ElecEng 2FL3

ASSIGNMENT #8

Magnetic Fields of Solenoids and Toroids

Jinyi Tan, Vartika Tyagi and Natalia K. Nikolova

Assignment 8 is based on Set 16 of "Electromagnetics I: MATLAB Experiments Manual for EE2FH3" by Dr. M. H. Bakr and Chen He. This Manual is freely available for download from the A2L course page. It is recommended that, before you start working on your MATLAB code, you study carefully the **Example** in Set. 16.

Your assignment is based on the **Exercise** part in Set 16. No analytical solution is required. Only your MATLAB code solving the problem is required. However, if you would like to provide a clear analytical outline of the formulas coded in your ***.m file, you can upload a supplementary PDF file with your explanations to the A2L course page. These explanations may help the TAs in marking your work fairly, especially if the output of your MATLAB code is incorrect.

1. Instructions

The MATLAB solution must be submitted through the A2L course website as a MATLAB ***.m file. Since only one ***.m file is required, please do not submit ZIP files.

2. Problem Statement

A toroid, the axis of which is along z, carries a current of $I = 5.0$ A and has K turns. The inner radius is R_{in} whereas the outer radius is R_{out}. Write a MATLAB program that computes and plots the magnetic field vector **H** in the xy plane ($z = 0$) in the region -4.0 cm $\le x \le 4.0$ cm and -4.0 cm $\le y \le 4.0$ cm. The values of K, R_{in} and R_{out} are based on **the last two digits of your student number**. Please refer to **Table I** for the K, R_{in} and R_{out} values that apply to you.

3. Help with Deriving Toroidal Coordinates in Cartesian Form

The *Analytical Part* of the **Example** in Set 16 explains how to obtain the Cartesian coordinates of a point residing on the solenoidal winding. You need to modify this approach for the case of a point residing on a toroidal winding.

Since the current element $I\Delta\mathbf{L}$ is a source, its position coordinates are primed, for example, (x', y', z'). This is in contrast to the observation point $P(x, y, z)$, the coordinates of which are not primed. The observation point P is where the field is calculated and plotted. In your assignment, the field must be calculated and plotted in the plane $z = 0$, i.e., at points $P(i\Delta x, j\Delta y, 0)$ where i and j are the grid indices along x and y, respectively, whereas Δx and Δy are the discrete steps along x and y, respectively.

Let us denote the parametric angle of a point on the toroid with α' to avoid confusion with ϕ', which is a coordinate in the global coordinate system centered at the center of the toroid as shown in Fig. 1. The angle α' is determined just like in the **Example** for the solenoid, i.e.,

$$\alpha' = \alpha + 2\pi(k-1),\ k = 1, 2, \ldots, K \tag{1}$$

where k is the turn index and K is the number of turns. The angle α is the local angle in each turn as illustrated in Fig. 2. As the parametric angle α' increases, the position of the current element $I\Delta\mathbf{L}$ advances along the axis of the toroid, which is a circle of radius

$$R_{\text{av}} = 0.5(R_{\text{in}} + R_{\text{out}})\,. \tag{2}$$

This circle is shown with a dash line in Fig. 1. The position l of the current element $I\Delta\mathbf{L}$ along the circular axis of the toroid (see Fig. 1) is a linear function of α' (just like the position of the solenoidal current element along the solenoid's axis z in the **Example**). Therefore, l can be found as

