Problem Set 8

1   Saha and the Redshift of Recombination

Based on Rybicki & Lightman 9.4.

The thermal de Broglie wavelength of electrons at temperature T is defined by $\lambda = h/(2\pi mkT)^{1/2}$. The degree of degeneracy of the electrions can be measured by the number of electrons in a cube λ on a side:

$$\xi \equiv n_e \lambda^3 \approx 4.1 \cdot 10^{-16} \; n_e T^{-\frac{3}{2}} \tag{1}$$

For many cases of physical interest, the electrons are very non-degenerate, with the quantity $\gamma \equiv \ln \xi^{-1}$ being of order 10 to 30. We want to investigate the consequences of the Boltzmann and Saha equations of γ being large and only weakly dependent on temperature. For the present purposes, assume that the partition functions are independent of temperature and of order unity.

1.1 Show that the value of temperature at which the stage of ionization passes from j to $j+1$ (i.e. when the $j+1$th electron is knocked off) is given approximately by

$$kT \sim \frac{\chi}{\gamma} \tag{2}$$

where χ is the ionization potential between stages j and $j+1$. Therefore, this temperature is much smaller than the ionization potential expressed in temperature units.

1.2 The rapidity with which the ionization stage changes is measured by the temperature range ΔT over which the ratio of populations N_j/N_{j+1} changes substantially. Show that

$$\frac{\Delta T}{T} \sim \left[\frac{d\log(N_{j+1}/N_j)}{d\log T} \right]^{-1} \sim \gamma^{-1} \tag{3}$$

Therefore, ΔT is much smaller than T itself, and the change occurs rapidly.

1.3 Using the Boltzmann equation and the result from §1.1, show that when γ is large, an atom or ion stays mostly in its ground state before being ionized.

2   Pulsar Dispersion Measure

A radio astronomer uses the Green Bank Telescope to observe a pulsar in a band from 1 to 2 GHz, recording a series of frequency power spectra for 1 s. The resulting (noisy) frequency spectrum is recorded as pulsar.dat (available on the course GitHub repo). It looks like noise, but I promise you that, for the right dispersion measure, there is a pulse in there.

2.1 De-disperse the measured power spectra to determine the DM of the pulsar. You may have to try many DMs to find the one that works. To help you, the beginning of the file is timed to the arrival time without dispersion delay. For bonus points, you could make your code not depend on this fact.

Just a warning: be careful about the difference between ω and ν. The reported frequencies in the data file are ν, of course.

2.2 For an assumed density of electrons in the interstellar medium of 0.03 cm^{-3}, calculate the distance to this pulsar. Does this seem reasonable (what scale would you use to judge "reasonable")?

2.3 For such an assumed electron density, is the radio astronomer safely observing above the plasma cut-off frequency?

2.4 Calculate the optical depth to Thomson scattering along the line-of-sight to this pulsar.