UNIVERSITY OF SYDNEY

Unit Commitment Modelling Toolbox with Graphical User Interface User Manual

by

Aaron Ramsden

and

Gregor Verbič, PhD

12th November, 2017

Abstract

This is a user manual to describe the functionality of the Unit Commitment (UC) Matlab toolbox and Graphical User Interface (GUI) that was developed by Aaron Ramsden and Gregor Verbič. This user manual is intended to be read in conjunction with "An Educational Open-Source Market Modeling Toolbox for Future Grid Studies" by Ramsden and Verbič, published in the IEEE PES ISGT Asia 2017[1] conference (available on the IEEE Xplore Digital Library[2] after December 2017).

The UC toolbox runs in Matlab, and can be used to simulate any UC problem that is presented in the required input format. For feasible problems, the solver returns the optimal scheduling and dispatch levels of each generator in the model in each time-step of the simulation period. If specified by the user, the solver can incorporate a full Direct Current (DC) load flow representation and return the active power flows across each bus connector in each time-step.

The model is presented as a Mixed Integer Linear Programming (MILP) optimisation problem and solved using the "IBM ILOG CPLEX Optimization Studio".

The toolbox includes the functionality of explicitly modelling any type of utility-scale energy storage plant as a Generic Energy Storage (GES) plant, as well as the ability to model Thermal Energy Storage (TES), which is used to model Concentrating Solar Thermal (CST) generators.

Chapter 1 explains how to use the UC toolbox and GUI. Chapter 2 discusses the six example case studies that are provided with the toolbox. Each case study is presented with a simple 5-bus network and extended to more complex 58-bus network. Chapters 3 and 4 explain some of the mathematical modelling that is incorporated into the toolbox.

[1] `http://ieee-isgt-asia.org`
[2] `http://ieeexplore.ieee.org/Xplore/home.jsp`

Contents

List of Figures

Abbreviations

AEMO Australian Energy Market Operator

CCGT Combined-Cycle Gas Turbine

CST Concentrating Solar Thermal

DC Direct Current

GES Generic Energy Storage

GUI Graphical User Interface

MILP Mixed Integer Linear Programming

NEM National Electricity Market

NREL National Renewable Energy Laboratory

NTNDP National Transmission Network Development Plan

OCGT Open-Cycle Gas Turbine

PV Photo-Voltaic

SRMC Short-Run Marginal Cost of production

TES Thermal Energy Storage

UC Unit Commitment

VOMC Variable Operation and Maintenance Cost

Units of Measure

$ Australian dollars normalised to 2014 value

h hours

MW Mega-Watts

MWh Mega-Watt hours

Ω Ohms

rad Radians

S Siemens

V Volts

Symbols

The convention used in this study is to indicate the bus number in the subscript. Variables or constants that refer to a value across two buses (such as voltage angle difference, power transfer, or series impedance) are indicated by two bus numbers. Time-step numbers are indicated in the superscript. Matrices and vectors are indicated by bold font and are not italicised.

$\mathbb{R}$	Set of real numbers	
$\mathbb{N}$	Set of natural numbers	
V	Voltage magnitude	V
θ	Voltage angle	rad
P	Active power	MW
P^{gen}	Active power output	MW
P^{use}	Active power input	MW
$P^{\text{gen,init}}$	Initial condition of active power output (P^{gen})	MW
$P^{\text{use,init}}$	Initial condition of active power input (P^{use})	MW
P^{dem}	Active power demand	MW
P^{max}	Active power transfer limit of a transmission line or transformer	MW
P^{cap}	Capacity (maximum active power output)	MW
P^{min}	Minimum active power output	MW
P^{wind}	Available active power input from wind	MW
P^{solar}	Available active power input from solar insolation	MW
M^{solar}	CST generator solar multiple	$M^{\text{solar}} \in \mathbb{R}$
P^{waste}	Renewable energy generator unused active power	MW
S^{gen}	Energy storage	MWh
S^{init}	Initial condition of energy storage (S^{g})	MWh
S^{cap}	Energy storage capacity	MWh

S^{min}	Minimum quantity of energy storage	MWh
σ	Binary variable to indicate energy storage charging	$\sigma \in \{0,1\}$
υ	Binary variable to indicate generator commitment	$\upsilon \in \{0,1\}$
γ	Binary variable to indicate generator turn-on decision	$\gamma \in \{0,1\}$
ζ	Binary variable to indicate generator turn-off decision	$\zeta \in \{0,1\}$
$\mathit{\Sigma}$	Initial condition of energy storage charging indicator (σ)	$\mathit{\Sigma} \in \{0,1\}$
$\mathit{\Upsilon}$	Initial condition of generator commitment indicator (υ)	$\mathit{\Upsilon} \in \{0,1\}$
$\mathit{\Gamma}$	Initial condition of generator turn-on indicator (γ)	$\mathit{\Gamma} \in \{0,1\}$
Z	Initial condition of generator turn-off indicator (ζ)	$Z \in \{0,1\}$
$\mathbf{I}$	Identity matrix	
$\mathbf{0}$	Zero matrix	
τ	Time-step length	h
t	Number of time-steps in a UC simulation	$t \in \mathbb{N}$
b	Number of buses in a power system	$b \in \mathbb{N}$
g	Number of generators in a power system	$g \in \mathbb{N}$
l	Number of bus interconnections in a power system	$l \in \mathbb{N}$
R	Series resistance of a transmission line	Ω
X	Series reactance of a transmission line	Ω
B	Series susceptance of a transmission line ($B = -{}^{1}/_{X}$, if $R = 0$)	S
$\mathbf{B}$	Bus susceptance matrix	S
$R^{\text{gen,up}}$	Maximum active power ramp-up limit	MW/h
$R^{\text{gen,down}}$	Maximum active power ramp-down limit	MW/h
$R^{\text{use,up}}$	Maximum charging power ramp-up limit	MW/h
$R^{\text{use,down}}$	Maximum charging power ramp-down limit	MW/h
T^{up}	Minimum generator up-time as number of time-steps	$T^{\text{up}} \in \mathbb{N}$
T^{down}	Minimum generator down-time as number of time-steps	$T^{\text{down}} \in \mathbb{N}$
C^{out}	Short-run marginal cost of production	\$
C^{up}	Generator start-up cost	\$
C^{down}	Generator shut-down cost	\$
$E^{\text{rate,max}}$	Energy storage maximum charge rate	MW
$E^{\text{rate,min}}$	Energy storage minimum charge rate	MW
Q^{spin}	Spinning reserve requirement	% of P^{dem}
η	Energy efficiency	$\eta \in [0,1]$

$\mathbf{x^r}$	Vector of real-valued MILP variables	$x_i^{\mathrm{r}} \in \mathbb{R}$
$\mathbf{x^b}$	Vector of binary-valued MILP variables	$x_i^{\mathrm{b}} \in \{0, 1\}$
B^{up}	Upper bound for MILP variable x^{r}	$B^{\mathrm{up}} \in \mathbb{R}$
B^{low}	Lower bound for MILP variable x^{r}	$B^{\mathrm{low}} \in \mathbb{R}$
$\mathbf{f}$	MILP objective function vector	
$\mathbf{A}$	Matrix used in MILP equality constraint	
$\mathbf{b}$	Vector used in MILP equality constraint	
$\mathbf{A^{ineq}}$	Matrix used in MILP inequality constraint	
$\mathbf{b^{ineq}}$	Vector used in MILP inequality constraint	

Chapter 1

Getting Started

1.1 Installation

Welcome to the Unit Commitment (UC) Graphical User Interface (GUI) Matlab toolbox.

All of the required Matlab files are available on GitHub[1]. The easiest way to install the files is:

- Click on "Clone or download" → "Download ZIP"
- Copy the downloaded folder and all contents into the local MATLAB directory
- Within Matlab, go to "Home" → "Set Path" → "Add with Subfolders...", and select the downloaded folder
- Install Cplex as per the instructions below
- Type "UCGUI" in the command window to run the UC GUI Matlab toolbox

Alternatively, clone the repository.

1.1.1 Back-end Solver

A Mixed Integer Linear Programming (MILP) back-end solver is required for the UC GUI Matlab toolbox.

[1] https://github.com/AaronRamsden/UCMatlabToolbox

1.1.1.1 Cplex

The IBM ILOG CPLEX Optimization Studio is recommended to be used as a back-end MILP solver. Cplex is available to academics and students for free as part of the IBM academic initiative program, more information is available on the IBM website[2].

Follow the installation instructions provided with the download. A useful tip for Mac users is to change the permission of the installation file to be executable before running it:

- In the terminal type "chmod +x file.bin"
  (e.g. "chmod +x cplex_studio123.acad.macos.bin")
- To run the .bin file (and start the installation process), type "./file.bin"
  (e.g. "./cplex_studio123.acad.macos.bin")

Detailed installation instructions that may be useful have been publised by Columbia University[3].

1.1.1.2 Matlab In-Built Optimisation Toolbox

The Matlab Optimisation Toolbox can be used as an alternative to Cplex. The benefit of this would be that the Matlab Optimisation Toolbox is included in most Matlab installs. The drawback is that the computational time required to perform simulations will significantly increase compared to using Cplex. This may not be an issue for simple models (on the order of 10 busses).

In order to use the Matlab Optimisation Toolbox, the Matlab function "UCGUI_cplex_solver.m" will have to be modified to use the Matlab function "intlinprog".

More information on the Matlab Optimisation Toolbox can be found on the MathWorks website[4], or by typing "help intlinprog" in the Matlab command window.

1.2 Using the GUI

To run the toolbox, type "UCGUI" in the Matlab command line. This will run the GUI that wraps around all of the code that makes up the solver. The GUI main window appears as shown in Figure 1.1.

[2] https://www.ibm.com/developerworks/community/blogs/jfp/entry/CPLEX_Is_Free_For_Students?lang=en

[3] http://www.columbia.edu/~jz2313/INSTALL

[4] http://au.mathworks.com/help/optim/index.html

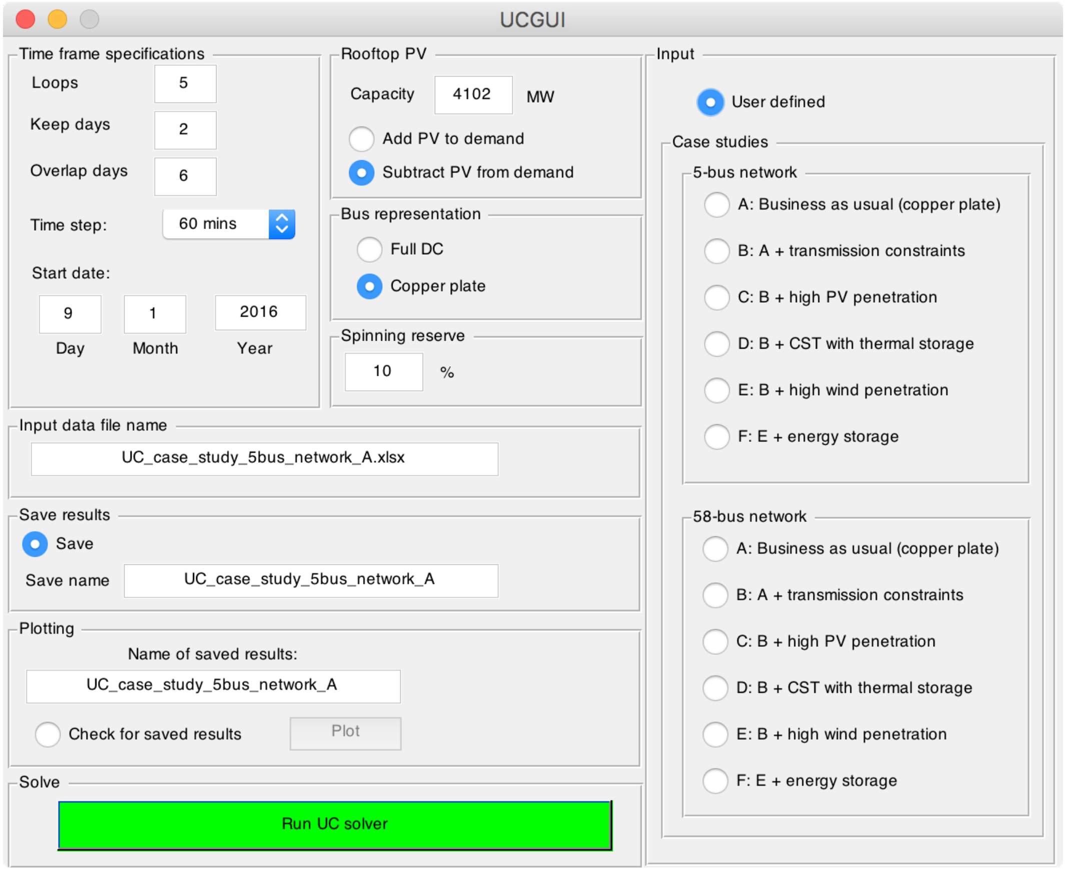

FIGURE 1.1: Graphical User Interface main window.

Depending on the resolution of the computer being used, it may be necessary to adjust the size of the components and the font in the GUI main window. To do this, type “guide” in the Matlab command line, choose “Open Existing GUI” and browse to the main folder → “GUI” → “UCGUI.fig”. It is then possible to drag and adjust the size of individual components in the GUI main window so that they are all visible on the display.

The GUI main window is the interface where the user specifies input data to be used. The GUI main window takes user inputs and runs the UC solver. Some of the user inputs are entered directly into the main GUI window, the remainder are entered into the input data file. The input data file is a spreadsheet of a predetermined format that specifies input data for the UC solver to use.

1.2.1 Input

The “Input” section of the GUI is used to either specify a custom input, or to use one of the 12 example case studies that are provided with the UC GUI toolbox.

The case studies are a good place to start as they provide examples of how to arrange the input data to generate a feasible UC problem that can be solved using this toolbox. Refer to Chapter 2 below for an explanation of the case studies.

1.2.2 Input Data Spreadsheet

Please refer to the example input files provided with the toolbox for working examples of how to enter data into the input data spreadsheet. Note that cells coloured light-orange are cells that are used by the UC toolbox, whereas the remaining cells are not read by the UC toolbox. There is no limit on the number of rows of data entered; the UC toolbox will read up to the last row of data entered.

1.2.2.1 Bus Index Sheet

The "Bus index" sheet specifies buses represented in the model. Each bus must be numbered sequentially starting at 1. Each bus is assigned to a node, which in the example data corresponds to a state in the National Electricity Market (NEM). Each bus requires a wind trace name and a solar trace name, which can be taken from the available data in the main folder → "input_traces" → "wind" or "solar". These traces are used by renewable generators, depending on the bus that the renewable generator is connected to.

The solar traces are also used to estimate the aggregated solar trace for the entire node, by averaging each solar trace within each of the nodes and multiplying by factor of 0.55.

The multiplication factor is used because the solar input power profiles available present a relatively high annual energy availability of the solar resource. State-wide rooftop Photo-Voltaic (PV) generation is likely to be characterised by a much lower annual energy availability because many small-scale residential and small commercial PV installations are installed in areas of comparatively lower suitability for PV generation. The averaged solar input profiles are multiplied by a factor of 0.55 so that the annual energy production from PV and PV capacity correlate in the same way that has been reported by Australian Energy Market Operator (AEMO) for installed rooftop PV in the NEM.

The relative weighting of demand within each node is required for each bus that has been included in the model. The relative weightings are used to disaggregate node-wide demand traces in proportion to the relative weighting given to each bus within the same node. The specified relative weightings are not required to add to 100, they are normalised within the UC toolbox.

1.2.2.2 Bus Connections Sheet

The “Bus connections” sheet specifies all of the transmission lines and transformers that connect two buses together. The bus numbers are required to correspond to the buses outlined in the “Bus index” sheet.

The series reactance is required for the power flow equation: $P_{ij} = \frac{\theta_{ij}}{X_{ij}} = |B_{ij}|\theta_{ij}$

The series reactances can be set arbitrarily unless the solutions for voltage angles are required. The active power transfer limits, specified in the column “Total MW limit” are used to limit power flows in the network when the full Direct Current (DC) power flow model is used. Voltage angles may be unrealistically large if the power transfer limits do not correspond to reasonable series reactances.

A boolean value is required to indicate if the connection will directly isolate a generator. When setting up the UC problem, the UC toolbox will overwrite the transfer limits of such connections if the generator rated capacity is greater than the connection limit.

