is $L_{IC,2}/L_{IC,1}$? That is, what is the ratio of second-generation to first-generation scattered power? Explain your answer.

2.7 Consider the situation after N scatterings, where $N \to \infty$. What is the critical f for which the total luminosity of the source becomes unbounded? Careful estimates will be rewarded.