Problem Set 6

1 Bremsstrahlung Spectrum

In deriving the spectrum of bremsstrahlung emission, we made an assumption that, in an individual collision of an electron moving at velocity v with an ion of charge Ze we could relate the impact parameter b to a unique frequency of emission, ν. The relationship we derived was $\nu \sim v/4b$. In this problem, we are going to numerically examine the validity of that assumption.

Starting from what we wrote for the quiz, write a python program that simulates a collision event. For this simulation, assume an ion of charge Ze is located at the origin and does not move. Inject an electron beginning at something like $\vec{x} = (-500a_0, 100a_0)$ (Bohr radii) moving with velocity $3 \cdot 10^7$ cm/s in the $\hat{x}$ direction. Simulate a time interval of 0.2 ps, with many thousands of time steps. At each time step, use the position of the electron to compute the force vector acting on the electron. Assuming that force vector acts for time Δt corresponding to the time resolution of your simulation, compute what the velocity will be in the next time step of your simulation. Similarly, use the current velocity to predict the position of the electron in the next step of your simulation.

1.1 Now plot, as a function of time, the acceleration along the $\hat{x}$ and $\hat{y}$ directions. In the style of the lecture on thermal bremsstrahlung, estimate the time interval corresponding to an oscillation period in the EM wave that results from accelerating the electron. Deduce a frequency of emitted radiation.

1.2 Use the Fourier transform of your acceleration profile versus time to deduce the power spectrum of the radiation. How should you combine emission from accelerations in the $\hat{x}$ and $\hat{y}$ directions? At what frequency does the power spectrum peak? To what degree is emission concentrated at a single frequency?

1.3 How well does your result motivate the approximation we made in lecture? For whatever deficiencies you see in our approximation, discuss how that might affect our estimate of the emissivity j_ν of free-free emission.

2 Blowing Strömgren Bubbles

Consider a lone O star emitting η Lyman limit photons per second. It sits inside hydrogen gas of infinite extent and of number density n. The star ionizes an HII region–a.k.a. a "Strömgren sphere," after Bengt Strömgren, who understood that such spheres have sharp boundaries inside of which hydrogen is nearly completely ionized.

Every Lyman limit photon goes towards ionizing a neutral hydrogen atom. That is, every photon emitted by the star goes towards maintaining the Strömgren bubble. Put yet another way, no Lyman limit photon emitted by the star travels past the radius of the Strömgren sphere.

The rate at which Lyman limit photons are emitted by the central star equals the rate of radiative recombinations in the ionized gas. The sphere is nearly completely ionized[1]. These facts of photo-ionization equilibrium determine the approximate radius of the Strömgren sphere.

[1] The sphere cannot be 100% ionized because then there would be no neutrals to absorb the Lyman limit photons that are continuously streaming out of the star.

The temperature inside the sphere is about 10,000 K. (A class on the interstellar medium can show you why.)

2.1 What is the timescale, t_{rec}, over which a free proton radiatively recombines in the sphere? That is, how long would a free proton have to wait before undergoing a radiative recombination? Give both a symbolic expression, and a numerical evaluation for $n = 1$ cm^{-3}.

2.2 If the star were initially "off," and the gas surrounding it initially neutral, what is the timescale for the Strömgren sphere to develop after the star were turned "on"? That is, how long does the star take to blow an ionized bubble? Think simply and to order-of-magnitude; you should get the same answer as 2.1.

2.3 How thick might the boundary of a Strömgren sphere be?

3 Time to Relax in the Strömgren Sphere

It is often assumed that velocity distributions of particles are Maxwellian. The validity of this assumption rests on the ability of particles to collide elastically with one another and share their kinetic energy. For a Maxwellian to be appropriate, the timescale for a collision must be short compared to other timescales of interest. This problem tests these assumptions for the case of the Strömgren sphere of nearly completely ionized hydrogen of problem 2.

3.1 Establishing the electron (kinetic) temperature: what is the timescale, t_e, for free electrons in the Strömgren sphere to collide with one another? Consider collisions occurring at relative velocities typical of those in an electron gas at temperature T_e. Work only to order-of-magnitude and express your answer in terms of n, T_e, and other fundamental constants.

3.2 Establishing the proton (kinetic) temperature: repeat 3.1, but for protons, and consider collisions at relative velocities typical of those in a proton gas of temperature T_p. Call the proton relaxation time t_p.

3.3 Establishing a common (kinetic) temperature: suppose that initially, $T_e > T_p$. What is the timescale over which electrons and protons equilibrate to a common kinetic temperature? This is not merely the timescale for a proton to collide with an electron. You must consider also the amount of energy exchanged between an electron and proton during each encounter. Estimate, to order-of-magnitude, the time it takes a cold proton to acquire the same kinetic energy as a hot electron. Call this time t_{ep}. Again, express your answer symbolically.

Hint: you might find it helpful to switch the charge on the electron and consider head-on collisions between the positive electron and positive proton.

3.4 Numerically evaluate t_e/t_{rec}, t_p/t_{rec}, and t_{ep}/t_{rec}, for $T_e \sim T_p \sim 10^4$ K (but $T_e \neq T_p$ so that $t_{ep} \neq 0$). Is assuming a Maxwellian distribution of velocities at a common temperature for both electrons and ions a good approximation in Strömgren spheres?