1.2.2.3 Node Index Sheet

The “Node index” sheet is required to specify information for each of the nodes that were previously referred to in the “Bus index” sheet. Each node is given a demand trace, found in the main folder → “input_traces” → “demand”.

The relative weightings of PV capacity are specified on this sheet. The relative weightings are not required to add to 1, they are normalised within the UC toolbox. For plotting purposes, each node is given a PV name and colour. To see a range of available colours, type “rgb chart” in the Matlab command line.

1.2.2.4 Generator Data Sheet

The “Generator data” sheet is used to specify the generation portfolio in the model. Each generator is described by 20 characteristics, one represented in each column.

“Name” and “Colour name” are used for plotting purposes. The colour names correspond to those displayed when typing “rgb chart” in the Matlab command line.

“Bus number” indicates the bus to which the generator is connected.

“Type (numeric key)” is a numeric key that is used by the UC toolbox when setting up the UC problem. This is important because different types of generators are treated differently when setting up the constraint equations that describe the MILP optimisation problem. The different types of generators are numbered as follows:

1. Brown coal generators,
2. Black coal generators,
3. Combined-Cycle Gas Turbine (CCGT) generators,
4. Open-Cycle Gas Turbine (OCGT) generators,
5. Concentrating Solar Thermal (CST) generators,
6. Generic Energy Storage (GES),
7. Wind farm generators,
8. Utility PV generators.

Each of the ramp rates must be set to a real number; the ramp rates cannot be set to “inf”. To specify no limit on the ramp rates of a generator, set the ramp rates to a value greater than the installed capacity divided by the time-step length (in units of MW/h).

The Short-Run Marginal Cost of production (SRMC) of each generator is assumed to have a linear relationship with production quantity. The SRMC can be estimated as follows:

$$\text{SRMC} = \text{VOMC} + (\text{Fuel cost})\frac{(\text{Heat rate})}{\eta}$$

The minimum up-time and minimum down-time are specified as a number of hours in the input spreadsheet, and are converted to the number of time-steps within the UC solver. The minimum up-time and minimum down-time should not exceed 16 hours, as only 15 hours are included in the initial conditions provided with each sub-problem presented to the UC solver.

The remainder of the columns from “Storage (MWh)” to “CST & PV solar multiple” apply to energy storage plant only (GES and CST generators). These columns are used to specify the quantity of energy storage, the maximum and minimum power rating of the plant, and the rate at which the plant can change its output power (rate of production) or input power consumption. The column “CST & PV solar multiple” specify the efficiency of the plant; this entry is used as a multiplication factor to scale the solar input power, which is otherwise normalised to the rated power of the plant.

The solar power profiles used for CST generators are scaled by the installed capacity of each generator, as well as the solar multiple of each plant. The solar multiple accounts for an increase in available solar power due to the field of heliostats that focus the sunlight onto the power tower. National Renewable Energy Laboratory (NREL) use a solar multiple of 2.0 for simulating CST plant with Thermal Energy Storage (TES) [1]. AEMO use a solar multiple of 2.5 for CST plant with TES in the the 2013 100 percent Renewables Study [2].

1.2.3 Time Frame Specifications

The user specifies the size of the problem by adjusting the values of the input variables in the "Time frame specifications" box. The total UC problem can be divided into sub-problems which are solved using a rolling horizon technique. "Loops" specifies the number of sub-problems. "Keep days" and "Overlap days" together specify the size of each sub-problem. For example, if "Keep days" is 1 and "Overlap days" is 6, then each sub-problem will span 7 days.

"Keep days" specifies the number of days to retain as the overall solution from each sub-problem solution. For example, if "Keep days" is 1 and "Overlap days" is 6, then only the solution to the first day of each sub-problem is retained as the solution to the overall UC problem.

The start date specifies the first date to be retained in the UC solution. For all simulations, an additional sub-problem is solved prior to the start of the first date in order to acquire initial conditions.

The time-step length will change the number of time-steps in each day. For hourly time-steps there are 24 time-steps in each day, for 30 minute time-steps there are 48 time-steps in each day, for 15 minute time-steps there are 96 time-steps in each day, and for 5 minute time-steps there are 288 time-steps in each day. The time taken for the solver to find an optimal solution to a problem will increase with the total number of time steps in each sub-problem.

Note that the input traces are presented as hourly. When choosing time-step lengths shorter than one hour, all input traces are approximated as having the same value throughout each hour. An improvement would be to approximate the input traces with a curve that takes a different value for each time step. The relevant functions to modify are "UCGUI_demand_rooftop_pv.m", "UCGUI_solar.m", and "UCGUI_wind.m".

The specified start date is required to be in range of 1/7/2015 to 30/6/2040. Note that the last available date for input data is 30/6/2040. Ensure that for all simulations the end date of the simulation (including the number of keep days) is at latest 30/6/2040.

Figure 1.2 shows the time frame specifications and how they relate to the the rolling horizon and size of the UC problem.

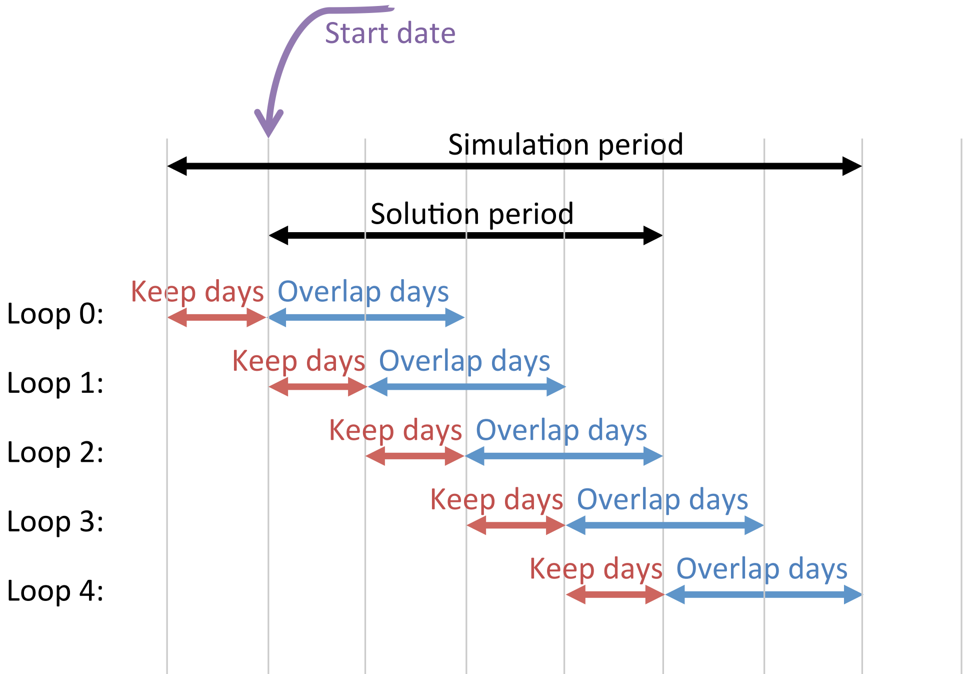

FIGURE 1.2: Time frame specifications.

1.2.4 Rooftop PV

The "Rooftop PV" box allows the user to adjust the total installed capacity of distributed rooftop PV in the entire power system being modelled. The total installed capacity of rooftop PV is specified by the user in the main GUI window, while the relative weightings of rooftop PV capacity are specified in the input data file.

Rooftop PV is modelled as aggregated generation for each node in the system. Rooftop PV is either subtracted or added to demand traces within each node, as specified by the user.

1.2.5 Bus Representation

The user has the option of including a full DC power flow model in the UC problem, or of ignoring network transmission constraints and using a more simple copper plate model.

1.2.6 Spinning Reserve

The "Spinning reserve" box allows the user to set the spinning reserve requirement. The spinning reserve requirement is set to a minimum percentage of total demand that is required to be online, available, and unused from synchronous generators in each time-step of the UC problem. CST generators are modelled as being able to provide spinning reserve, and are required to maintain an hour of rated capacity in storage in order to be able to provide spinning reserve at all times.

1.2.7 Save Results

The "Save results" box allows the user to save results from the simulation by entering a name to be used for the MAT-File that will contain the solution struct. If the option to save the results is not selected, the results will not be available to the user after the simulation.

1.2.8 Plotting

The "Plotting" box allows the user to check the results from any previously saved simulation. The name of the MAT-File must be entered in the text box before the user can verify the existence of the file by clicking the check-box "Check for saved results". If the MAT-File exists, the user will be able to view a plot of the results by clicking "Plot".

The active power generation that is presented in the plot is shown to give the user an idea of what the solution contains rather than to provide an exhaustive examination of the results. The user is encouraged to examine other aspects of the saved results. For

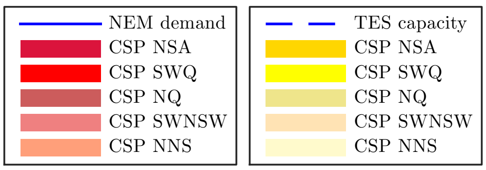

FIGURE 1.3: Legend for Figure 1.4.

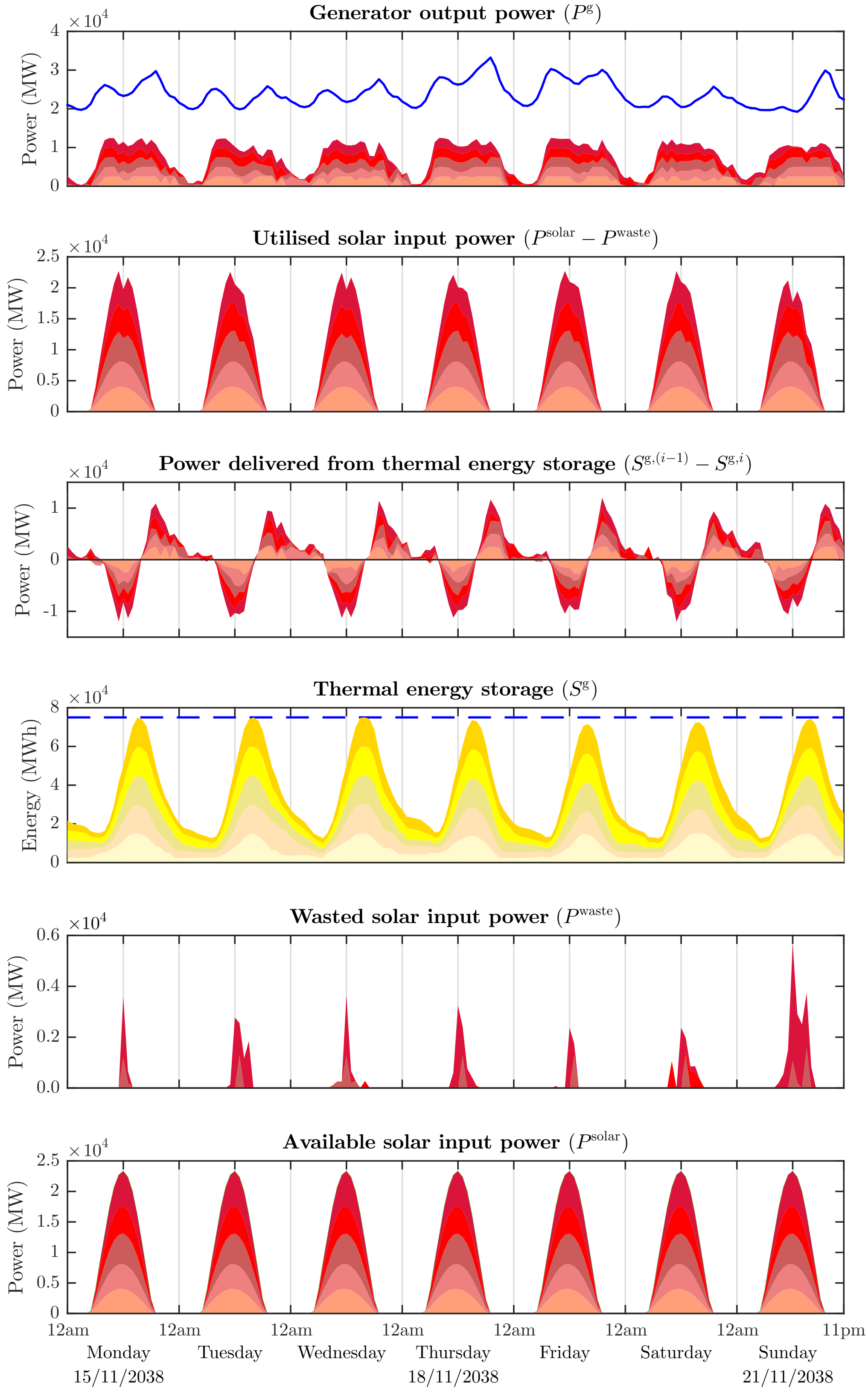

FIGURE 1.4: Week of CST generation in November 2038.

example, Figures 1.3 and 1.4 show a plot of some of the key behavioural characteristics of a fleet of CST generators compared to the total demand and available solar insolation, from a user-defined UC problem that presents a hypothetical future scenario. Note that the plots shown in Figures 1.3 and 1.4 are generated using simulation results from the UC toolbox, however these plots are not automatically generated by the GUI.

1.2.9 Solve

Once the input data has been correctly entered, the user can click "Run UC solver" to find out whether the problem is feasible and if so, to determine the optimal solution.

Figure 1.5 shows a simplified representation of how the GUI main window works and interacts with other code in the toolbox.

When the solver is run, the GUI main window runs the script "UCGUI_simulation.m" and provides all of the user specified input data. Some of the user input data details the names of traces to be used, and so "UCGUI_simulation.m" reads the relevant sections of those traces and rearranges the data to present a standard input to the solver, "UCGUI_cplex_solver.m".

"UCGUI_cplex_solver.m" is a Matlab function that will solve a generic UC problem when given an input in the required format. "UCGUI_cplex_solver.m" is run on each of the sub-problems to determine the optimal solution for each sub-problem.

After acquiring solutions for each sub-problem, "UCGUI_simulation.m" arranges the relevant sections of the solutions to each sub-problem by concatenating them together

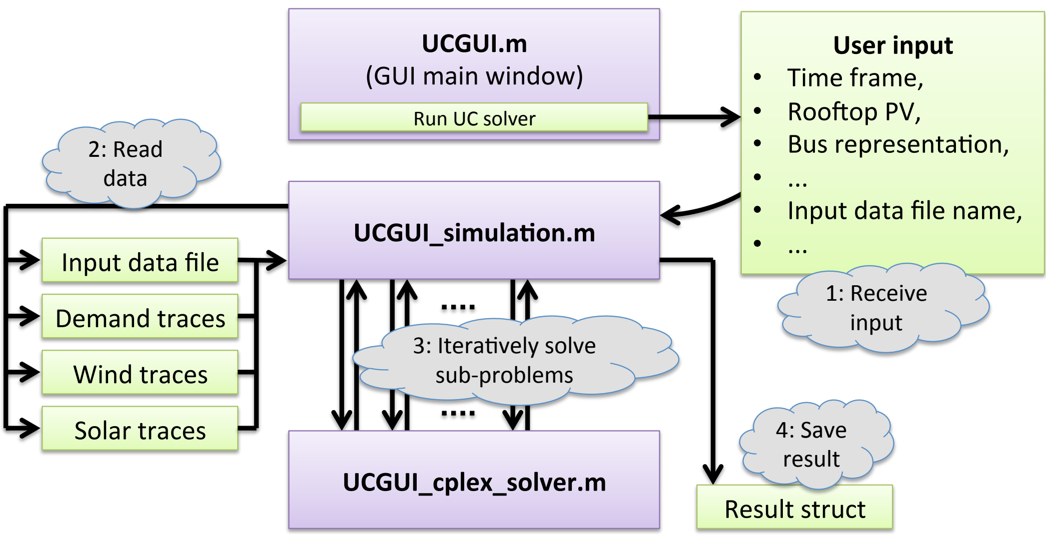

FIGURE 1.5: Block diagram of code flow with GUI.

to present the solution to the overall UC problem. The overall solution is then saved using the save name specified by the user.

For more information about any of the functions or scripts that make up the UC GUI toolbox, type "help <*name of function/script*>" in the Matlab command line. For example, type "help UCGUI_cplex_solver.m" in the Matlab command line for additional information about the function "UCGUI_cplex_solver.m".

