

**Impedance Transformation by a Transmission Line**

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Assignment 5 aims at extending your understanding of how electrically long transmission lines transform the impedance of a load attached at one end into an input impedance at the other end. The assignment will also deepen your understanding of the concepts of reflection coefficient  $\Gamma$  and standing wave ratio (*SWR*).

**1. Instructions**

It is assumed that at this stage the students have gained sufficient knowledge of the MATLAB programming environment in order to be able to complete the MATLAB portion of this Assignment without having the benefit of an example `***.m` file.

You must solve the analytical problem first. It will provide you with an answer, which will aid you in validating the output of your MATLAB code. You must write this code in the second (MATLAB) portion of your Assignment.

The analytical solution must be submitted through the A2L course website in PDF file format. The MATLAB solution must be submitted through the A2L course website as a MATLAB `***.m` file.

**2. Problem Statement****Analytical Problem**

A loss-free transmission line of characteristic impedance  $Z_0$  and length  $l$  is connected to a load  $Z_L$ , which consists of the following elements connected in series:  $R_s = Z_0$ ,  $L_s = 1.0$  nH, and  $C_s = 1.8$  pF. The frequency is 3 GHz and the phase velocity  $c$  is that in vacuum. (a) Calculate the reflection coefficient  $\Gamma$  at the load. (b) Calculate the *SWR* in the line. (c) Calculate the input impedance  $Z_{in} = R_{in} + jX_{in}$  of the transmission line. The values of  $Z_0$  and  $l$  are based on the **last two digits of your student number**. Please refer to **Table I** for the  $Z_0$  and  $l$  values.

**MATLAB Problem**

Write a MATLAB program which calculates in a given frequency range  $f_{\min} \leq f \leq f_{\max}$  the following:

- the impedance  $Z_L$  of a load, which consists of the following components: a resistor  $R_s$ , a capacitor  $C_s$ , and an inductor  $L_s$ , all connected in series;
- the input impedance  $Z_{in}$  of a transmission line when the line is loaded with a known load impedance  $Z_L$ , provided the following is given: (i) the physical length  $l$  of the line, (ii) the characteristic impedance  $Z_0$  of the line (assume it is constant in frequency), (iii) the attenuation constant in dB/m  $\alpha_{\text{dB/m}}$  (assume it is constant in frequency), and (iv) the phase constant  $\beta = \omega / c$  where  $c$  is the speed of light in vacuum and  $\omega = 2\pi f$ ;
- the reflection coefficient  $\Gamma$  at the load when the line is loaded with  $Z_L$  in (a);
- the *SWR* when the line is loaded with  $Z_L$ .

The MATLAB program must plot the following responses versus frequency: (a)  $R_{in} = \text{Re} Z_{in}$ , (b)  $X_{in} = \text{Im} Z_{in}$ , and (c) *SWR*.

Use this MATLAB program to analyze the following problem:

A loss-free transmission line of characteristic impedance  $Z_0$  and length  $l$  is connected to a load  $Z_L$ , which consists of the following elements connected in series:  $R_s = Z_0$ ,  $L_s = 1.0$  nH, and  $C_s = 1.8$  pF. Calculate and plot  $R_{in}$ ,  $X_{in}$ , and the  $SWR$  versus frequency in the range from 1 GHz to 5 GHz with a step of 5 MHz. The values of  $Z_0$  and  $l$  are based on the **last two digits of your student number**. Please refer to **Table I** for the  $Z_0$  and  $l$  values.

TABLE I  
VARIATIONS OF  $Z_0$  AND  $l$  FOR ANALYTICAL, MATLAB PROBLEMS

Last 2 digits	$Z_0$ ( $\Omega$ )	$l$ (m)
00	50	0.2136
01	50	0.2628
02	50	0.1644
03	50	0.1152
04	50	0.1111
05	50	0.066
06	50	0.1603
07	50	0.2669
08	50	0.2095
09	50	0.2177
10	50	0.2587
11	50	0.1685
12	50	0.1193
13	50	0.107
14	50	0.12
15	75	0.2628
16	75	0.2136
17	50	0.0701
18	75	0.1644
19	75	0.1111
20	75	0.1152
21	75	0.1603
22	75	0.066
23	50	0.1562
24	75	0.2095
25	75	0.2669
26	75	0.2587
27	75	0.2177
28	50	0.2054
29	75	0.1685
30	50	0.2218
31	75	0.107
32	75	0.1193
33	50	0.2546
34	75	0.1562
35	75	0.0701
36	50	0.1726
37	75	0.2054

38	75	0.2546
39	75	0.2218
40	50	0.1234
41	50	0.1029
42	75	0.1726
43	50	0.0742
44	75	0.1029
45	75	0.1234
46	50	0.1521
47	75	0.1521
48	75	0.0742
49	75	0.2013
50	50	0.2013
51	75	0.2505
52	50	0.2259
53	75	0.2259
54	50	0.2505
55	75	0.1767
56	50	0.1767
57	75	0.0988
58	75	0.1275
59	50	0.1275
60	75	0.148
61	75	0.0783
62	50	0.0988
63	75	0.1972
64	50	0.0783
65	75	0.2464
66	50	0.148
67	75	0.23
68	75	0.1808
69	50	0.1972
70	75	0.0947
71	75	0.1316
72	75	0.1439
73	50	0.23
74	75	0.0824
75	50	0.2464
76	75	0.1931
77	75	0.2423
78	50	0.1808
79	75	0.2341
80	75	0.1849
81	75	0.0906
82	75	0.1357
83	75	0.1398
84	75	0.0865

85	50	0.1316
86	75	0.189
87	75	0.2382
88	50	0.0947
89	50	0.0824
90	50	0.1439
91	50	0.1931
92	50	0.2341
93	50	0.2423
94	50	0.1849
95	50	0.1357
96	50	0.0906
97	50	0.0865
98	50	0.1398
99	50	0.189