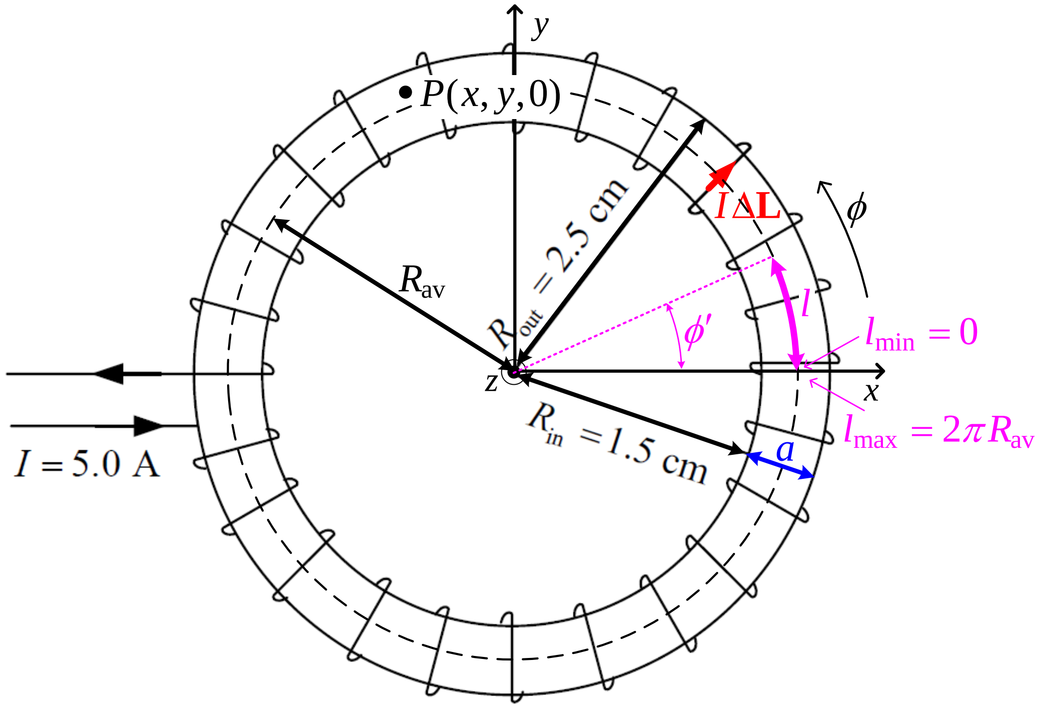

Fig. 1. Two-dimensional (2D) cut through the toroid in the z = 0 plane.

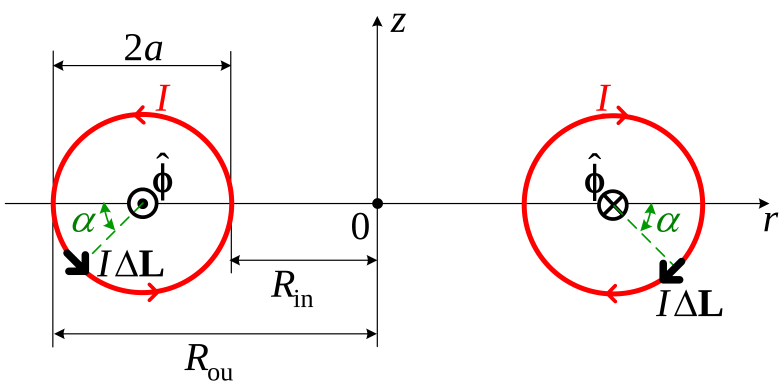

Fig. 2. 2D cut through the toroid in a ϕ = *const*. plane.

$$l(\alpha') = l_{min} + \left(\frac{l_{max} - l_{min}}{\alpha'_{max} - \alpha'_{min}}\right)(\alpha' - \alpha'_{min}). \tag{3}$$

The axis of a full toroid goes all the way from $l_{min} = 0$ to $l_{max} = 2\pi R_{av}$. Also, the angle α in each full turn goes all the way from 0 to 2π. Assuming the toroid consists of full turns only, the last turn included, the maximum value of α in (1) is $\alpha_{max} = 2\pi$ whereas $\alpha_{min} = 0$. Since the maximum index value for k is K, it follows from (1) that

$$\alpha'_{max} = 2\pi K \tag{4}$$

where K is the number of turns. At the same time,

$$\alpha'_{min} = 0. \tag{5}$$

Finally, we conclude that (3) simplifies to

$$l(\alpha') = \left(\frac{2\pi R_{av}}{2\pi K}\right)\cdot\alpha' = \frac{R_{av}}{K}\cdot\alpha'. \tag{6}$$

Next, if the total length of the toroid's wire is divided into N equal segments, we can find the parametric angle of each n-th segment as

$$\alpha'_n = \alpha'_{\min} + \frac{(\alpha'_{\max} - \alpha'_{\min})}{N} \cdot (n-1) = \frac{2\pi K}{N} \cdot (n-1),\ n = 1, 2, \ldots, N+1. \tag{7}$$

Substituting (7) into (6) allows for expressing the axial position of the n-th current element:

$$l(\alpha'_n) = \frac{R_{\text{av}}}{K} \cdot \frac{2\pi K}{N} \cdot (n-1) = \frac{2\pi R_{\text{av}}}{N} \cdot (n-1),\ n = 1, 2, \ldots, N+1. \tag{8}$$

This is an expected result because, with a uniformly wound toroid, the axial position of the current element is increasing linearly with its index.

At the same time, the linear position l of the current element $I\Delta\mathbf{L}$ along the toroid's circular axis relates to the position angle ϕ' of the current element in a cylindrical coordinate system as (see Fig. 1)

$$l(\phi') = R_{\text{av}}\phi'. \tag{9}$$

Equating (8) to (9) yields the expression for the angular position ϕ'_n of the n-th current segment of the toroidal wire as

$$\phi'_n = \frac{2\pi}{N}(n-1),\ n = 1, 2, \ldots, N+1. \tag{10}$$

The radial position r' of the current element in the global coordinate system (i.e., relative to the center of the toroid) can be determined from the parametric angle α' as well (see Fig. 2):

$$r'_n = R_{\text{av}} + a\cos\alpha_n = R_{\text{av}} + a\cos\alpha'_n,\ n = 1, 2, \ldots, N+1. \tag{11}$$