Chapter 2

Example Case Studies

Six case studies are provided with the toolbox, each of which is repeated for a simple 5-bus network based on [3, Example 12.8] and a 58-bus model of the NEM in Australia based on [4]. All of the case studies can be easily augmented by the user. The input data used for these case studies, including demand, wind, and solar traces, is based on publicly available data published by AEMO [5].

2.1 5-Bus Network

The case studies are presented using a 5-bus network in Section IV of “An Educational Open-Source Market Modeling Toolbox for Future Grid Studies” by Ramsden and Verbič. The 5-bus network is shown in Figure 2.1. The results, shown in Figure 2.2, are not discussed in this user manual. Refer to “An Educational Open-Source Market Modeling Toolbox for Future Grid Studies” by Ramsden and Verbič for a detailed discussion on the results.

The 5-bus network, shown in Figure 2.1, is based on an educational example by Glover et al. [3, Example 12.8]. Demand from regions of the NEM is used so that the network model represents a modified NEM with interconnections from Queensland to South Australia and New South Wales to South Australia.

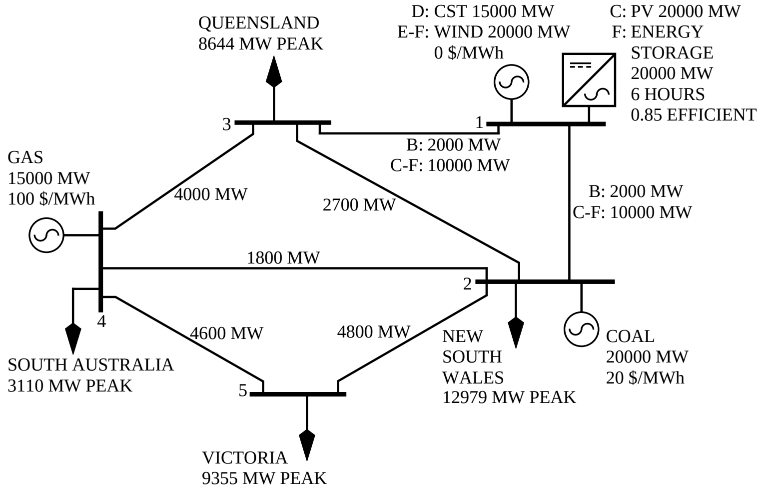

FIGURE 2.1: 5-bus network used for case studies.

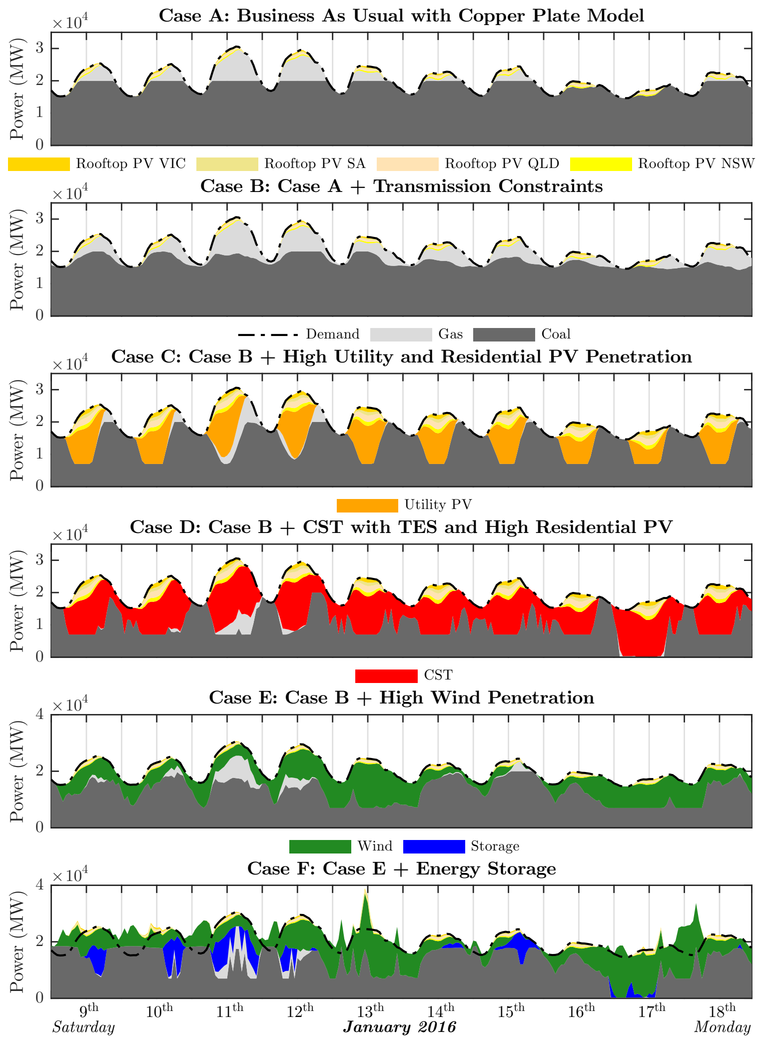

FIGURE 2.2: Case studies A–F results with 5-bus network.

2.2 58-Bus Network

The toolbox includes a full set of input files where each of the six case studies is extended to a more complex 58-bus model. The 58-bus transmission network model is a simplified representation of the NEM and is based on a 14-generator test system originally developed for small-signal stability studies [4].

Table 2.1 shows the generator portfolio for each of the six case studies. The bus numbers correlate to the single line diagram shown in Figure 2.3. Refer to Appendix A for supplementary information on the generator portfolios used in each case study.

This 58-bus network is used to analyse the six case studies labelled 'A'–'F'. Refer to "An Educational Open-Source Market Modeling Toolbox for Future Grid Studies" by Ramsden and Verbič for a detailed explanation of each of the six scenarios. The results are shown in Figure 2.4 and are briefly discussed below.

2.2.1 Case Study A: Business As Usual with Copper Plate Model

The "business as usual" scenario includes coal and gas generators only. This case study assumes a copper plate model, which ignores bus interconnections and transmission network power limits. The results show that the coal generators serve full demand, or are dispatched at maximum capacity when demand exceeds the total installed capacity of coal. Gas generation is used to supply daily peaks in demand that exceed the coal generators' capacity. Gas is also dispatched due to the spinning reserve requirement, where demand approaches the maximum capacity of the coal generators.

2.2.2 Case Study B: Business As Usual with Transmission Constraints

This case study examines the "business as usual" fossil fuel scenario with transmission constraints imposed on all lines between buses. A conventional dc load flow model is used to represent the network.

The transmission constraints limit the ability of coal power stations to serve the network demand.

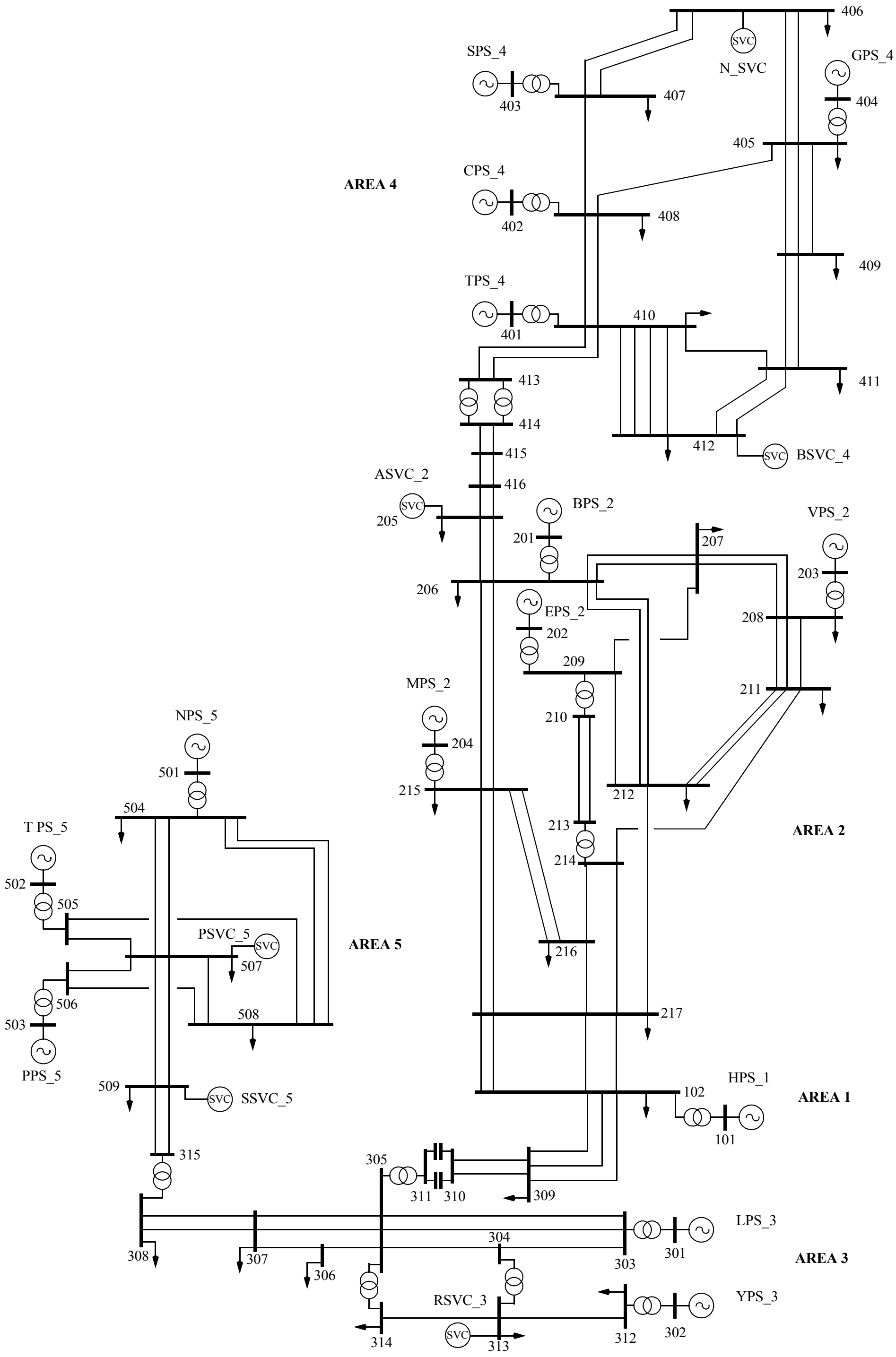

FIGURE 2.3: 58-bus network used for case studies [6].

Table 2.1: Generators included in 58-bus model.

Type	**Bus**	**Capacity MW**	**Case Study** A	B	C	D	E	F
Brown Coal	302	4992	•	•	•	•	•	•
	501	650	•	•	•		•	•
Black Coal	201	3320	•	•	•		•	•
	202	2880	•	•	•	•	•	•
	402	4567	•	•	•	•	•	•
CCGT	203	2231	•	•	•	•	•	•
	305	1000	•	•	•	•	•	•
	404	1556	•	•	•	•	•	•
	503	678	•	•	•	•	•	•
OCGT	204	3700	•	•	•	•	•	•
	301	2362	•	•	•	•	•	•
	401	3479	•	•	•	•	•	•
	502	2131	•	•	•	•	•	•
CST	201	3000				•		
	217	2500				•		
	403	2500				•		
	413	3000				•		
	501	2500				•		
Utility Storage	206	2500						•
	209	1500						•
	211	2000						•
	306	2000						•
	313	2000						•
	405	1250						•
	412	2000						•
	508	1250						•
Wind	207	2500					•	•
	217	2500					•	•
	307	2500					•	•
	309	2500					•	•
	403	2500					•	•
	411	2500					•	•
	507	2500					•	•
	509	2552	•	•	•	•	•	•
Utility PV	212	1000	•	•		•	•	•
		1500			•			
	217	1000			•			
	306	2000			•			
	313	2000			•			
	409	1000	•	•		•	•	•
		1500			•			
	411	2000			•			
	508	1500			•			

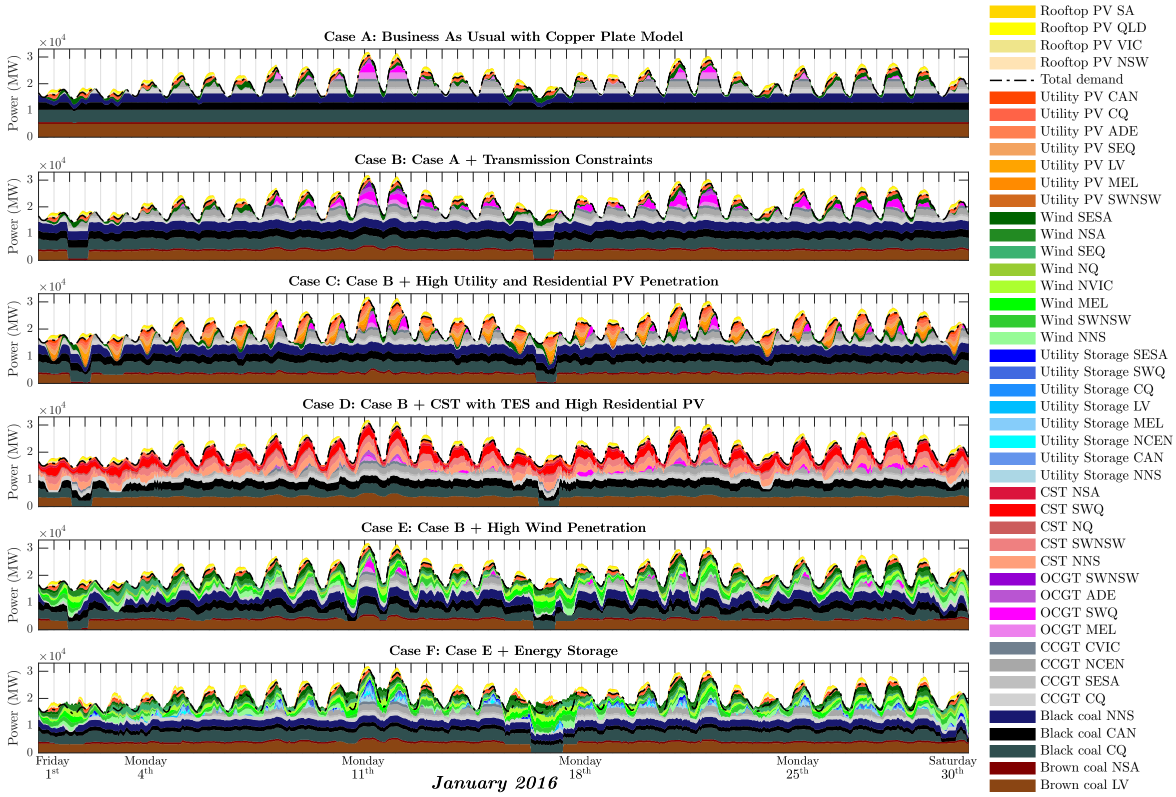

FIGURE 2.4: Case studies A–F results with 58-bus network.

2.2.3 Case Study C: High Penetration of Solar Photovoltaic

This case study demonstrates the effect of increasing solar PV in a traditional fossil fuel grid. Case B is modified to include five additional grid-scale utility PV generators. Distributed residential PV in each state, which is modeled as negative demand, is increased from the values in Case B.

The reduction of demand during daylight hours, coupled with the availability of comparatively low-cost generation from utility PV generators results in coal generators following a more exaggerated daily cyclic output pattern than Case B at times when daily peak total demand is below 25 000 MW. When the daily peak total demand exceeds 25 000 MW, PV generation displaces gas generation, with negligible differece in coal dispatch compared to Case B.

2.2.4 Case Study D: High Penetration of Concentrated Solar Thermal with Thermal Energy Storage

This case study explores the dispatchable renewable technology of CST with TES. Case B is modified to include grid-scale CST generators, with six hours' energy storage at rated generation capacity. Residential PV capacity is the same as Case C.

The type of CST plant modeled in this study is a central receiver power tower surrounded by a field of heliostats that focus the solar energy onto the tower to heat molten salt. The molten salt provides TES, and is used to drive a conventional synchronous steam turbine generator that is able to provide spinning reserve to the grid. The results show that the coal generators are operated at their minimum stable output for a greater proportion of the time compared to Case C. The ramping behavior of the coal generators is somewhat improved compared to Case C.

2.2.5 Case Study E: High Penetration of Intermittent Wind Generation

This case study demonstrates the effect of increasing intermittent wind generation in a traditional fossil fuel grid. Case B is modified to include grid-scale wind generation.

