Problem Set 7 (part 1)

1 Pulsar Dispersion Measure

A radio astronomer uses the Green Bank Telescope to observe a pulsar in a band from 1 to 2 GHz, recording a frequency power spectrum about every 10 ms. The resulting (noisy) frequency spectrum is recorded as pulsar.dat (available on the course GitHub repo). It looks like noise, but I promise you that, for the right dispersion measure, there is a pulse in there.

1.1 De-disperse the measured power spectra to determine the DM of the pulsar. You may have to try many DMs to find the one that works. To help you, the beginning of the file is timed to the arrival time without dispersion delay. For bonus points, you could make your code not depend on this fact.

Just a warning: be careful about the difference between ω and ν. The reported frequencies in the data file are ν, of course.

1.2 For an assumed density of electrons in the interstellar medium of 0.03 cm^{-3}, calculate the distance to this pulsar. Does this seem reasonable (what scale would you use to judge "reasonable")?

1.3 For such an assumed electron density, is the radio astronomer safely observing above the plasma cut-off frequency?

1.4 In an earlier problem set (the Eddington limit), we made use of the Thomson cross-section, $\sigma_T = 6.65 \cdot 10^{-25}$ cm^2, for a photon scattering off an electron. We will discuss Thomson scattering in more detail later, but for now, calculate the optical depth to Thomson scattering along the line-of-sight to this pulsar.

2 Faraday Rotation

Consider the propagation of light thorugh a magnetized plasma. The magnetic field is uniform: $\vec{B}_0 = B_0\hat{z}$. Light travels parallel to $\hat{z}$.

An electron in the plasma feels a force from the electromagnetic wave and a force from the externally imposed B-field. Its equation of motion reads

$$m\dot{\vec{v}} = -e\vec{E} - \frac{e}{c}\vec{v} \times \vec{B}_0 \tag{1}$$

where the electric field $\vec{E}$ can be decomp[osed into right-circularly polarized (RCP) and left-circularly-polarized (LCP) waves:

$$\vec{E} = E_0(\hat{x} \mp i\hat{y})e^{i(k_\mp z - \omega t)} \tag{2}$$

where it is understood that the real part of $\vec{E}$ should be taken. The upper sign ($-$) corresponds to RCP waves, while the lower sign ($+$) corresponds to LCP waves.

In the equation of motion above, we have neglected the Lorentz force from the wave's B-field, since it is small (by v/c) compared to the force from the wave's E-field.

2.1 Prove that the solution of the equation of motion reads

$$\vec{v} = \frac{-ie}{m(\omega \pm \omega_{cyc})}\vec{E} \tag{3}$$

where $\omega_{cyc} \equiv eB_0/mc$. This is the hardest part of the derivation for the dispersion relation of RCP and LCP waves.

2.2 Based on Rybicki & Lightman Problem 8.3.

The signal from a pulsed, polarized source is measured to have an arrival time delay that varies with frequency as $dt_p/d\omega = 1.1 \cdot 10^{-5}$ s^2, and a Faraday rotation that varies with frequency as $d\Delta\theta/d\omega = 1.9 \cdot 10^{-4}$ s. The measurements are made around the frequency $\omega = 10^8$ s^{-1}, and the source is at unknown distance from the earth. Find the mean magnetic field, $\langle B_\parallel \rangle$, in the interstellar space between the earth and the source:

$$\langle B_\parallel \rangle \equiv \frac{\int n B_\parallel \; ds}{\int n \; ds} \tag{4}$$

Problem Set 7 (part 2)

3 The Orion Nebula

3.1 Sketch the flux density, F_ν, for the Orion Nebula due to its free-free emission from a wavelength of $\lambda = 100\ \mu m$ to $\lambda = 100\ cm$. Express F_ν in Jys (10^{-23} is cgs units). Plot the spectrum on a log-log plot, and indicate power-law indices where appropriate.

Take the density of electrons to be $n_e = 2 \cdot 10^3$ cm^{-3}, the electron and proton temperatures to be $T = T_e = T_p = 8 \cdot 10^3\ K$, the dimension of the ionized cloud to be $R = 1pc$, and the distance to the Orion Nebula to be $d = 500pc$. Take whatever geometry (cube, sphere) for the cloud is most convenient.

Please do not forget free-free self-absorption.

3.2 OPTIONAL Overlay on your plot F_ν from pure electron-electron scatterings. Here aim only for order-of-magnitude accuracy. Keep in mind that electron-electron collisions involve time-varying quadrupole moments, not time-varying dipole moments, and recall the quadrupole vs. dipole scalings from our discussion of Einstein A's.

For this part, consider the emissivity (j_ν) purely from electron-electron scatterings. Drop the emissivity from electron-proton scatterings. However, for the absorptivity (α_ν), consider the contributions from both electron-proton and electron-electron scatterings. If you think one absorption process is more important than the other, justify why you think that is the case and proceed by including just the one.