Problem Set 8

1 Pulsar Dispersion Measure

A radio astronomer uses the Green Bank Telescope to observe a pulsar in a band from 1 to 2 GHz, recording a series of frequency power spectra for 1 s. The resulting (noisy) frequency spectrum is recorded as pulsar.dat (available on the course GitHub repo). It looks like noise, but I promise you that, for the right dispersion measure, there is a pulse in there.

1.1 De-disperse the measured power spectra to determine the DM of the pulsar. You may have to try many DMs to find the one that works. To help you, the beginning of the file is timed to the arrival time without dispersion delay. For bonus points, you could make your code not depend on this fact.

Just a warning: be careful about the difference between ω and ν. The reported frequencies in the data file are ν, of course.

1.2 For an assumed density of electrons in the interstellar medium of 0.03 cm^{-3}, calculate the distance to this pulsar. Does this seem reasonable (what scale would you use to judge "reasonable")?

1.3 For such an assumed electron density, is the radio astronomer safely observing above the plasma cut-off frequency?

1.4 Calculate the optical depth to Thomson scattering along the line-of-sight to this pulsar.

2 Faraday Rotation

Consider the propagation of light thorugh a magnetized plasma. The magnetic field is uniform: $\vec{B}_0 = B_0\hat{z}$. Light travels parallel to $\hat{z}$.

An electron in the plasma feels a force from the electromagnetic wave and a force from the externally imposed B-field. Its equation of motion reads

$$m\dot{\vec{v}} = -e\vec{E} - \frac{e}{c}\vec{v} \times \vec{B}_0 \tag{1}$$

where the electric field $\vec{E}$ can be decomp[osed into right-circularly polarized (RCP) and left-circularly-polarized (LCP) waves:

$$\vec{E} = E_0(\hat{x} \mp i\hat{y})e^{i(k_\mp z - \omega t)} \tag{2}$$

where it is understood that the real part of $\vec{E}$ should be taken. The upper sign ($-$) corresponds to RCP waves, while the lower sign ($+$) corresponds to LCP waves.

In the equation of motion above, we have neglected the Lorentz force from the wave's B-field, since it is small (by v/c) compared to the force from the wave's E-field.

2.1 Prove that the solution of the equation of motion reads

$$\vec{v} = \frac{-ie}{m(\omega \pm \omega_{cyc})}\vec{E} \tag{3}$$

where $\omega_{cyc} \equiv eB_0/mc$. This is the hardest part of the derivation for the dispersion relation of RCP and LCP waves.

2.2 Based on Rybicki & Lightman Problem 8.3.

The signal from a pulsed, polarized source is measured to have an arrival time delay that varies with frequency as $dt_p/d\omega = 1.1 \cdot 10^{-5}$ s^2, and a Faraday rotation that varies with frequency as $d\Delta\theta/d\omega = 1.9 \cdot 10^{-4}$ s. The measurements are made around the frequency $\omega = 10^8$ s^{-1}, and the source is at unknown distance from the earth. Find the mean magnetic field, $\langle B_\parallel \rangle$, in the interstellar space between the earth and the source:

$$\langle B_\parallel \rangle \equiv \frac{\int n B_\parallel \, ds}{\int n \, ds} \tag{4}$$