Upon increasing intermittent wind generation capacity, gas generation follows less regular patterns of dispatch, and is used to serve peaking loads at times of low wind generation. The system relies on traditional fossil fuel generation when the available wind power is comparatively low. Coal generation output cycles over greater limits compared to Case B, as low-cost intermittent wind generation is dispatched with a higher priority than coal.

2.2.6 Case Study F: Energy Storage with High Penetration of Intermittent Wind Generation

This case study examines the effect of including large-scale energy storage near intermittent wind generation. Case E is modified to include grid-scale GES plant, which in this case represents battery storage plant that do not provide spinning reserve to the grid. Each energy storage plant has energy storage capacity equivalent to six hours' generation at rated capacity.

Results show that GES plant dispatch at times when gas generation would otherwise be dispatched. Coal and wind generation are utilized for the energy storage plant charging at times of lower network demand.

Chapter 3

Modelling of Generators

3.1 Fossil Fuel Generators

Aggregated generators in the market model are defined by the following characteristics:

Name

Name of generator to appear on plots.

Colour

Colour associated with generator to be used for plots (list of colours from "rgb chart").

Location

Geographic location of generator, presented as a bus number in the transmission network model (bus number $\in \mathbb{N}$).

Type

Numeric key to indicate the type of generator technology (type number $\in \mathbb{N}$).

Capacity (P^{cap})

Generator nameplate capacity, also generator maximum output power (MW).

Minimum stable output (P^{min})

Minimum output power that generator can provide when on-line (MW).

Ramp-up rate ($R^{\text{gen,up}}$)

Maximum rate of increase in generator output power (MW/h).

Ramp-down rate ($R^{\text{gen,down}}$)

Maximum rate of decrease in generator output power (MW/h).

Start-up cost (C^{up})

Cost incurred by generator when brought on-line ($).

Shut-down cost (C^{down})

Cost incurred by generator when brought off-line ($).

SRMC (C^{out})

Short-Run Marginal Cost of production, cost incurred by generator in production of energy ($/MWh).

Minimum up-time (T^{up})

Minimum time that generator must remain on-line once brought on-line (h).

Minimum down-time (T^{down})

Minimum time that generator must remain off-line once brought off-line (h).

Energ storage capacity (S^{cap})

Maximum quantity of energy that can be retained in storage (MWh). Applies to GES and CST generators only.

Energy storage maximum charge and discharge rate ($E^{\text{rate,max}}$)

Maximum rate of change in quantity of energy in storage (MW). Applies to GES and CST generators only.

Energy storage minimum charge and discharge rate ($E^{\text{rate,min}}$)

Minimum rate of change in quantity of energy in storage (MW). Applies to GES and CST generators only.

Energy storage charge ramp-up rate ($R^{\text{use,up}}$)

Maximum rate of increase in energy storage plant input power (MW/h). Applies to GES generators only.

Energy storage charge ramp-down rate ($R^{\text{use,down}}$)

Maximum rate of decrease in energy storage plant input power (MW/h). Applies to GES generators only.

Energy storage efficiency (η)

Energy efficiency ($\eta \in [0, 1]$). Applies to GES generators only.

Solar multiple (M^{solar})

Scaling factor applied to available solar input power, to account for increase or decrease in available power from physical structure of plant (a field of heliostats that focus the solar insolation onto a CST power tower, or a reduction in available input power to a utility PV plant relative to the scaled solar trace) ($M^{\text{solar}} \in \mathbb{R}$). Applies to CST and utility PV generators only.

Note that the maximum allowable minimum up-time or minimum down-time is 16 hours as initial conditions are not included beyond 15 hours prior to the start of each sub-problem.

3.2 Renewable Generation Technologies

3.2.1 Wind Generators

Wind generators are modelled in the same way as fossil fuel generators, with the exception that the available energy resource imposes an upper limit on power output. The active power output of wind generators is limited by the generation capacity of the plant or the available wind power at each time interval, which ever is lower.

3.2.2 Utility PV Generators

Utility PV generators are modelled in the same way as fossil fuel generators, with the exception that the available energy resource imposes an upper limit on power output. The active power output of utility PV generators is limited by the generation capacity of the plant or the available solar power at each time interval, which ever is lower.

FIGURE 3.1: Torresol Gemasolar CST plant, Spain [7].

3.2.3 CST Generators with TES

All CST generators modelled in this study are central receiver units, as shown in Figure 3.1. All CST generators include TES. The active power output of CST generators is limited by the generation capacity of the plant or the available thermal power at each time interval, which ever is lower. Available thermal power is derived from both solar insolation and TES due to prior accruement of stored energy.

It is assumed that CST generators are able to provide spinning reserve when synchronised to the grid. In order to provide spinning reserve, CST generators are required to maintain a minimum energy reserve, which is chosen to be one hour at generation nameplate capacity.

CST generators with TES require additional governing equations that are not required for other generators, as well as the introduction of a storage variable, S^{gen}. The maximum rate of change of S^{gen} is limited by applying a mathematical limit on TES charge and discharge rates:

$$|S^{\text{gen},i} - S^{\text{gen},(i-1)}| \leqslant \tau E^{\text{rate,max}}$$

$E^{\text{rate,max}}$ can be chosen to be equal to the generator nameplate capacity so that CST generators are able to provide full power output at times when there is no solar insolation available provided they have sufficient energy in storage.

The governing equation for power output of CST generators is derived by modelling a typical CST plant from an external point of view of all power flows into and out of the plant. Each power flow is accounted for and included in a power balance equation.

Figure 3.2 shows a representative block diagram of a CST facility with a TES capability, S^{gen}, in a time interval, i. Power input from solar insolation is indicated by P^{solar}. Two sources of power output are included; generated power delivered to the grid, P^{gen}, and unused power, P^{waste}. For the time interval, i, the power taken from TES is $\frac{1}{\tau}(S^{\text{gen},(i-1)} - S^{\text{gen},i})$, a value that will be positive if energy from storage is being delivered to the grid, and negative if the plant's energy storage is increasing.

The variable P^{waste} is required because it is feasible that the plant may not deliver all of the available energy. This may be the case if power system constraints limit the power output of the plant and storage reaches capacity while power from solar insolation is available. The wasted power is included so that in such a case the available solar input power or power from TES can be dissipated without violating the power balance equation.

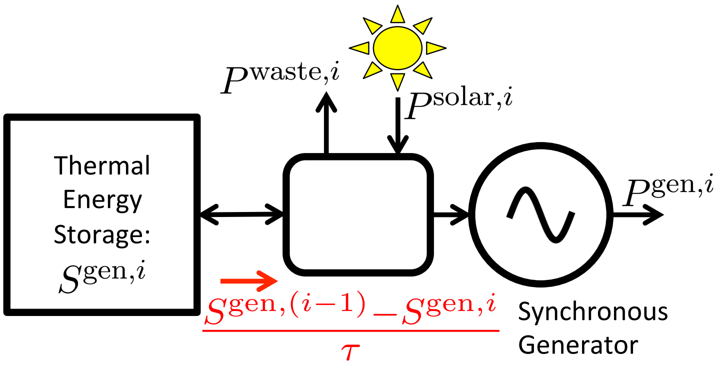

FIGURE 3.2: CST power flow diagram.

The power balance equation for Figure 3.2 is then:

$$P^{\text{gen},i} = \frac{S^{\text{gen},(i-1)} - S^{\text{gen},i}}{\tau} + P^{\text{solar},i} - P^{\text{waste},i}$$

The variable P^{waste} can be eliminated from the power balance equation by forming an inequality equation:

$$P^{\text{gen},i} \leqslant \frac{S^{\text{gen},(i-1)} - S^{\text{gen},i}}{\tau} + P^{\text{solar},i}$$

Both the equality and inequality power output governing equations are equally valid, however, the inequality equation is more efficient from the perspective of implementing a model as it includes one less variable. Chapter 4, §4.11 and §4.15, repeat the CST governing equations as they are used in the market model.

3.2.4 Generic Energy Storage Generators

GES generators are included to account for a range of different types of energy storage plant, including battery storage and pumped hydro-electric generation. The user is able to describe a GES generator by the maximum and minimum rate of energy production and consumption, the total energy storage capacity and energy storage efficiency, and the maximum and minimum rate of change of power output and input. A binary variable is included in the model to describe the charge state of energy storage plant, refer to item 5 in §4.2.

Energy storage plant are modelled by the same constraints that apply to conventional generators, with additional energy storage equtions that are specific to GES plant. The energy storage equations include a simple power balance equation that incorporates energy efficiency, as well as charge and discharge limits and limits on the rate of change of charging and discharging. Energy efficiency is modelled as a fixed value for each plant for all energy stored by the plant. For further details, refer to §4.11, §4.9, §4.7, and §4.6.

Chapter 4

Mathematical Representation of Market Model

4.1 Mixed Integer Linear Programming Notation

The MILP optimisation problem used in this toolbox is expressed mathematically as follows:

$$\text{Minimise} \quad \mathbf{f}^{\mathsf{T}} \begin{bmatrix} \mathbf{x^r} \\ \mathbf{x^b} \end{bmatrix}$$

Subject to:

$$\mathbf{A} \begin{bmatrix} \mathbf{x^r} \\ \mathbf{x^b} \end{bmatrix} = \mathbf{b}$$

$$\mathbf{A^{ineq}} \begin{bmatrix} \mathbf{x^r} \\ \mathbf{x^b} \end{bmatrix} \leqslant \mathbf{b^{ineq}}$$

The objective function is defined by $\mathbf{f}$, variables are defined by $\mathbf{x^r}$, $\mathbf{x^b}$:

$$\mathbf{f} = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{bmatrix}, \quad \begin{bmatrix} \mathbf{x^r} \\ \mathbf{x^b} \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

Variables are defined as either real or binary-valued. Fixed bounds are given to each variable so it is constrained to a value between the specified bounds, binary-valued variables are constrained to either 0 or 1:

$$\mathbf{x^r} \in \mathbb{R}, \quad B_i^{\text{low}} \leqslant x_i^{\text{r}} \leqslant B_i^{\text{up}}, \quad \mathbf{x^b} \in \{0,1\}$$

4.2 Decision Variables

Key decision variables are chosen in order to allow a mathematical formulation of the constraints:

1. Active power produced at each bus, $P^{\text{gen}} \in \mathbb{R}$,
2. Active power from the grid used for energy storage at each bus, $P^{\text{use}} \in \mathbb{R}$,
3. Voltage angle at each bus, $\theta \in \mathbb{R}$,
4. Energy storage at each bus, $S^{\text{gen}} \in \mathbb{R}$,
5. Energy storage charging state at each bus, $\sigma \in \{0,1\}$,
6. Generator on-state at each bus, $\upsilon \in \{0,1\}$,
7. Generator turn-on indicator at each bus, $\gamma \in \{0,1\}$,
8. Generator turn-off indicator at each bus, $\zeta \in \{0,1\}$.

These variables are unique in each time-step of a simulation; for a system of b buses and a simulation period of t time-steps, there are $8bt$ unique variables.

P^{gen} represents the quantity of power produced continuously for a single time-step. P^{gen} is expressed in MW in all results. At each bus, P^{gen} is bounded by zero and the generator installed capacity or available renewable input power:

$$0 \leqslant P_i^{\text{gen},j} \leqslant P_i^{\text{wind},j} \quad \text{for wind generators,}$$

$$0 \leqslant P_i^{\text{gen},j} \leqslant P_i^{\text{solar},j} \quad \text{for solar PV generators,}$$

$$0 \leqslant P_i^{\text{gen},j} \leqslant P_i^{\text{cap}} \quad \text{otherwise.}$$

Where:

$$i \in \{1,\ldots,b\}, \quad j \in \{1,\ldots,t\}$$

If a bus does not have a connected generator, P^{gen} is bounded above and below by zero. For each bus that has a connected wind or solar PV generator, the bounds are modified to account for the available input power from the renewable resource, which takes a different value at each time-step, i. Note that for renewable generators, P^{gen} cannot exceed the generator capacity due to the generator output constraints described below in §4.8.

P^{use} represents the quantity of power consumed continuously for a single time-step by an energy storage plant. P^{use} is expressed in MW in all results. At each bus, P^{use} is bounded by zero and the energy storage plant maximum charge rate:

$$0 \leqslant P_i^{\text{use},j} \leqslant E_i^{\text{rate,max}}$$

Where:

$$i \in \{1, \ldots, b\}, \quad j \in \{1, \ldots, t\}$$

If a bus does not have a connected energy storage plant, P^{use} is bounded above and below by zero.

The voltage angle at each bus, θ, is expressed in radians in order to implement the DC power flow formulation. Voltage angle requires a reference angle, which is chosen to be zero at Bus 1. The voltage angle at Bus 1 is forced to zero, all other angles are unbounded:

$$0 \leqslant \theta_1^j \leqslant 0, \quad -\infty \leqslant \theta_i^j \leqslant \infty$$

Where:

$$i \in \{2, \ldots, b\}, \quad j \in \{1, \ldots, t\}$$

Voltage angle is included in the model in order to impose limits on power flows across interconnectors, according to the DC power flow formulation incorporated into the market model defined in this toolbox.

S^{gen} represents the total energy storage at the end of each time-step. The energy storage at each bus, S^{gen}, is expressed in MWh in all results. At each bus, S^{gen} is bounded by a minimum level of energy storage and the installed storage capacity:

$$S_i^{\min} \leqslant S_i^{\text{gen},j} \leqslant S_i^{\text{cap}}$$

Where:

$$i \in \{1, \dots, b\}, \quad j \in \{1, \dots, t\}$$

For CST geneartors, the minimum allowable level of TES is taken to be the equivalent of 1 time step of generation at rated capacity. This minimum level of storage is included so that CST generators are able to provide spinning reserve when they are synchronised and connected to the grid:

$$S_i^{\text{min}} = (1 \text{ time-step}) P_i^{\text{cap}}$$

If a bus does not have a connected CST generator or energy storage plant, S^{gen} is bounded above and below by zero.

Binary variables, σ, υ, γ, ζ, are forced to take the value 0 or 1. These variables are used to indicate commitment decisions that are taken by generators, where 0 is interpreted as false and 1 is interpreted as true.

The decision variables are included in the MILP formulation of the UC problem by expressing the variables in a single vector:

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}^{\mathbf{r},1} \\ \mathbf{x}^{\mathbf{b},1} \\ \mathbf{x}^{\mathbf{r},2} \\ \mathbf{x}^{\mathbf{b},2} \\ \vdots \\ \mathbf{x}^{\mathbf{r},i} \\ \mathbf{x}^{\mathbf{b},i} \\ \vdots \\ \mathbf{x}^{\mathbf{r},h} \\ \mathbf{x}^{\mathbf{b},h} \end{bmatrix}, \text{ where } \begin{bmatrix} \mathbf{x}^{\mathbf{r},i} \\ \mathbf{x}^{\mathbf{b},i} \end{bmatrix} = \begin{bmatrix} \mathbf{P}^{\mathbf{gen},i}_{(b\times1)} \\ \mathbf{P}^{\mathbf{use},i}_{(b\times1)} \\ \boldsymbol{\theta}^{i}_{(b\times1)} \\ \mathbf{S}^{\mathbf{gen},i}_{(b\times1)} \\ \boldsymbol{\sigma}^{i}_{(b\times1)} \\ \boldsymbol{\upsilon}^{i}_{(b\times1)} \\ \boldsymbol{\gamma}^{i}_{(b\times1)} \\ \boldsymbol{\zeta}^{i}_{(b\times1)} \end{bmatrix}$$

4.3 Cost Functions

All generators are modelled as offering energy at the SRMC. The SRMC is assumed to be linearly related to the active output power of each generator, an assumption used by AEMO in all planning reports [8], including the 2014 National Transmission Network Development Plan (NTNDP) report.

