

Compeng 3SK3

Project 1
Aaron Pinto
pintoa9
400190637

The pseudocode and simulation program are included in the MATLAB file.

4)

a) No, you cannot because the IEEE-754 32-bit floating point number has an inherent max precision of 6-9 significant decimal digits. Therefore, at some point, $1/N$, will become too small to be representable by a 32-bit floating point number.

b) Using an IEEE 32-bit floating point number, the minimum N value such that $1/N$ becomes too small to be representable is $N = 2^{128}$, as any integer greater than or equal to 2^{128} is rounded to infinity. The minimum N value such that $1/(N - 1) - 1/N$ becomes too small to be representable is $N = 2^{24} + 4$. When I plugged this value - 1 into the equation represented in MATLAB, I got 0.0000000000000071054273576010018587112426757812, so $2^{24} + 3$ is still representable. This is the code I used:

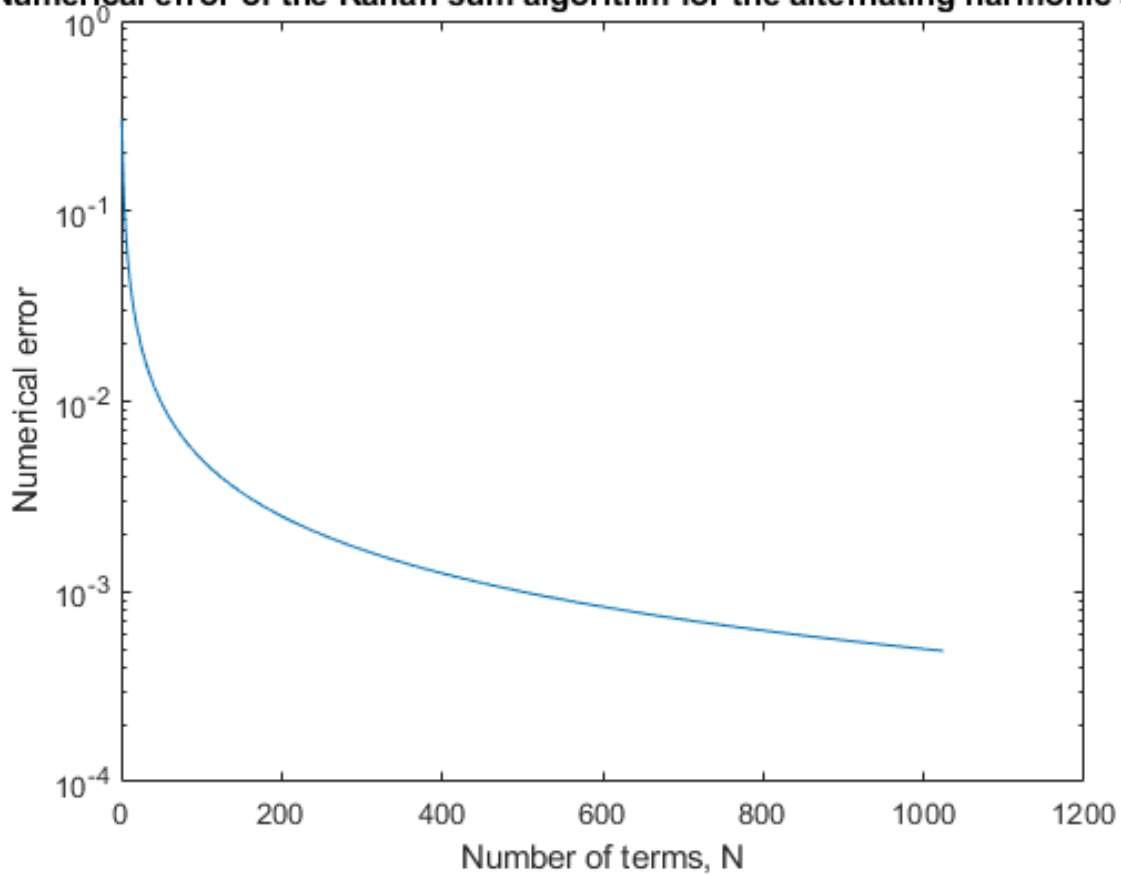
```
N = 2^24 + 3;  
test = vpa(single(single(1.0) / single(N - 1) - single(1.0) / single(N)))
```

The vpa() does not change the actual value of the answer, without vpa() I get 7.1054e-15.

The minimum N value such that $1/N$ is smaller than machine precision is $N = 2^{23} + 1$ because there are only 23 bits of mantissa in a 32-bit float. The minimum value of N such that $1/(N - 1) - 1/N$ is smaller than machine precision is $N = 2898$, which gives a result of 0.00000011912197805941104888916015625, and machine precision is 0.00000011920928955078125. I got this value by doing a linear search in MATLAB for values of N between 2^{11} and 2^{12} .

c)

Numerical error of the Kahan sum algorithm for the alternating harmonic series



d) It does not. I only achieved a precision of $1.9047\text{e-}09$ with an N value of 2^{30} . One way to improve the precision of the algorithm would be to use [arbitrary-precision arithmetic](#) or use a [second-order "iterative Kahan–Babuška algorithm"](#) to increase the accuracy some more.