Here,

$$a = \frac{R_{\text{out}} - R_{\text{in}}}{2} \tag{12}$$

is the radius of one full turn of the coil. Similarly, the vertical position z'_n of the n-th current element can be determined as

$$z'_n = -a\sin\alpha_n = -a\sin\alpha'_n,\ n = 1, 2, \ldots, N+1. \tag{13}$$

The last step is to transform the cylindrical coordinates to Cartesian ones:

$$x'_n = r'_n\cos\phi'_n = \left(R_{\text{av}} + a\cos\alpha'_n\right) \cdot \cos\left[\frac{2\pi}{N}(n-1)\right] \tag{14}$$

$$y'_n = r'_n\sin\phi'_n = \left(R_{\text{av}} + a\cos\alpha'_n\right) \cdot \sin\left[\frac{2\pi}{N}(n-1)\right] \tag{15}$$

$$z'_n = -a\sin\alpha'_n \tag{16}$$

where α'_n is calculated from the rightmost expression in (7), i.e.,

$$\alpha'_n = \frac{2\pi K}{N} \cdot (n-1),\ n = 1, 2, \ldots, N+1. \tag{17}$$

Formulas (14)-(17) provide the discrete positions along the toroid's wire in rectangular coordinates. With these coordinates, you can compute the vector $\Delta\mathbf{L}_n$ as well as the vector $\mathbf{R}_n$ (the vector that points from the center of the n-th current element $I\Delta\mathbf{L}_n$ to the observation point $P(i\Delta x, j\Delta y, 0)$) in the same way as explained in the **Example**.

The magnetic field vector **H** is calculated using Biot-Savar's law and the superposition principle by adding vectorially the contributions of all wire segments in the toroid:

$$\mathbf{H}(P) = \sum_{n=1}^{N} \frac{I\Delta\mathbf{L}_n \times \mathbf{R}_{n\to P}}{|\mathbf{R}_{n\to P}|^3}$$

where $P(i\Delta x, j\Delta y, 0)$ is an observation point where the vector must be plotted and $\mathbf{R}_{n\to P}$ is the vector pointing from the n-th segment to P.

TABLE I

VARIATIONS OF K, R_{in} AND R_{out} FOR THE MATLAB PROBLEM

Last two digits	K	R_{in} (cm)	R_{out} (cm)
00	50	0.5	2.25
01	60	0.5	2.45
02	155	0.8	2.45
03	170	0.5	2.3
04	320	0.5	2.9
05	365	0.5	3.3
06	455	0.5	2.5
07	55	1.5	2.3
08	75	0.7	2.65
09	185	0.6	2.65
10	275	0.6	2.7
11	320	0.6	3.05
12	410	0.6	2.6
13	455	0.6	3.65
14	95	0.7	2.95
15	140	0.7	2.7
16	170	0.7	2.55
17	275	0.7	2.9
18	365	0.7	2.85
19	410	0.7	3.35
20	500	0.7	3.45
21	55	1.6	2.95
22	95	0.8	2.4
23	185	0.8	2.85
24	230	0.8	2.35
25	320	0.8	2.4
26	365	0.8	2.8
27	455	0.8	2.5
28	500	0.8	2.55
29	50	0.9	2.85
30	75	0.9	2.85
31	185	0.9	2.75
32	275	0.9	2.9
33	320	0.9	2.45
34	410	0.9	3.3
35	455	0.9	3.1
36	95	1	2.3
37	140	1	2.3
38	230	1	3.3
39	275	1	3.65
40	410	1	2.4
41	500	1	2.35
42	50	1.1	3.7

43	95	1.1	2.5
44	185	1.1	2.4
45	230	1.1	2.8
46	320	1.1	2.7
47	365	1.1	2.7
48	500	1.1	3
49	50	2	2.65
50	140	1.2	3
51	185	1.2	3
52	320	1.2	2.6
53	410	1.2	2.5
54	140	1.3	2.35
55	230	1.3	3.4
56	275	1.3	2.85
57	365	1.3	3.15
58	410	1.3	3.05
59	500	1.3	2.35
60	50	1.4	2.25
61	95	1.4	2.4
62	230	1.4	3.1
63	320	1.4	3.1
64	365	1.4	2.95
65	50	1.5	3.55
66	140	1.5	3.65
67	185	1.5	2.9
68	275	1.5	3.25
69	320	1.5	2.35
70	410	1.5	3.7
71	95	1.6	3.1
72	140	1.6	3.5
73	230	1.6	3
74	275	1.6	3.15
75	365	1.6	3.1
76	410	1.6	2.3
77	50	1.7	2.3
78	95	1.7	2.7
79	185	1.7	3.3
80	230	1.7	2.4
81	365	1.7	3.55
82	50	1.8	2.55
83	140	1.8	3.4
84	185	1.8	2.4
85	275	1.8	3.4
86	320	1.8	3.2
87	410	1.8	2.4
88	95	1.9	3.6
89	140	1.9	3.25

90	230	1.9	3.05
91	275	1.9	2.45
92	365	1.9	2.65
93	410	1.9	3.05
94	50	2	3.75
95	95	2	2.3
96	185	2	2.75
97	230	2	2.4
98	320	2	2.6
99	365	2	3.7