Ancillary service markets are not explicitly modelled in this toolbox. Other costs that are included are start-up and shut-down costs, one of which are incurred upon each start-up or shut-down of a generator. The total cost of operation for a generator during a single time interval is shown in Equation 4.1.

$$\text{Cost} = \tau C^{\text{out}} P^{\text{gen}} + C^{\text{up}}\gamma + C^{\text{down}}\zeta \tag{4.1}$$

Cost functions are included in the MILP objective function so that each cost will be incurred in proportion to the associated system variable:

$$\mathbf{f} = \begin{bmatrix} \mathbf{f}^1 \\ \mathbf{f}^2 \\ \vdots \\ \mathbf{f}^t \end{bmatrix}, \quad \mathbf{f}^1 = \mathbf{f}^2 = \cdots = \mathbf{f}^t = \begin{bmatrix} \tau\mathbf{C}^{\mathbf{out}}_{(b\times1)} \\ \mathbf{0}_{(b\times1)} \\ \mathbf{0}_{(b\times1)} \\ \mathbf{0}_{(b\times1)} \\ \mathbf{0}_{(b\times1)} \\ \mathbf{0}_{(b\times1)} \\ \mathbf{C}^{\mathbf{up}}_{(b\times1)} \\ \mathbf{C}^{\mathbf{down}}_{(b\times1)} \end{bmatrix}$$

Costs incurred by energy storage plants that consume energy during a time step are not explicity modelled because the energy consumed is dispatched in a later time-step. The cost of energy consumption by energy storage plants is implicitly included by the nature of the optimisation problem, which minimises the total cost of production.

4.4 Active Power Balance Requirements

Power balance requirements are incorporated into the UC problem. Power balance equations stem from the principle of conservation of energy. The user has the option of including a full DC power flow model or using a copper plate model.

4.4.1 Full DC Power Flow

To include a full DC power flow model, equations are written at each bus in the system stating that the sum of all generated active power equals the sum of all demand plus the sum of all active power transferred to other buses, as shown in Equation 4.2.

$$P_i^{\text{gen}} - P_i^{\text{use}} = P_i^{\text{dem}} + \sum_j \frac{\theta_{ij}}{X_{ij}} \tag{4.2}$$

The power balance requirements are formulated in the MILP equality constraint matrix:

$$\begin{bmatrix} \mathbf{A}^1_{(b\times 8b)} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{A}^2_{(b\times 8b)} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{A}^t_{(b\times 8b)} \end{bmatrix} \mathbf{x} = \begin{bmatrix} \mathbf{b}^1 \\ \mathbf{b}^2 \\ \vdots \\ \mathbf{b}^i \\ \vdots \\ \mathbf{b}^t \end{bmatrix}$$

Where:

$$\mathbf{A}^1 = \mathbf{A}^2 = \cdots = \mathbf{A}^t = \begin{bmatrix} \mathbf{I}_{(b\times b)} & -\mathbf{I}_{(b\times b)} & \mathbf{B}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} \end{bmatrix},$$

$$\mathbf{b}^i = \mathbf{P}^{\mathbf{dem},i}_{(b\times 1)}$$

The bus susceptance matrix is used in the full DC power flow representation. The bus susceptance matrix is represented as follows:

$$\mathbf{B} = \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1b} \\ B_{21} & B_{22} & \cdots & B_{2b} \\ \vdots & \vdots & \ddots & \vdots \\ B_{b1} & B_{b2} & \cdots & B_{bb} \end{bmatrix}$$

The diagonal terms of $\mathbf{B}$ at indices (ii) are the sum of all susceptances connected to bus i. The off-diagonal terms of $\mathbf{B}$ at indices (ij), with $i \neq j$, are the negated sum of susceptances connecting buses i and j. Note that if the series resistance between two buses, R_{ij}, is zero, then by definition of susceptance; $B_{ij} = -{}^1\!/_{X_{ij}} < 0$.

The bus susceptance matrix is used in the MILP formulation of the UC problem to capture the physical transmission network layout as well as transmission line and transformer impedances. The transmission network representation of the NEM used in the provided example inputs has been acquired from Gibbard and Vowles [6], with some further modifications as required.

4.4.2 Copper Plate Model

If the user has specified a copper plate model, Equation 4.2 is modified to ignore the transmission network representation as shown in Equation 4.3.

$$\sum_{i=1}^{g} P_i^{\text{gen}} - P_i^{\text{use}} = \sum_{i=1}^{b} P_i^{\text{dem}} \tag{4.3}$$

Equation 4.3 states that, in any time-step, the sum of all generated power must equal the sum of all demand.

4.5 Generator Start-Up and Shut-Down Enforcement

Start-up and shut-down constraints are included in the model to ensure that each generator can only start up from an off-line state, and can only shut down from an on-line state. This seemingly obvious constraint is required so that the model provides realistic solutions.

The governing equation for these constraints is expressed for each time-step, i, in Equation 4.4 [9].

$$\gamma^i - \zeta^i = \upsilon^i - \upsilon^{(i-1)} \tag{4.4}$$

Note that when the associated generator is either off or on for two consecutive time-steps, Equation 4.4 alone does not explicitly force both γ and ζ to be equal to 0. In such a case a mathematically correct solution to Equation 4.4 is that γ and ζ are both equal to 1. However, there is an implicit enforcement to ensure that both γ and ζ will be 0 in such a case; both are included in the MILP objective function. The minimum value to the objective function is used in the solution to the UC problem, the implication of which is that all γ and ζ variables are 0 unless the associated generator is required to be brought on-line or off-line during a particular time-step of the simulation period.

The MILP formulation of these constraints are included below. Note that the initial state of each turn on indicator is included for the first time-step, labelled Υ:

$$\begin{bmatrix} \mathbf{A}^1_{(b\times 8b)} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{A}^2_{(b\times 8b)} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{A}^t_{(b\times 8b)} \end{bmatrix} \mathbf{x} = \begin{bmatrix} \boldsymbol{\Upsilon} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix}$$

Where:

$$\mathbf{A}^1 = \mathbf{A}^2 = \cdots = \mathbf{A}^t = \begin{bmatrix}\mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{I}_{(b\times b)} & -\mathbf{I}_{(b\times b)} & \mathbf{I}_{(b\times b)}\end{bmatrix},$$

$$\boldsymbol{\Upsilon} = \begin{bmatrix}\Upsilon_1 \\ \Upsilon_2 \\ \vdots \\ \Upsilon_b\end{bmatrix}$$

The above matrix equations for generator start-up and shut-down enforcement are further simplified by eliminating rows corresponding to buses that do not have a connected generator, resulting in all $\mathbf{A}$ matrices of size $(g \times 8b)$.

4.6 Generic Energy Storage Plant Active Power Balance Requirements

Active power balance requirements apply to all energy storage plant, and enforce a limitation on input and output power due to energy storage capacity. This constraint incorporates energy efficiency by applying the efficiency of the plant to the charging power. The total energy in storage is then the available energy that can be produced and sent back to the grid. Equation 4.5 stipulates the power balance requirement for a single storage plant that dictates the relationship between active power output, active power input, and change in energy storage in time-step i.

$$\frac{S^{\text{gen},(i-1)} - S^{\text{gen},i}}{\tau} = P^{\text{gen},i} - \eta P^{\text{use},i} \tag{4.5}$$

$$\begin{bmatrix}
\mathbf{A}^{1,1}_{(b\times 8b)} & \mathbf{0} & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\
\mathbf{A}^{2,2}_{(b\times 8b)} & \mathbf{A}^{1,2}_{(b\times 8b)} & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\
\mathbf{0} & \mathbf{A}^{2,3}_{(b\times 8b)} & \mathbf{A}^{1,3}_{(b\times 8b)} & & & \vdots \\
\mathbf{0} & \mathbf{0} & \ddots & \ddots & & \vdots \\
\vdots & \vdots & & \ddots & \ddots & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{A}^{2,t}_{(b\times 8b)} & \mathbf{A}^{1,t}_{(b\times 8b)}
\end{bmatrix} \mathbf{x} = \begin{bmatrix}\frac{1}{\tau}\mathbf{S}^{\mathbf{init}} \\ \mathbf{0} \\ \vdots \\ \mathbf{0}\end{bmatrix}$$

Where:

$$\mathbf{A}^{1,1} = \mathbf{A}^{1,2} = \cdots = \mathbf{A}^{1,t} =$$

$$\begin{bmatrix} -\mathbf{I}_{(b\times b)} & \eta\mathbf{I}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \frac{-1}{\tau}\mathbf{I}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} \end{bmatrix},$$

$$\mathbf{A}^{2,1} = \mathbf{A}^{2,2} = \cdots = \mathbf{A}^{2,t} =$$

$$\begin{bmatrix} \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \frac{1}{\tau}\mathbf{I}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} \end{bmatrix}$$

The above matrix equations for the energy storage plant power balance enforcement are further simplified by eliminating rows corresponding to buses that do not have a connected generator, resulting in all $\mathbf{A}$ matrices of size $(g \times 8b)$.

4.7 Charging and Discharging Mutually Exclusive Enforcement

Charging and discharging constraints are included in the model to ensure that charging and discharging states are mutually exclusive.

The governing equation for these constraints is expressed for each time-step, i, in Equation 4.6.

$$\sigma^i + \upsilon^i \leqslant 1 \tag{4.6}$$

The MILP formulation of these constraints are included below.

$$\begin{bmatrix} \mathbf{A}^1_{(b\times 8b)} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{A}^2_{(b\times 8b)} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{A}^t_{(b\times 8b)} \end{bmatrix} \mathbf{x} \leqslant \begin{bmatrix} \mathbf{1}_{(b\times 1)} \\ \mathbf{1}_{(b\times 1)} \\ \vdots \\ \mathbf{1}_{(b\times 1)} \end{bmatrix}$$

Where:

$$\mathbf{A}^1 = \mathbf{A}^2 = \cdots = \mathbf{A}^t = \begin{bmatrix} \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{I}_{(b\times b)} & \mathbf{I}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} \end{bmatrix}$$

The above matrix equations for charging and discharging are further simplified by eliminating rows corresponding to buses that do not have a connected generator, resulting in all $\mathbf{A}$ matrices of size $(g \times 8b)$.

4.8 Active Power Output Limits

Active power output is confined by technical limitations of the plant. Each generator is limited to produce power between the minimum practicable output and maximum installed capacity.

The mathematical representation of the output limits include the constraint that the output power of a generator should be 0 MW when the generator is off by including the binary variable, υ. The generator output limits are shown in Equation 4.7, which applies to each bus at each time-step.

$$P^{\text{min}}\upsilon \leqslant P^{\text{gen}} \leqslant P^{\text{cap}}\upsilon \tag{4.7}$$

Generator active power output limits are formulated in the MILP inequality constraint matrix:

$$\begin{bmatrix} \mathbf{A}^1_{(2b\times 8b)} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{A}^2_{(2b\times 8b)} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{A}^t_{(2b\times 8b)} \end{bmatrix} \mathbf{x} \leqslant \mathbf{0}$$

Where:

$$\mathbf{A}^1 = \mathbf{A}^2 = \cdots = \mathbf{A}^t = \begin{bmatrix} -\mathbf{I}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{P}^{\mathbf{min}}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} \\ \mathbf{I}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & -\mathbf{P}^{\mathbf{cap}}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} \end{bmatrix},$$

$$\mathbf{P}^{\mathbf{min}} = \begin{bmatrix} P_1^{\text{min}} & 0 & 0 & \cdots & 0 \\ 0 & P_2^{\text{min}} & 0 & \cdots & 0 \\ 0 & 0 & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & 0 \\ 0 & 0 & \cdots & 0 & P_b^{\text{min}} \end{bmatrix}, \quad \mathbf{P}^{\mathbf{cap}} = \begin{bmatrix} P_1^{\text{cap}} & 0 & 0 & \cdots & 0 \\ 0 & P_2^{\text{cap}} & 0 & \cdots & 0 \\ 0 & 0 & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & 0 \\ 0 & 0 & \cdots & 0 & P_b^{\text{cap}} \end{bmatrix}$$

The above matrix equations for the generator output limits are further simplified by eliminating rows corresponding to buses that do not have a connected generator, resulting in all $\mathbf{A}$ matrices of size $(2g \times 8b)$.

4.9 Generic Energy Storage Active Power Input Limits

Active power input is confined by technical limitations of the plant. Each GES plant is limited to consume power between the minimum and maximum charging rates.

The mathematical representation of the input limits include the constraint that the input power should be 0 MW when the plant is not charging by including the binary variable, σ. The input limits are shown in Equation 4.8, which applies to each bus at each time-step.

$$E^{\text{rate,min}}\sigma \leqslant P^{\text{use}} \leqslant E^{\text{rate,max}}\sigma \tag{4.8}$$

Active power input limits are formulated in the MILP inequality constraint matrix:

$$\begin{bmatrix} \mathbf{A}^1_{(2b\times 8b)} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{A}^2_{(2b\times 8b)} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{A}^t_{(2b\times 8b)} \end{bmatrix} \mathbf{x} \leqslant \mathbf{0}$$

Where:

$$\mathbf{A}^1 = \mathbf{A}^2 = \cdots = \mathbf{A}^t =$$

$$\begin{bmatrix} \mathbf{0}_{(b\times b)} & -\mathbf{I}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{E}^{\mathbf{rate,min}}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} \\ \mathbf{0}_{(b\times b)} & \mathbf{I}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & -\mathbf{E}^{\mathbf{rate,max}}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} \end{bmatrix},$$

$$\mathbf{E}^{\mathbf{rate,min}} = \begin{bmatrix} E_1^{\text{rate,min}} & 0 & 0 & \cdots & 0 \\ 0 & E_2^{\text{rate,min}} & 0 & \cdots & 0 \\ 0 & 0 & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & 0 \\ 0 & 0 & \cdots & 0 & E_b^{\text{rate,min}} \end{bmatrix},$$

$$\mathbf{E}^{\mathbf{rate,max}} = \begin{bmatrix} E_1^{\text{rate,max}} & 0 & 0 & \cdots & 0 \\ 0 & E_2^{\text{rate,max}} & 0 & \cdots & 0 \\ 0 & 0 & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & 0 \\ 0 & 0 & \cdots & 0 & E_b^{\text{rate,max}} \end{bmatrix}$$

The above matrix equations for the energy storage plant input limits are further simplified by eliminating rows corresponding to buses that do not have a connected generator, resulting in all $\mathbf{A}$ matrices of size $(2g \times 8b)$.

4.10 Active Power Output Ramping Limitations

Generators have physical limitations on the rate at which they are able to increase and decrease output power. These limitations are included in the market model by ensuring that the output power of a generator in a given time-step is not greater than the output power in the previous time-step by more than the ramp-up limit, and is not less than the output power in the previous time-step by more than the ramp-down limit [9].

In this toolbox, ramp limits are assumed to be constant regardless of generator output level, with the exception of ramp limits associated with start-up or shut-down.

Equation 4.9 and Equation 4.10 express the ramping requirements for a generator at time-step i.

$$\frac{P^{\text{gen},i} - P^{\text{gen},(i-1)}}{\tau} \leqslant R^{\text{gen,up}} + (\max\{R^{\text{gen,up}}, P^{\text{min}}\} - R^{\text{gen,up}})\gamma^i \tag{4.9}$$

$$\frac{P^{\text{gen},(i-1)} - P^{\text{gen},i}}{\tau} \leqslant R^{\text{gen,down}} + (\max\{R^{\text{gen,down}}, P^{\text{min}}\} - R^{\text{gen,down}})\zeta^i \tag{4.10}$$

Equation 4.9 and Equation 4.10 account for a variable time-step length. Equation 4.9 and Equation 4.10 also allow a generator to transition from an output of 0 MW to the minimum output when being brought on-line if the minimum output is greater than the ramp up rate, and to transition from the minimum output to 0 MW output when being brought off-line if the minimum output is greater than the ramp down rate. This requirement is important for generators that have a minimum output that is greater than the ramp-up or ramp-down limit.

The mathematical formulation of these constraints in the MILP optimisation problem is described below. Note that for both the ramp-up and ramp-down constraints, the initial output power of each generator, specified by P^{init}, is included for the first time-step.

