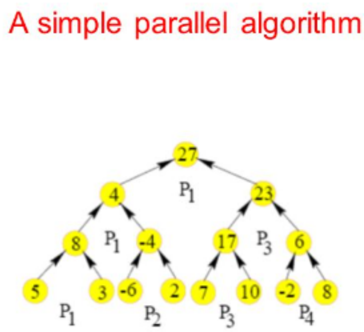
Project 1. Numerical precision (Due date: January 31, 2023)

The following diagram illustrates how 4 parallel processors, P_1, P_2, P_3, P_4, can add 8 numbers in 3 steps.



Your task is to design a parallel addition algorithm to evaluate the sum of the alternating harmonic series using the 4 parallel processors of the IEEE 32-bit floating point format.

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \cdots = ln2 \tag{1}$$

Your design goal is to obtain **the highest precision possible** while speeding up the task by the 4 parallel processors to your best ability. You do not need to implement your algorithm on an actual parallel computer, instead are required to

1. Use pseudo code to describe all necessary details of your parallel algorithm, in particular the assignment of the series terms to the 4 processors and the sequence of carrying out all intermedium additions.
2. Simulate your parallel addition algorithm on a serial computer, using Matlab or C. Submit your simulation program with detailed documentation.
3. Submit a written project report to justify your design decisions, and include both the pseudo code of your parallel algorithm and the simulation program.
4. In your report, answer the following questions and address the requests.
   (a) Can you achieve, in the IEEE 32-bit floating number system, any high precision you desire by summing up the first N terms of the series and by running the summation program for a sufficiently large N? Explain your answer.
   (b) With respect to the IEEE 32-bit floating number system, what is the minimum N value such that 1/N becomes too small to be representable? What is the minimum N value such that 1/(N-1)-1/N becomes too small to be representable? What is the minimum N value such that 1/N is smaller than machine precision? What is the minimum N value such that 1/(N-1)-1/N is smaller than machine precision.
   (c) For your algorithm to sum the first N terms of the alternating harmonic series, plot the curve of numerical error against N. Explain the behavior of the error curve. To compute the numerical error, the reference (ground truth) is the Matlab double precision value ln2 = Matlab function log(2) = 0.693147180559945....

(d) Does your algorithm achieve the highest possible precision? If not, suggest a possible way to improve the precision.

Do not use any built-in functions of matlab, i.e., use only addition, subtraction, multiplication, and division for calculations.

5. This assignment is an individual work.

Bonus: up to 50% bonus will be given to students who can reach a precision near 10^{-9} or higher and can be independently verified.