Sixth Semester B. E. (Computer Science and Engineering) Examination

DESIGN AND ANALYSIS OF ALGORITHMS

Time : 3 Hours] [Max. Marks : 60

Instructions to Candidates :—

(1) All questions carry equal marks.

(2) Solve any **Two** sub – questions from each question.

(3) Mention comments properly before writing the algorithms.

1. (a) Solve the following recurrence exactly :

$$t_n = t_{n-1} + 2\,t_{n-2} - 2t_{n-3} \quad n > 2$$

subject to $t_n = 9n^2 - 15n + 106$ for $0 \leqslant n \leqslant 2$

Express your answer using θ notation. 5 (CO 1)

(b) Solve the following recurrence and provide asymptotic bound. :

$$T(n) = 1 \text{ if } n = 1$$

$$T(n) = 2T(n/2) + n/\lg n \text{ (otherwise)}$$ 5(CO1)

(c) USe a recursion tree to determine a good asymptotic upper bound on the recurrence

$$T(n) = 3T(n/2) + n$$

Use the substitution method to verify your answer. 5 (CO 1)

2. (a) Illustrate the importance and applicability of bitonic sequence. Design a 8 bit bitonic sorting network for the following data :—

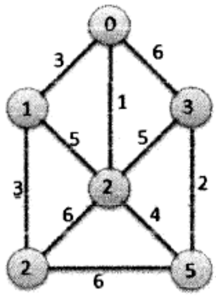
1 , 1 , 0 , 0 , 0 , 0 , 0 , 1 5(CO1, 2)

(b) A sequence of stack operations is performed on a stack whose size never exceeds k. After every k operations, a copy of entire stack is made for backup purposes. Demonstrate that the cost of n operations, including copying the stack, is O (n) by assigning suitable amortized costs to various stack operations. 5 (CO 1, 2)

(c) Illustrate insertion sort on the following data to arrange the elements in descending sequence. State the algorithm along with the worst case time complexity.

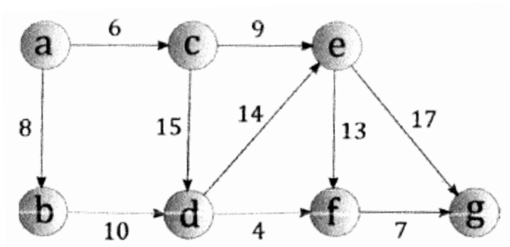
12 , 18 , 56 , 36 , 42 , 13 5(CO1,2)

3. (a) Apply the Prim's Algorithm for constructing minimum cost spanning tree on the following graph. Also state how greedy methodology is applicable to this problem ?



5 (CO 2 , 3)

(b) Construct the shortest paths from source vertex a to all other destination vertices by implementing single source shortest path algorithm on the following connected graph. Write all intermediate steps also.

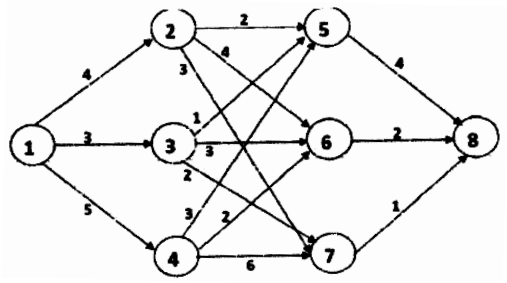


5 (CO 2 , 3)

(c) How divide and conquer strategy can be applicable to merge sort ? Apply the algorithm on the following set of data :—

23 , 45 , 18 , 89 , 12 , 79 , 77 , 11 , 40 , 80 5(CO2)

4. (a) Evaluate the minimum cost path from source (first stage) to destination (last stage) on the following multistage graph. Use necessary algorithm for solving multistage graph problem. Also state the time complexity of the used algorithm.



5(CO2,3)

(b) How can you justify the elements of dynamic programming i. e. optimal substructure and overlapping subproblem in the context of formulation of matrix chain multiplication problem ? Construct an optimal order of parenthesization for the following chain of matrices.

M_1 (100 * 1) , M_2 (1 * 100) , M_3 (100 * 1) , M_4 (1 * 100) 5 (CO 2)

(c) Implement the LCS problem for the following two strings and compute the LCS. Also state the underlying algorithms and their complexity equation.

String A : AABAABABAA

String B : BABAABAB 5 (CO 2)

5. (a) Let w = { 5 , 7 , 10 , 12 , 15 , 18 , 20 } and m = 35. Find all possible subsets of w that sum to m. Draw the portion of the state space tree that is generated. 5 (CO 2 , 5)

(b) Construct a formulation for printing all the Hamiltonian cycles present in an undirected connected graph. Explain how backtracking is implemented in this problem. 5 (CO 2 , 3 , 5)

(c) How can we estimate the efficiency of a backtracking algorithm ? State the algorithm for the same. Give an example using 8 – queen problem. 5 (CO 2 , 3 , 5)

6. (a) Explain the necessity of Approximation algorithms. Solve TSP by applying any one of the approximation strategy. Also comment on the optimality of the solution. 5 (CO 4)

(b) What do you mean by Independent set problem ? How can you reduce a Clique problem into an Independent set problem ? 5 (CO 4)

(c) Explain polynomial reduction in the context of NP problems. Draw the inferences along with the justification from the following statements :

A and B have been asked to show that a certain problem L is NP – Complete. A shows a polynomial reduction from 3 – satisfiability problem to L and B shows a polynomial reduction from L to 3 – satisfiability problem. 5 (CO 4)