The ramp-up constraints are expressed in standard MILP matrix form as follows:

$$\begin{bmatrix} \mathbf{A}^{1,1}_{(b\times 8b)} & \mathbf{0} & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \mathbf{A}^{2,2}_{(b\times 8b)} & \mathbf{A}^{1,2}_{(b\times 8b)} & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{A}^{2,3}_{(b\times 8b)} & \mathbf{A}^{1,3}_{(b\times 8b)} & & & \vdots \\ \mathbf{0} & \mathbf{0} & \ddots & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{A}^{2,t}_{(b\times 8b)} & \mathbf{A}^{1,t}_{(b\times 8b)} \end{bmatrix} \mathbf{x} \leqslant \begin{bmatrix} \tau\mathbf{R}^{\mathbf{gen,up}} + \mathbf{P}^{\mathbf{gen,init}} \\ \tau\mathbf{R}^{\mathbf{gen,up}} \\ \vdots \\ \tau\mathbf{R}^{\mathbf{gen,up}} \end{bmatrix}$$

Where:

$$\mathbf{A}^{1,1} = \mathbf{A}^{1,2} = \cdots = \mathbf{A}^{1,t} =$$

$$\begin{bmatrix} \mathbf{I}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \tau\mathbf{D}^{\mathbf{up}}_{(b\times b)} & \mathbf{0}_{(b\times b)} \end{bmatrix},$$

$$\mathbf{A}^{2,1} = \mathbf{A}^{2,2} = \cdots = \mathbf{A}^{2,t} =$$

$$\begin{bmatrix} -\mathbf{I}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} \end{bmatrix},$$

$$\mathbf{R}^{\mathbf{gen,up}} = \begin{bmatrix} R^{\text{gen,up}}_1 \\ R^{\text{gen,up}}_2 \\ \vdots \\ R^{\text{gen,up}}_b \end{bmatrix}, \quad \mathbf{P}^{\mathbf{gen,init}} = \begin{bmatrix} P^{\text{gen,init}}_1 \\ P^{\text{gen,init}}_2 \\ \vdots \\ P^{\text{gen,init}}_b \end{bmatrix},$$

$\mathbf{D}^{\mathbf{up}}$ is a diagonal matrix:

$$D^{\text{up}}_{ij} = 0 \text{ for } i \neq j,\ D^{\text{up}}_{ii} = (R^{\text{gen,up}}_i - \max\{R^{\text{gen,up}}_i, P^{\text{min}}_i\}).$$

The ramp-down constraints are expressed in standard MILP matrix form as follows:

$$\begin{bmatrix} \mathbf{A}^{1,1}_{(b\times 8b)} & \mathbf{0} & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \mathbf{A}^{2,2}_{(b\times 8b)} & \mathbf{A}^{1,2}_{(b\times 8b)} & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{A}^{2,3}_{(b\times 8b)} & \mathbf{A}^{1,3}_{(b\times 8b)} & & & \vdots \\ \mathbf{0} & \mathbf{0} & \ddots & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{A}^{2,t}_{(b\times 8b)} & \mathbf{A}^{1,t}_{(b\times 8b)} \end{bmatrix} \mathbf{x} \leqslant \begin{bmatrix} \tau\mathbf{R}^{\mathbf{gen,down}} - \mathbf{P}^{\mathbf{gen,init}} \\ \tau\mathbf{R}^{\mathbf{gen,down}} \\ \vdots \\ \tau\mathbf{R}^{\mathbf{gen,down}} \end{bmatrix}$$

Where:

$$\mathbf{A}^{1,1} = \mathbf{A}^{1,2} = \cdots = \mathbf{A}^{1,t} =$$

$$\begin{bmatrix} -\mathbf{I}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \tau\mathbf{D}^{\mathbf{down}}_{(b\times b)} \end{bmatrix},$$

$$\mathbf{A}^{2,1} = \mathbf{A}^{2,2} = \cdots = \mathbf{A}^{2,t} =$$

$$\begin{bmatrix} \mathbf{I}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} \end{bmatrix},$$

$$\mathbf{R}^{\mathbf{gen},\mathbf{down}} = \begin{bmatrix} R_1^{\text{gen,down}} \\ R_2^{\text{gen,down}} \\ \vdots \\ R_b^{\text{gen,down}} \end{bmatrix},$$

$\mathbf{D}^{\mathbf{down}}$ is a diagonal matrix:

$$D_{ij}^{\text{down}} = 0 \text{ for } i \neq j,\ D_{ii}^{\text{up}} = (R_i^{\text{gen,down}} - \max\{R_i^{\text{gen,down}}, P_i^{\text{min}}\}).$$

The above matrix equations for the ramping limitations are further simplified by eliminating rows corresponding to buses that do not have a connected generator, resulting in all $\mathbf{A}$ matrices of size $(g \times 8b)$.

4.11 Energy Storage Charge and Discharge Limitations

The charge and discharge rates for GES plants are imposed on the variable P^{use} in Equation 4.8. However, Equation 4.8 does not apply to CST generators, which are not constrained by the variable P^{use} and therefore require a separate equation to limit the rate at which they are able to charge and discharge their energy storage reserves. Equation 4.11 expresses the maximum charge and discharge limit for CST generators with TES, S^{gen}, in time-step i.

$$|S^{\text{gen},i} - S^{\text{gen},(i-1)}| \leqslant \tau E^{\text{rate,max}} \tag{4.11}$$

Note that the initial condition of energy storage is included for the first time-step of the simulation period.

The charge rates are expressed as follows:

$$\begin{bmatrix} \mathbf{A}^{1,1}_{(b\times 8b)} & \mathbf{0} & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \mathbf{A}^{2,2}_{(b\times 8b)} & \mathbf{A}^{1,2}_{(b\times 8b)} & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{A}^{2,3}_{(b\times 8b)} & \mathbf{A}^{1,3}_{(b\times 8b)} & & & \vdots \\ \mathbf{0} & \mathbf{0} & \ddots & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{A}^{2,t}_{(b\times 8b)} & \mathbf{A}^{1,t}_{(b\times 8b)} \end{bmatrix} \mathbf{x} \leqslant \begin{bmatrix} \tau \mathbf{E}^{\mathbf{rate,max}} + \mathbf{S}^{\mathbf{init}} \\ \tau \mathbf{E}^{\mathbf{rate,max}} \\ \vdots \\ \tau \mathbf{E}^{\mathbf{rate,max}} \end{bmatrix}$$

Where:

$$\mathbf{A}^{1,1} = \mathbf{A}^{1,2} = \cdots = \mathbf{A}^{1,t} =$$

$$\begin{bmatrix} \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{I}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} \end{bmatrix},$$

$$\mathbf{A}^{2,1} = \mathbf{A}^{2,2} = \cdots = \mathbf{A}^{2,t} =$$

$$\begin{bmatrix} \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & -\mathbf{I}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} \end{bmatrix},$$

$$\mathbf{E}^{\mathbf{rate,max}} = \begin{bmatrix} E_1^{\text{rate,max}} \\ E_2^{\text{rate,max}} \\ \vdots \\ E_b^{\text{rate,max}} \end{bmatrix}, \quad \mathbf{S}^{\mathbf{init}} = \begin{bmatrix} S_1^{\text{init}} \\ S_2^{\text{init}} \\ \vdots \\ S_b^{\text{init}} \end{bmatrix}$$

The discharge rates are expressed as follows:

$$\begin{bmatrix} \mathbf{A}^{1,1}_{(b\times 8b)} & \mathbf{0} & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \mathbf{A}^{2,2}_{(b\times 8b)} & \mathbf{A}^{1,2}_{(b\times 8b)} & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{A}^{2,3}_{(b\times 8b)} & \mathbf{A}^{1,3}_{(b\times 8b)} & & & \vdots \\ \mathbf{0} & \mathbf{0} & \ddots & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{A}^{2,t}_{(b\times 8b)} & \mathbf{A}^{1,t}_{(b\times 8b)} \end{bmatrix} \mathbf{x} \leqslant \begin{bmatrix} \tau \mathbf{E}^{\mathbf{rate,max}} - \mathbf{S}^{\mathbf{init}} \\ \tau \mathbf{E}^{\mathbf{rate,max}} \\ \vdots \\ \tau \mathbf{E}^{\mathbf{rate,max}} \end{bmatrix}$$

Where:

$$\mathbf{A}^{1,1} = \mathbf{A}^{1,2} = \cdots = \mathbf{A}^{1,t} =$$

$$\begin{bmatrix} \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & -\mathbf{I}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} \end{bmatrix},$$

$$\mathbf{A}^{2,1} = \mathbf{A}^{2,2} = \cdots = \mathbf{A}^{2,t} =$$

$$\begin{bmatrix} \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{I}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} \end{bmatrix}$$

The above matrix equations for the energy storage charge and discharge limitations are further simplified by eliminating rows corresponding to buses that do not have a connected generator, resulting in all $\mathbf{A}$ matrices of size $(g \times 8b)$.

4.12 Generic Energy Storage Charging Ramping Limitations

GES plants may have physical limitations on the rate at which they are able to increase and decrease input power. These limitations are included in the market model by ensuring that the input power of an energy storage plant in a given time-step is not greater than the input power in the previous time-step by more than the ramp-up limit, and is not less than the output power in the previous time-step by more than the ramp-down limit.

Equation 4.12 and Equation 4.13 express the ramping requirements for a GES plant at time-step i.

$$\frac{P^{\text{use},i} - P^{\text{use},(i-1)}}{\tau} \leqslant R^{\text{use,up}} \tag{4.12}$$

$$\frac{P^{\text{use},(i-1)} - P^{\text{use},i}}{\tau} \leqslant R^{\text{use,down}} \tag{4.13}$$

Equation 4.12 and Equation 4.13 account for a variable time-step length.

The mathematical formulation of these constraints in the MILP optimisation problem is described below. Note that for both the ramp-up and ramp-down constraints, the initial input power of each energy storage plant, specified by $P^{\text{use,init}}$, is included for the first time-step.

The ramp-up constraints are expressed in standard MILP matrix form as follows:

$$\begin{bmatrix} \mathbf{A}^{1,1}_{(b\times 8b)} & \mathbf{0} & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \mathbf{A}^{2,2}_{(b\times 8b)} & \mathbf{A}^{1,2}_{(b\times 8b)} & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{A}^{2,3}_{(b\times 8b)} & \mathbf{A}^{1,3}_{(b\times 8b)} & & & \vdots \\ \mathbf{0} & \mathbf{0} & \ddots & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{A}^{2,t}_{(b\times 8b)} & \mathbf{A}^{1,t}_{(b\times 8b)} \end{bmatrix} \mathbf{x} \leqslant \begin{bmatrix} \tau \mathbf{R}^{\mathbf{use,up}} + \mathbf{P}^{\mathbf{use,init}} \\ \tau \mathbf{R}^{\mathbf{use,up}} \\ \vdots \\ \tau \mathbf{R}^{\mathbf{use,up}} \end{bmatrix}$$

Where:

$$\mathbf{A}^{1,1} = \mathbf{A}^{1,2} = \cdots = \mathbf{A}^{1,t} =$$

$$\begin{bmatrix} \mathbf{0}_{(b\times b)} & \mathbf{I}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} \end{bmatrix},$$

$$\mathbf{A}^{2,1} = \mathbf{A}^{2,2} = \cdots = \mathbf{A}^{2,t} =$$

$$\begin{bmatrix} \mathbf{0}_{(b\times b)} & -\mathbf{I}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} \end{bmatrix},$$

$$\mathbf{R}^{\mathbf{use,up}} = \begin{bmatrix} R_1^{\text{use,up}} \\ R_2^{\text{use,up}} \\ \vdots \\ R_b^{\text{use,up}} \end{bmatrix}, \quad \mathbf{P}^{\mathbf{use,init}} = \begin{bmatrix} P_1^{\text{use,init}} \\ P_2^{\text{use,init}} \\ \vdots \\ P_b^{\text{use,init}} \end{bmatrix}$$

The ramp-down constraints are expressed in standard MILP matrix form as follows:

$$\begin{bmatrix} \mathbf{A}^{1,1}_{(b\times 8b)} & \mathbf{0} & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \mathbf{A}^{2,2}_{(b\times 8b)} & \mathbf{A}^{1,2}_{(b\times 8b)} & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{A}^{2,3}_{(b\times 8b)} & \mathbf{A}^{1,3}_{(b\times 8b)} & & & \vdots \\ \mathbf{0} & \mathbf{0} & \ddots & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{A}^{2,t}_{(b\times 8b)} & \mathbf{A}^{1,t}_{(b\times 8b)} \end{bmatrix} \mathbf{x} \leqslant \begin{bmatrix} \tau \mathbf{R}^{\mathbf{down}} - \mathbf{P}^{\mathbf{use,init}} \\ \tau \mathbf{R}^{\mathbf{down}} \\ \vdots \\ \tau \mathbf{R}^{\mathbf{down}} \end{bmatrix}$$

Where:

$$\mathbf{A}^{1,1} = \mathbf{A}^{1,2} = \cdots = \mathbf{A}^{1,t} =$$

$$\begin{bmatrix} \mathbf{0}_{(b\times b)} & -\mathbf{I}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} \end{bmatrix},$$

$$\mathbf{A}^{2,1} = \mathbf{A}^{2,2} = \cdots = \mathbf{A}^{2,t} =$$

$$\begin{bmatrix} \mathbf{0}_{(b\times b)} & \mathbf{I}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} \end{bmatrix},$$

$$\mathbf{R^{down}} = \begin{bmatrix} R_1^{\text{down}} \\ R_2^{\text{down}} \\ \vdots \\ R_b^{\text{down}} \end{bmatrix}$$

The above matrix equations for the energy storage charging and discharging ramping limitations are further simplified by eliminating rows corresponding to buses that do not have a connected generator, resulting in all $\mathbf{A}$ matrices of size $(g \times 8b)$.

4.13 Minimum Up-Time and Minimum Down-Time

The minimum up-time and minimum down-time for a generator are each specified as a number of hours in the input data, however this is converted to the corresponding number of time-steps in the solver. Equation 4.14 and Equation 4.15 show the mathematical expressions that state that in any time-step, i, upon turning on, a generator must remain on for T^{up} time-steps, and upon turning off, a generator must remain off for T^{down} time-steps.

$$-\upsilon^i + \sum_{k=1}^{T^{\text{up}}-1} \gamma^{(i-k)} \leqslant 0 \tag{4.14}$$

$$\upsilon^i + \sum_{k=1}^{T^{\text{down}}-1} \zeta^{(i-k)} \leqslant 1 \tag{4.15}$$

For Equation 4.14, the value of the turn-on indicator is required for $(T^{\text{up}}-1)$ prior time-steps. For Equation 4.15, the value of the turn-off indicator is required for $(T^{\text{down}}-1)$ prior time-steps. The maximum minimum up-time and maximum minimum down-time used in this model is 16 hours. Initial conditions are included for 15 hours prior to the start of the simulation interval. The initial conditions of the generator decision variables are included so that the minimum up-time and minimum down-time constraints can be enforced for the initial 15 hours. Note that 15 hours may be more than 15 time-steps if time-steps shorter than one hour are used.

For the MILP standard matrix equations, information from times steps prior to the start of the simulation period are included as Γ and Z.

The minimum up-time constraints are expressed as follows:

$$\begin{bmatrix} \mathbf{A}^{1,1}_{(b\times 8b)} & \mathbf{0} & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \mathbf{A}^{2,2}_{(b\times 8b)} & \mathbf{A}^{1,2}_{(b\times 8b)} & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \mathbf{A}^{3,3}_{(b\times 8b)} & \mathbf{A}^{2,3}_{(b\times 8b)} & \mathbf{A}^{1,3}_{(b\times 8b)} & & & \vdots \\ \vdots & \vdots & \ddots & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \ddots & \mathbf{0} \\ \mathbf{A}^{t,t}_{(b\times 8b)} & \mathbf{A}^{(t-1),t}_{(b\times 8b)} & \cdots & \cdots & \mathbf{A}^{2,t}_{(b\times 8b)} & \mathbf{A}^{1,t}_{(b\times 8b)} \end{bmatrix} \mathbf{x} \leqslant \begin{bmatrix} \mathbf{b}^0 \\ \mathbf{b}^1 \\ \vdots \\ \mathbf{b}^i \\ \vdots \\ \mathbf{b}^t \end{bmatrix}$$

Where:

$$\mathbf{A}^{1,1} = \mathbf{A}^{1,2} = \cdots = \mathbf{A}^{1,t} =$$

$$\begin{bmatrix} \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & -\mathbf{I}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} \end{bmatrix},$$

$$\mathbf{A}^{k,1} = \mathbf{A}^{k,2} = \cdots = \mathbf{A}^{k,t} =$$

$$\begin{bmatrix} \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{D}^{\mathbf{up},k}_{(b\times b)} & \mathbf{0}_{(b\times b)} \end{bmatrix},$$

$\mathbf{D}^{\mathbf{up},k}$ is a diagonal matrix:

$$D^{\mathrm{up},k}_{ij} = 0 \text{ for } i \neq j,\ D^{\mathrm{up},k}_{ii} = \begin{cases} 1 & \text{if} \quad T^{\mathrm{up}}_i > k, \\ 0 & \text{otherwise.} \end{cases}$$

$$\mathbf{b}^i = \begin{bmatrix} b^i_1 \\ b^i_2 \\ \vdots \\ b^i_j \\ \vdots \\ b^i_b \end{bmatrix}, \quad b^i_j = \begin{cases} \displaystyle\sum_{k=1}^{T^{\mathrm{up}}_j - i - 1} -\Gamma^{-k}_j & \text{if} \quad T^{\mathrm{up}}_j - i - 1 > 0, \\ 0 & \text{otherwise.} \end{cases}$$

Γ is the initial condition of the generator turn-on indicator; Γ contains information about generator turn-on decisions for time-steps prior to the start of the simulation period. Time-steps are numbered $(\cdots, -3, -2, -1)$, with -1 referring to the time-step immediately prior to the first time-step in the simulation period.

The minimum down-time constraints are expressed as follows:

$$\begin{bmatrix} \mathbf{A}^{1,1}_{(b\times 8b)} & \mathbf{0} & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \mathbf{A}^{2,2}_{(b\times 8b)} & \mathbf{A}^{1,2}_{(b\times 8b)} & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \mathbf{A}^{3,3}_{(b\times 8b)} & \mathbf{A}^{2,3}_{(b\times 8b)} & \mathbf{A}^{1,3}_{(b\times 8b)} & & & \vdots \\ \vdots & \vdots & \ddots & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \ddots & \mathbf{0} \\ \mathbf{A}^{t,t}_{(b\times 8b)} & \mathbf{A}^{(t-1),t}_{(b\times 8b)} & \cdots & \cdots & \mathbf{A}^{2,t}_{(b\times 8b)} & \mathbf{A}^{1,t}_{(b\times 8b)} \end{bmatrix} \mathbf{x} \leqslant \begin{bmatrix} \mathbf{b}^0 \\ \mathbf{b}^1 \\ \vdots \\ \mathbf{b}^i \\ \vdots \\ \mathbf{b}^t \end{bmatrix}$$

Where:

$$\mathbf{A}^{1,1} = \mathbf{A}^{1,2} = \cdots = \mathbf{A}^{1,t} =$$

$$\begin{bmatrix} \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{I}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} \end{bmatrix},$$

$$\mathbf{A}^{k,1} = \mathbf{A}^{k,2} = \cdots = \mathbf{A}^{k,t} =$$

$$\begin{bmatrix} \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{D}^{\mathbf{down},k}_{(b\times b)} \end{bmatrix},$$

$\mathbf{D}^{\mathbf{down},k}$ is a diagonal matrix:

$$D^{\text{down},k}_{ij} = 0 \text{ for } i \neq j,\ D^{\text{down},k}_{ii} = \begin{cases} 1 & \text{if} \quad T^{\text{down}}_i > k, \\ 0 & \text{otherwise.} \end{cases}$$

$$\mathbf{b}^i = \begin{bmatrix} b^i_1 \\ b^i_2 \\ \vdots \\ b^i_j \\ \vdots \\ b^i_b \end{bmatrix}, \quad b^i_j = \begin{cases} 1 - \displaystyle\sum_{k=1}^{T^{\text{down}}_j - i - 1} Z^{-k}_j & \text{if} \quad T^{\text{down}}_j - i - 1 > 0, \\ 1 & \text{otherwise.} \end{cases}$$

Z is the initial condition of the generator turn-off indicator; Z contains information about generator turn-off decisions for time-steps prior to the start of the simulation period. Time-steps are numbered in the same convention that is used for Γ.

The above matrix equations for the minimum up time and minimum down time enforcement are further simplified by eliminating rows corresponding to buses that do not have a connected generator, resulting in all $\mathbf{A}$ matrices of size $(g \times 8b)$.

4.14 Active Power Transfer Limits

Active power flow limits are used with the full DC power flow model. When using the copper plate model, the active power flow limits are ignored. For the DC power flow formulation used in this study, the active power transferred over a transmission line or transformer is expressed as:

$$P_{ij} = \frac{\theta_{ij}}{X_{ij}}$$

Power transfer limits are implemented by applying an inequality constraint to the difference in voltage angle over a transmission line or transformer, as shown in Equation 4.16 for a connection between buses i and j.

$$|\theta_{ij}| \leqslant X_{ij} P_{ij}^{\max} \tag{4.16}$$

For the purposes of this study, both the series susceptance and the active power transfer limit of each transmission line and transformer in the system are assumed constant.

The MILP mathematical formulation of these constraints is described below for a system with l bus interconnections:

$$\begin{bmatrix} \mathbf{A}^1_{(2l\times 8b)} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{A}^2_{(2l\times 8b)} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{A}^t_{(2l\times 8b)} \end{bmatrix} \mathbf{x} \leqslant \begin{bmatrix} \mathbf{b}^1 \\ \mathbf{b}^2 \\ \vdots \\ \mathbf{b}^t \end{bmatrix}$$

Where:

$$\mathbf{A}^1 = \mathbf{A}^2 = \cdots = \mathbf{A}^t =$$

$$\begin{bmatrix} \mathbf{0}_{(2l\times b)} & \mathbf{0}_{(2l\times b)} & \mathbf{L}_{(2l\times b)} & \mathbf{0}_{(2l\times b)} & \mathbf{0}_{(2l\times b)} & \mathbf{0}_{(2l\times b)} & \mathbf{0}_{(2l\times b)} & \mathbf{0}_{(2l\times b)} \end{bmatrix}$$

$\mathbf{L}$ and $\mathbf{b}$ depend on the network topography, active power transfer limits, and series reactance of each bus interconnector. Some generic bus connections are indicated as

(cd), (ef), and (ij), in order to show the structure of $\mathbf{L}$ and $\mathbf{b}$:

$$\mathbf{L} = \begin{bmatrix} \mathbf{L}_{cd(2\times b)} \\ \mathbf{L}_{ef(2\times b)} \\ \vdots \\ \mathbf{L}_{ij(2\times b)} \end{bmatrix},$$

$$\mathbf{L}_{ij} = \begin{array}{c} \begin{array}{ccccccccccc} 1 & & (i-1) & i & (i+1) & & (j-1) & j & (j+1) & & b \end{array} \\ \begin{bmatrix} 0 & \cdots & 0 & 1 & 0 & \cdots & 0 & -1 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & -1 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \end{bmatrix} \end{array},$$

$$\mathbf{b}^1 = \mathbf{b}^2 = \cdots = \mathbf{b}^t = \begin{bmatrix} X_{cd}P_{cd}^{\max} \\ X_{cd}P_{cd}^{\max} \\ X_{ef}P_{ef}^{\max} \\ X_{ef}P_{ef}^{\max} \\ \vdots \\ X_{ij}P_{ij}^{\max} \\ X_{ij}P_{ij}^{\max} \end{bmatrix}$$

4.15 TES Active Power Balance Requirements

TES active power balance requirements apply to all CST generators, and enforce a limitation on output power due to available solar insolation and existing energy storage. Equation 4.17 stipulates the power balance requirement for a single CST generator that dictates the relationship between active power output, power input from solar insolation, and change in energy storage in time-step i.

$$P^{\text{gen},i} \leqslant \frac{S^{\text{gen},(i-1)} - S^{\text{gen},i}}{\tau} + P^{\text{solar},i} \tag{4.17}$$

Equation 4.17 includes an allowance for energy available to a CST generator to be unused. Note that the initial condition of thermal energy storage is included for the first time-step of the simulation period.

$$\begin{bmatrix} \mathbf{A}^{1,1}_{(b\times 8b)} & \mathbf{0} & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \mathbf{A}^{2,2}_{(b\times 8b)} & \mathbf{A}^{1,2}_{(b\times 8b)} & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{A}^{2,3}_{(b\times 8b)} & \mathbf{A}^{1,3}_{(b\times 8b)} & & & \vdots \\ \mathbf{0} & \mathbf{0} & \ddots & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{A}^{2,t}_{(b\times 8b)} & \mathbf{A}^{1,t}_{(b\times 8b)} \end{bmatrix} \mathbf{x} \leqslant \begin{bmatrix} \mathbf{P}^{\mathbf{solar}} + \frac{1}{\tau}\mathbf{S}^{\mathbf{init}} \\ \mathbf{P}^{\mathbf{solar}} \\ \vdots \\ \mathbf{P}^{\mathbf{solar}} \end{bmatrix}$$

Where:

$$\mathbf{A}^{1,1} = \mathbf{A}^{1,2} = \cdots = \mathbf{A}^{1,t} =$$

$$\begin{bmatrix} \mathbf{I}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \frac{1}{\tau}\mathbf{I}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} \end{bmatrix},$$

$$\mathbf{A}^{2,1} = \mathbf{A}^{2,2} = \cdots = \mathbf{A}^{2,t} =$$

$$\begin{bmatrix} \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \frac{-1}{\tau}\mathbf{I}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} & \mathbf{0}_{(b\times b)} \end{bmatrix},$$

$$\mathbf{P}^{\mathbf{solar}} = \begin{bmatrix} P^{\text{solar}}_1 \\ P^{\text{solar}}_2 \\ \vdots \\ P^{\text{solar}}_b \end{bmatrix}$$

The above matrix equations for the TES power balance enforcement are further simplified by eliminating rows corresponding to buses that do not have a connected generator, resulting in all $\mathbf{A}$ matrices of size $(g \times 8b)$.

4.16 Spinning Reserve

The spinning reserve requirement is summarised by a constraint that references variables at each bus. The constraint states that at each time-step, for all synchronous generators that are on-line, the sum of additional capacity beyond the active power currently produced should always be greater than a user-specified percentage of the total demand,

Q^{spin}. This constraint is shown in Equation 4.18 for a single time-step.

$$\sum_{\text{Sync. Gen.}} (P^{\text{cap}} \upsilon - P^{\text{gen}}) \geqslant Q^{\text{spin}} \sum_{b} P^{\text{dem}} \tag{4.18}$$

Note that the summation of unused capacity of connected generators is over synchronous generators only, whereas the summation of demand is over all buses in the power system.

The MILP formulation of the spinning reserve constraint is detailed below:

$$\begin{bmatrix} \mathbf{A}^1_{(1\times 8b)} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{A}^2_{(1\times 8b)} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{A}^t_{(1\times 8b)} \end{bmatrix} \mathbf{x} \leqslant \begin{bmatrix} b^1 \\ b^2 \\ \vdots \\ b^i \\ \vdots \\ b^t \end{bmatrix}$$

Where:

$$\mathbf{A}^1 = \mathbf{A}^2 = \cdots = \mathbf{A}^t =$$

$$\begin{bmatrix} \mathbf{a}^{\mathbf{sync}}_{(1\times b)} & \mathbf{0}_{(1\times b)} & \mathbf{0}_{(1\times b)} & \mathbf{0}_{(1\times b)} & \mathbf{0}_{(1\times b)} & -\mathbf{c}^{\mathbf{sync}}_{(1\times b)} & \mathbf{0}_{(1\times b)} & \mathbf{0}_{(1\times b)} \end{bmatrix},$$

$$a_j^{\text{sync}} = \begin{cases} 1 & \text{if synchronous generator at bus } j, \\ 0 & \text{otherwise.} \end{cases}$$

$$c_j^{\text{sync}} = \begin{cases} P_j^{\text{cap}} & \text{if synchronous generator at bus } j, \\ 0 & \text{otherwise.} \end{cases}$$

$$b^i = -Q^{\text{spin}} \sum_{k=1}^{b} P_k^{\text{dem},i}$$

Appendix A

Case Studies Supplementary Information

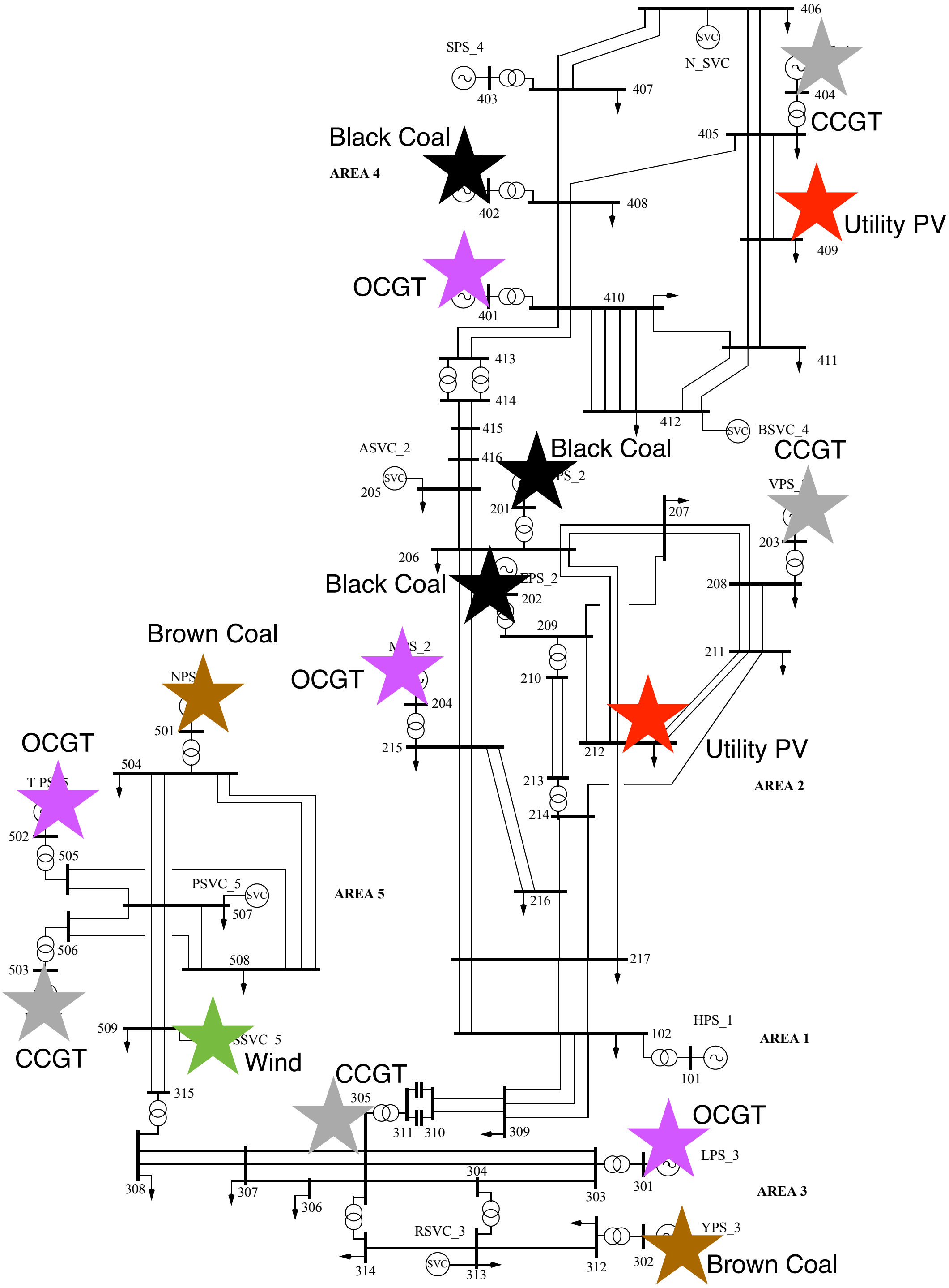

FIGURE A.1: Case Study A & B Generator Portfolio *"Business as usual."*

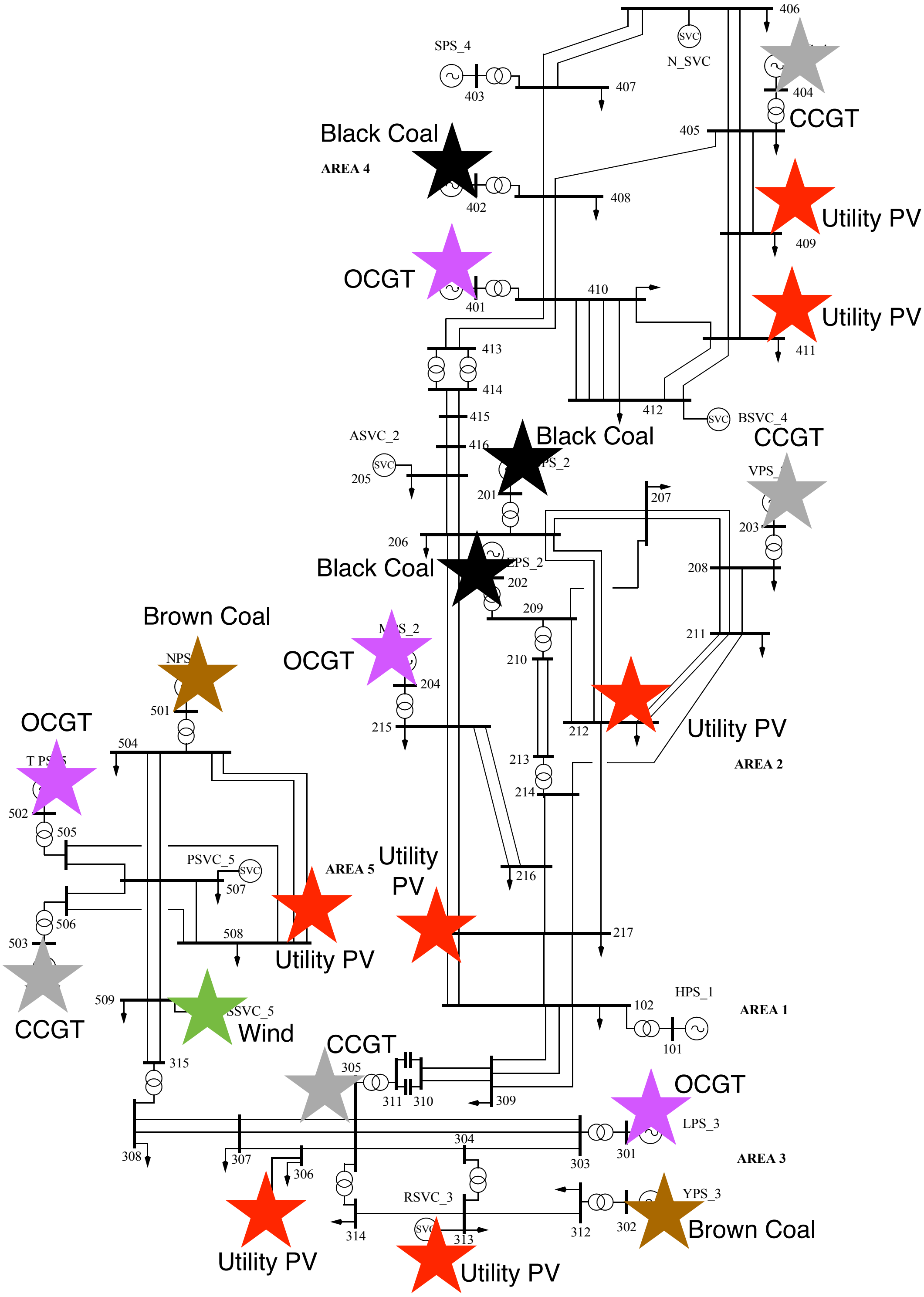

FIGURE A.2: Case Study C Generator Portfolio *"High penetration of solar PV."*

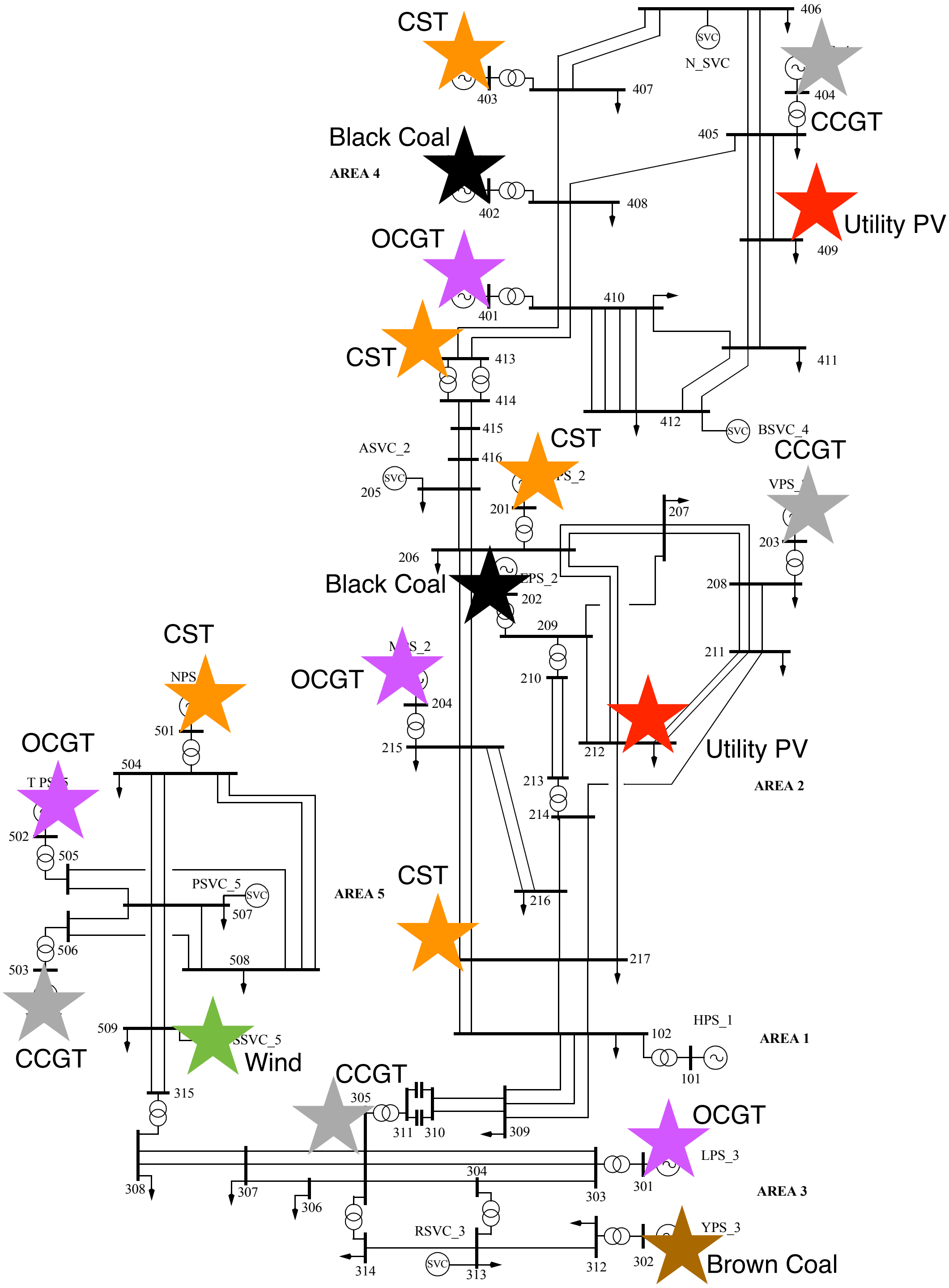

FIGURE A.3: Case Study D Generator Portfolio *"High penetration of CST with TES."*

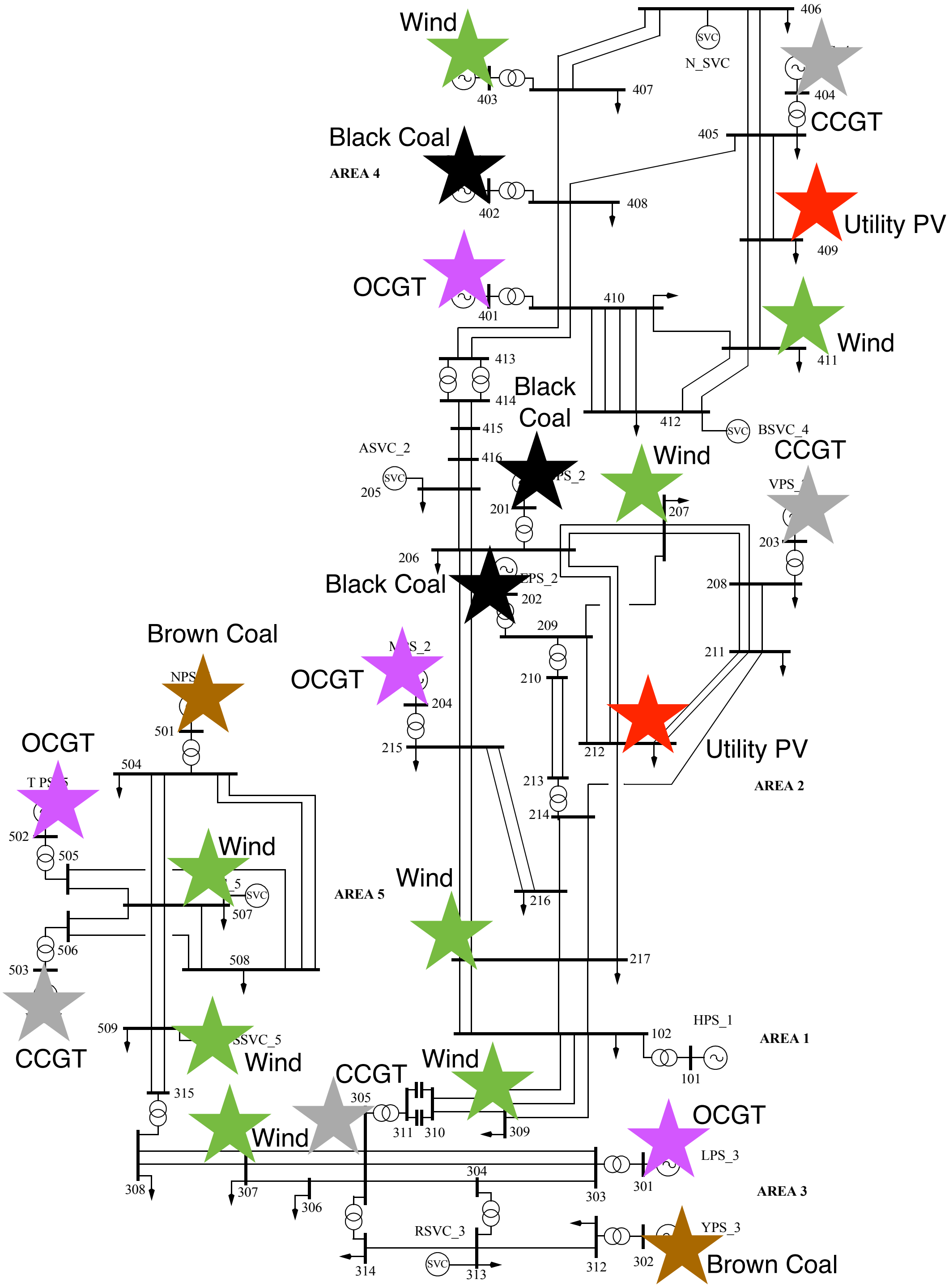

FIGURE A.4: Case Study E Generator Portfolio *"High penetration of intermittent wind."*

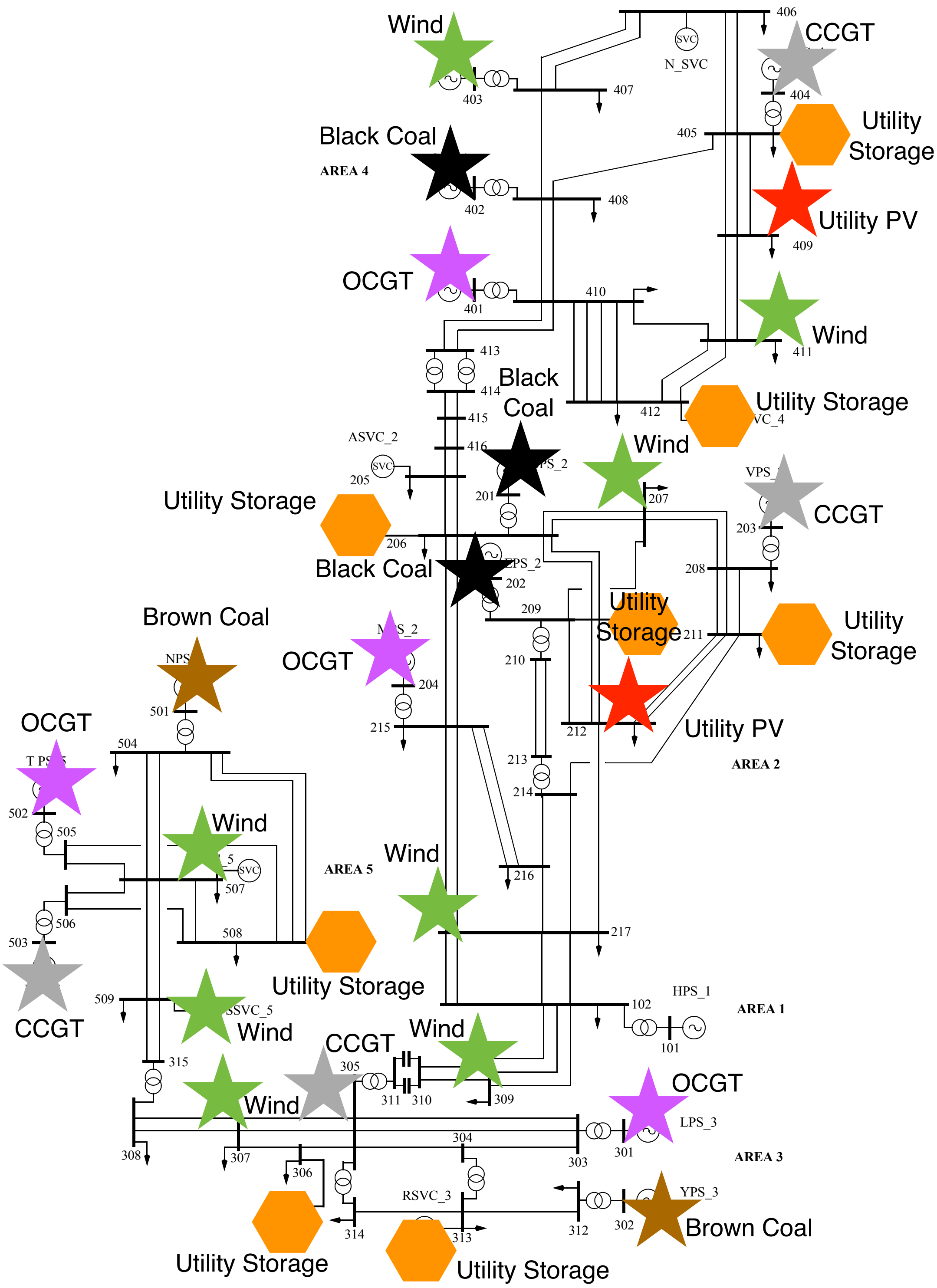

FIGURE A.5: Case Study F Generator Portfolio *"Energy storage with high penetration of intermittent wind."*

Bibliography

[1] P. Denholm, G. Brinkman, D. Lew, and M. Hummon, “Operation of concentrating solar power plants in the western wind and solar integration phase 2 study,” National Renewable Energy Laboratory (NREL), Golden, CO., Tech. Rep., 2014.

[2] “100 per cent renewables study - modelling outcomes,” Australian Energy Market Operator, Tech. Rep. Final, Jul. 2013.

[3] J. Glover, M. Sarma, and T. Overbye, *Power System Analysis and Design*, 5th ed. Cengage Learning, 2012, ISBN: 9781111425777.

[4] M. Gibbard and D. Vowles, “Simplified 14-generator model of the se australian power system,” The University of Adelaide, Report, 2010.

[5] Australian Energy Market Operator (AEMO), “National transmission network development plan 2016,” Report, 2016.

[6] M. Gibbard and D. Vowles, “Simplified 14-generator model of the SE Australian power system,” School of Electrical and Electronic Engineering, The University of Adelaide, South Australia, Tech. Rep., Jun. 2010, Revision 3.

[7] S. G. Bardón. (2011). Gemasolar, fuentes de andalucia. Licensed under a Creative Commons Attribution 2.0 Generic (CC BY 2.0), [Online]. Available: https://www.flickr.com/photos/sagabardon/6214463554.

[8] “2014 planning methodology and input assumptions,” Supply Forecasting, Australian Energy Market Operator, Tech. Rep. SF-PC v1.0, Jan. 2014.

[9] E. Castillo, A. J. Conejo, P. Pedregal, R. Garcia, and N. Alguacil, *Building and solving mathematical programming models in engineering and science*. John Wiley & Sons, 2011, vol. 62.